2003

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Laser Physics, Vol. 13, No. 6, 2003, pp. 892–896.
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A Microscopic Stern–Gerlach Magnet for Electrons?

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Invited paper: Annual International Laser Physics Workshop
Received September 28, 2002

Abstract—We discuss the possibility of realizing a microscopic Stern–Gerlach magnet for electrons using counter-propagating bichromatic laser light. Absorption of two photons with frequency ω combined with stimulated emission of one photon with frequency 2ω allows for the conservation of energy, momentum, and angular momentum. The possibility of constructing such a device appears to be an open question.

1. INTRODUCTION

At the 1930 Solvay conference, Pauli and Bohr stated that the magnetic moment of an electron cannot be determined with the help of macroscopic electromagnetic fields [1, 2]. The consequences of this statement have found their way into many contemporary textbooks [3] and imply that the construction of electron Stern–Gerlach magnets is impossible. Although Pauli and Bohr's dictum is a currently debated issue [4–8], we would like to attempt to sidestep this issue altogether by asking the question, "Can an electron beam be split completely into its two spin components with microscopic fields?" In Mott scattering, microscopic fields are used to obtain good polarization at certain scattering angles. This good polarization can only be obtained for a marginal fraction of the incident electron beam. Indeed, no polarizing beam splitters for electrons appear to exist to this date. It is unknown to the authors whether such a device is possible or perhaps a physical principle forbids this.

As an example of a microscopic field, consider a standing wave of light. When electrons pass through such a field, it is possible for the electron wave to diffract from the periodic light structure, just as light waves can diffract from the periodic structure of a material grating. This effect, known as the Kapitza–Dirac effect, was proposed in 1933 [9], and we recently realized this experiment [10]. The motion of the electron has to be described by a wave to predict the outcome of the experiment correctly. In this case, it is clear that the standing wave of light does constitute a microscopic electromagnetic field. Because the diffracted beams are coherent with each other, the Kapitza–Dirac effect is, in effect, a beam splitter for electrons. In relation to the question posed above, it appears natural to wonder whether or not the electron spin will flip while it is diffracted into different beams and as such constitutes a Stern–Gerlach magnet. We will first show that the electron spin is not influenced in the diffracted beams of the Kapitza–Dirac effect. Then, we will proceed to consider a modified laser field configuration involving counter-propagating laser beams with frequencies ω and 2ω, which is more interesting in this respect.

2. THE KAPITZA–DIRAC EFFECT

2.1. Photon Picture

Consider an electron that is approached by two photons from opposing sides. One photon will be absorbed, while the other stimulates an emission. This is the basic process behind the Kapitza–Dirac effect and is called stimulated Compton scattering (Fig. 1a). In the rest of the paper, we will ignore all possibilities where photon scattering is involved with frequencies other than the specified laser frequencies (i.e., to or from unoccupied modes). Processes such as spontaneous Compton scattering are ignored. In stimulated Compton scattering, energy and momentum can be conserved as illustrated in Figs. 1b and 1c, in contrast to the scattering of a free electron from a free photon. We are free to choose the polarization of the photons. When both photons carry the same angular momentum with respect to the lab frame, the angular momentum carried by the photons

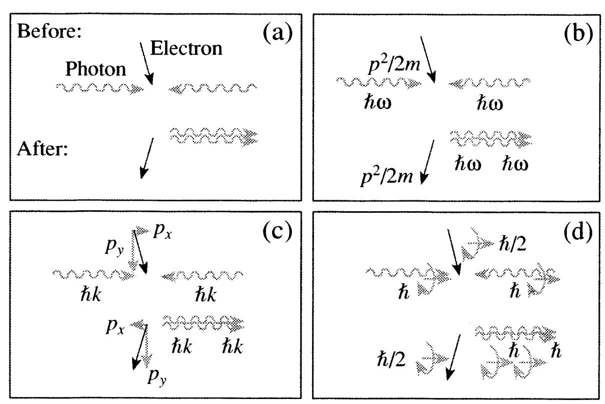

Fig. 1. (a) Stimulated Compton scattering is the basis for the Kapitza–Dirac effect. (b) Energy is conserved in this process. (c) Momentum is conserved in this process. (d) Angular momentum is conserved when the electron spin does not flip.

