Author:
Wu, Xin

Title:
Vulnerability analysis of structural systems.

VULNERABILITY ANALYSIS

OF

STRUCTURAL SYSTEMS

by

Xin Wu

A thesis submitted to the University of Bristol in accordance with the requirements for the degree of Doctor of Philosophy in the Faculty of Engineering, Department of Civil Engineering , April 1991

ABSTRACT

The objective of the research work described here has been to develop a theory of structural vulnerability for 2-D frame structures. The purpose of the theory is to enable the identification of the most vulnerable parts of a structural system so that they may be suitably protected and monitored.

Structural vulnerability analysis is concerned with the identification of various failure scenarios of structural rings at various hierarchical levels of definition. A method is developed to help in the identification of the most critical structural rings together with the critical failure scenarios.

The concept of a deteriorating event is presented and a measure of the damage demand for a failure scenario is defined.

The "robustness" of a structure is described in terms of structural vulnerability.

In this thesis a graph model of a structure is developed in which various load paths and loops are analysed. The single most important concept presented is that of a structural ring. A structural ring is an over-stiff or just-stiff structure which can transmit forces around a closed loop. A structural system then can be represented at various hierarchical levels of definition in terms of sets of interconnected structural rings.

The concept of a structural cluster is presented and an algorithm is developed to form structural clusters at various levels of definition.

A measure to evaluate the quality of well-formedness of a structural ring is developed. The measure is also used to provide a quantitative estimate of the structural vulnerability.

The deteriorating hierarchy of structural rings, **DHSR**, is presented which shows all possible ways in which a structural ring can deteriorate into a mechanism.

DEDICATION

to My father

ACKNOWLEDGEMENTS

I am indebted to Professor D. I. Blockley and Dr. N. J. Woodman for their help and guidance in carrying out this research and in producing this thesis.

I would like to thank all the former and present members of the Civil Engineering Systems Group for the interesting discussions which have contributed to this work. I would also like to express my thanks to Professor D. G. Elms for his helpful discussions.

I would like to express my special thanks to my wife, Zuying, and my parents and friends for their support and interest during the last three years

Finally, financial supports from a joint scholarship of British Council and Chinese Government are greatly acknowledged.

DECLARATION

This thesis entitled: Vulnerability Analysis of Structural Systems, is submitted for the Degree of Doctor of Philosophy, in the Faculty of Engineering, at the University of Bristol.

The research, on which this thesis is based was carried out between November 1988 and April 1991 under the joint supervision of Professor D. I. Blockley and Dr N. J. Woodman. It is due entirely to the author except where otherwise acknowledged in the text and has not formed the basis of a submission for any other degree.

Signed..Xin...Wu 吴辛

Date..16 - 06 - 1991

CONTENTS

LIST OF FIGURES

NOTATIONS

Symbols	Meaning
G	Graph
L	Link set in graph
N	Node set in graph
l_i	A link in G
n_i	A node in G
A	Associated Matrix
a_{ij}	Element in row i, column j of A
B	Symbol matrix
b_{ij}	Element in row i, column j of B
S	Structural system
M	Member set in S
J	Joint set in S
m_i	A member in S
j_i	A joint in S
$\mathbf{K}$	Structure stiffness matrix
$\mathbf{D}_{ii}$	Submatrix of $\mathbf{K}$ associated with a joint j_i
$det(\mathbf{D}_{ii})$	Determinant of $\mathbf{D}_{ii}$
λ	Eigenvalue
q_i	Quality of the well-formedness of a joint j_i
R^l_i	Structural ring at a level of definition l
$q(R^l_i)$	Quality of the well-formedness of a ring R^l_i
S^l_i	Structural cluster at a level of definition l
M^l_i	Member set in S^l_i
J^l_i	Joint set in S^l_i
$T(S^l_i)$	Tightness of a cluster S^l_i

$Q(S^l_i)$	Structural tightness of a cluster S^l_i
$\eta(S^l_i)$	Relative tightness of a cluster S^l_i
D^l_i	A degree of freedom set at a joint j^l_i
$d^l_{i,j}$	A degree of freedom in D^l_i
$s^l_{i,j}$	The capacity of a cluster S^l_i to transmit a degree of freedom
R^l_k	A deteriorated ring of R^l
$F_h(R^l)$	A failure scenario of R^l
$F(R^l)$	A failure scenario set of R^l
$f^l_{i,j,k}$	The k^{th} deteriorating event cause the loss of a degree of freedom $d^l_{i,j}$
$g^l_{i,j,k}$	The k^{th} deteriorating event to cause the loss of the capacity $s^l_{i,j}$ of a cluster S^l_i to transmit a degree of freedom.
$e(f^l_{i,j,k})$	Damage demand for a deteriorating event $f^l_{i,j,k}$
$e(g^l_{i,j,k})$	Damage demand for a deteriorating event $g^l_{i,j,k}$
$E[F_h(R^l)]$	Damage demand for a failure scenario $F_h(R^l)$
$\gamma[\mathrm{F}_h(R^l)]$	Separateness of a ring R^l with respect to $\mathrm{F}_h(R^l)$
S^l_r	A reference cluster at a level of definition l
$\xi[\mathrm{F}_h(R^l)]$	Effective consequence of a failure scenario $\mathrm{F}_h(R^l)$

Mathematical symbols

min	Minimal value
max	Maximal value
Σ	Summation
Σ_i	Summation to variable i
$\cap$	Intersection
$\cup$	Union
$\in$	Membership of set

Subscripts

i, j, k .	Designates ith, jth, kth elements

Superscripts

l	Level of definition

Chapter 1

Introduction

1.1 Objectives

The objectives of this thesis are:

(1). To present a theory of structural vulnerability, the purpose of which is to identify the most vulnerable parts of a structural system.

(2). To define the robustness of a structure in terms of structural vulnerability.

(3). To develop an analytical method to identify the various failure scenarios for a structural ring.

(4). To present the concept of a deteriorating event and to define a measure of the damage demand for a failure scenario.

(5). To examine the use of the graph theory in the development of the object oriented graph model of a structure and to analyse the various load paths and loops within it.

(6). To develop a structural ring model. To describe a structural system at various hierarchical levels of definition in terms of sets of interconnected structural rings.

(7). To develop a measure of the well-formedness of structural rings.

(8). To present the concept of a structural cluster and to develop an algorithm of cluster formation.

(9). To show how a structural system may be represented in a form of hierarchy.

(10). To develop the deterioration hierarchy of structural rings which consists of a set of all possible failure scenarios for a single structural ring.

(11). To describe the potential application of the theory of structural vulnerability in the analysis of structural system reliability.

1.2 General Introduction

In this thesis a theory is presented the purpose of which is to identify the most vulnerable parts of a structural system so that they may be suitably protected and monitored. A graph model of a structure will be developed in order to analyse the various load paths and loops within it. The model includes some new concepts, the single most important of which is that of a structural ring. A structure will be described at various levels of definition in terms of sets of interconnected structural rings and this will provide a basis for structural vulnerability analysis. The emphasis of structural vulnerability analysis is not the usual one of analysing a structure under some given loading condition, rather it is to examine the quality of the well-formedness of the structural rings at various levels of definition within a structure and to identify those structural rings which are the most vulnerable or critical together with the actions which might cause failure. The concept of the "robustness" of a structure will also be explored.

The theory of structural vulnerability involves a variety of subjects such as: graph theory, linear algebra, clustering techniques, structural analysis, and systems theory. Some relevant aspects of these subjects are discussed in chapters 2 to 7. The relevant literature is discussed in each chapter rather than a separate review chapter.

The material in this thesis is presented in eight chapters together with the references and a appendix. This chapter is of an introductory nature, describing the purposes and general outlines of this research and introducing the key concepts used in this thesis.

Chapter 2 aims to present a critical review of the theory of graphs and the theory of matrices which form the foundation of the mathematics on which the further development can be placed. The object oriented graph model (OOGM) of structural systems is developed.

In chapter 3, different types structural paths and loops within an OOGM are examined and the structural ring model is developed. The deterioration hierarchy of structural rings is generated to illustrate a set of all possible failure scenarios for a single structural ring. An algorithm is presented to identify structural rings of a structure.

The main objective of Chapter 4 is to develop a measure to evaluate the quality of the well-formedness of a structural ring. The study starts with exploring the well-formedness of joint objects contained in a structural ring. The theory of linear algebra is used as mathematical tools for the analysis, such as determinants, eigenvalues, eigenvectors, etc. The concept of well-formedness of structural rings is examined and the measurement of well-formedness of structural rings is quantitatively defined.

Chapter 5 examines some of the important clustering techniques in dealing with complex systems. The concept of structural cluster is presented. The tightness and structural tightness of clusters are defined to evaluate the connectivity and structural quality of clusters quantitatively. An algorithm is developed to implement the process of cluster formation. An example is given to illustrate the whole process of cluster formation step by step.

Chapter 6 discusses the concepts of hierarchy and holon and their roles in the representation of structural systems. A structural system is represented in a form of hierarchy and described at various levels of definition in terms of sets of interconnected structural rings.

Chapter 7 brings together all results from previous chapters and develop the basic principles of structural vulnerability analysis. An analytical method is presented to identify the various failure scenarios of a structural ring. The "robustness" of a structural system is explored.

Chapter 8 summarises the conclusions drawn from this research. Some possible directions for further research are also suggested.

1.3 Key Concepts

The key concepts used in this thesis are presented in this section, together with details of the subsequent chapter in which the concept will be discussed more fully.

Holon: A *holon* is a concept which is both a part and a whole. It is a part of a wider system and is itself a system(of subsystem) (see Chapter 2).

Object: An *object* is a computer data structure which is a holon and has characteristics. Characteristics are either external (public) or internal (hidden). Characteristics may include features (attributes), behaviours (transformations) and constraints (conditions) (see Chapter 2).

Degree of freedom: A *degree of freedom* is the capacity of a structural joint to permit the transmission of movement in a defined co-ordinate direction (separate and independent of other movements) (see Chapter 2).

Object oriented graph model (OOGM): The basic mathematical model to represent a structural system $S=(M,J)$ is the *object oriented graph model* (OOGM) which consists of two sets: a finite set of structural member objects M and a finite set of structural joint objects J (see Chapter 2).

Joint object: A *joint object* in an OOGM is a node or reference point where member objects connect. Its features includes at least its co-ordinate positions and the degrees of freedom of every member connecting into the joint. Other features could be velocities and accelerations. Behaviour may be modelled by an appropriate physics of motion (see Chapter 2).

Member object: A *member object* is a relation, linking and communicating object which connects at least two joint objects. It is a communication channel between joints transmitting movements (disturbances) along the degrees of freedom. Its features include its geometrical and physical properties such as length, areas, second moment of area, elastic modulus etc. Its behaviour may be modelled by an appropriate physics of material behaviour (see Chapter 2).

Structural cluster: A *structural cluster* S^l_i at a level of definition l is a sub-set $S^l_i=(M^l_i,J^l_i)$ of S in which a set of overlapping structural rings are more densely connected to each other within the cluster than to other structural rings outside of the cluster (see Chapter 5).

Primitive structural cluster: A *primitive structural cluster* is one which contains only one single member object (see Chapter 4).

Connected clusters: Two structural clusters are said to be *connected* when one or more joint objects are contained in both clusters (see Chapter 6).

Complex joint: A *complex joint* is the intersection of any two connected clusters. It may be either (i) a set of primitive cluster and/or (ii) a set of joints not directly connected but indirectly connected through the clusters which form the intersection.

Structural path: A *structural path* is a sequence of connected clusters (see Chapter 3).

Structural loop: A *structural loop* is a self connected structural path. It is a sequence of connected clusters which starts and ends with the same cluster (see Chapter 3).

Structural ring: A *structural ring* R^l at a level of definition l is a structural loop which is either (i) structurally over-stiff; or (ii) structurally just-stiff. A structural ring is the basic object of a structure which is capable of resisting any arbitrary equilibrium set of applied forces from any direction (see Chapter 3).

Well-formedness: The well-formedness of a structural ring is a measure of its ability to resist damage or loading from any arbitrary direction (see Chapter 4).

Hierarchy:
(i) A structural system can be represented by successively subordinate structural rings at each level of the hierarchy.
(ii) A structural ring R^l at the level of definition l in the hierarchy represents a

substructural system.

(iii) Given a structural ring R^l in the hierarchy, any of its arcs can be regarded as the cluster containing a set of structural rings at lower levels of definition and itself can be an (or part of) arc of a structural ring at a higher level of definition.

(iv) Structural rings at lower levels of the hierarchy are a more detailed description of a structure than those at higher levels of the hierarchy (see Chapter 6).

Deteriorating event: *A deteriorating event* is the result of actions which would cause the loss, by a structural ring, of the capacity to transmit a degree of freedom (see Chapter 7).

Failure scenario: *A failure scenario* is a sequence of deteriorating events which transforms a structural ring into a mechanism (see Chapter 7).

Deterioration hierarchy of structural rings (DHSR): The DHSR is a set of all possible failure scenarios for a single structural ring (see Chapter 3).

Damage demand: The *damage demand* is a measure of the effort which is required to make the occurrence of a deteriorating event. The damage demand of a failure scenario is thus equal to the sum of the damage demands of all deteriorating events contained in that failure scenario (see Chapter 7)

Minimal failure scenario: The *minimal failure scenario* of a structural ring at level of definition l is the one in which the damage demand required to transform the structural ring into a mechanism is a minimum (see Chapter 7).

Separateness: The *separateness* of a structural ring at a level of definition is a description of the consequence of a failure scenario. It is the number of structural clusters structurally disconnected from a reference cluster contained in that ring (see Chapter 7).

Reference cluster: A *reference cluster* at a level of definition may be any cluster chosen for its importance or because it has the highest value of structural tightness. On earth a reference cluster would normally be the ground cluster S_G or a cluster which contains S_G (see Chapter 7).

Effective consequence: The *effective consequence* of a failure scenario at level of definition l is measured by the ratio of the separateness of a structural ring caused by that failure scenario to the total required damage demands (see Chapter 7).

Maximal failure scenario: The *maximal failure scenario* of a structural ring at level of definition is one in which the effective consequence is maximal. The maximal failure scenario of whole structure is that for which the effective consequence over all levels of definition is maximal (see Chapter 7).

Structural vulnerability analysis: *Structural vulnerability analysis* is concerned with the identification of:

(i) the minimal failure scenario;
(ii) the maximal failure scenario;
(iii) any particular interesting failure scenarios with respect to a given reference cluster (see Chapter 7).

Robustness: The *robustness* of a ring is measured by the size of the damage demand. The most robust ring is the one with maximal damage demand. For a ring the robustness is the same as the damage demand and for a structure it is the minimal damage demand over all levels of definition, i.e. there is one level of definition which is the weakest (see Chapter 7).

Chapter 2

Object Oriented Graph Model of Structural Systems

2.1 Objectives

The objectives of this chapter are

1. To present a critical review of the theory of graphs which form the foundation of the mathematics of this research.
2. To introduce the matrix representations of graphs -- associated matrices and symbol matrices.
3. To develop the object oriented graph model of structural systems.

2.2 Introduction

As the title suggests, one of the main objectives of this chapter is to develop the object oriented graph model (OOGM) of a structure. The term object is used as in objected oriented programming (OOPs) (Meyer, 1988). This graph model of a structure consists of a set of joint objects and a set of member objects. Joints (or nodes) and members (or links) are in fact basic elements of graph theory. This motivates us to explore the use of graph theory in the structural vulnerability analysis and beginning our study with an introduction in this chapter to some basic concepts and definitions in the theory of graphs. The concepts and definitions to be used will generally follow that of Berge(1962) and Swanmy & Thulasirman(1981).

For a graph, which consists of large number of interacting nodes and links, a mathematical representation becomes essential for the computer manipulation. One way is by the use of matrices. In this chapter, we introduce the associated matrices and the symbol matrices and some of their properties. These two matrices provide a simple technique for clarifying the interrelationships between the objects of a graph. They can be used to identify the paths and loops, which is very useful in the analysis of the connectivity of a graph.

Finally, we develop the object oriented graph model (OOGM) of a structure. The elements of an OOGM are objects -- joint objects and member objects, and therefore it has structural characteristic which represents the specific structure being studied in the sense of its own features, behaviours and constraints. Secondly, the objects in a structure can be represented by a graph model-- nodes and links. This enables us to use the theory of graphs and the theory of matrices to analyze the vulnerability of a structure.

