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# Distributed Beamforming with Imperfect Phase Synchronization for Cognitive Radio Networks

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*Abstract*—**In this paper, we present the analysis and simulation evaluation of a cognitive radio network employing a distributed beamforming technique with imperfect phase synchronization in the presence of a primary receiver. Our system model consists of a group of cognitive transmitters, each with an ideal isotropic antenna and equal transmit power, communicating with a secondary receiver in the far-field. The objective of the network of cognitive transmitters is to optimize its beampattern in the direction of the secondary receiver while minimizing the beampattern in the direction of the primary receiver to a certain threshold. The phases of the transmitted signals determine the beampattern, and we demonstrate that an optimization problem can be formulated to determine the phases of the transmitters that satisfy the constraints. We then evaluate the beampattern under imperfect phase synchronization and present how the phase error can impact the performance of beamforming and cause protection to the primary receiver to suffer. The results bring some interesting insights to distributed beamforming with imperfect phase synchronization for cognitive radio networks.**

**Keywords** - ***Signal processing, distributed beamforming, cognitive radio networks, beampattern.***

## I. INTRODUCTION

Cognitive radio can be defined as a device or network that dynamically adapts to its environment based on the detection of environmental conditions [1, 2]. Research in cognitive radio has emerged due to the promise of efficient use of the radio spectrum, which is a limited resource. Experiments have shown that much of the licensed radio spectrum is unoccupied by the user to which it is licensed or what is referred to in the literature as the primary user (PU) [1]. Cognitive radio systems have been proposed that allow secondary users (SU) to communicate with each other using licensed spectrum when it is not occupied by the primary user [2]. This is predicated on ensuring that interference with primary communications is kept below a certain threshold or the incidence of interfering is kept below a certain probability requirement. As the licensed radio spectrum becomes more saturated, the hope of cognitive radio providing more efficient use of this scarce resource has sparked a growth in related research and potential implementations of cognitive radio networks [3].

In [4], it was shown that a linear array of $N$ transmitting nodes could achieve a directivity asymptotically approaching $N$ by phasing each node such that the signals combine constructively in the direction of the target receiver. This scenario was expanded to a distributed network of randomly placed nodes [5] and cooperative radio networks [6, 7, 8]. In accordance with linear arrays, the authors showed that the directivity of randomly placed nodes also approaches $N$ asymptotically. Thus, distributed beamforming can lend itself to cognitive radio networks in order to exploit spatial diversity by steering a beam towards a SU receiver and at the same time to create a null in the direction of a PU receiver. The objective of our distributed cognitive radio network is to maximize the power of the beampattern in the direction of the secondary receiver while limiting the power in the direction of the primary receiver, which is accomplished by determining the optimal phase of each transmitter.

As discussed in [5], distributed beamforming by randomly placed nodes requires each node to phase itself to the receiver or to know its relative location in the network. In this paper, we assume that each transmitting node knows its location in the network relative to a reference location. This requires collaboration among the transmitting nodes to attain their relative locations which is a reasonable assumption [9, 10].

Because the phase of each node determines the overall beampattern of the cognitive radio network, imperfect phase synchronization is an important consideration. The authors in [11] demonstrated the effect of phase errors caused by phase jitter in the phase-locked loop (PLL) on the beam formed by the distributed network. It was shown that phase error can reduce the strength of the beampattern in the target direction and offset the main lobe from the target direction if the signal-to-noise ratio (SNR) in the PLL is below a certain level. We expand on the analysis presented in [11] by adding a primary receiver to the system model, which adds a constraint to the problem and requires that the beam-weight of each transmitter is found using an optimization method. We then evaluate the effect of imperfect phase synchronization using our system model.

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The remainder of the paper is organized as follows. The system model of the cognitive radio network is presented in section II. In section III, we analyze the problem of finding the complex beam-weight of each node subject to the constraints. In section IV, we investigate the effect of imperfect phase synchronization on the beamforming solution. Our simulation results are presented in section V and our conclusions are drawn in section VI.