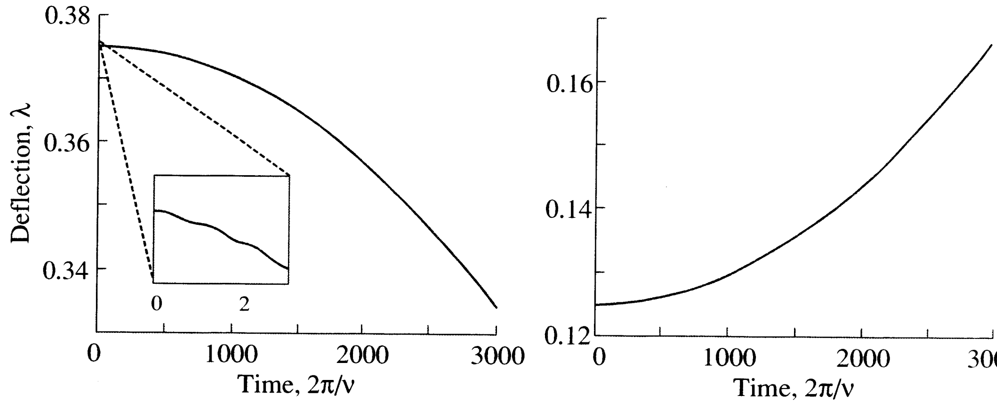

Fig. 2. Deflection of an electron by a standing wave of light starting at different positions, $x = \lambda/8$ (left) and $x = 3\lambda/8$ (right), in 3000 oscillators of the light field. The inset shows the deflection for three field oscillations, and the quiver motion can be seen.

before and after the interaction is unchanged. We can conclude directly that the spin of the electron in the Kapitza–Dirac effect should not have changed (Fig. 1d). For completeness, we should point out that in our experiment [10] we did not attempt to measure the spin dependence of the effect. When one photon carries an angular momentum opposite to that of the other photon, the electron should absorb two units of angular momentum, which is not possible. We did verify this prediction experimentally.

2.2. Field Picture

It is interesting to compare the photon picture with that of electrons interacting with a standing wave of light. Consider a charged particle (ignoring spin) in an arbitrary electromagnetic field. The classical equation for the evolution of the momentum is given by

$$\frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \tag{1}$$

where q is the charge of an electron, **E** is the electric field, **B** is the magnetic induction, and **v** is the electron velocity. Now, calculate the motion of the electron through a standing wave formed by one color of laser light described by the vector potential

$$\mathbf{A} = A_y\hat{y} = A_0\cos(kx)\sin(\omega t)\hat{y}, \tag{2}$$

where k is the light wavevector, ω is laser light frequency, x is the laser light propagation direction, and A is directed along y. This yields the electric field and magnetic induction as

$$\mathbf{E} = E_y\hat{y} = -A_0\omega\cos(kx)\cos(\omega t)\hat{y}, \tag{3}$$

$$\mathbf{B} = B_z\hat{z} = -kA_0\sin(kx)\sin(\omega t)\hat{z}. \tag{4}$$

Such a standing wave of light is obtained by counter-propagating two travelling waves. The equation of motion can be solved both analytically [11] and numerically. For our present argument, the numerical solution will turn out to be more valuable. The resulting acceleration of the electron depends on where it starts in the standing wave. At the nodes or antinodes of the standing waves, the electron only experiences the usual quiver motion due to the fast oscillations of the fields but no net time-averaged acceleration. The electron experiences a net acceleration to the left or right just between these points, indicating a force grating. This force can be expressed as a periodic potential with a periodicity of half the optical wavelength, which turns out to be the ponderomotive potential [11]. Note that the standing wave in the electric field coincides in position with the average potential and for that reason the two are often not explicitly mentioned separately. The result of the numerical integration of the classical equation of motion [1] is given in Fig. 2. For a laser intensity of 10^{10} W/cm^2 and an interaction time of 10^{-11} s (which is comparable to the experimental parameters used in reference [10]), electrons starting at $x = \lambda/8$ and $x = 3\lambda/8$ are deflected in opposite directions.

One can now use the above found potential and solve the Schrodinger equation [11]. It is interesting to observe that if the laser intensity in this calculation is sufficient to cause some appreciable scattering into the first order (at a transverse momentum kick of $2\hbar k$ = 2000 m/s), then the classically accelerated electron also has reached a velocity of several thousand meters per second. Within both pictures where the electron is treated as a wave and the picture where the classical deflection is calculated, we may now observe the following qualitative behavior. When the counter-propagating waves have the same polarization (their photons carry the same angular momentum), they interfere and form a standing wave from which the electron wave diffracts (or classically the electron deflects). When the two waves have opposite polarization, they cannot interfere and no diffraction (or deflection) is found, which is in very nice agreement with the photon picture discussed above. Based on our previous argument of angular momentum conservation, the electron does not flip sign even when the diffraction takes place.