2.3 Some Concepts and Definitions in the Theory of Graphs

2.3.1 Graphs, Links and Nodes

A *graph* $G=(N, L)$ (Swanmy & Thulasiraman 1981) consists of two sets: a finite set N of elements called *nodes* and a finite set L of elements called links. Each link is identified with a pair of nodes. If the links of a graph are identified with order, then G is called a *directed* or an *oriented* graph. Otherwise G is called an *undirected* or a *non-oriented* graph. The discussions in this chapter are mainly concerned with undirected graphs.

We use the symbols $n_1, n_2, n_3, \ldots$, to represent the nodes and the symbols $l_1, l_2, l_3, \ldots$ to represent the links of a graph. The nodes n_i and n_j associated with a link l_k are called the end nodes of the link l_k. The link is then denoted as $l_k=(n_i, n_j)$. Note that while the elements of L are distinct, more than one link in

L may have the same pair of end nodes. All links having the same pair of end nodes are called *parallel links*. Further, the end nodes of a link need not be distinct. If $l_k = (n_i, n_i)$, then the link l_k is call a *self-loop* at node n_i. A graph is called a *simple graph* if it has no parallel links or self-loops. All graphs studied in this thesis are simple graphs except where specified.

Pictorially a graph can be represented by a diagram in which a node is represented by a dot or a circle and a link is represented by a line segment connecting the dots or the circles which represent the end nodes of the link.

For example, if

$$N = \{n_1, n_2, n_3, n_4, n_5\}$$

and

$$L = \{ l_1, l_2, l_3, l_4, l_5, l_6, l_7 \}$$

such that

$$l_1 = (n_1, n_5), \qquad l_2 = (n_1, n_2),$$
$$l_3 = (n_1, n_3) \qquad l_4 = (n_4, n_5),$$
$$l_5 = (n_2, n_4) \qquad l_6 = (n_2, n_3)$$
$$l_7 = (n_3, n_4)$$

then the graph $G = (N, L)$ is represented as in Fig.2.1. G is an undirected graph.

In an undirected graph, a single line segment connecting two nodes n_i and n_j actually denotes two equal links in opposite directions, one is from n_i to n_j, another from n_j to n_i. For the reason of simplicity, normally we only use l_k to denote both of these two links. Fig.2.2 has shown this case. Therefore, in an undirected graph, a link l_k is said to be symmetric.

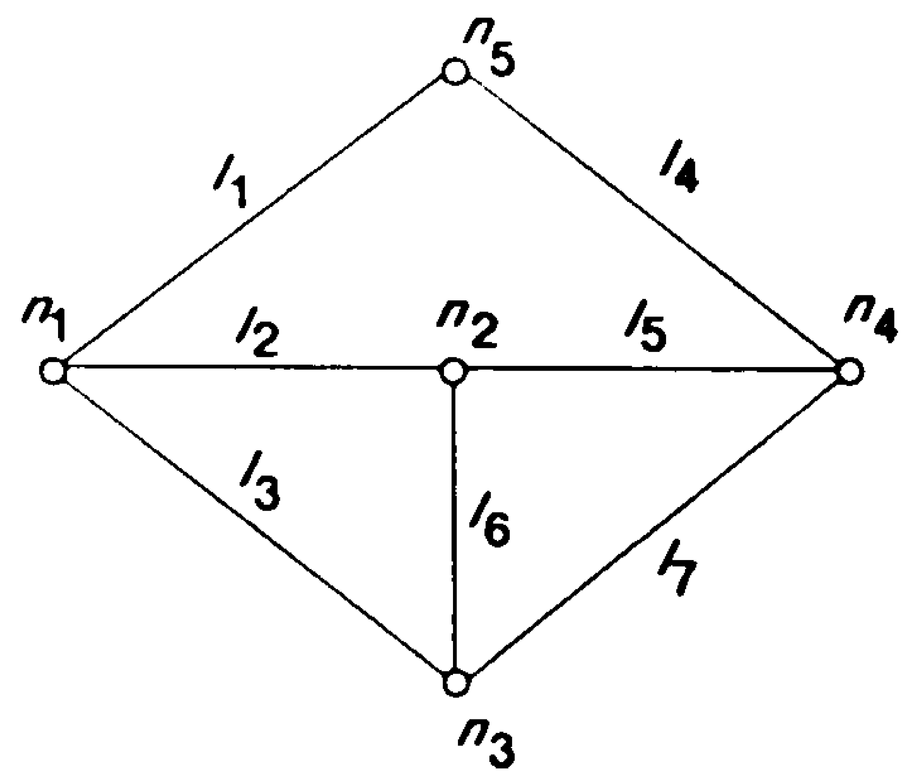


Fig. 2.1 A Graph G = (N, L)

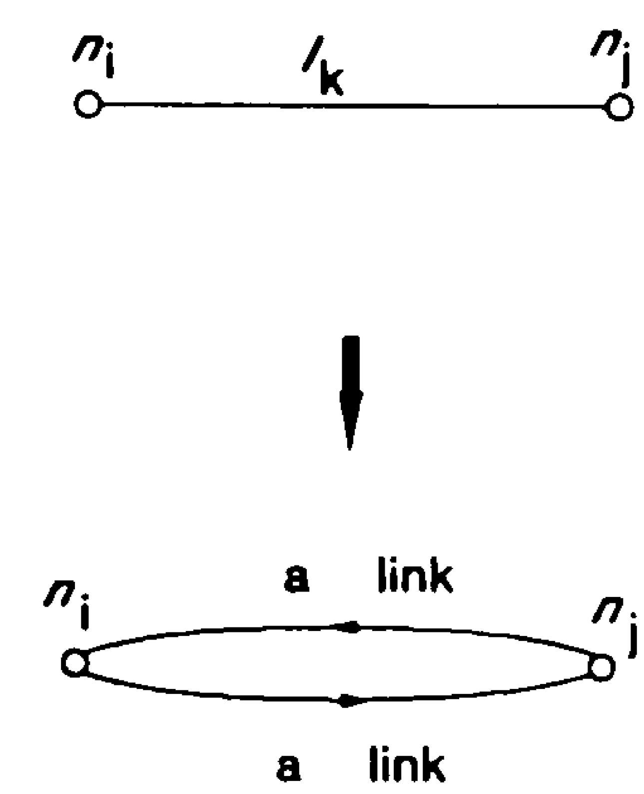


Fig. 2.2 A link

A link is said to be *incident* on its end nodes. Two nodes are *adjacent* if they are the end nodes of same link. If two links have a common end node, then these links are said to be adjacent.

For example, in the graph of Fig.2.1, link l_2 is incident on nodes n_1 and n_2, n_1 and n_3 are two adjacent nodes, while l_1 and l_2 are two adjacent links.

2.3.2 Paths and Loops

A *path* in a graph G is a finite alternating sequence of nodes and links $n_1, l_1, n_2, l_2, ..., n_{k-1}, l_k, n_k$ beginning and ending with nodes such that n_{i-1} and n_i are the end nodes of the link l_i ($2 \leq i \leq$ k). Alternatively, a path can be considered as a finite sequence of nodes $n_1, n_2, ..., n_k$, such that (n_{i-1}, n_i), ($2 \leq i \leq$ k), is a link in the graph G. This path is usually called a n_1-n_k path with n_1 and n_k referred to as the end or terminal nodes of this path. All other nodes are internal nodes of this path. Note that in a path, links and nodes can only appear once. This definition of a path is slight different from the one made by Swanmy &

Thulasiraman (1981) in which links and nodes in a path can appear more than once.

In the undirected graph G of Fig.2.1, the sequence n_1, n_2, n_3, n_4, n_5, is an open path, whereas n_1, n_2, n_3, n_1 is a closed path.

A closed path is a *loop* if all its nodes, except the end nodes, are distinct. Again, apart from the terminal node, links and nodes can only appear once in a loop.

For example, in Fig.2.1, the sequence n_1, n_2, n_4, is a path, whereas the sequence n_1, n_2, n_4, n_3, n_1 is a loop.

The number of links in a path is called the *length of the path*. Similarly the *length of a loop* is defined. A primitive path is one link path, i.e a single link.

2.3.3 Subgraphs

Consider a graph $G=(N, L)$. $G_i=(N_i, L_i)$ is a *subgraph* of G if N_i and L_i are, respectively, subsets of N and L such that a link (n_i, n_j) is in L_i only if n_i and n_j are in N_i.

For example, consider the graph G shown in Fig.2.3. The graph G_1 in (b) is a subgraph of G in (a). Another subgraph G_2 is shown in (c).

Consider two graphs G_1, G_2, $G_1=(N_1, L_1)$ *and* $G_2=(N_2, L_2)$. The *union* of G_1 and G_2, denoted as $G_1 \cup G_2$, is the graph $G=(N_1 \cup N_2,\ L_1 \cup L_2)$; that is, the node set G is the union of N_1 and N_2, and the link set of G is the union of L_1 and L_2

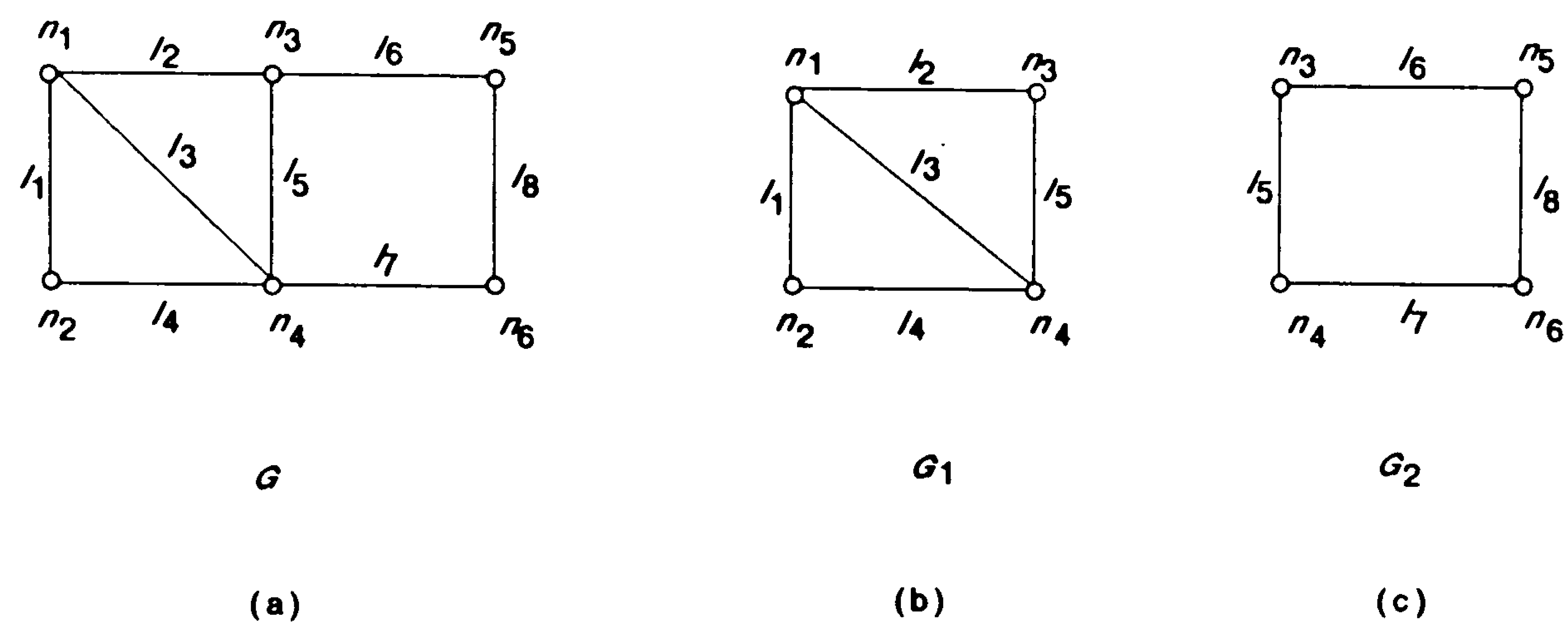
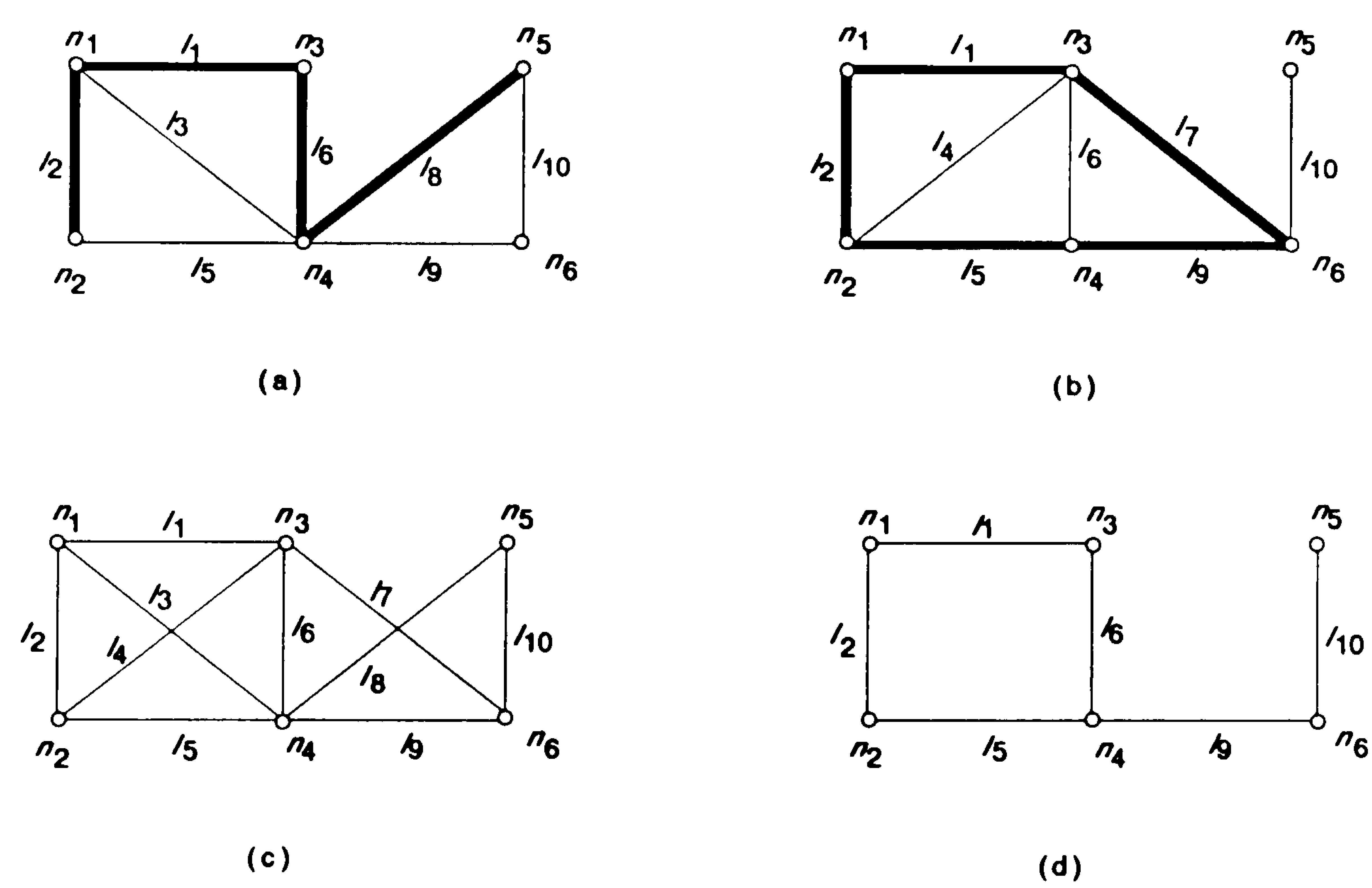


Fig.2.3 A graph and some of its subgraphs G_1, G_2



(a) Graph G_1 and a 4-link path between n_2 and n_5 (b) Graph G_2 and a 5-link loop

(c) Union of G_1 and G_2 (d) Intersection of G_1 and G_2

Fig. 2.4 Paths and loops in a graph

For example, the union of two graphs G_1 and G_2 of Fig.2.4 (a) and (b) is the graph of Fig.2.4(c).

The *intersection* of G_1 and G_2, denoted as $G_1 \cap G_2$, is the graph $G=(N_1 \cap N_2, L_1 \cap L_2)$, that is, the node set of G consists of only those nodes present in both G_1 and G_2, and the link set of G consists only those links present in both G_1 and G_2.

The intersection of G_1 and G_2 in Fig.2.4(a) and (b) is shown in Fig.2.4(d).

2.3.4 Connected Graphs

An important concept in graph theory is that of connectivity. Two nodes n_i and n_j are said to be *connected* in a graph G if there exists a n_i-n_j path in G. A node is connected to itself.

A graph G is connected if there exists a path between every pair of nodes in G.