## II. SYSTEM MODEL

A two-dimensional network model with cognitive capability is shown in Figure 1. Our system model consists of a group of transmitting nodes placed randomly in a circle of radius *R*. All transmitters use an ideal isotropic antenna and equal transmit power. The location of the *n*-th transmitting node is denoted in polar coordinates as ($d_n$, $\Phi_n$), where the origin is taken at the center of the circle of radius *R*. A secondary receiver is placed outside of the circle at a distance *K* from the center of the network and a primary receiver is placed outside of the circle at a distance *A* from the center of the network. For simplicity, we assume that there is a single primary receiver, which would be the case if the cognitive network was operating in the presence of a single primary transmit-receive pair. The location of the primary receiver and secondary receiver are denoted as ($A$, $\Phi_p$) and ($K$, $\Phi_s$) respectively.

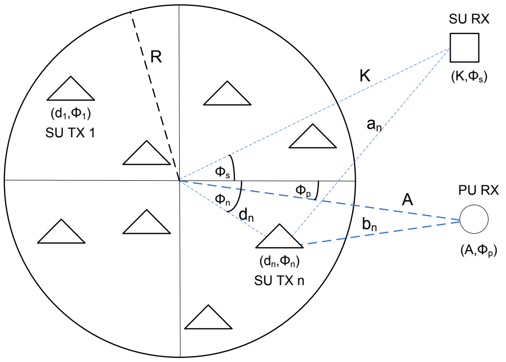

Figure 1: System Model

In order to determine how the signal radiated from each transmitter will combine at the secondary and primary receivers, it is of interest to determine the distance between each transmitter and each receiver. The distance, $a_n$, between the *n*-th transmitter and the secondary receiver can be found simply using Euclidean geometry and was determined to be:

$$a_n = \sqrt{K^2 + {d_n}^2 - 2 d_n K \cos(\Phi_s - \Phi_n)}. \tag{1}$$

If the far-field condition, $K \gg d_n$, is met, then the distance, $a_n$, can be approximated as [5]:

$$a_n \approx K - d_n \cos(\Phi_s - \Phi_n). \tag{2}$$

Similarly, the distance, $b_n$, between the *n*-th node and the primary receiver can be approximated as:

$$b_n \approx A - d_n \cos\left(\Phi_p - \Phi_n\right) \tag{3}$$

Using the distance $a_n$, the antenna array factor at the azimuth angle of the secondary receiver will be

$$S_s = \sum_{n=1}^{N} w_n e^{j\frac{2\pi}{\lambda} a_n} \tag{4}$$

where $N$ is the number of transmitting nodes, $\lambda$ is the wave-length, and $w_n$ is the complex beam-weight with the form

$$w_n = \frac{1}{N} \sum_{n=1}^{N} e^{j\beta_n} \quad . \tag{5}$$

where the phase of each transmitter, $\beta_n$, for $n = 1, 2, \ldots, N$, can then be obtained from the solution in Section III such that the signals from all transmitters arrive in phase at the secondary receiver and combine constructively. The expression in (4) is derived by finding the phase of the signal from each transmitter as it would arrive at the secondary receiver. This is accomplished by dividing the distance, $a_n$, by the wavelength and multiplying by the number of radians in a signal period. The magnitude of each transmitter's beam-weight is $1/N$, such that the total power is normalized.

The array factor at the azimuth angle of the primary receiver is similarly found as

$$S_p = \sum_{n=1}^{N} w_n e^{j\frac{2\pi}{\lambda} b_n}. \tag{6}$$

Finally, the far-field beam-patterns in the direction of the secondary receiver, $P_s$, and in the direction of the primary receiver, $P_p$, respectively, are

$$\begin{aligned} P_s &= S_s \times {S_s}^* = |S_s|^2 \\ &= \frac{1}{N^2} \sum_{k=1}^{N} \sum_{l=1}^{N} e^{j\frac{2\pi}{\lambda}[(a_k+\beta_k)-(a_l+\beta_l)]} \end{aligned} \tag{7}$$

and

$$\begin{aligned} P_p &= S_p \times {S_p}^* = |S_p|^2 \\ &= \frac{1}{N^2} \sum_{k=1}^{N} \sum_{l=1}^{N} e^{j\frac{2\pi}{\lambda}[(b_k+\beta_k)-(b_l+\beta_l)]} \quad . \end{aligned} \tag{8}$$

From (7) and (8), we can determine the desired values of $\beta_n$ as discussed in the following section.

## III. ANALYSIS OF BEAM-WEIGHT OPTIMIZATION

The objective of the network of cognitive transmitters is to determine the phase of each transmitter, $\beta_n$, for $n = 1, 2, \ldots N$, that gives maximum power in the direction of the secondary receiver under the constraint that power in the direction of the primary receiver is limited to the threshold, $\gamma_p$.

