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Aaron J. Engel
University of Nebraska-Lincoln, aaronjengel@gmail.com

Gregory R. Bashford
University of Nebraska-Lincoln, gbashford2@unl.edu

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Enabling Real-Time Ultrasound Imaging of Soft Tissue Mechanical Properties by Simplification of the Shear Wave Motion Equation

Aaron J. Engel, *Student Member, IEEE* and Gregory R. Bashford, *Senior Member, IEEE*

***Abstract*— Ultrasound based shear wave elastography (SWE) is a technique used for non-invasive characterization and imaging of soft tissue mechanical properties. Robust estimation of shear wave propagation speed is essential for imaging of soft tissue mechanical properties. In this study we propose to estimate shear wave speed by inversion of the first-order wave equation following directional filtering. This approach relies on estimation of first-order derivatives which allows for accurate estimations using smaller smoothing filters than when estimating second-order derivatives. The performance was compared to three current methods used to estimate shear wave propagation speed: direct inversion of the wave equation (DIWE), time-to-peak (TTP) and cross-correlation (CC). The shear wave speed of three homogeneous phantoms of different elastic moduli (gelatin by weight of 5%, 7%, and 9%) were measured with each method. The proposed method was shown to produce shear speed estimates comparable to the conventional methods (standard deviation of measurements being 0.13 m/s, 0.05 m/s, and 0.12 m/s), but with simpler processing and usually less time (by a factor of 1, 13, and 20 for DIWE, CC, and TTP respectively). The proposed method was able to produce a 2-D speed estimate from a single direction of wave propagation in about four seconds using an off-the-shelf PC, showing the feasibility of performing real-time or near real-time elasticity imaging with dedicated hardware.**

I. Introduction

Biomechanical properties of soft tissues can change in pathological conditions, and several useful diagnoses have been shown possible from knowledge of these changes [1, 2]. Soft tissue mechanical properties can be characterized and imaged with ultrasound based shear wave elastography (SWE) [3]. Shear waves can be generated by mechanical excitation applied at the surface of the tissue, or by acoustic radiation force applied within the tissue. Because shear wave propagation speed is directly related to tissue elasticity, methods to estimate tissue elasticity often rely on estimation of shear wave speed. These methods include inversion of the wave equation [4], time-of-flight (TOF) measurements [5, 6], and estimation of the phase gradient from the propagating shear wave [7].

Reliable estimates from data with low signal-to-noise-ratio (SNR) often require increased computational time or increased smoothing which degrades spatial resolution. For modern imaging systems the increase in computational time may be costly; moreover, real-time or near real-time elastography imaging remains a challenge while working with general-purpose equipment found in most academic laboratories. Therefore, it is of interest to develop a reliable method which requires less computational time while reducing the need for resolution-degrading data smoothing.

Current methods used in SWE include direct inversion of the wave equation (DIWE), time-to-peak (TTP), and cross-correlation (CC). DIWE inversion requires two spatial derivatives, increasing noise. TTP and CC in general require a five- to ten-fold interpolation, which increases the computational complexity significantly. In this study we implement a method of shear wave speed estimation based on the inversion of the first-order wave equation. Our hypothesis is that directional filtering effectively reduces the second-order wave equation to a first-order wave equation form. Shear wave speed can be reliably and quickly estimated by using finite difference methods for derivative approximation, and robustness can be obtained by integrating through time the result of the inversion for each frame of the shear wave motion estimation. This method was validated using homogeneous tissue-mimicking phantoms with differing nominal elasticity values. The purpose of this study was to show the feasibility and time improvements of using this method.

II. Methods

A. The Inverse Problem for Shear Wave Speed

In this study, shear waves were imaged. Navier's equation describing motion in an elastic media can be written as

$$(\lambda+2\mu)\nabla(\nabla\cdot\boldsymbol{u}) - \mu\nabla\times(\nabla\times\boldsymbol{u}) = \rho\boldsymbol{u}_{tt}, \tag{1}$$

where $\boldsymbol{u} = \boldsymbol{u}(x,y,z,t)$ is the displacement vector field, λ and μ are Lamé's first parameter and the shear modulus, ρ the material density, ∇ the gradient, $\nabla\cdot$ the divergence, and $\nabla\times$ the curl. Because soft tissues are composed mostly of water, they can be assumed incompressible and divergence free ($\nabla\cdot\boldsymbol{u}=\boldsymbol{0}$). The equation of motion for shear waves becomes

$$\mu\nabla^2\boldsymbol{u} = \rho\boldsymbol{u}_{tt}, \tag{2}$$

where the shear modulus μ and the shear wave speed c are related by the equation

$$\mu = \rho c^2, \quad c > 0. \tag{3}$$

A solution to (2) can be written as the summation of waves propagating along all possible paths through the media [8]

$$\boldsymbol{u} = \Sigma_\theta\, \boldsymbol{U}(\theta,t), \tag{4}$$

A. J. Engel and G. R. Bashford are with the Department of Biological Systems Engineering at the University of Nebraska-Lincoln, Lincoln, NE 68583 USA (email: aaronjengel@gmail.com, gbashford2@unl.edu).