Consider a graph $G=(N, L)$ which is not connected. Then the node set N of G can be partitioned into subsets N_1, N_2,...,N_p such that the node-induced subgraphs $<N_i>$, $i=1,2, ...,p$, are connected and no node in subset N_i is connected to any node in subset N_j, $j \neq i$. Therefore, a non-connected graph can always be divided into a number of connected subgraphs which can be analyzed separately. In this thesis, all graphs being studied are connected.

2.4 Matrix Representations of Graphs

In the previous section it was noted that pictures of graphs are an aid to the comprehension of graph theoretical problems. For large problems however, the use of a computer becomes essential and the question arises of how best to represent a graph for computer manipulation. One way is by the use of matrices, with perhaps the simplest of these being the associated matrices and symbol matrices.

2.4.1 Associated Matrices

Let us consider a graph $G=(N, L)$, and let $n_1, n_2, n_3, \ldots, n_k$ be its nodes. Let a_{ij} be the number of links of L going from n_i to n_j. The square matrix (a_{ij}) with k rows and k columns is called the matrix associated with the graph G -- *Associated Matrix*, denoted by A, according to standard practice, the coefficient a_{ij} is the element located at the intersection of the i^{th} row and the j^{th} column. The i^{th} row vector will be denoted by $\mathbf{a}_i=(a_{i1}, a_{i2}, \ldots, a_{ik})$ and the j^{th} column vector by $\mathbf{a}_j=(a_{1j}, a_{2j}, \ldots, a_{kj})$

For a simple graph, the associated matrix becomes

$$A = \begin{cases} a_{ii}=0 \\ a_{ij}=1 \text{ if there is a link from nodes } i \text{ to } j \quad (i=1,\ldots,n) \\ a_{ij}=0 \text{ otherwise} \end{cases} \tag{2.1}$$

where n is the number of nodes in the node set N of the graph G.

Since we are only discussing the undirected graphs, the links in the undirected graph are symmetric. So we have the conclusion that the associated matrix A of an undirected graph G is symmetric $a_{ij}=a_{ji}$.

Theorem (Berge, 1962). If G is a graph and A its associated matrix, the element P_{ij} of the matrix $P=A^k$ (obtained by taking the product of A with itself k times) is equal to the number of distinct paths of length k which go from n_i to n_j.

This theorem enables us to identify certain problems concerning graphs and structural graph model later in the theory of matrices. We shall now examine some of these in detail. We are especially interested in A^2 and A^3

According to the definition of associated matrix and **Theorem 1**, a term a_{ij} of A represent the number of single link paths from node n_i to n_j. A term $a_{ij}^{(2)}$ of the matrix A^2 formed by multiplying A by itself, that is

$$A^2 = AA \tag{2.2}$$

represent the number of 2-link paths from i to j, and similarly a term $a_{ij}^{(3)}$ of A^3:

$$A^3 = AA^2 \tag{2.3}$$

denotes the number of 3-link paths from n_i to n_j.

More importantly, a diagonal term $a_{ii}^{(2)}$ of A^2 represents the number of 2-link loops from node i back to itself. It is also a measure of the number of links adjacent to n_i. Similarly, a diagonal term $a_{ii}^{(3)}$ is a measure of the number of 3-link loops associated with node n_i.

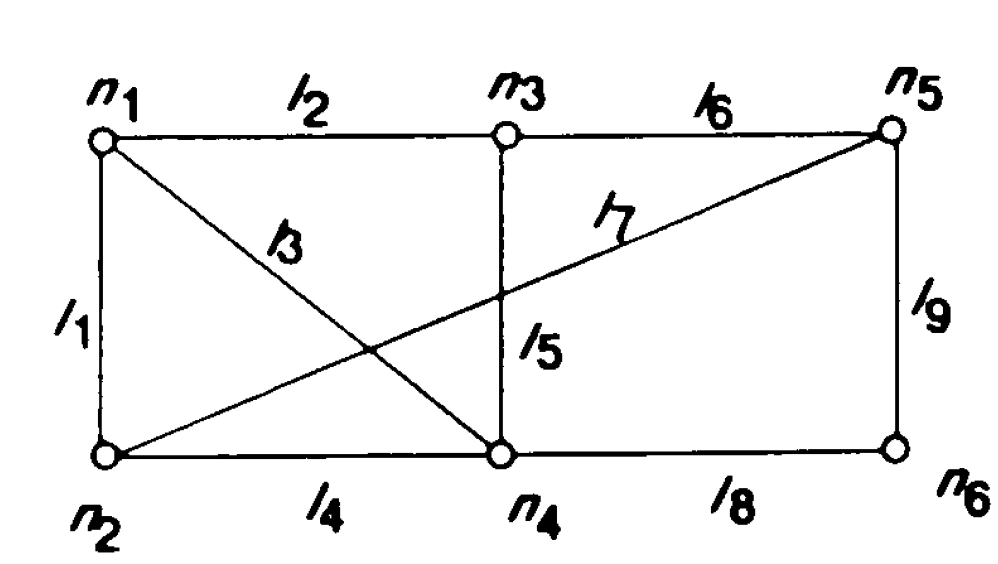


Fig. 2.5 A graph

As an example consider the graph G of Fig.2.5. We form its associated matrix A and compute A^2 and A^3

$$
A = \begin{array}{c c}
 & \text{Node} \\
\text{Node} & \begin{array}{cccccc} n_1 & n_2 & n_3 & n_4 & n_5 & n_6 \end{array} \\
\begin{array}{c} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \end{array} &
\left[\begin{array}{cccccc}
0 & 1 & 1 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 0
\end{array} \right]
\end{array}
$$

Multiply A by itself gives

$$
A^2 = \begin{array}{c c}
 & \text{Node} \\
\text{Node} & \begin{array}{cccccc} n_1 & n_2 & n_3 & n_4 & n_5 & n_6 \end{array} \\
\begin{array}{c} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \end{array} &
\left[\begin{array}{cccccc}
3 & 1 & 1 & 2 & 2 & 1 \\
1 & 3 & 3 & 1 & 0 & 2 \\
1 & 3 & 3 & 1 & 0 & 2 \\
2 & 1 & 1 & 4 & 3 & 0 \\
2 & 0 & 0 & 3 & 3 & 0 \\
1 & 2 & 2 & 0 & 0 & 2
\end{array} \right]
\end{array}
$$

and

$$
A^3 = \begin{array}{c c}
 & \text{Node} \\
\text{Node} & \begin{array}{cccccc} n_1 & n_2 & n_3 & n_4 & n_5 & n_6 \end{array} \\
\begin{array}{c} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \end{array} &
\left[\begin{array}{cccccc}
4 & 7 & 7 & 6 & 3 & 4 \\
7 & 2 & 2 & 9 & 8 & 1 \\
7 & 2 & 2 & 9 & 8 & 1 \\
6 & 9 & 9 & 4 & 2 & 7 \\
3 & 8 & 8 & 2 & 0 & 6 \\
4 & 1 & 1 & 7 & 6 & 0
\end{array} \right]
\end{array}
$$

It can be easily verified from Fig.2.5 that, for example,

$a_{11}^{(2)} = 3$, there are three 2-link loops associated with node n_1, or there are three links adjacent to node n_1;

$a_{36}^{(2)}=2$ there are two 2-link paths form nodes n_3 to n_6, one is n_3, n_4, n_6, another is n_3, n_5,n_6; and

$a_{44}^{(3)}=4$ there are four 3-link loops associated with node n_4;

$a_{55}^{(3)}=0$ there is no 3-link loops associated with node n_5.

2.4.2 Symbol Matrices

For a graph $G=(N, L)$, the symbol matrix B of G is defined as (Boffey, 1982)

$$B = \begin{cases} b_{ii}=0 \\ b_{ij}=ij \quad \text{if there is a link from nodes } i \text{ to } j \quad (i=1,\dots,n) \\ b_{ij}=0 \quad \text{otherwise} \end{cases} \tag{2.4}$$

where n is the number of nodes in the node set N of the graph G.

We see that the definition of symbol matrix is almost same as the definition of associated matrix except that "ij" is to be read as a character string rather then a number. It would be natural to look for a rule of 'multiplication' such that $b^{(2)}_{ij}=i\alpha j$ if there is a path $i\alpha j$ from node i to node j containing just two links, and that $b^{(2)}_{ij}$ is a 'sum' of all such paths if there is more than one. This can be achieved as follows.

Let $\xi(uv)$ and $\eta(vw)$ be two strings of symbols which end and start respectively with v. Then if $\times$ denotes the multiplication symbol

$$\xi(uv) \times 0 = \eta(vw) \times 0 = 0 \tag{2.5}$$

and $\xi(uv) \times \eta(vw)$ is formed by concatenating $\xi(uv)$ with the string that results from $\eta(vw)$ by removing the first symbol 'v' (for example, $ubv \times vebw=ubvebw$).

This is extended to 'sums' of string by defining

$$\Sigma_i\ \xi_i(uv) \times \Sigma_i\ \eta_i(vw) = \Sigma_i\ \xi_i(uv) \times \eta_i(vw) \tag{2.6}$$

The product of powers of B is then defined as

$$B^{r+t}{}_{ij} = (B^r \times B^t)_{ij} = \Sigma_\alpha\ B^r{}_{i\alpha} \times B^t{}_{\alpha j} \tag{2.7}$$

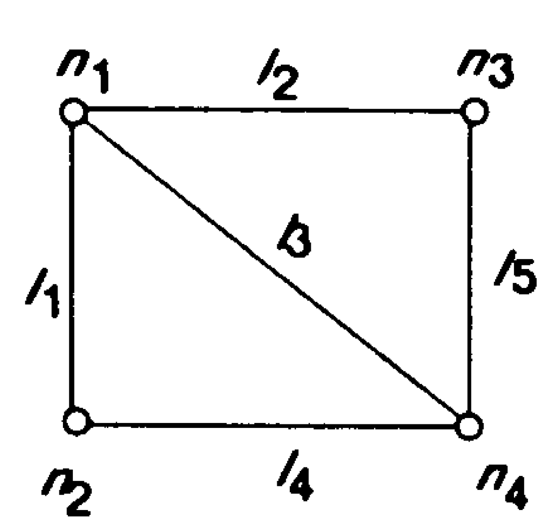


Fig. 2.6 A graph

As an example, find B and B^2 for the graph of Fig.2.6. Also find all distinct loops containing exactly three links.

The symbol matrix B of Fig.2.6 is given by

$$B = \begin{array}{cc} & \text{Node} \\ \text{Node} & \begin{array}{cccc} n_1 & n_2 & n_3 & n_4 \end{array} \\ \begin{array}{c} n_1 \\ n_2 \\ n_3 \\ n_4 \end{array} & \left(\begin{array}{cccc} 0 & l_1 & l_2 & l_3 \\ l_1 & 0 & 0 & l_4 \\ l_2 & 0 & 0 & l_5 \\ l_3 & l_4 & l_5 & 0 \end{array} \right) \end{array}$$

Multiplying B by itself gives

$$
\begin{array}{cc}
 & \text{Node} \\
\text{Node} & \begin{array}{cccc} n_1 & n_2 & n_3 & n_4 \end{array}
\end{array}
$$

$$
B^2 = \begin{array}{c} n_1 \\ n_2 \\ n_3 \\ n_4 \end{array}
\begin{bmatrix}
l_1l_1+l_2l_2+l_3l_3 & l_3l_4 & l_3l_5 & l_1l_4+l_2l_5 \\
l_4l_3 & l_1l_1+l_5l_5 & l_1l_2+l_4l_5 & l_1l_3 \\
l_5l_3 & l_2l_1+l_5l_4 & l_2l_2+l_5l_5 & l_2l_3 \\
l_4l_1+l_5l_2 & l_3l_1 & l_3l_2 & l_3l_3+l_4l_4+l_5l_5
\end{bmatrix}
$$

it is easily checked that this gives all paths containing just two links. We might expect that paths containing three links will be given by the element $b^{(3)}{}_{ii}$, and, since only loops are asked for, only the elements on the principal diagonal are shown below.

$$
\begin{array}{cc}
 & \text{Node} \\
\text{Node} & \begin{array}{cccc} n_1 & n_2 & n_3 & n_4 \end{array}
\end{array}
$$

$$
B^3 = \begin{array}{c} n_1 \\ n_2 \\ n_3 \\ n_4 \end{array}
\begin{bmatrix}
l_1l_4l_3+l_2l_3l_5 +l_3l_4l_1+l_3l_5l_2 & . & . & . \\
. & l_1l_3l_4+l_4l_3l_1 & . & . \\
. & . & l_2l_3l_5+l_5l_3l_2 & . \\
. & . & . & l_3l_1l_4+l_3l_2l_5 +l_4l_1l_3+l_5l_2l_3
\end{bmatrix}
$$

There are two distinct loops with three links each namely $l_1l_3l_4$ and $l_2l_3l_5$ and each is given three times (starting from different links).

From the above example we might infer the following result that if B is the symbol matrix of a graph G, then the element $b^{(r)}{}_{ij}$ of $B^{(r)}$, $r > 0$ gives all paths between node i and node j containing exactly r links. The diagonal term $b^{(r)}{}_{ii}$ tells us not only the total number of r-link loops associated with joint i but also gives us how the routes through the paths. That is very useful tool when we try to identify valid structured loops in a system which will be discussed more fully in Chapter 3.

2.5 Structural Systems

In this thesis we are concerned with the vulnerability analysis of structural systems. Before attempting to develop a theory of structural vulnerability, it is necessary to examine the structural systems themselves, because it is through such systems that the methodology of vulnerability analysis has evolved.

The word "system" has many interpretations depending on the context in which it is used (Wilson, 1984). It can mean, for example, a procedure, a process or its control, a network, or a computer-based data processing package. While these are all valid uses of the word, a definition is needed which will allow a particular interpretation to be placed in this research.

A useful starting point in arriving at a precise definition is to take a general definition that includes all of the interpretation mentioned above, i.e. the dictionary definition: a *system* is a structured set of objects and/ or attributes with the relationships between them.

This definition leads to the definition of structural system. A *structural system* is first of all a set of objects, i.e it contains parts, called elements, that have some reason for being taken together rather than with some other elements. But it is more than just a set of objects, it also includes the relationships that exist between the objects of that set. This relationship actually is a kind of connectivity within the structural system. The elements are interconnected and assembled into a whole, i.e. a system, in such manner that certain desired functions are obtained.

Attention is initially focused on two-dimensional skeletal structural systems with the aim of firmly establishing the basic procedure of structural vulnerability analysis before moving on to the consideration of three-dimensional or other types of structures. Here the skeletal structures are defined in a conventional manner. A line member will be regarded as a part of a structure which is clearly distinct in a physical sense from neighbouring parts, e.g. by being inclined at a different angle to that of its neighbour(s) in a structure, or by having different cross-sectional dimensions to its neighbour(s); a joint then also has a clear physical interpretation as the junction of two or more members.

2.6 Object Oriented Graph Model of Structural Systems

Fundamental to this section is the concept of "modelling". That is, when given a real structural system to analyze, it is always necessary to represent it by some "model" of it which characterises the features of the system in which we are interested in. Before we define the mathematical model of structural systems, we need to introduce three important concepts -- object, holon and degree of freedom.

An *object* is a computer data structure which is a holon and has characteristics. Characteristics are either external (public) or internal (hidden). Characteristics may include features (attributes), behaviours (transformations) and constraints (conditions).

A *holon* (Koestler, 1968) is a concept which is both a part and a whole. It is a part of a wider system and is itself a system(of subsystem).

A *degree of freedom* is the capacity of a structural joint to permit the transmission of movement in a co-ordinate direction (separate and independent of other movements) along a given member to another joint. The total number of separate degrees of freedom in a structural system defines the total number of separate movements necessary to define the behaviour of the system.

The basic mathematical model to represent a structural system $S = (M, J)$ is the *object oriented graph model* (OOGM) which consists of two sets: a finite set of structural member objects M and a finite set of structural joint objects J. Each member object is identified with at least two joint objects.

A *joint object* in an OOGM is a node or reference point where member objects connect. Its features includes at least its co-ordinate positions and the degree of freedom of every member connecting into the joint. Other features could be velocities and accelerations. Behaviour may be modelled by an appropriate physics of motion.

A *member* or *element object* is a relation, linking and communicating object which connects at least two joint objects. It is a communication channel between joints transmitting movements (disturbances) along the degrees of freedom. Its features include its geometrical and physical properties such as length l, areas **A**, second moment of area **I**, elastic modulus **E** and etc. Its behaviour may be modelled by an appropriate physics of material behaviour.