Therefore, we formulate the following optimization problem and constraints as [12, 13]:

$$\max_{\{\beta_n,\ n=1\ldots N\}} \frac{1}{N^2}\sum_{k=1}^{N}\sum_{l=1}^{N} e^{j\frac{2\pi}{\lambda}[(a_k+\beta_k)-(a_l+\beta_l)]} \quad (9)$$

Subject to

$$\frac{1}{N^2}\sum_{k=1}^{N}\sum_{l=1}^{N} e^{j\frac{2\pi}{\lambda}[(b_k+\beta_k)-(b_l+\beta_l)]} \le \gamma_p . \quad (10)$$

Without the constraint imposed by the presence of the primary receiver, selection of the phase of each transmitter is straightforward as in [5]. The work in [11] did not account for interference to a PU, which is a critical consideration in cognitive radio networks. We have added a PU receiver to our system model, which must be protected by the SU transmitters. This presents an optimization problem involving a function of multiple variables and a nonlinear constraint. This optimization can be performed using various algorithms and we have used the interior-point method to solve this problem [15]. Figure 2 illustrates a solution to a particular configuration of transmitters where the number of cognitive users, $N = 16$. It can be observed that the normalized beampattern is maximized in the direction of the secondary receiver and is kept below the threshold in the direction of the primary receiver. Details of the simulation will be discussed further in section V. Figure 2 demonstrates that this method of distributed beamforming can be used in cognitive radios to avoid interference to primary users.

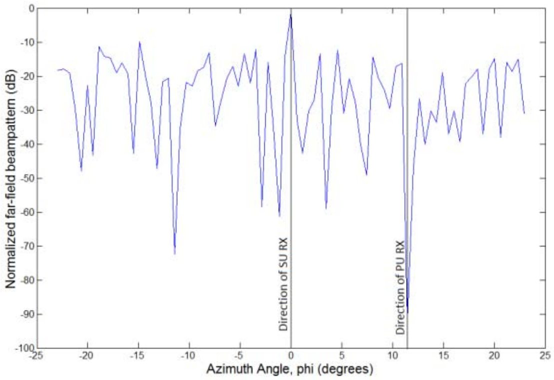

Figure 2: Realization of far-field beampattern, $N = 16$, $\Phi_s = 0°$, $\Phi_p = 11.5°$

## IV. EFFECT OF IMPERFECT PHASE SYNCHRONIZATION ON BEAMPATTERN

As shown in [5], the effect of imperfect phase synchronization among beamforming transmitters can have a significant effect on the desired beampattern. In [11], the distribution of the main lobe power level under imperfect phase synchronization for various PLL's was investigated. This effect of imperfect phase synchronization is of particular interest in cognitive radio networks, because of the constraint of limiting the far-field beampattern in the direction of the primary receiver. The distribution of the phase error at each transmitter is directly related to the SNR in the PLL [5].

The distribution of the phase error due to phase jitter in the PLL has a variance equal to the inverse of the SNR in the PLL as follows:

$$\rho = \frac{1}{\sigma^2} \quad (11)$$

where $\rho$ is the loop SNR in the PLL, and $\sigma^2$ is the variance of the phase error distribution. To generate the phase error in our simulation, we have used a Tikhonov distribution of the form [14]

$$f(x) = \frac{1}{2\pi I_0(\rho)} e^{(\rho\cos(x))} , \quad (12)$$

where $I_0(x)$ is the zero-th order Bessel function of the first kind. With the phase error at the $n$-th transmitter denoted by $\alpha_n$, the array factor at the azimuth angle of the secondary receiver becomes,

$$S_s = \sum_{n=1}^{N} w_n e^{j\left(\frac{2\pi}{\lambda}a_n + \alpha_n\right)} , \quad (13)$$

and the array factor at the azimuth angle of the primary receiver becomes,

$$S_p = \sum_{n=1}^{N} w_n e^{j\left(\frac{2\pi}{\lambda}b_n + \alpha_n\right)} . \quad (14)$$

As shown in (13) and (14), the phase error following a Tikhonov distribution with variance, $\sigma^2$, is applied to the phase of each transmitter. This offset cannot be accounted for in the optimization of the beam-weight and will be applied to the overall beampattern of the distributed beamforming network.