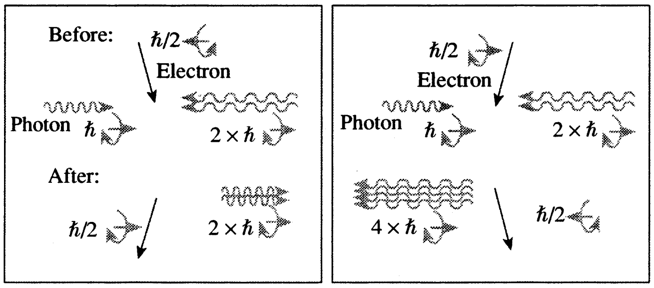

Fig. 3. When two counter-propagating travelling waves, one with frequency ω and one with 2ω, interact with a free electron, energy, momentum, and angular momentum can be conserved. Two photons of frequency ω are absorbed, while one photon of frequency 2ω stimulates an emission. Angular momentum can only be conserved when the electron flips its spin.

3. THE MICROSCOPIC STERN–GERLACH EFFECT

3.1. Photon Picture

The basic idea of this paper is to change the light field in such a way that one would expect a spin flip. We would like to consider the following arrangement as a candidate to do just that: an electron interacts with counter-propagating laser beams, one with frequency ω and the other with frequency 2ω (Fig. 2). Now, assume that two photons from the laser with frequency ω are absorbed while a photon from the laser with frequency 2ω causes a stimulated emission. When the electron does not change its kinetic energy in this process, the total energy is conserved. Momentum is also conserved when the electron enters the light field at the Bragg angle $\theta = \hbar k/p_e$. The recoil momentum of the photon is given by $\hbar k$, and the electron momentum is given by p_e. Again, we are free to choose the polarization of the photons. Assume that, before the interaction, all photons carry the same angular momentum (totaling $3\hbar$) with respect to the lab frame. After the interaction, the remaining two photons carry $2\hbar$ of angular momentum. This interaction can only conserve angular momentum when the electron absorbs one $\hbar$ of angular momentum and spin flips (Fig. 2a). In the same light field, it is also possible that one photon from the laser with frequency 2ω will be absorbed and two photons from the other laser will both cause a stimulated emission (Fig. 2b). In this case, the angular momentum can only be conserved when the electron supplies one unit of angular momentum in a spin-flip process. This implies that incident spin-up electrons can only be kicked one way, while spin-down electrons can only be kicked the other way. In the Bragg scattering regime, one polarization of electrons will be scattered away from the beam, while in the unscattered beam some polarization mixture remains. Only when the Bragg scattering is complete will the incident beam be separated into two orthogonally polarized beams. In the diffraction regime, the electron does not need to enter at the Bragg angle. The diverging laser light will provide the necessary angles between the electrons and photons. The resulting unpolarized incident electron beam is split into two spin-polarized components and a nonscattered component. Blocking this latter part of the beam gives effectively a microscopic Stern–Gerlach magnet for electrons.

At this point, it is perhaps interesting to compare this situation with the existing optical Stern–Gerlach magnets for atoms [12]. In this experiment, a 2 μm atomic beam crosses a standing wave of light with a period of 15 μm. The force on the atom depends on its internal state, and, because the atoms are localized within the optical periodicity, the external motion can be treated classically. This is a strong analogy to the usual Stern–Gerlach magnet and as such not a microscopic Stern–Gerlach magnet as discussed above.

Also, we may consider our above example for atoms instead of electrons. For the above process, we require a one-photon absorption followed by a two-photon stimulated emission. For a two-level system this necessitates, for example, an electric dipole transition followed by a magnetic dipole transition. Although this is possible, the weakness of such a transition would require intense laser light for its observation.