We use the symbols $j_1, j_2, j_3, \ldots,$ to represent the joint objects and $m_1, m_2, m_3, \ldots,$ to represent the member objects. The joints j_i and j_j associated with a member m_k are called the end joints of the member m_k, denoted as $m_k = (j_i, j_j)$.

Comparing this definition with the definition of graph in Section 2.2, we can see that these two definitions are very similar. The joint objects in a structural system corresponds to the nodes in a graph and member objects to links. The main difference is that a graph is a mathematical model, the links and nodes in a graph are not assigned any physical meanings. But the joint objects and member objects in a structural system have their specific physical interpretation although they are denoted by nodes and links pictorially. This is why we have defined the object oriented graph model -- OOGM as the basic mathematic model for structural systems. An OOGM is a graph so that all the definitions and methodologies in the theory of graphs we have discussed in previous sections can be applied to it. Meanwhile the elements of it are objects, which have their features and attributes. The physical interpretation of these features and attributes depends on the structural system we are studying.

The following two types of terminology may be used interchangeably for the study throughout this thesis:

Structural terminology	*Mathematical terminology*
Structural system(System)	Graph model(OOGM)
Structural cluster(Subsystem)	Subgraph
Joint object(Joint)	Node
Member object(Member)	Link

2.7 Summary and Conclusions

As we have mentioned in Chapter 1, one of the main objectives of this thesis is to provide a graph model of a structure in order to analyse the various load paths and loops within it. In this sense, the work in this chapter can be regarded as a mathematical foundation of this thesis.

This chapter has covered some basic concepts and definitions in the theory of graphs. It has also presented two very useful matrices in the mathematical representation of graphs. These are:

Associated matrices

Symbol matrices

These matrices provide a simple technique for clarifying the interrelationships between objects of a graph. It can be used to identify various paths and loops and to reveal the hidden structure of a graph model. We will see from the following chapters that the identification of different load paths, loops and rings is the very fundamental step in the process of structural vulnerability analysis.

This chapter has also presented the object oriented graph model (OOGM) of a structure. The OOGM developed here is particularly beneficial in the development of the theory of structural vulnerability. This is due to the fact that

(i) the elements of an OOGM are joint objects and member objects; thus the OOGM of a structure represents its specific characteristics (features, behaviours and constraints) which the process of the structural vulnerability analysis should be based on.

(ii) an OOGM itself is a graph model; therefore, the theory of graphs and the theory of matrices introduced in this chapter can be used in the development of the theory of structural vulnerability.

Chapter 3

Structural Rings

3.1 Objectives

The objectives of this chapter are:

1. To introduce the concepts of structural paths and structural loops;
2. To develop a structural ring model;
3. To generate a deterioration hierarchy of structural rings - **DHSR**;
4. To define a string pattern of structural rings;
5. To present an algorithm to identify structural rings in a structure.

3.2 Introduction

The analysis and identification of various paths and loops in a graph model are of fundamental importance in different network and engineering problems. Many systematic methodologies have been developed to enumerate all paths and loops in a system or to identify a specific path and loop in a graph model. For example, shortest path problems (Boffey, 1982) have obvious relevance when a shortest (or quickest or least cost) path is required between two points of a transportantation system. Similar problems arise when it is required to enumerate all possible flow paths and find out a maximal flow (Williams, H. 1973) or least cost flow between two points.

Some path-finding algorithms are concerned with the identification of a specific type of path or loop. Elms (1983) used an algorithm to identify all 3-link loops of a graph model, which can be used to explore the connectivity of that graph. Gandhi and Agrawal (1990) suggested a qualitative estimate of the reliability of mechanical and hydraulic systems by identifying the most critical subsystems, paths and loops which could directly cause the failure of the systems.

The OOGM of a structure consists of member objects and joints. Member objects and joint objects connect to each other to form many load paths and loops, and we call them structural paths and structural loops. A structural path or loop is a sequence of member and joint objects and therefore each structural path or loop has its own structural characteristic. As far as structural vulnerability analysis concerned, we are particularly interested in those which themselves are capable of resisting any arbitrary equilibrium set of applied forces. We call them structural rings. Thus, one of the main objectives of this chapter is to develop the structural ring model.

With the concept of a structural ring, a structure can be described in terms of a set of interconnected structural rings. We will see in Chapter 5 and Chapter 6 that structural rings also exist at various levels of hierarchy. The essential issue of structural vulnerability analysis is therefore to identify the most critical structural rings together with the actions which could directly cause the failure of a structure.

3.3 Structural Paths and Structural Loops

In the OOGM of a structure, *a structural path* is defined as sequence of joint objects $j_1, j_2, ..., j_k$, such that j_{i-1} and j_i are two end joints of a member object in the structure. Remember that in a path nodes and links can only appear once, this restriction is also applied to a structural path. The number of member objects in a structural path is called the *length* of that path.

For example, in the structure of Fig.3.1, j_1, j_2, j_7, j_{11} is a structural path, where j_i refers to joint i.

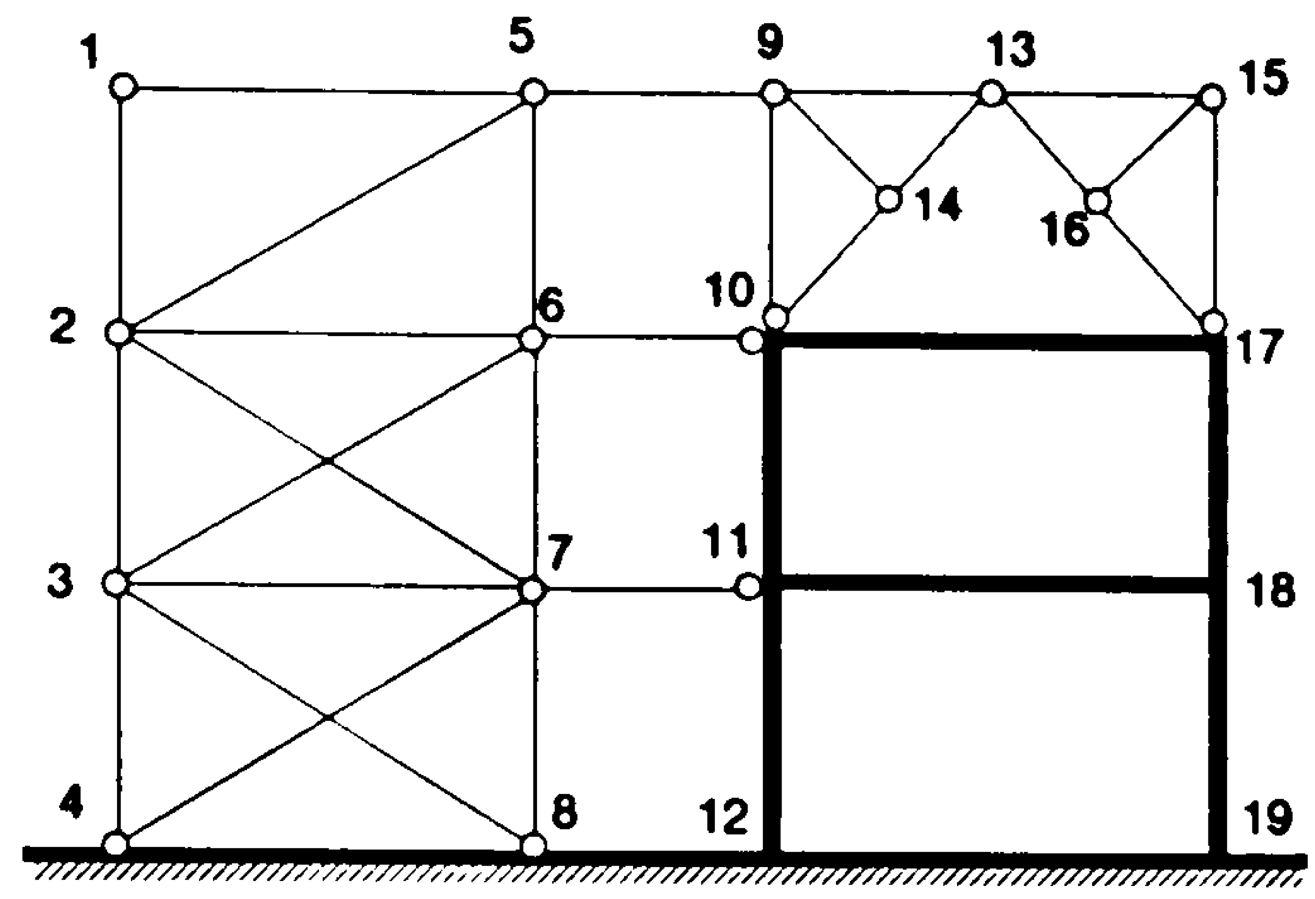


Fig. 3.1 A structural system

A *structural loop* is a closed structural path beginning and ending with the same joint object.

For example, Fig.3.2 has shown some of the structural loops contained in the structure of Fig.3.1. Obviously, there are many other structural loops in the structure.

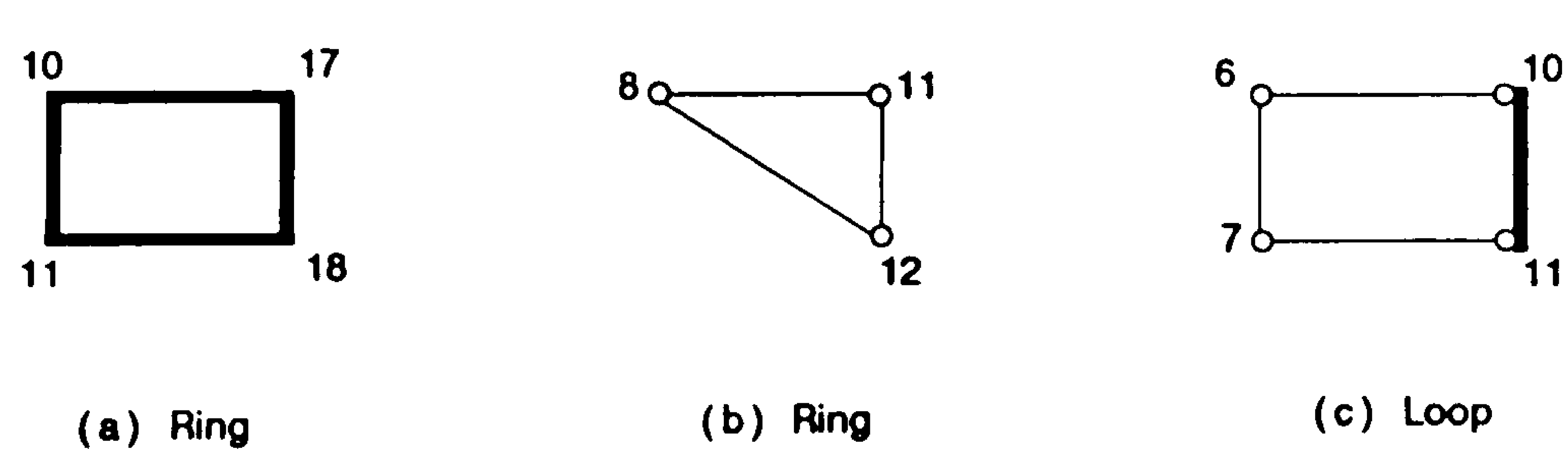


Fig. 3.2 Structural Loops and Rings

The joints and members in a structural loop are all objects which have their structural characteristics (features, behaviours and constraints). A structural loop thus is not only a description of the connectivity between joints and members (say, member m_i connects to member m_j) but also indicates some structural characteristic.

For instance, the structural loop of Fig.3.2(a) consists of four members connected by four fixed joints. This structural loop itself is an over-stiff structure. The structural loop of Fig.3.2(b) is a just-stiff triangle frame. These two structural loops are capable of resisting any arbitrary equilibrium set of applied forces and thus are structural rings. The structural loop of Fig.3.2(c), however, is not a ring but a mechanism.

3.4 Structural Rings

Generally, in a structural system, members are connected to each other by joints to form many structural loops. Different structural loops have different structural characteristics. The structural loops of major interest are those which themselves are complete structural systems (just-stiff or over-stiff structures), that is, structural rings.

A *structural ring* *R* is a structural loop which is either

(1) structurally over-stiff
(2) structurally just-stiff

A structurally *over-stiff ring* is statically indeterminate and is one where if any one of the degrees of freedom is released, either in a member or adjacent to a joint contained in the ring then the ring remains stiff.

Therefore, the structural loop of Fig.3.2(a) is an over-stiff structural ring. If one of the degrees of freedom is released in the structure, the ring remains stiff.

A structurally *just-stiff ring* is statically determinate and is one such that if any one of the degrees of freedom is released either adjacent to a joint or in a member contained in the ring then the ring becomes a mechanism. A just-stiff ring is a *primitive ring.*

The just-stiff structural ring of Fig.3.2(b) thus is a primitive ring. If any one of the degrees of freedom is released in the structure then the ring becomes a mechanism.

A structural loop is a mechanism if it is unable to sustain any arbitrary loading, as for example Fig3.2(c).

A structural ring is denoted pictorially by a circle with a number of joints along it. For example, the structural rings of Fig.3.2(a) and (b) can be represented by Fig.3.3(a) and (b) respectively. The type of joints are designated in the conventional manner, shown in Fig.3.4. A single member is denoted by a portion of arc in a ring.

Fig. 3.3 Structural Rings

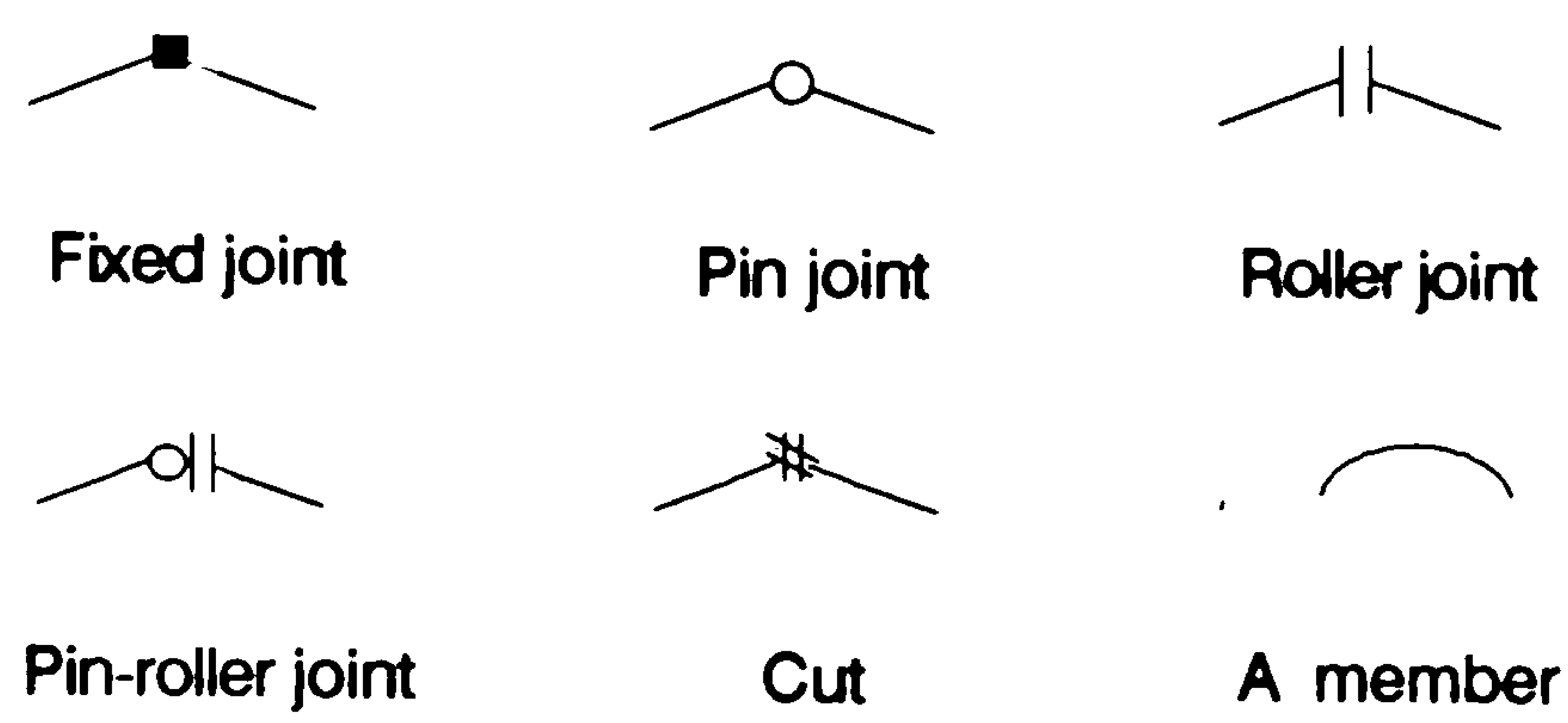


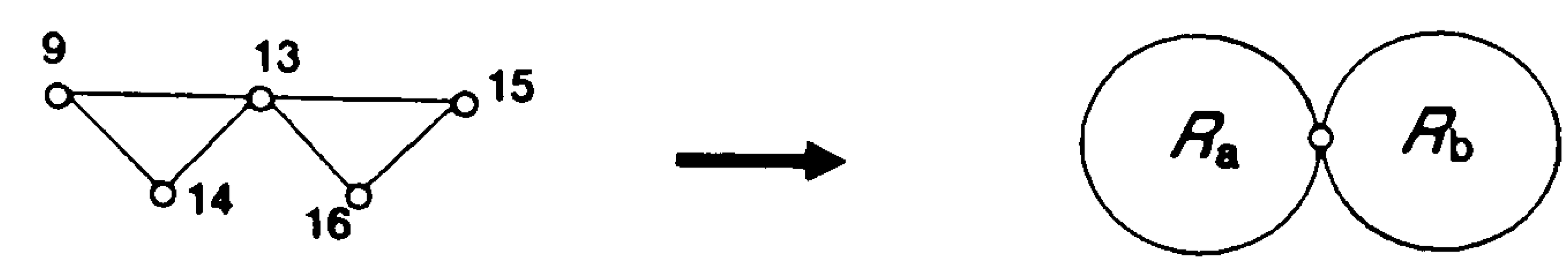
Fig.3.4 Structural joints

A fixed joint allows adjacent members to restrain each others' end translation and rotation. Therefore, two members which are connected by a fixed joint can be treated as a continuous element. For the sake of simplicity and generality, fixed joints are normally not notified along a ring. An arc in a structural ring can contain a series of members connected by fixed joints.