The phase error, $\alpha_n$, will therefore have an effect on the beampattern in the direction of the secondary and primary receivers. The beampattern in the direction of the secondary receiver with the phase error becomes

$$P_s = \frac{1}{N^2}\sum_{k=1}^{N}\sum_{l=1}^{N} e^{j\frac{2\pi}{\lambda}[(a_k+\beta_k)-(a_l+\beta_l)+(\alpha_k-\alpha_l)]} . \quad (15)$$

Similarly, the beampattern in the direction of the primary receiver with phase error becomes

$$P_p = \frac{1}{N^2}\sum_{k=1}^{N}\sum_{l=1}^{N} e^{j\frac{2\pi}{\lambda}[(b_k+\beta_k)-(b_l+\beta_l)+(\alpha_k-\alpha_l)]} . (16)$$

The contribution of the phase offset will be dependent on the variance of the distribution, $\sigma^2$, which is determined by the loop SNR of the PLL [5]. In the next section, we will discuss the effect of the loop SNR on the beampattern in the direction of the secondary and primary receivers and illustrate the effect of system parameters on the beamforming performance.

## V. SIMULATION RESULTS

In this section, we present the results of our MATLAB simulation, which was used to solve the optimization problem and analyze the network performance. Our proposed system model consists of $N$ cognitive transmitters randomly distributed within a circular area of radius $R$. To achieve a uniform, random configuration of cognitive transmitters, the distribution of the radius of the $n$-th user from the center of the network, $d_n$, is given [5]

$$f(d_n) = \frac{2d_n}{R^2}, 0 < d_n < R . \qquad (17)$$

The distribution of the polar angle, $\Phi_n$, of the $n$-th cognitive transmitter is

$$f(\Phi_n) = \frac{1}{2\pi}, -\pi \le \Phi_n < \pi . \qquad (18)$$

These distributions were used in our simulation to generate each particular configuration of transmitters, which affects the solution to the optimal beam-weight problem. The simulations were performed for a frequency of 2 GHz and the secondary receiver was placed at a distance $K = 2000\lambda = 300$ m at an azimuth angle of $\Phi_s = 0°$. The primary receiver was placed at the same distance, $A = K = 300$ m at an azimuth angle of $\Phi_p = 28.65°$. The radius of the network, $R$, was set to $500\lambda = 75$ m.

Because the configuration of transmitters affects the solution to the optimization problem, and subsequently the power that can be achieved in the direction of the secondary receiver, it was important to observe the performance over many trials. The Monte Carlo method was used to execute the trials. We first wanted to observe the effect of the threshold for power in the direction of the primary receiver, $\gamma_p$, on the performance of the beampattern in the direction of the secondary receiver, $P_s$. The maximum far-field power that can be achieved is 0 dB due to normalization and any reduction from the maximum of $P_s$ is caused by the constraint on the primary receiver. Figure 3 shows the cumulative distribution function (CDF) of the far-field power in the direction of the secondary receiver, $P_s$, for different values of $\gamma_p$ with $N = 8$. It can be seen that as the constraint on the primary receiver is loosened, the statistical distribution $P_s$ is improved. This is characterized by more realizations of $P_s$ being closer to the maximum of 0 dB for higher values of $\gamma_p$. Figure 4 shows the CDF for $P_s$ for $N = 16$. Compared to Figure 3, Figure 4 shows that increasing the number of cognitive transmitters improves the statistical distribution of $P_s$, because a larger percentage of realizations fall closer to the maximum possible power.

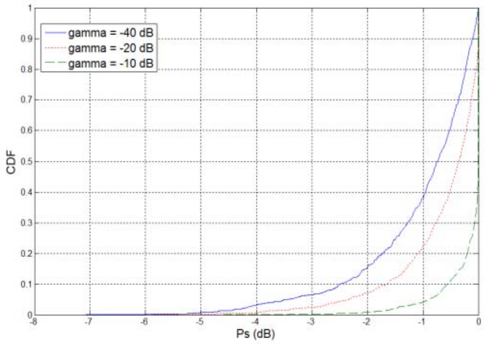

Figure 3: Cumulative distribution function of $P_s$, $\Phi_p = 28.65°$, $\Phi_s = 0°$, N=8