3.2. Field Picture

Let us compare this photon picture with that of electrons interacting with waves of light, analogous to the line of reasoning we followed in our discussion of the Kapitza–Dirac effect. At the same time, such an approach may also provide us with an estimate of how much intensity would be needed to realize a microscopic Stern–Gerlach magnet for electrons. Consider a charged particle including spin in an arbitrary electromagnetic field. The classical equation for the evolution of the momentum is given by

$$\frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \nabla(\boldsymbol{\mu}(t) \cdot \mathbf{B}), \qquad (5)$$

where q is the charge of an electron, $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic induction, $\mathbf{v}$ is the electron velocity, and $\boldsymbol{\mu}$ is the electron's magnetic moment. In turn, the magnetic moment of the electron is evolving in the magnetic field and has a time dependence given by the Bargman, Michell, and Telegdi (BMT) equation [13] (for $g = 2$ and $\gamma = 1$):

$$\frac{d\boldsymbol{\mu}}{dt} = \frac{e}{m}\boldsymbol{\mu} \times \left(\mathbf{B} - \frac{1}{2}\frac{\mathbf{v}}{c^2} \times \mathbf{E}\right), \qquad (6)$$

where the first term is the usual Larmor precession and the second term is the Thomas precession. Both of these sets of differential equations are coupled and should be integrated simultaneously. We calculate the motion of the electron through a laser field formed by one travelling beam of laser light of frequency ω counter-propagating with a laser beam of frequency 2ω. The vector potential for such a field is given by