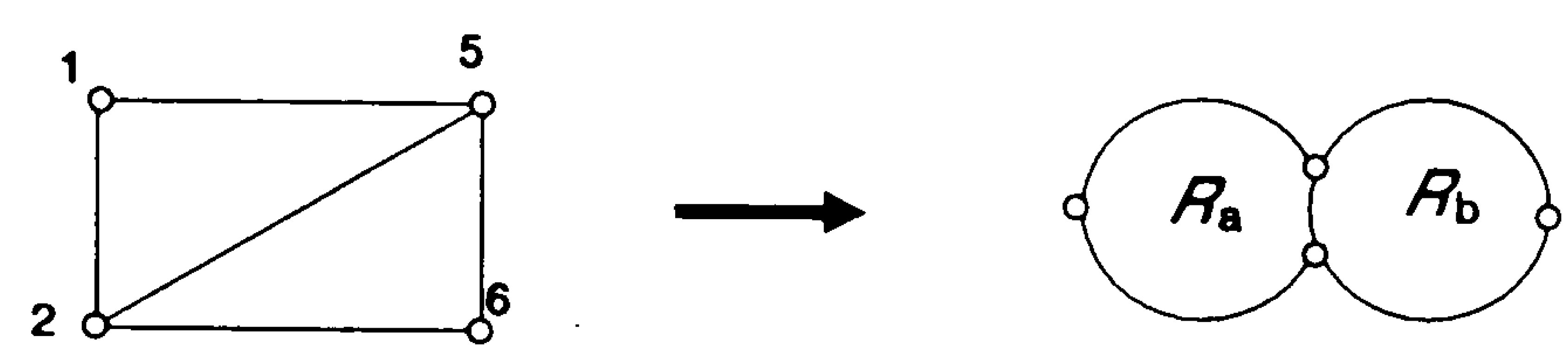
Each joint in a structural ring has three or less degrees of freedom which can transmit corresponding movements from one of its adjacent members to another adjacent member.

In Chapter 6, we will see that a structural ring does not necessarily consist of a sequence of connected members and joints. A sequence of connected structural clusters and complex joints can also form a structural ring. Structural rings exists at each level of hierarchy.

Assume that R_a and R_b are two structural rings, if there is at least one joint contained in both rings, then R_a and R_b are said to be *connected*. For example, in the structure of Fig.3.1, structural rings j_9, j_{14}, j_{13}, j_9 and $j_{13}, j_{15}, j_{16}, j_{13}$ are two connected rings, shown in Fig.3.5(a).



(a) Connected Rings



(b) Overlapping Rings

Fig. 3.5 Connected and overlapping rings

Assume that R_a and R_b are two structural rings, if there is at least one member contained in both rings, then R_a and R_b are said to be *overlapped*. For instance, in the structure of Fig.3.1, structural rings j_1, j_2, j_5, j_1 and j_2, j_5, j_6, j_2 are two overlapping rings, shown in Fig.3.5(b).

A similar concept of a ring was used by Henderson and Bickley (1955) to deal with statical indeterminacy of a structure. Two types of rings were developed. One is a fully fixed ring which has completed stiffness throughout the ring. This implies that the structure has no points (or segments) where any of its stress-resultants, for any loading whatsoever, are always zero. Another is not completely stiff, the corresponding structure has pin-jointed nodes, cable members, etc. They recommended that a statically determinate structure may be formed by making point release in a fully fixed ring. Each release provides additional information for the evaluation of the stress resultant in the form of an equilibrium equation, which later has been used in topological aspects of structural linear analysis by Henderson (1960).

The structural ring model we have developed here has a different basis. The structural ring is considered as a very basic structural unit (instead of members and joints) to resist loading and transmit movements (displacements) along the ring. Any complex structural system is made of different structural rings at various levels of definition. From the structural vulnerability analysis point of view, we are more interested in the structural capability (stiffness or well-formedness) of a ring. A release of a degree of freedom in a ring is considered as a deterioration to the ring. This will be discussed in the next section.

3.5 Deterioration Hierarchy of Structural Rings

Fig.3.6(a) shows three portal frames with different configurations. There is, however, one thing in common and that is that these three frame structures can be denoted by a single over-stiff structural ring (a fully fixed ring). The structural ring R_a reflects this general structural attribute of the three frames.

If one of degrees of freedom is released (e.g. a fixed joint becomes a pinned joint) in the three frames respectively, then they are degenerated into new structures of Fig.3.6(b), the corresponding structural ring is denoted as R_b.

Following the same procedure at each step, so that only one degree of freedom is released, then the three frames deteriorate into the structures of Fig.3.6(d). At this point, all three frames become statically determinate and the corresponding structural ring R_d is a just- stiff ring. If any one of the degrees of freedom is released in the frames, the structures will then become mechanisms as shown in Fig.3.6(e)

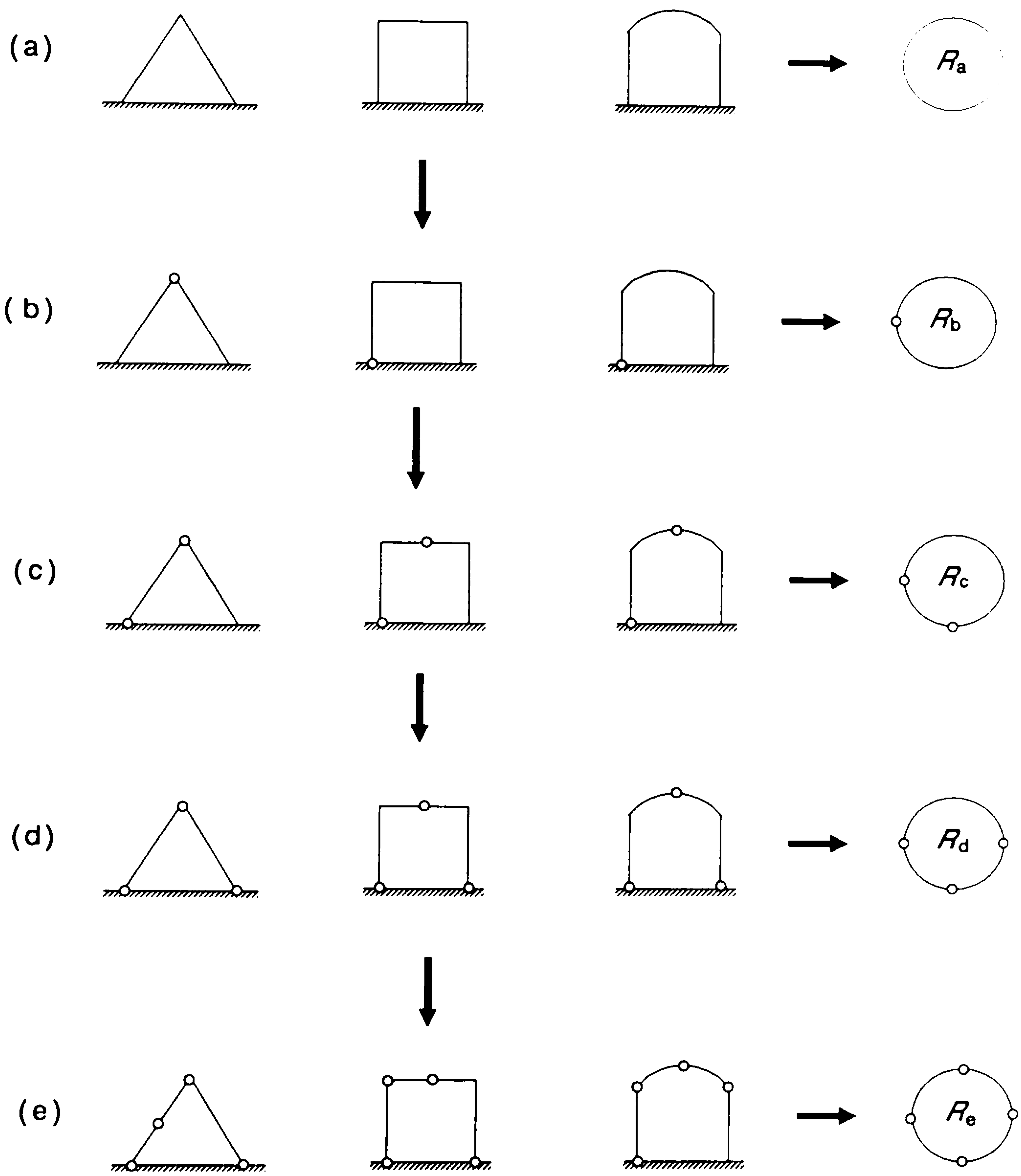


Fig. 3.6 Deterioration of structural rings

For even more generality, we develop the deterioration hierarchy of structural rings--**DHSR**, shown in Fig.3.7. The **DHSR** includes all possible types of structural ring patterns. A simple portal frame is used to illustrate the type of structures which a structural ring denotes. Some of the structural rings are probably not practical but for the reason of completeness they are still included in the **DHSR**. The principle for generating the **DHSR** is that at each step one degree of freedom is released either adjacent to a joint or in an arc contained in the ring. The ring degenerates into a new ring. The procedure is carried through until a ring deteriorates into a mechanism.

The structural ring at the highest level of the **DHSR** is a fully fixed ring. All joints in that ring are fixed joints which have three degrees of freedom. If one of the degrees of freedom is released a fixed joint becomes either a pinned joint or a roller joint, with two degrees of freedom. The ring is then degenerated into two structural rings at next lower level. These two rings can also degenerate into new rings at next lower level by releasing one of the degrees of freedom, either adjacent to a joint or in an arc. Following the same principle, the whole **DHSR** has been generated.

A structural ring at a higher level in the **DHSR** is more tightly connected than those at lower levels. When moving from higher level to lower level structural rings gradually become more loosely connected. At the two extreme ends of the **DHSR**, the structural ring at the highest level is the most tightly and fully connected structural loop. The corresponding structure is most highly structurally over-stiff. Those at bottom level are all mechanisms.

The **DHSR** shows all of the possible ways in which a fully fixed ring can deteriorate into a mechanism. A path through the **DHSR** is an ordered subset of the power set of **DHSR** and this is a failure scenario. A failure scenario indicates the particular way in which a structural ring deteriorates into a mechanism. If we can model a structural system as a structural ring and match it in the **DHSR** then we can find out a set of all possible failure scenarios to fail the system. The details how to use the **DHSR** will be discussed more fully in Chapter 7

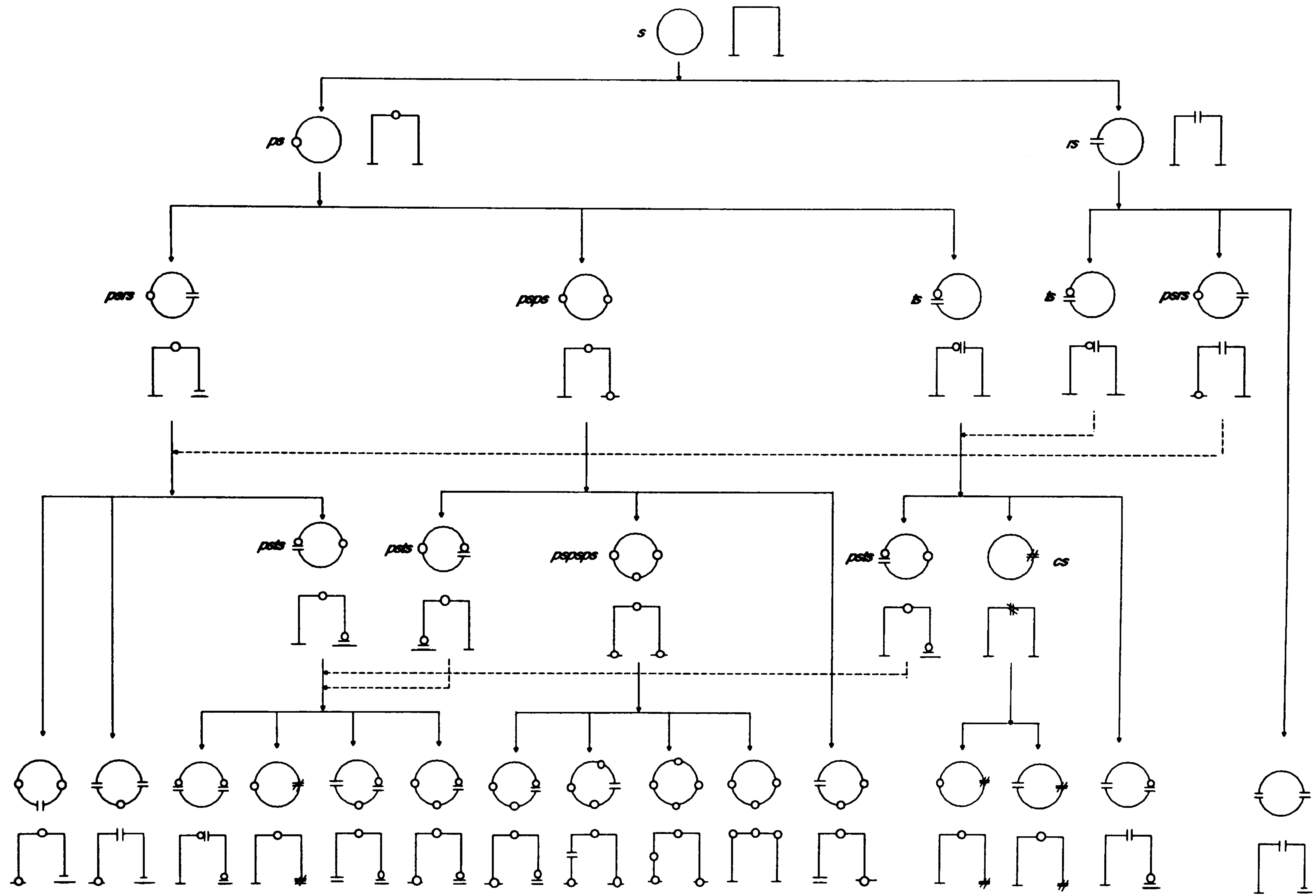


Fig. 3.7 Deterioration hierarchy of structural rings

3.6 String Pattern of Structural Rings

In the DHSR, all structural rings are described graphically by a circle with different type joints along its arc. This graphical pattern of a structural ring is vivid and very easy for a reader to understand but not easy for a computer to search specified type of rings automatically and efficiently. We need to develop another pattern which makes it easy for a computer to do the analysis, that is a string pattern of structural rings.

An *alphabet* is a finite set of elements called letters; the alphabet is denoted by X, the letters by a,b,c.... A *string* on X is an ordered sequence of elements of X and is represented by simple juxtaposition or concatenation of these elements.

For example

Alphabet: $X = \{a,b,c,\}$
String on X: $x = b\ c\ a\ a\ b$

The set of all strings on X is written X^*.