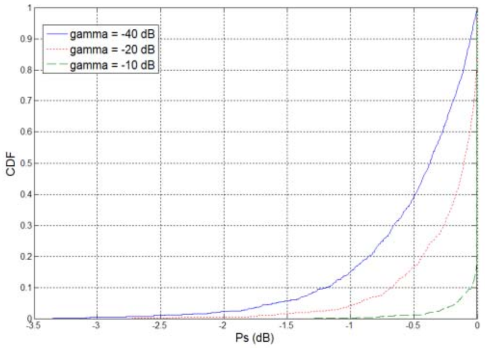

Figure 4: Cumulative distribution function of $P_s$, given $\Phi_p = 28.65°$, $\Phi_s = 0°$, N=16

Our next objective was to determine the effects of imperfect phase synchronization on the beampattern. Figure 5 shows the beampattern around the direction of the secondary receiver, $\Phi_s = 0°$. The 3D plot shows that as the SNR in the PLL increases, the beampattern in the direction of the secondary receiver improves. This is because as the SNR increases, the variance of the distribution of the phase error decreases. Figure 5 confirms that phase error can reduce $P_s$ from the maximum value and can shift the main lobe from the desired angle. Figure 6 is indicative of the problems that phase error can cause in cognitive radio beamforming networks. Figure 6 shows the beampattern around the primary receiver with $\Phi_p = 28.65°$. As shown, if the variance of the error is large enough, protection to the primary is compromised. As the SNR in the PLL improves, we can see that at around 14 dB, the primary receiver is given consistent protection as shown by the depression in the beampattern. These results demonstrate that phase synchronization among transmitters in our model is of critical importance if protection to the primary receiver is to be ensured

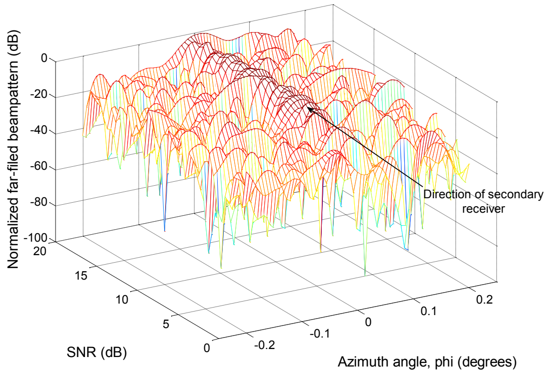

Figure 5: Far-field beampattern at secondary receiver, $\Phi_p = 28.65^\circ$, $\Phi_s = 0^\circ$, $N = 8$

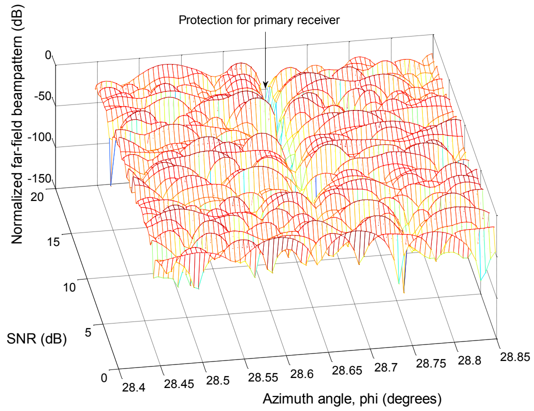

Figure 6: Far-field beampattern at primary receiver, $\Phi_p = 28.65^\circ$, $\Phi_s = 0^\circ$, $N = 8$

## VI. CONCLUSION

This paper presents the analysis and simulation evaluation of a cognitive radio network employing a distributed beamforming technique with imperfect phase synchronization in the presence of a primary receiver. Our results show that the constraint on the primary receiver affects the power in the direction of the secondary receiver. As this constraint is loosened, the statistical distribution of the power in the direction of the secondary receiver improves as more realizations become closer to the maximum power that can be achieved. Furthermore, the number of transmitters improves the statistical distribution of the power directed at the secondary receiver due to increased directivity. We demonstrated that imperfect phase synchronization can reduce far-field power in the direction of the secondary receiver and can compromise protection to the primary receiver. As the phase error of the transmitters improves, these problems are alleviated as this causes the variance of the distribution of phase error to decrease. Further research will evaluate a system model which incorporates multiple primary receivers. These results demonstrate some critical insights into distributed cognitive radio networks.

## ACKNOWLEDGMENT

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