where U is an arbitrary waveform propagating in the direction θ, its form governed by the initial conditions. By rotating and redefining the axis as the characteristic curve $\xi - ct = C$, where C is some constant, the arbitrary waveform becomes

$$\boldsymbol{U}(\xi,t) = \boldsymbol{U}_0(\xi - ct)e^{-\lambda t}, \quad \lambda \geq 0, \tag{5}$$

where $\boldsymbol{U}_0 = \boldsymbol{U}(\xi,0)$ and λ is introduced as a coefficient containing any attenuation of the waveform U_0. Equation (6) is the solution to the first-order wave equation with attenuation which is written

$$\boldsymbol{U}_t + c\boldsymbol{U}_\xi = -\lambda \boldsymbol{U}. \tag{6}$$

Rearrangement of (6) can be used to form an estimate for shear wave speed:

$$c = (\boldsymbol{U}_t + \lambda \boldsymbol{U}) / \boldsymbol{U}_\xi. \tag{7}$$

For weakly attenuating waves, $U_t = O(\omega U) >> O(\lambda U)$. The temporal derivative caused by wave propagation dominates that caused by wave attenuation. The leading order approximation for a directionally filtered shear wave becomes

$$c \sim | \boldsymbol{U}_t / \boldsymbol{U}_\xi |, \tag{8}$$

where the absolute value is taken to force c to be positive. Because spatial sampling was performed in the xz-plane, the gradient magnitude was used as the spatial derivative,

$$c \sim |\boldsymbol{U}_t| / \mathrm{sqrt}(\boldsymbol{U}_x^2 + \boldsymbol{U}_z^2). \tag{9}$$

B. Shear Wave Motion Estimation

In this study, shear waves were imaged using a Verasonics Vantage System using a high frequency linear array transducer (L12-3v, Verasonics Inc., Kirkland WA, USA) with a transmit center frequency of 10 MHz and a pulse repetition frequency of 6000 Hz. To generate shear waves, a 20 cycle, 400 Hz signal was used to drive a mechanical shaker (Type 4810, Brüel and Kjaer, Nærum, Denmark) placed on the surface of the phantom. To limit the effects of out of plane wave propagation, the transducer was placed perpendicular to the assumed direction of wave propagation. Ultrasound imaging began following cessation of the mechanical excitation. Axial and lateral spatial sampling was set to 0.1 mm. The final 3-D in-

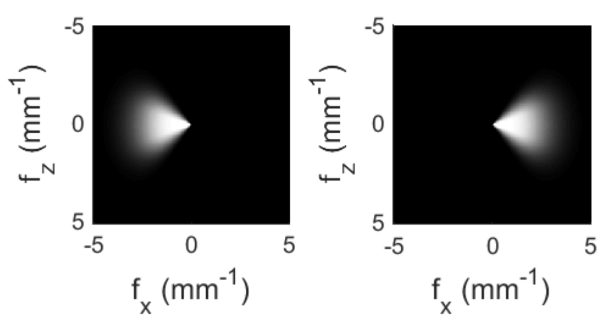

Figure 1. Directional filters in the spatial frequency domain used to separate opposite directions of wave propagation.

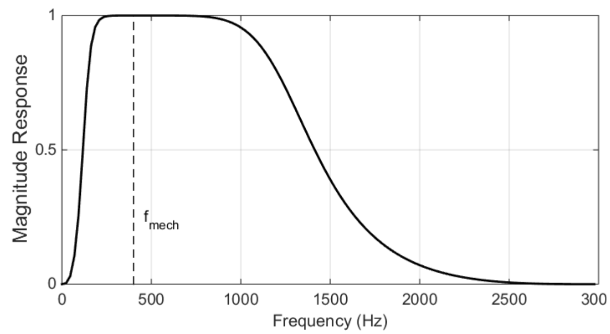

Figure 2. Butterworth filter used to filter about the mechanical excitation frequency of 400 Hz.

phase/quadrature (IQ) data size was 256 × 256 spatial pixels and 129 frames in time. Shear wave motion was estimated from the axial velocity component of the shear wave displacement [9]. To improve signal-to-noise-ratio (SNR) in the shear wave motion estimate while maintaining frame rate, the shear wave was imaged using plane waves transmitted at three angles (-4°, 0°, 4°) and motion was estimated from consecutive frames of the same transmit angle and averaged over the angles [10].