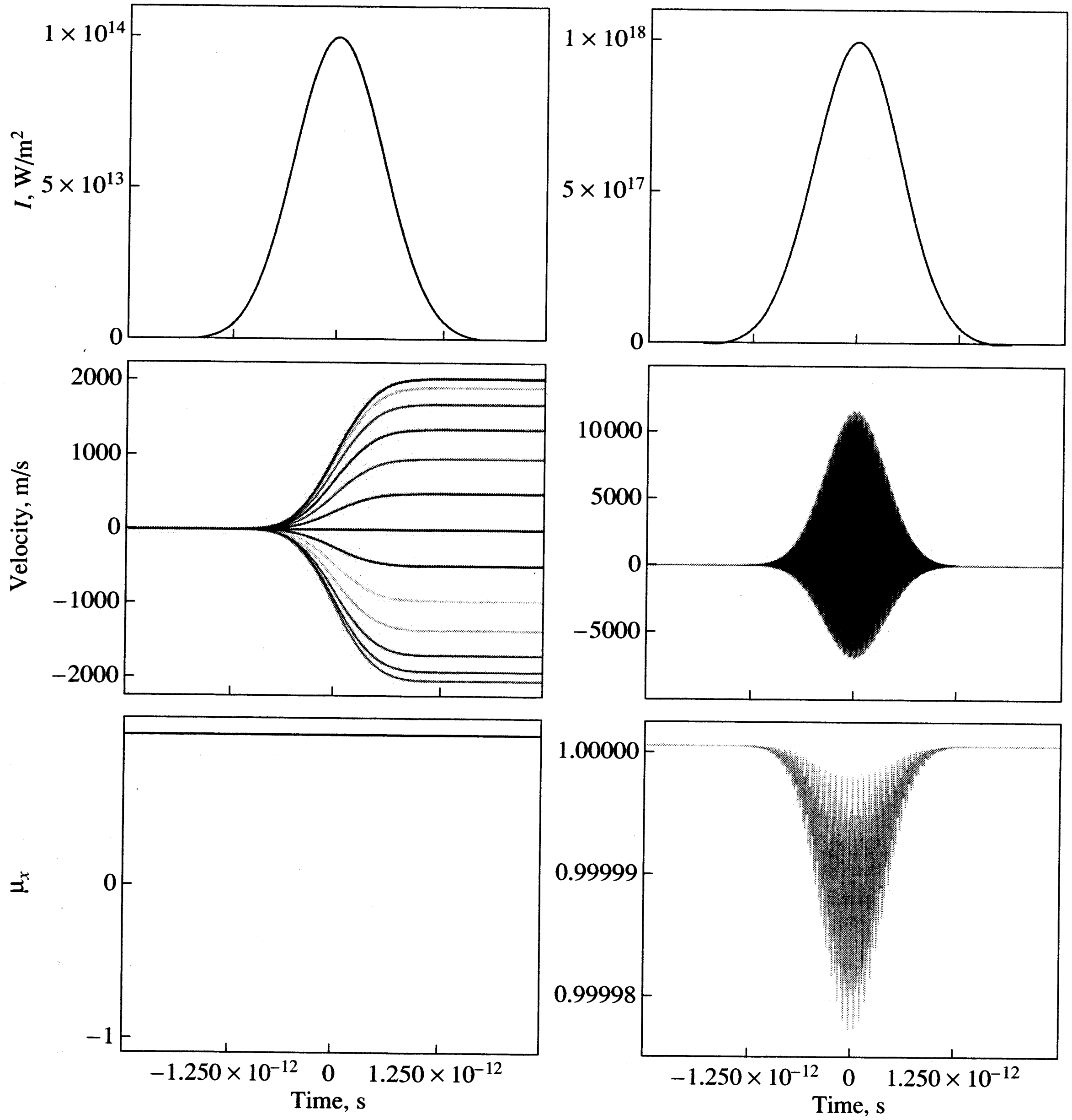

Fig. 4. The evolution of the velocity and the magnetic moment of an electron in two counter-propagating laser beams both of frequency ω (left) at 10^{14} W/m^2. The same for two laser frequencies ω and 2ω (right) at 10^{18} W/m^2.

$$\mathbf{A} = A_y\hat{y} = [A_1\cos(kx-\omega t) + A_2\cos(-2kx-2\omega t)]\hat{y}, \tag{7}$$

which yields the electric field and magnetic induction as

$$\mathbf{E} = E_y\hat{y} = -\omega A_0[\sin(kx-\omega t) + 2\sin(-2kx-2\omega t)]\hat{y}, \tag{8}$$

$$\mathbf{B} = B_z\hat{z} = kA_0[-\sin(kx-\omega t) + 2\sin(-2kx-2\omega t)]\hat{z}. \tag{9}$$

It is important to note that no standing wave in the electromagnetic field is formed. However, this does not appear to exclude the formation of a time-averaged potential given the coupled dynamics of the magnetic moment and the momentum. We also used the vector potential for circular polarization,

$$A_x = 0, \tag{10}$$

$$A_y = A_1\sin(kx-\omega t) + A_2\cos(-2kx-2\omega t), \tag{11}$$

$$A_z = A_1\cos(kx-\omega t) + A_2\cos(-2kx-2\omega t), \tag{12}$$

which leads to qualitatively identical results. The equations of motion can now be numerically integrated starting with an electron at rest at different starting positions x in the laser field. Laser intensities from 10^{14}

up till 10^{18} W/m^2 give no spin flip and no time-averaged force in the laser propagation direction (see Fig. 3, right column). The same calculation is compared for the Kapitza–Dirac situation at 10^{14} W/m^2, where a force dependent on starting position accelerates the electrons to several thousand meters per second (Fig. 3, left column). This calculation differs from the calculation in Section 2.2 in that here the laser field is turned on and off smoothly, as indicated in Fig. 3 (top left). For the low intensity result, the difference is negligible between a slow turn on or an instantaneous turn on. For the high intensity case, an instantaneous turn on gives the electron an initial kick where the direction depends on the phase of the electric field. For the present paper, we were interested to find out whether or not a time-averaged potential existed. For the case considered here, the answer is negative.

4. SUMMARY AND CONCLUSION

We raised the question if it is possible to construct a beam-splitting polarizer for electron beams with microscopic electromagnetic fields. The exchange with photons from two counter-propagating laser beams allows for such a process to conserve energy, momentum, and angular momentum, even though a classical integration of the equation of motion gives neither a position-dependent force nor spin flip. We view this result as a first step in a series of calculations. The dependence of our result on laser intensity should be carefully studied. A comparison between this simple classical calculation and a full quantum calculation may indicate if there are some purely quantum mechanical effects. Different configurations of laser polarization, k vectors, and laser frequencies could be studied easily with the current approach. For instance, the combination of three laser frequencies ω, 2ω, and 3ω also allows the conservation of energy, momentum, and angular momentum but involves only exchange of even numbers of photons. This case would not require the electron to spin flip, but still there is no standing wave in the electric field! Without performing any calculation, we find it hard to predict if there will be a time-averaged potential in this case. We also plan to extend the present calculation to the full BMT equation and the relativistic equations of motion to investigate the dependence of this result on the g factor and relativistic effects. In turn, this can be compared to QED calculations, which in some cases are available [14].

ACKNOWLEDGMENTS

We would like to thank Olga Smirnova for helpful discussions. This work has been funded by NSF, the Research Corporation, and DOD-EPSCoR.

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