Block Notes of Math

The Collatz's problem (3x+ 1)
The forms 4n+3 and the "bizarre" odd numbers

eng. Rosario Turco, prof. Maria Colonnese

Abstract

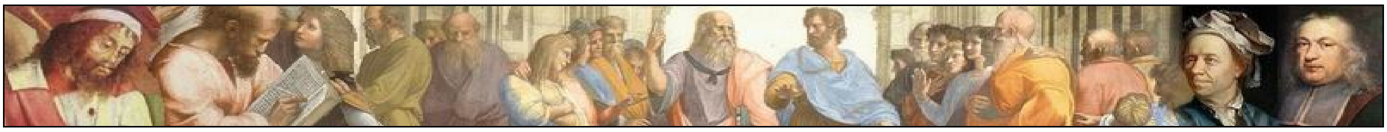
In the paper "Il problema di Collatz in N ", on database of CNR Solar (Italy), the authors have used the method of C-equivalence " to analyze the data for the algorithm of Collatz to deduce if it is convergent to 1 or, equivalently, if the algorithm has got a criteria for arrest.

With the "C-equivalence" the authors observed, however, that from one form 4m+3 you can get other forms 4m+3, which gave no clues to the criteria for arrest algorithmic.

This work, however, is a study on the forms 4m+3, where the authors show that they aren't a problem to the convergence of the sequence 1, through the introduction of specific numbers, baptized "bizarre" odd numbers.

Email

mailto:rosario_turco@virgilio.it



INDEX

TABLES

Introduction

With the Collatz’s problem we have the following definition of algorithm "Since a number any positive integer n, where n equals the divide by 2; if, instead, n is odd multiply it by 3 and add 1 (or apply the formula 3n 1) And so on. "

The fundamental problem is understanding whether the succession of values obtained by this algorithm always converges to 1.

In fact we know that a succession is an application (or function)-type f: N -> N. In particular, the function f in the game is like this:

$$f(n) = \begin{cases} 3n+1, & \text{if n is odd} \\ n/2, & \text{if n is even} \end{cases}$$

The numbers obtained in succession are remembered as "Hailstone numbers."

The weaker Collatz conjecture says: "Not a positive integer is divergent".

The strong Collatz conjecture says: "All the positive integers are converging".

If you determine that there may be a value n≠1, for which you determine that there may be a value n cyclical, then it would be true only the weaker conjecture; if there is no counter-example then are true the weak and strong conjectures and it would be true the theorem: "Given the sequence obtained by applying f: N -> N, where:

$$f(n) = \begin{cases} 3n+1, & \text{if n is odd} \\ n/2, & \text{if n is even} \end{cases}$$

The sequence converges always to integer 1 (excluding 1 and odd numbers, otherwise the succession oscillates indefinitely). "

Problem analysis

If it were true the previous theorem, then the algorithm proposed in Appendix will stop itself, without a stop condition, which avoids the loop indefinitely.

But from the type of data in input N (seed of the sequence) we can reach that conclusion? Let's see.

We define T(n) the **glide** (or trajectory or steps associated with f (n)), are true then the following properties:

Property on power of 2

If the integer n is even and power of 2, ie $n = 2^k$, then the glide T (n) is:

$$T(2^k) = k$$

$$T(2^{k+1})= k+1$$

...

Property on "the Collatz's numbers"

If the integer n odd is expressed as $n = (2^k -1) / 3$ with $k\geq4$ (Collatz's numbers) then the glide T (n) is:

$$T(n) = k+1$$

K is even, otherwise n isn't a integer.

The Collatz's number are of form 4m+1.

Property of the "form 4m+1"

In this case the glide T(n) is:

$$T(4n+1) = T(n) + 2$$

Property of the "form 4m+3"

We propose to call "*bizarre odd numbers*" any odd number generated from $(2^k-4)/4$ for each k, which , in the sequence of Collatz, generates forms 4m+3 not consecutive.

If we indicate with the symbol #b the number of odd numbers of form 4m+3 generated by Collatz with seed equal to an odd number bizarre b, we note that this number increases by 1 the number prior b ',

$$\#b = \#b' + 1.$$

The bizarre numbers are derived as b = 2b ' + 1 and attach, two by two, all bizarre, the **glide** of a pair differs only 1:

$$T(b) = T(b') + 1$$

Starting with a bizarre sequence b Collatz after m = k-3 numbers form 4m +3, such that Mod (b-3, 4) = Mod (0.4), joins a number of form 4q 1 . So the total is glide:

$$T(b) = 2*(k-3) + T(4q+1)$$

Proof

The probability can help us immediately give a first insight of the theorem. If n is even, n / 2 may be an odd or even (2 option 3). If n is odd, 3n +1 is equal (1 option 3). So the probability that the values of succession grow is less than the values decreased to 1, so the succession at the end is more likely to converge to 1. This does not prove that the sequence does not differ whether or not converge or if it is oscillating.

To use the verification program APPENDIX without stopping criterion and the demonstration will show the same property that is never necessary.

Power of 2
$256=2^8$
T(256)=8

Collatz's numbers
$85= (2^8 -1)/3$
T(85)=k+1=8+1=9

$5 = 2^4-1/3$ and 5=4*1+1 (4m+1).