Directional filtering similar to the process described in [11] was performed in the frequency domain, with left-to-right propagating waves separated from right-to-left propagating waves. The window function used for directional filtering was a Tukey window with a tapered-to-constant ratio of 0.75 with bounds at $\pm\pi/3$ radians about the desired direction of wave propagation. Figure 1 shows the spatial frequency directional filters used in this study. Shear wave motion was bandpass filtered around the mechanical excitation frequency. Figure 2 shows the temporal frequency filter.

C. Homogeneous Phantom Experiments

Three homogeneous gelatin phantoms were made with gelatin concentrations of (5%, 7% and 9%) by weight. All phantoms contained 1% propanol by weight and 6.5% glycerol for tissue-mimicking properties and durability. Shear waves were generated and imaged as described above. For every frame in both directions of the shear wave motion estimate, shear wave speed was estimated by (9) using a robust method for finite differences [12], and the resulting shear wave energy calculated. A final shear wave speed map was recovered by weighted averaging of all shear wave speed maps, where shear wave energy was used as the weights.

To check for homogeneity and robustness of the imaging method, data were collected at three spatially different locations throughout each phantom. For comparison with conventional methods, shear wave speed was estimated by DIWE, 2-D TTP, and 2-D CC. The same smoothed directionally filtered data was used for each estimate. For comparison to the nominal speed value, immediately following SWE imaging the Young's modulus was measured using a compression test on samples taken from three separate regions in each phantom using a commercial texture analyzer (TMS-Pro, Food Tech. Corp., Sterling, VA, USA). For soft tissues, the Young's Modulus is related to the shear wave speed by the equation

$$E = 3\rho\, c^2, \tag{11}$$

where the density ρ was assumed 1000 kg/m^3 for all phantoms.

III. Results

Figure 3 shows the representative 2-D speed maps recovered using the proposed method. The black dashed box indicates the ROI used to estimate shear wave speed within each image. Representative shear wave speed estimates from within the ROI were 1.98±0.20, 2.87±0.10, and 3.54±0.23 m/s for phantoms of 5%, 7%, and 9% gelatin concentration respectively. Here the mean was taken from a single ROI and the standard deviation was calculated from all the pixels within the ROI. Calculation time for each image took an average of four seconds per direction for a 256×256 image with 129 frames in time. Calculation times for DIWE, 2D-CC and 2D-TTP were four seconds, 52 seconds, and 80 seconds respectively. Calculations were performed on a general computing desktop with an Intel i5 CPU at 2.8 GHz.

Figure 4 shows the results from the homogeneous phantom experiments. The means were calculated from each of the shear wave speed estimates from each phantom, and the standard deviation from these means was calculated. Speed estimates are comparable between each method used for estimation of shear wave speed. Nominal shear wave speed was calculated from the measured Young's Modulus using the compression test. The Young's Moduli for the phantoms were measured as 8.59±1.27, 14.8±0.45, and 20.3±3.08 kPa.

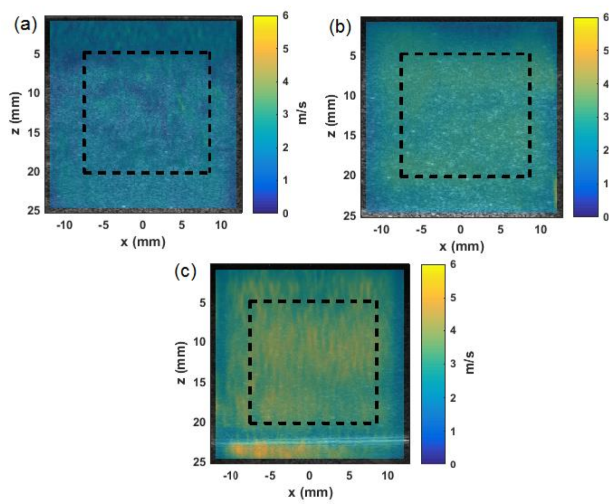

Figure 3. 2-D Shear wave speed estimates from phantoms of (a) 5%, (b) 7%, and (c) 9% gelatin concentration. The black dashed box indicates the ROI used for average speed estimate comparison.