In order to develop the string pattern of the structural rings R in the DHSR, an alphabet X is defined as

$$X = \{f, p, r, t, c, s\}$$

where the elements in X represent:

f: fixed joint
p: pinned joint
r: roller joint

t: pin-roller joint

c: cut

s: section of arc

Then the string pattern of a structural ring can be generated by forming an ordered sequence of elements of *X*.

For example, a structural ring with two pin joints may represented by a string pattern *R* =*psps*, shown in Fig.3.8

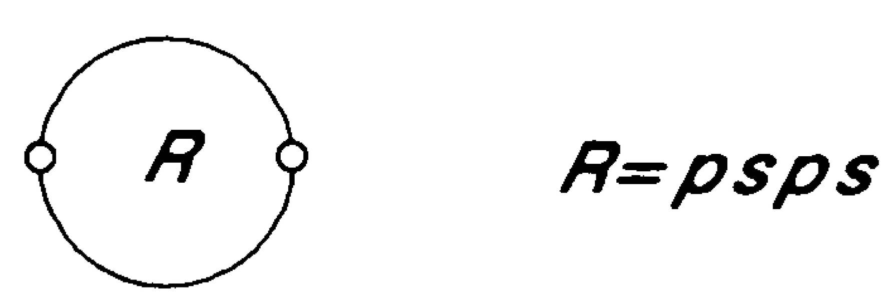


Fig. 3.8 String pattern of a ring

Some points should be noted when string patterns are generated

(1). The element *s* with any one of other elements in *X* forms a very basic unit which is non-separated unit in generation of strings. Obviously there are five basic units in the language: *fs*, *ps*, *rs*, *ts*, *cs*.

(2). All these five basic units in a string can swop their position without influencing the graph pattern which the string describes.

(3). Since an arc can contain a series of members connected by fixed joints, therefore we have: $s = fs = fsfs = fsfsfs$.

According to what we have defined above, the strings of all structural rings can be generated. The set of all strings of the structural rings in DHSR of Fig.3.7 is denoted by R^*

$$R^{*} = \left\{ \begin{array}{l} s \\ ps \\ rs \\ psrs \\ psps \\ ts \\ psts \\ pspsps \\ cs \end{array} \right\}$$

If the string pattern of a structural loop can not be found in R^{*}, then this structural loop is not a structural ring.

3.7 Identification of Structural Rings

Structural vulnerability analysis requires an algorithm for identifying and enumerating all the structural rings of a structure. Such an algorithm is desirable in cases where the structure is complex or where the analysis is to be done with the aid of a computer.

In this section we focus our attention on a general method which leads directly to the explicit definition of every structural ring of a structure and gives the result in an easily usable form. The method is based on a symbol matrices approach to the problem of detecting structural rings rather than a purely mathematical one. The first point to clarify is that a structural ring is really just a special type of structural loop, the string pattern of which can be found in R^{*}, therefore, the general problem of finding loops can be considered.

A structural loop is a sequence of members, connected through joints. More basically it is a combination of members. If all of the combinations of members are determined we can then determine all of the possible combinations of these members which can form structural rings. The symbol matrix B which was developed in Section 2.4.2 can be used for this purpose because the diagonal term $b_{ii}^{(r)}$ gives all loops associated with joint i containing exactly r members.

When we apply this method to identify structural rings, we will find that the structural rings would be lost in a multitude of information. This would be composed of invalid combination of members, or loops in which one or more joints appears more than once, or loops the string pattern of which can not be found in R^*. The trick then, is to form a method such that no useless information is generated.

First of all, the length of structural rings should be limited. A fully fixed ring in the DHSR could theoretically consist of an infinite number of fixed joints but in practice the number of joints along the ring would be finite. However we might compress the number of adjacent fixed joints to 4, so that the description of a fully fixed ring always contains a maximum of 4 links. Therefore, the length of any other rings degenerated from the fully fixed ring in the DHSR is less or equal to four.

Secondly, from the connectivity point of view, a structural ring is more closely or tightly connected if it has a shorter length. Among all rings with different length, a 3-link ring, i.e. a triangle, is the most tightly and closely self connected structure. A ring with more than 5-links is considered as very loosely and weakly self connected.

Having done this, the procedure of identifying structural rings is described below

(1) Form the symbol matrix B of a structure S;
(2) Calculate B^3 diagonal terms $b_{ii}^{(3)}$;
(3) Remove unnecessary duplicated loops and obtain all 3-link loops in S;
(4) Generate string patterns of these loops;
(5) Match these string patterns to R^* and identify all valid structural rings.
(6) Compute $B^{(4)}$ diagonal terms $b_{ii}^{(4)}$;
(7) Similarly identify all alternative 4-link rings;
(8) For those left, each member becomes a single-member ring.

The above algorithm has been implemented on a computer and works very quickly and efficiently.

It would be instructive at this point to consider the structure of Fig.3.1 to see how the procedure discussed above can be used to yield all structural rings in the system.

Firstly, form the symbol matrix B of Fig.3.1, the element of B is denoted as

$b_{ij} = i\text{-}j$ if i and j are two end joints of a member

$b_{ij} = 0$ otherwise

where $i\text{-}j$ is a string symbol.

According to the operation rules of string symbols (2.5) (2.6) and (2.7), B^3 is computed. Then step (3) to step (5) is straight forward to find out all structural rings containing three members. The results, rings R_1 to R_{12}, are shown in Fig.3.9. The graphical patterns and string patterns of those structural rings are also listed.

Then the next step is to compute $B^{(4)}$ diagonal terms $b_{ii}^{(4)}$ and carry out similar analysis and two 4-link rings are identified, R_{13} and R_{14} of Fig.3.9. The remaining three members form three single-link rings, R_{15} to R_{17}.

Having looked at the structural rings in Fig.3.9, we see that each ring is either an over-stiff structure such as R_{14} or a just-stiff structure such as R_1. Some rings connect to each other such as R_{10} and R_{11}, and some rings are overlapping to each other such as R_1 and R_2. These structural rings are the basic elements of the structure of Fig.3.1 which are capable of resisting any arbitrary equilibrium set of applied forces. The structure of Fig.3.1 is, therefore, built up with these structural rings in a certain manner such that the desired function has been achieved.

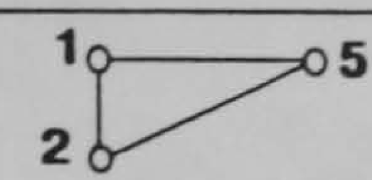
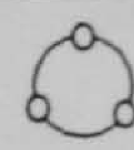
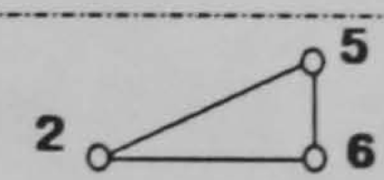
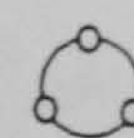
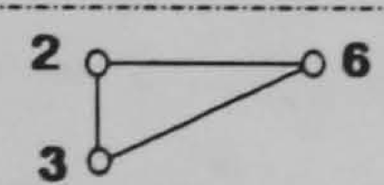
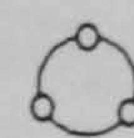
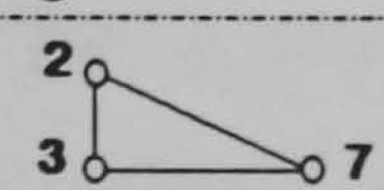
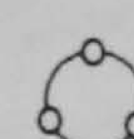
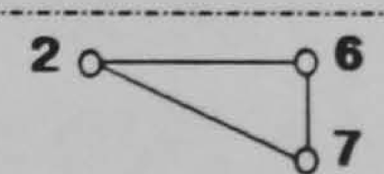
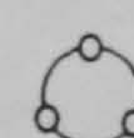
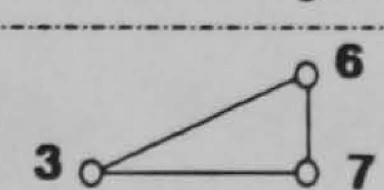
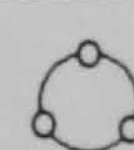
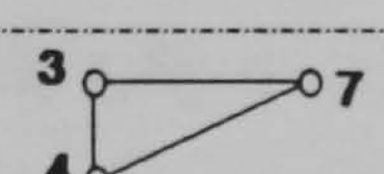
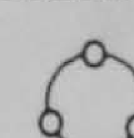
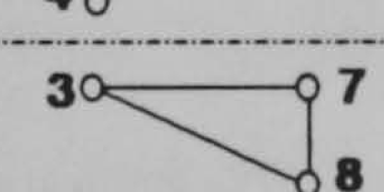
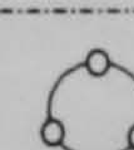
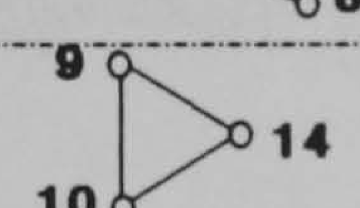
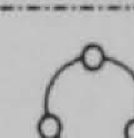
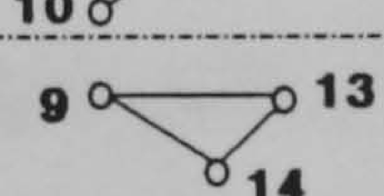
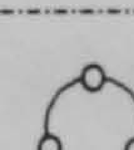
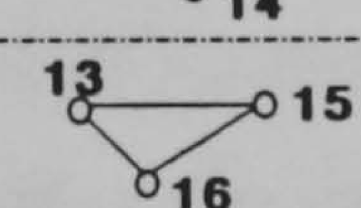
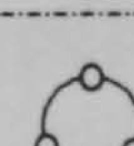
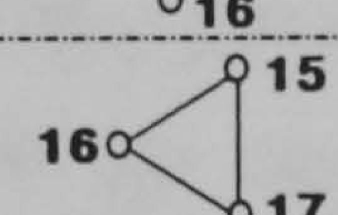
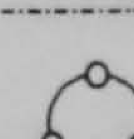
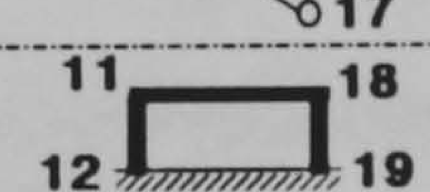
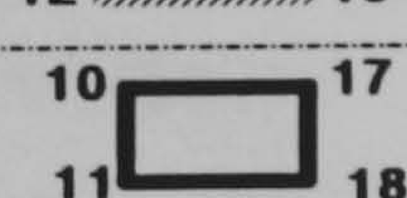
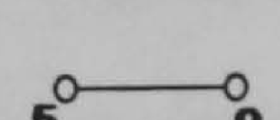
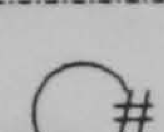
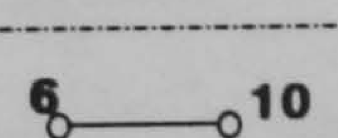
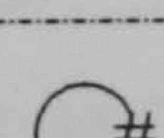
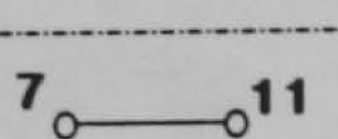
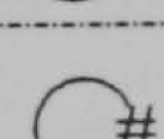
Ring R_m	Joints	Structure	Graphical Pattern	String Pattern
1	1-2-5-1			pspsps
2	2-5-6-2			pspsps
3	2-3-6-2			pspsps
4	2-3-7-2			pspsps
5	2-6-7-2			pspsps
6	3-6-7-3			pspsps
7	3-4-7-3			pspsps
8	3-7-8-3			pspsps
9	9-10-14-9			pspsps
10	9-13-14-9			pspsps
11	13-15-16-13			pspsps
12	15-16-17-15			pspsps
13	12-11-18-19			s
14	10-11-18-17-10			s
15	5-9-5			cs
16	6-10-6			cs
17	7-11-7			cs

Fig.3.9 Structural Rings Table

3.8 Summary and Conclusions

This chapter has examined two closely related graphical descriptions of the interaction of objects in a structure: structural paths and structural loops. A structural path or a structural loop, first of all, is a description of the connectivity between members and joints within a structure. Secondly, since a structural path or loop consists of a sequence of member objects and joint objects, each path or loop also implies a specific structural characteristic.

For a structural system consisting of many structural paths and loops, the particular interest are those loops which are themselves either just-stiff or over-stiff structures, i.e. structural rings. A structural ring model which has been developed in this chapter is considered as a very basic structural element which is capable of resisting any arbitrary equilibrium set of applied forces. The development of the structural ring model is the very fundamental work in this thesis.

We have presented the deterioration hierarchy of structural rings--DHSR. The DHSR includes all possible types of structural ring patterns. A path through the DHSR is a failure scenario. Thus, the DHSR also shows all possible failure scenarios for a structural ring. If we could model a structural system as a structural ring and match it in the DHSR then we would find out a number of possible failure scenarios to fail the system.

In this chapter an algorithm has been developed to identify all structural rings of a structure. An example is given to illustrate the procedure of the algorithm. The results have shown that a complex structural system is built up with a set of interconnected structural rings.

Well-Formedness of Structural Rings

4.1 Objectives

The objectives of this chapter are:

(1). To present a measure of the well-formedness of structural rings.
(2). To present a measure of the well-formedness of joint objects.
(3). To examine the concepts of eigenvalue and eigenvector of a structure stiffness matrix and the application to the analysis of structures.
(4). To discuss the meaning of well-formedness in terms of the principal stiffness coefficients and principal displacement axes of structures.

4.2 Introduction

In Chapter 3 the basic philosophy of structural paths and loops has been described and a structural ring model has been developed and used to represent a type of structure with common structural attributes. In the structural ring model members are simplified into arcs and joints become connecting points along arcs. The example, shown in Fig.4.1, shows this case. The three structures S_1, S_2, and S_3 in Fig.4.1 are all pin-jointed triangle frames which can be denoted by a single structural ring. This ring reflects the common structural attributes of those three structures. They all consist of three members connected by three pinned joints.

From the structural point of view, however, these three frames are different. Obviously, structure S_1 is very well formed but S_2 and S_3 are not. It seems that a measure should be developed to test the quality of well-formedness of structural rings. The measure should tell us how well formed a ring is, in terms of its stiffness, configuration and connectivity. The well-formedness of a structural ring is a measure of its ability to resist loading from any arbitrary direction and to perform its desired function. It has shown in Chapter 2 that a structure is been built up with a set of interconnected structural rings. A robust structure will therefore consist of a set of well formed structural rings.

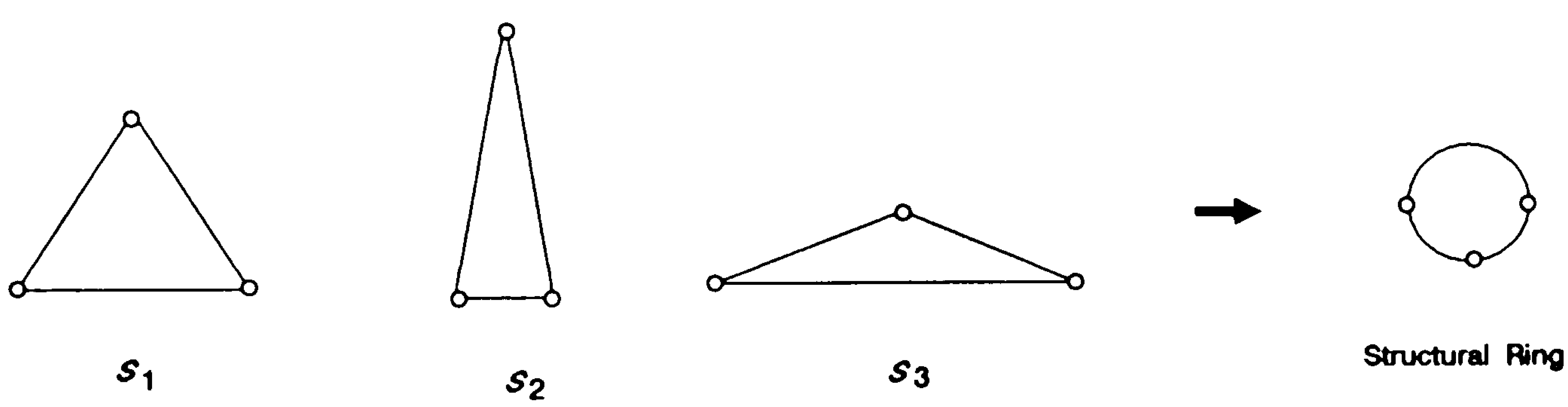


Fig. 4.1 Structures and their corresponding ring

The early part of this chapter is concerned with the introduction of the concepts of eigenvalue and eigenvector and their application to the analysis of structures. Then it proceeds to analyze the well-formedness of joints using similar principle. Finally, a measure is developed to evaluate the quality of the well-formedness of structural rings.