Examples on 4m+1
n=7, T(7)=16
4*7+1=29, T(29)=16+2=18

4*29+1=117, T(117)=18+2=20

4*117+1=469, T(469)=20+2=22

469-1/4=117-1/4=29-1/4=7: 3 iterazioni per arrivare a 7
T(469)=16+2*3=22

4m+3
We see the next table with the bizarre numbers with $(2^k-4) / 4$ and the associated numbers 4m +3 (including the bizarre).

k	Bizzarre	#4m+3	T(n)
5	7	**2**	16
6	15	**3**	17
7	31	**4**	106
8	63	**5**	107
9	127	**6**	46
10	255	**7**	47
11	511	**8**	61
12	1023	**9**	62

Table 1 – Bizzarre numbers and Collatz's problem

The table shows that for every k, odd or belonging to N:

- To moving from a bizarre to he next b is: b = 2b '1. Example if b '= 7 b = 2 * 7 +1 = 15.
- The colored bands in Table 1 underline twin bizarre numbers and you see that the glide of a pair differs only by 1.
- To moving from a bizarre to the next it will increase by 1 the number of forms 4m+3 consecutive obtained in the problem of Collatz
- The number of steps in more than a number of form 4m +1, is given by 2*(k-3), for example k = 10 is k-3 = 7 bizarre numbers.

To quickly verify that there are numbers of form 4m+3 generated in a sequence of Collatz with PARI / GP we will exploit Mod (n-3, 4): if equals Mod(0,4) then it is a form 4m+3; otherwise if it is a form 4m+1 we will obtain Mod(2,4) and stops the sequence of 4m+3.

Examples:

7 22 11 34 <u>17</u> 52 26 13 40 20 10 5 16 8 4 2 1 #step: 16

15 46 23 70 35 106 <u>53</u> 160 80 40 20 10 5 16 8 4 2 1 #step: 17

31 94 47 142 71 214 107 322 <u>161</u> 484 242 121 364 182 91 274 137 412 206 103 310 1
55 466 233 700 350 175 526 263 790 395 1186 593 1780 890 445 1336 668 334 167 50
2 251 754 377 1132 566 283 850 425 1276 638 319 958 479 1438 719 2158 1079 3238
1619 4858 2429 7288 3644 1822 911 2734 1367 4102 2051 6154 3077 9232 4616 2308 1
154 577 1732 866 433 1300 650 325 976 488 244 122 61 184 92 46 23 70 35 106 53 1
60 80 40 20 10 5 16 8 4 2 1 #step: 106

63 190 95 286 143 430 215 646 323 970 <u>485</u> 1456 728 364 182 91 274 137 412 206 10
3 310 155 466 233 700 350 175 526 263 790 395 1186 593 1780 890 445 1336 668 334
167 502 251 754 377 1132 566 283 850 425 1276 638 319 958 479 1438 719 2158 107
9 3238 1619 4858 2429 7288 3644 1822 911 2734 1367 4102 2051 6154 3077 9232 4616
2308 1154 577 1732 866 433 1300 650 325 976 488 244 122 61 184 92 46 23 70 35 1
06 53 160 80 40 20 10 5 16 8 4 2 1 #step: 107

Conclusions
This paper shows that there is no possibility 4m+3 loop infinitely and if that increases the size of odd number certainly increases the number of steps but it is a finite number of steps.

That concludes that from any integer n in N, the sequence of Collatz always ends in a finite number of steps and an algorithm does not need a criterion to stop, because the problem whatever the given input is N stops by itself and therefore the theorem is true of departure

References

- **J. C. Lagarias**: "*The 3x+1 Problem: An annotated Bibliography*"
- **C. A. Feinstein**: "*The Collatz 3n+1 Conjecture is Unprovable*
- **Sergio Faccia** (blog Eureka! di kataweb http://serghej.blog.kataweb.it/) "Sul problema 3n+1 o di Collatz" del 14 Giugno 2006
- **R. Turco, M. Colonnese** "Il problema di Collatz in N" CNR Solar
- **Gruppo ERATOSTENE** - Vari articoli (tra cui "Dimostrazione della Congettura di Collatz - rivista METODO n. 22 - 2006") sul sito gruppo ERATOSTENE e su CNR Solar.

Sites

CNR SOLAR
http://150.146.3.132/

Prof. Matthew R. Watkins
http://www.secamlocal.ex.ac.uk

Aladdin's Lamp (eng. Rosario Turco)
www.geocities.com/SiliconValley/Port/3264 menu MISC section MATEMATICA

ERATOSTENE group
http://www.gruppoeratostene.com or http://www.gruppoeratostene.netandgo.eu

Dr. Michele Nardelli
http://xoomer.alice.it/stringtheory/

Appendix

```
/***********************************************************
* Collatz

* Rosario Turco

***********************************************************/
{printCollatz(n) = local (j, k, f);

 if( n<2 , error("printCollatz(n): You must insert an integer n>1"));

 print1(n, " ");

 j=0;
 k=0;
 f=0;

 while( n>1,

        f=0;
        if( n%2 != 0 & f==0, n=3*n+1; f=1;);
        if(n%2 == 0 & f==0, n=n/2; f=1;);
        j++;

        if( (n-1)%4 == 0, k++;);

        print1(n, " ");

      );
      print(" #step: ", j, "\n");
      if( k>0, print("4m+3: ", k, "\n"););
}

{getCollatz(n) = local (j);

 if( n<2 , error("getCollatz(n): You must insert an integer n>1"));

 j=0;
 while( n>1,

        if( n%2 != 0, n=3*n+1;);
        if( n%2 == 0, n=n/2;);
        j++;
      );
      return(j);
}
```