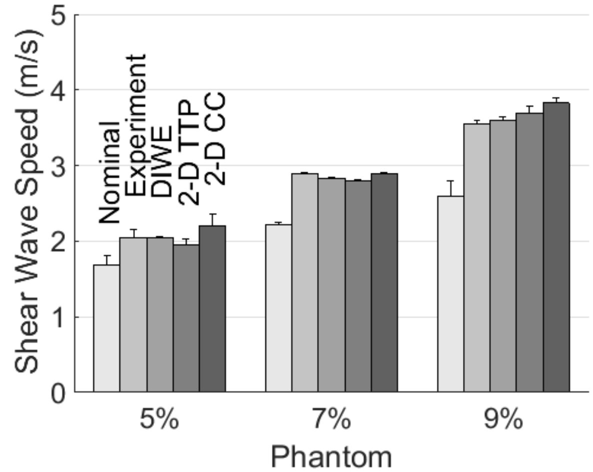

Figure 4. Shear wave speed estimates from directionally filtered data on phantoms containing gelatin percentages of (5%, 7%, 9%).

IV. Discussion

A. Homogeneous Phantom Experiments

In Fig. 3, Homogeneity within the imaging region can be seen. The low standard deviation within the ROI suggests the phantom can be considered as homogeneous.

The computation time for TTP estimation was slowed because the method requires peaks to be estimated, and the continuously sinusoidal nature of the waves requires tracking of multiple peaks. Computation time for DIWE was equal to the proposed method however relied on second order derivatives, which are assumed to be less robust in noise. This speed of the proposed method suggests it is capable of real-time or near real-time imaging with using laboratory equipment. Further decreases in computation time may be possible by reducing the spatial sampling within the imaging frame, or by performing calculations only in the direction with the highest energy.

A difference can be seen between the speed estimated from the multiple SWE estimates and the speed calculated from the compression test. Because the compression test operates near 0 Hz while the shear waves generated in the SWE experiments used 400 Hz, the differences can be explained by viscosity within the phantom, which causes wave dispersion where higher frequency waves experience a stiffer media.

Lowering the driving frequency from 400 Hz may cause the estimates from the SWE methods to converge with the compression test. Because spatial wavelength is inversely proportional to temporal frequency, spatial frequencies will tend towards DC and directional filtering will start to fail. A solution is to increase the imaging field of view to provide a more robust Fourier Transform for directional filtering, which could be accomplished by changing the plane wave transmit delays in the L12-3v transducer to mimic that of a curvilinear transducer.

B. The Inverse Problem for Shear Wave Speed

In the derivation of the shear wave speed estimate given by (9), no assumption was made on the directionality of the shear wave in the imaging media. Shear waves can be filtered from a 3-D volume signal. Because conventional ultrasound imaging systems acquire data in a 2-D plane, generation of the initial shear wave should be controlled to reduce the shear wave propagation through the elevational direction which may bias the shear wave speed estimate higher than the true value.

Simplification to (9) requires directional filtering; however this is only to remove waves propagating in significantly different directions. In this study, two directions were used with the assumption that the primary direction of wave propagation was laterally through the phantom. When calculating the derivative in a propagation direction, if the wave is locally unidirectional, the gradient magnitude will point in the direction of propagation and give the proper estimate for the spatial derivative. This study used directional filters where the radial directional component was defined by a Tukey window. This allowed more wave energy to remain when the direction of propagation was not aligned with the desired filtering direction. It is assumed that more wideband directional filters may be used in the case where an initial direction is unknown, spherical, or when refraction at boundary layers may cause a change in the direction of propagation. Narrowband filters may be more preferable when there is significant wave propagation in several directions, which may be the case when using mechanical excitation to image structures deep within tissue.

The derivation of the proposed method shows similarities with other TOF methods. From (5), two TOF methods for speed estimation can be deduced (TTP and CC). A point of interest on U_0 can be chosen and tracked as is moves along the characteristic curve, or because the waveform U_0 propagates distorted only by attenuation, the entirety of U_0 can be tracked for speed estimation. Both the TTP and the CC methods have previously been shown to be successful in recovering accurate shear wave speed.

V. Conclusion

This study introduced a fast method for visualization of soft tissue mechanical properties using ultrasound based shear wave elastography. This method estimates shear wave speed by inversion of the first-order wave equation. Using homogeneous phantoms, this method was found to provide estimates comparable to other current methods used for shear wave speed estimation in terms of accuracy, but with greatly increased speed and/or less computational complexity. Reduction of the second order wave equation to the first-order wave equation by way of directional filtering allows for the first derivative calculations to be a viable option for real-time shear wave elastography using laboratory equipment.

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