4.3 Principal Stiffness Coefficients

For a structural system with n degrees of freedom, by applying the displacement boundary conditions, the completed reduced set of structural stiffness equations under a predefined global co-ordinate system is:

$$\mathbf{F} = \mathbf{KX} \tag{4.1}$$

where **F** is global load vector; **K** is global stiffness matrix; and **X** is global displacement vector.

Since the global stiffness **K** of a structure in (4.1) is symmetric, referring to **Theorems 2** and **3** of Appendix A, there is an orthogonal matrix **P** such that

$$\mathbf{H} = \mathbf{P}^{-1}\mathbf{KP} \tag{4.2}$$

and $\mathbf{H} = [\ \lambda_i\ ]$ is a diagonal matrix; and we also have

$$det(\mathbf{K}) = det(\mathbf{PHP}^{-1}) = det(\mathbf{H}) = \lambda_1 \times \lambda_2 \times, \ldots, \times \lambda_n \tag{4.3}$$

where λ_i is the eigenvalue of **K** ($i = 1,2,\ldots,n$)

If we substitute (4.2) to (4.1), we have

$$\mathbf{F} = \mathbf{KX} = \mathbf{PHP}^{-1}\mathbf{X} \tag{4.4}$$

or $$\mathbf{P}^{-1}\mathbf{F} = \mathbf{HP}^{-1}\mathbf{X} \tag{4.5}$$

let $$\mathbf{F}' = \mathbf{P}^{-1}\mathbf{F} \quad \text{and} \quad \mathbf{X}' = \mathbf{P}^{-1}\mathbf{X} \tag{4.6}$$

then (4.1) becomes

$$\mathbf{F}' = \mathbf{HX}' \tag{4.7}$$

(4.7) represents a set of linear equations

$$\begin{cases} F'_1 = \lambda_1 \; x_1' \\ F'_2 = \lambda_2 \; x_2' \\ \cdots\cdots\cdots \\ F'_n = \lambda_n \; x_n' \end{cases} \tag{4.8}$$

For an eigenvalue λ_i of **K**, when the x_i' is given a unit displacement, the value of force F_i' required is equal to the eigenvalue λ_i

$$F'_i = \lambda_i \tag{4.9}$$

Here λ_i is called the *principal stiffness coefficient*. The eigenvector corresponding to λ_i defines the *principal displacement axis*. F_i' and x_i' are the force and displacement along the principal displacement axis.

From **Theorem 2** of Appendix A, we also know that all principal displacement axes of **K** are linearly independent. That allows us to study the behaviour of a structure along specific principal displacement axis independently. If the principal stiffness coefficient of a structure along a specific axis is very small, then a small force acting on the structure along that direction will cause a large displacement. The principal stiffness coefficient of a structure therefore indicates the capability of the structure to resist loading along the corresponding principal displacement axis.

For an even more general case (Dhatte & Touzot, 1984), solving a matrix eigenvalue problem consists in finding values λ_i and $\{x_i\}$ satisfying the following equation:

$$\mathbf{K}\,\{x_i\} = \lambda_i\,\mathbf{M}\,\{x_i\} \tag{4.10}$$

and a normalization condition for the vectors:

Either $<x_i>\{x_i\}=1$ or $<x_i>M\{x_i\}=1$

The physical meaning of eigenvalue λ_i of this system equation varies with different disciplines; for example:

Structural Vibrations

If

K is the stiffness matrix of a structure;

M is the mass matrix;

and $\{x_i\}$ is the displacement vector of the structure for the i^{th} mode of vibration;

then $\lambda_i = w_i^2$ is the square of the corresponding frequency.

Buckling Load

If

K is the stiffness matrix of a structure;

M is the geometric or initial stress matrix of the structure;

and $\{x_i\}$ is the displacement vector of the structure for its ith mode of buckling;

then $\lambda_i = w_i^2$ defines the value of the critical load to cause the i^{th} mode of buckling.

Summarising our results so far, some chief properties of eigenvalues and eigenvectors of the stiffness matrix of a structure are now given. Some of the characteristics have already been mentioned earlier but will be included again here for the sake of completeness.

(1). An eigenvalue λ_i of a reduced structure stiffness matrix **K** is a principal stiffness coefficient whose value is equal to the value of the force F_i' required when the displacement x_i' is given the value unity and all other displacements have the value zero.

(2). The eigenvector corresponding to λ_i defines the direction of the principal displacement axis. The force F_i' and displacement x_i' are along the principal displacement axis.

(3). All principal displacement axes of **K** are linearly independent.

(4). All eigenvalues of **K** i.e. principal stiffness coefficients must be positive since a positive force cannot produce a negative corresponding displacement.

(5). The principal stiffness coefficient of a structure indicates the capability of the structure to resist loading along the corresponding principal displacement axis.

(6). The determinant of **K** is equal to the product of all eigenvalues of **K**.

(7). The sum of all eigenvalues of **K** is a constant.

(8). The stiffness matrix of an unrestrained structure is singular, i.e. the determinant of the stiffness matrix has the value of zero. The physical explanation of this is that until some valid boundary conditions are applied the structure is free to move with an arbitrary rigid-body motion in addition to deforming elastically.

4.4 Well-formedness of Structural Joints

In this section we extend the method discussed above to the analysis of the well-formedness of joint objects. We confine our attention initially to the determinant of a single joint, such as joint 2 in the structure of Fig.4.2.

In Fig.4.2(a) the members of the structure are referred to by A and B and structural joints by numbers 1-3. The global co-ordinate system has been set up. The arrow shown on the members are there to indicate the direction of the local axis of the members, i.e. the arrows run form end joint *i* to end joint *j* of each member.

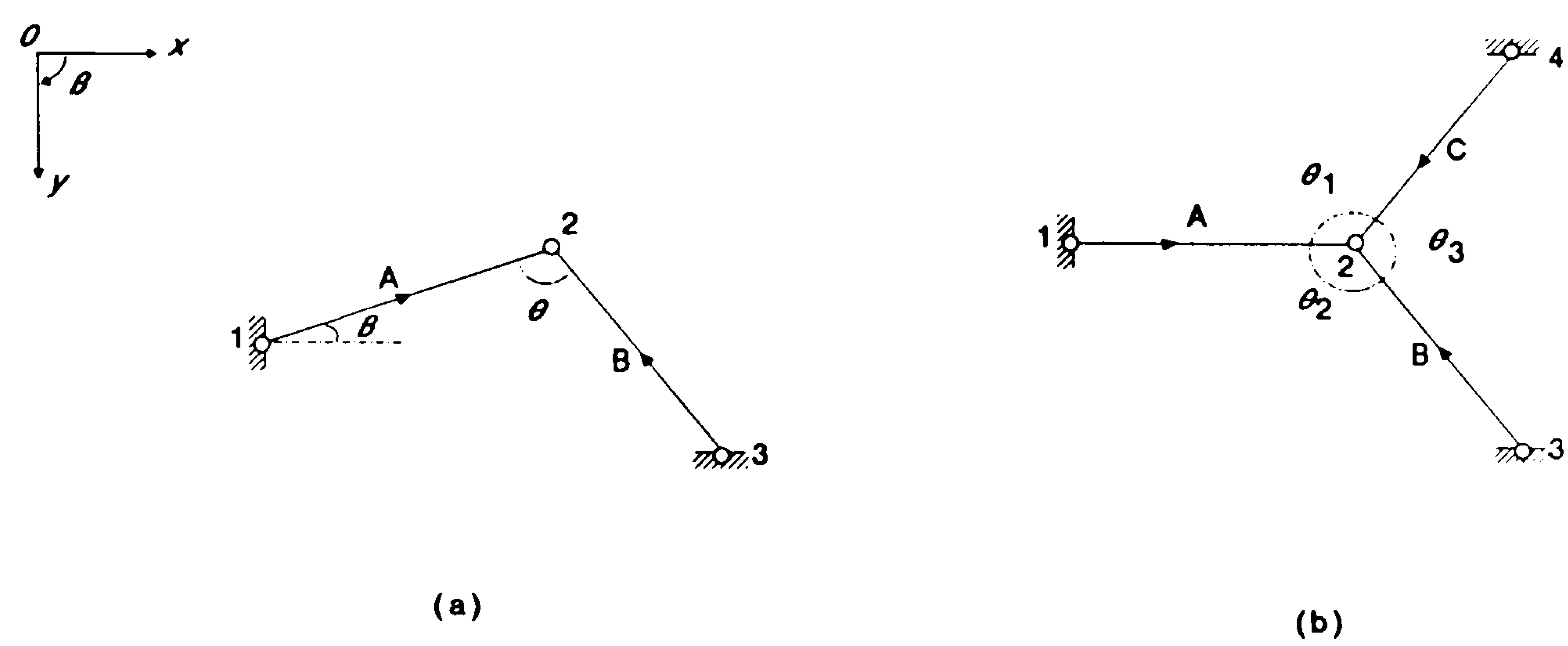


Fig. 4.2 Structures

The relationship between the individual member joints and the structure joints is clearly as follows:

Members	i	j	
			← member joints
A	1	2	structure joint numbers
B	3	2	

Correspondingly the stiffness matrix for the members A,B can be conveniently expressed in general form as

$$\mathbf{k} = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} i & j \\ \end{array}$$

$$\mathbf{k} = \begin{bmatrix} \mathbf{k}_{ii} & \mathbf{k}_{ij} \\ \mathbf{k}_{ji} & \mathbf{k}_{jj} \end{bmatrix} \qquad (4.11)$$

The application of the direct stiffness procedure gives the global structure stiffness matrix **K** as

$$\begin{array}{cccc} & 1 & 2 & 3 \\ \mathbf{K} = & \left[\begin{array}{c} \mathbf{k}_{11}^{A} \\ \mathbf{k}_{21}^{A} \\ \mathbf{0} \end{array} \right. & \begin{array}{c} \mathbf{k}_{12}^{A} \\ \mathbf{k}_{22}^{A}+\mathbf{k}_{22}^{B} \\ \mathbf{k}_{32}^{B} \end{array} & \left. \begin{array}{c} \mathbf{0} \\ \mathbf{k}_{23}^{B} \\ \mathbf{k}_{33}^{B} \end{array} \right] \end{array} \qquad (4.12)$$

Here **0** denotes a null matrix.

The numbers 1-3 at the tops of the columns of the structure stiffness matrix are there to identify joints 1-3 of the structure. We also know that **K** is a symmetric matrix.

Having done this, the following points should be noted.

(1). The submatrices of a global structure stiffness matrix lying on the leading diagonal in row j_2 and column j_2, say, will be the sum of the submatrices $\mathbf{k}_{ii}$ or $\mathbf{k}_{jj}$ of all members which meet at joint j_2 (Submatrix $\mathbf{k}_{ii}$ applies if joint i of the member corresponds to joint j_2 of the structure, $\mathbf{k}_{jj}$ applies if joint j of the member corresponds to joint j_2 of the structure).
(2). There will be a (non-zero) contribution in an off-diagonal location of the structure matrix such as j_1, j_2 (i.e. row j_1, column j_2) only if joints j_1 and j_2 are connected by a member; if two such joints are not directly connected then the associated structure stiffness submatrix is zero.
(3). The submatrix of the global structure stiffness matrix lying on the leading diagonal in row j_i and column j_i is called the submatrix associated with joint j_i.

An important point to realize is that the actual process of assembly of a global structure stiffness matrix from the member stiffness matrix depends solely on the manner in which the individual members are connected together. The detailed characteristics of the members are taken account of in deriving the member stiffness matrices but once this is done, the formation of a structure stiffness matrix by superposition proceeds independently of any knowledge of the structural properties of the members.

Referring to the global structure stiffness matrix **K** of (4.12), a few general points about these diagonal submatrices can be obtained immediately.

(1). The dimension of the submatrix associated with a joint j_i, denoted by $\mathbf{D}_{ii}$, depends on the structural characteristic of that joint j_i, i.e. the degrees of freedom of that joint. If j_i is a pinned joint, the $\mathbf{D}_{ii}$ is 2×2 matrix. If j_i is a rigid joint, then $\mathbf{D}_{ii}$ is 3×3 matrix.
(2). The submatrix $\mathbf{D}_{ii}$ is still symmetric.
(3). The dimension of the submatrix $\mathbf{D}_{ii}$ is independent of the number of members adjacent to joint j_i.

Let us take a close look at one single submatrix associated with joint 2 in the stiffness matrix of (4.12) and assume that $\mathbf{k}^A = \mathbf{A}_1 \times \mathbf{E}/l_1$ and $\mathbf{k}^B = \mathbf{A}_2 \times \mathbf{E}/l_2$ are structure stiffness coefficients of members A and B respectively. We take the submatrix $\mathbf{D}_{22}$, associated with joint j_2 out of the global stiffness matrix. It is as follows:

$$\mathbf{D}_{22} = \mathbf{k}_{22}^A + \mathbf{k}_{22}^B = \begin{pmatrix} \mathbf{k}^A \cos^2\theta + \mathbf{k}^B \cos^2(\theta+\beta) & \begin{matrix} -\mathbf{k}^A \cos\theta \sin\theta \\ -\mathbf{k}^B \sin(\theta+\beta)\cos(\theta+\beta) \end{matrix} \\ \begin{matrix} -\mathbf{k}^A \cos\theta \sin\theta \\ -\mathbf{k}^B \sin(\theta+\beta)\cos(\theta+\beta) \end{matrix} & \mathbf{k}^A \sin^2\theta + \mathbf{k}^B \sin^2(\theta+\beta) \end{pmatrix}$$

The characteristic equation of $\mathbf{D}_{22}$ is:

$$det(\mathbf{D}_{22} - \lambda I_n) =$$
$$[\mathbf{k}^A \cos^2\theta + \mathbf{k}^B \cos^2(\theta+\beta) - \lambda] \times [\mathbf{k}^A \sin^2\theta + \mathbf{k}^B \sin^2(\theta+\beta) - \lambda] -$$
$$[-\mathbf{k}^A \cos\theta \sin\theta - \mathbf{k}^B \sin(\theta+\beta) \cos(\theta+\beta)]^2$$
$$= \lambda^2 - \lambda (\mathbf{k}^A + \mathbf{k}^B) + \mathbf{k}^A \mathbf{k}^B \sin^2\theta = 0 \qquad (4.13)$$

The eigenvalues of $\mathbf{D}_{22}$ must therefore satisfy (4.13) and the solution of this equation are

$$\lambda_i = \{(\mathbf{k}^A + \mathbf{k}^B) \pm [(\mathbf{k}^A + \mathbf{k}^B)^2 - 4 \times \mathbf{k}^A \times \mathbf{k}^B \times \sin^2\theta]^{1/2}\}/2 \quad (i=1,2) \qquad (4.14)$$

the eigenvalues of the submatrix associated with a joint is simply called the eigenvalues of that joint.

The determinant of $\mathbf{D}_{22}$ is then equal to the product of two eigenvalues

$$det(\mathbf{D}_{22}) = \lambda_1 \times \lambda_2 = \mathbf{k}^A \times \mathbf{k}^B \times \sin^2\theta \qquad (4.15)$$

and from (4.14), it can be easily proved that the summation of two eigenvalues is a constant.

$$\lambda_1 + \lambda_2 = \mathbf{k}^A + \mathbf{k}^B \qquad (4.16)$$

And again the determinant of the submatrix associated with a joint is simply called the determinant of that joint.

Having looked at (4.14) and (4.15), some conclusions can be drawn.

(1). The eigenvalues(or determinant) of a joint j_i is independent of the global co-ordinate system. Variable β does not appear in both (4.14) and (4.15).

(2). The eigenvalues, i.e. principal stiffness coefficients, of a joint is related to the stiffness of all members connecting to that joint, in other words, it is a function of the physical and geometric features of all members connecting to the joint.

(3). The eigenvalues of a joint also depends on the angles between members connecting to that joint. The angles actually reflect the way in which the members frame in to that joint.

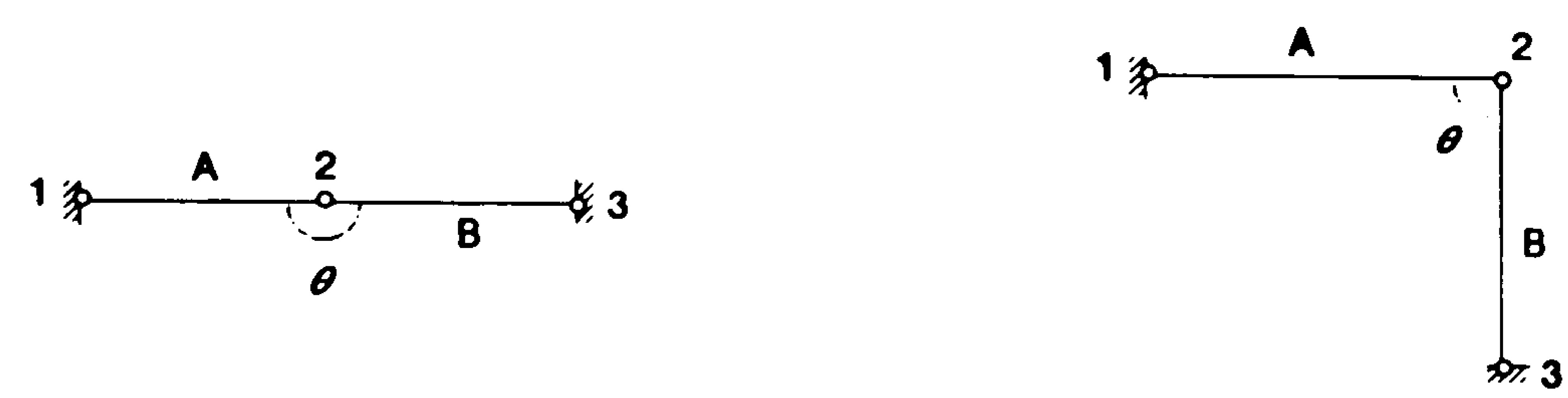
Once the stiffness coefficients of all members are settled, the only factor which will influence the eigenvalues of a joint is the angles between the members adjacent to that joint. Referring to the structure of Fig.4.2(a) and (4.14) and (4.15), by varying θ -- the angle between two members, we can easily prove that

(1) when $\theta=0^{\circ}$ or 180°, $\lambda_1=\mathbf{k}^A+\mathbf{k}^B$, $\lambda_2=0$
and $det(\mathbf{D}_{22})=det(\mathbf{D}_{22})_{min}=0$.

It implies that there is one principal stiffness coefficient having zero value. A tiny force acting on joint 2 along the corresponding principal axis will cause infinite displacement along that axis, and in this case the structure itself is said to be a mechanism, shown in Fig.4.3(a).

(2) When $\theta=90^{\circ}$, $\lambda_1=\mathbf{k}^A$, $\lambda_2=\mathbf{k}^B$
and $det(\mathbf{D}_{22})=det(\mathbf{D}_{22})_{max}=\mathbf{k}^A\mathbf{k}^B$.

Intuitively, we know that the structure reaches its best quality of well-formedness, shown in Fig.4.3(b).



When θ = 180 deg. or 0 deg.
$det(D_{22}) = det(D_{22})_{min} = 0$

(a)

When θ = 90 deg.
$det(D_{22}) = det(D_{22})_{max} = k^A k^B$

(b)

Fig. 4.3 Well-formedness of a structure

(a)

Similarly, we derive the submatrix of joint 2 in the structure of Fig.4.2(b), it is as follows

$$\mathbf{D}_{22} = \mathbf{k}_{22}^A + \mathbf{k}_{22}^B + \mathbf{k}_{22}^C = \begin{bmatrix} \mathbf{k}^A + \mathbf{k}^B \cos^2\theta_2 + \mathbf{k}^C \cos^2\theta_3 & -\mathbf{k}^B \sin\theta_2 \cos\theta_2 + \mathbf{k}^C \sin\theta_3 \cos\theta_3 \\ -\mathbf{k}^B \cos\theta_2 \sin\theta_2 + \mathbf{k}^C \cos\theta_3 \sin\theta_3 & \mathbf{k}^A + \mathbf{k}^B \sin^2\theta_2 + \mathbf{k}^C \sin^2\theta_3 \end{bmatrix} \tag{4.17}$$

and then calculate the determinate of $\mathbf{D}_{22}$, we get

$$det(\mathbf{D}_{22}) = \mathbf{k}^A \mathbf{k}^C \sin^2\theta_1 + \mathbf{k}^A \mathbf{k}^B \sin^2\theta_2 + \mathbf{k}^B \mathbf{k}^C \sin^2\theta_3 \tag{4.18}$$

Referring to (4.18) and the structure of Fig.4.2(b), the conclusions are as follows

(1). $det(\mathbf{D}_{22}) = det(\mathbf{D}_{22})_{min} = 0$ when $\theta_1 = \theta_2 = 180^\circ$ and $\theta_3 = 0^\circ$, in this case, the structure of Fig.4.2(b) is a mechanism.

(2). if $k^A = k^B = k^C$, $det(\mathbf{D}_{22}) = det(\mathbf{D}_{22})_{max}$ when $\theta_1 = \theta_2 = \theta_3 = 120^\circ$, there are three symmetric axes in the structure.

(3). if $k^A > k^B = k^C$, $det(\mathbf{D}_{22}) = det(\mathbf{D}_{22})_{max}$ when $\theta_1 = \theta_2 > 120^\circ$, there is one symmetric axis in the structure;

(4). if $k^A < k^B = k^C$, $det(\mathbf{D}_{22}) = det(\mathbf{D}_{22})_{max}$ when $\theta_1 = \theta_2 < 120^\circ$, there is one symmetric axis in the structure.

So far we have only considered the determinant of pinned joints but fixed joints can also be explored with similar approach. The stiffness submatrix of a fixed joint is 3x3. Thus there are three eigenvalues associated with a fixed joint.

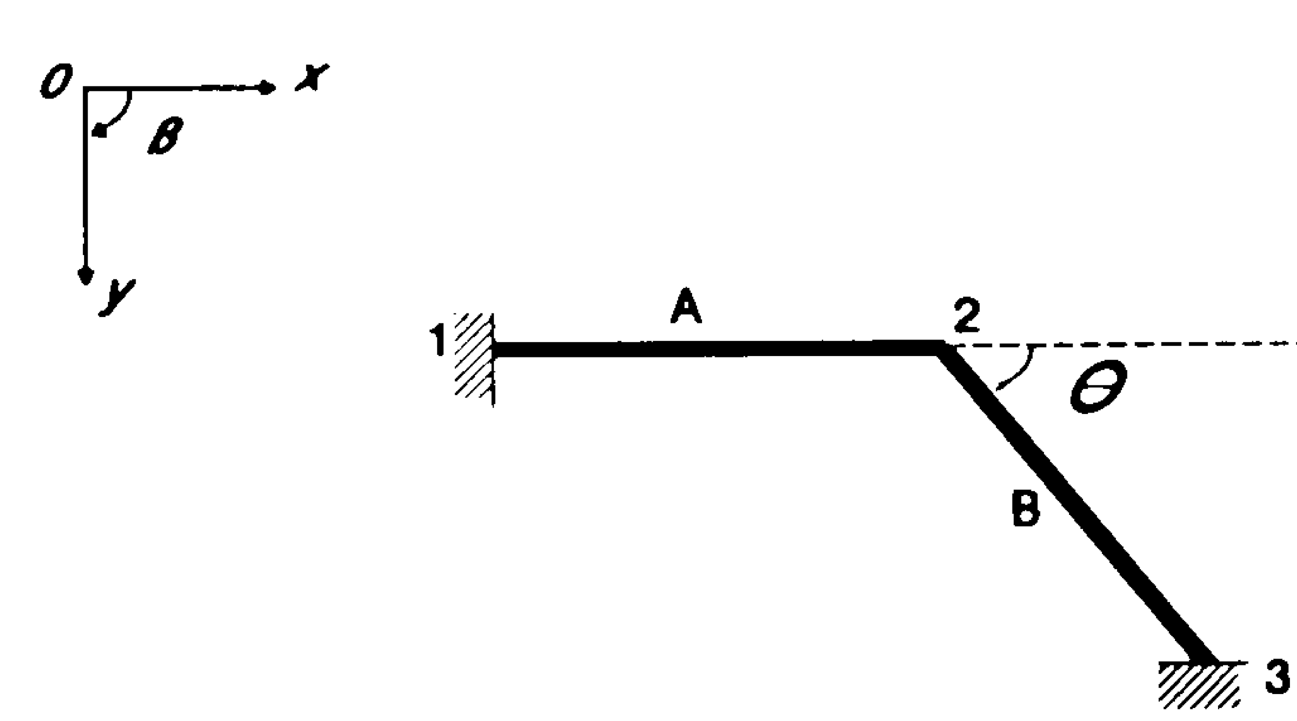


Fig. 4.4 A structure

Fig.4.4 shows a plane frame in which two member are rigidly connected together at joint 2 and supports at joints 1 and 3 are fully fixed. The bending, axial rigidities and geometric length of the members are: $\mathbf{AE} = 50 \times 10^4$ kN, $\mathbf{EI} = 10^4$ kNm^2, and $l = 5$m.

The submatrix associated with joint 2 can be set up using similar method as above.

$$D_{22} = 10^4 \left[\begin{array}{c|c|c} 10 + 10\sin^2\theta + 0.096\cos^2\theta & \text{Symmetric} & \\ \hline 0 + 9.904\cos\theta\sin\theta & 0.096 + 0.096\cos^2\theta + 10\sin^2\theta & \\ \hline 0 \; -0.24\sin\theta & -0.24 \; 0.24\cos\theta & 0.8 \; 0.8 \end{array} \right] \tag{4.19}$$

Having obtained the submatrix, the eigenvalues of this matrix can be conveniently calculated using the method described by Alexander,J.M. (1981,pp254-255). The three eigenvalues and the determinant of (4.24) are given by Fig.4.5 in which the horizontal axis denotes the angle between two members -- θ.

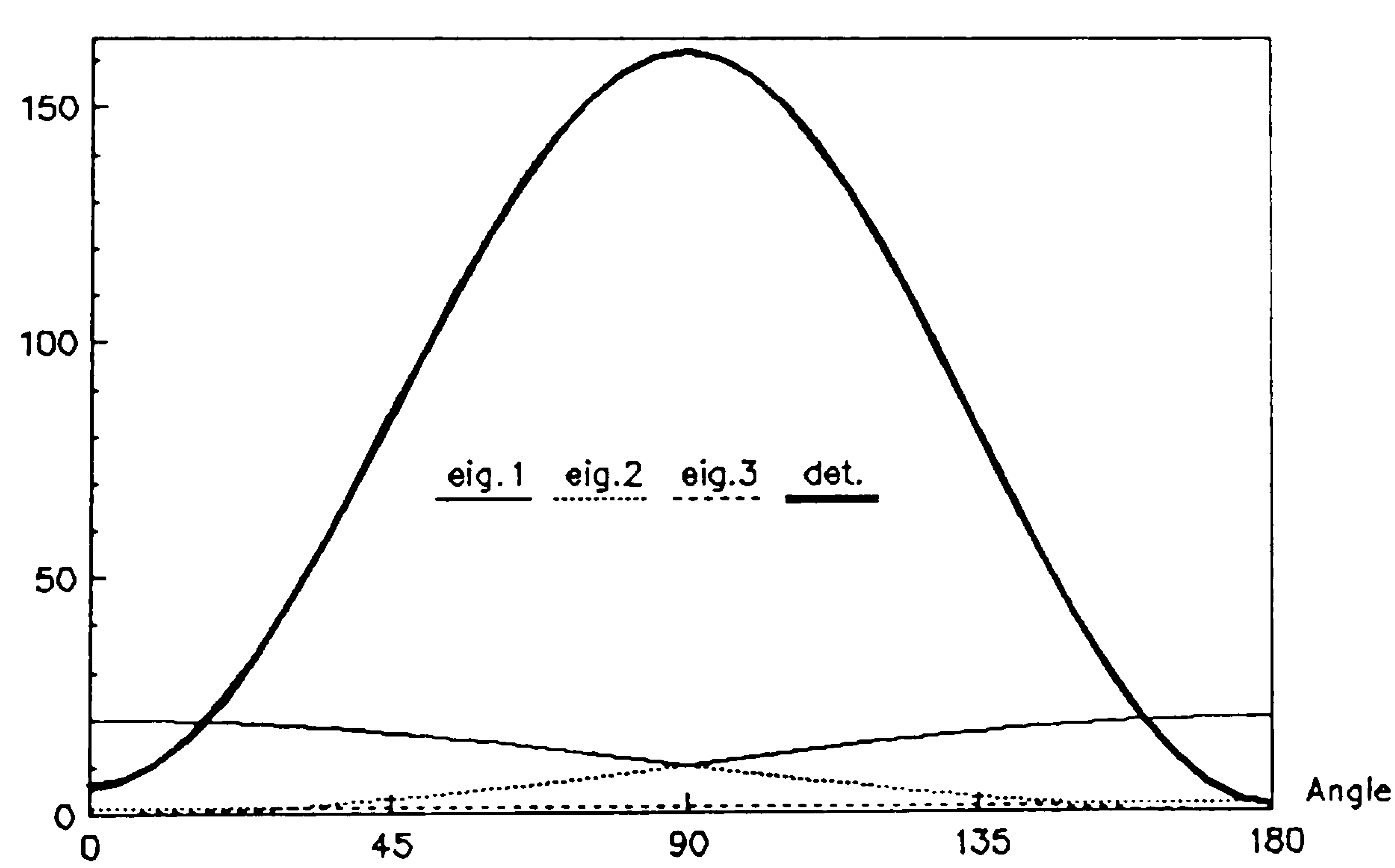


Fig.4.5 Determinant and Eigenvalues

Initially, as shown in the figure, the determinant exhibits a result from the combination of three eigenvalues. The determinant reaches its maximum value when the angle between two member is 90°. At two extreme ends of the horizontal axis, i.e. when θ is equal to 0° or 180°, the determinant decreases to its minimum value.

Having looked at three eigenvalue curves, interestingly, we have found that one eigenvalue (eig.3) almost remains unchanged when θ changes from 0° to 180°. The physical interpretation of this is that the principal stiffness coefficient corresponding to rotation displacement for a fixed joint is only a function of the stiffness of members connecting to that joint. The orientation of the members adjacent to the joint will not influence the ability of the joint to resist the rotation displacement. However, the other two eigenvalues (eig.1 and eig.2) which correspond to two translation displacements are noticeably influenced by θ--the orientation of the members framing in to the joint and the product of three eigenvalues--determinant, therefore, is mainly decided by two translation stiffness coefficients.

Summarizing the results so far, we see that the determinant of a joint is dependent on the following three factors:

(1). the structural characteristic of that joint(pinned or fixed);
(2). the stiffness of the members adjacent to that joint;
(3). the way in which the member objects frame in to that joint.

These three factors characterize the well-formedness of a joint. The well-formedness of a joint implies locally physical or structural condition with reference to capability to resist loading in any arbitrary direction and to perform its desired function. A robust joint should have good quality of well-formedness. If a joint is said to be badly formed, it implies that the joint itself has bad structural capability to resist damage or deterioration. The well-formedness of joints have significant influences on structural systems. The badly formed joints are the potential vulnerable components to the safety and reliability of structures.

It appears logical that the value of the determinant of a joint can be a measure of the well-formedness of that joint. The well-formedness of a joint i can be defined quantitatively by the following expression:

$$q_i = det(\mathbf{D}_{ii}) \tag{4.20}$$

where $\mathbf{D}_{ii}$ is the stiffness submatrix associated with joint j_i.

Having defined the measure of well-formedness of structural joints, some points should be noted.

(1). The measure q_i is totally independent of co-ordinate system.
(2) q_i is a measure of the capability of a structural joint to resist loading from any arbitrary direction.
(3) The quality of well-formedness of a structural joint could be improved either by

(i) changing the stiffness of that joint, i.e. from pinned joint to fixed joint;
(ii) or increasing the stiffness of the members connecting to that joint;
(iii) or changing the orientation of the members framing in to that joint.

4.5 Well-formedness of Structural Rings

A structural ring consists of a sequence arcs and joints. For a structural ring R_m with k joints, we can use (4.20) to calculate q_i for each joint contained in the ring. Then we postulate another measure which is a property of a structural ring. This measure implies the quality of well-formedness of a structural ring R_m which is defined quantitatively as

$$q(R_m) = \Sigma_i q_i \quad (i=1,\ldots,k) \tag{4.21}$$

where k is the total number of joints in the ring R_m.

To illustrate concept of this measurement, we consider the application of (4.20) and (4.21) to the evaluation of well-formedness of structural rings, as shown in Fig.4.6. All members have $\mathbf{AE} = 50\times10^4$ kN, $\mathbf{EI} = 10^4$ kN m^2 and $l = 5$m.

The structural ring R_1 is a fully fixed ring with value $q(R_1) = 97.29$. If we deteriorate R_1 by releasing one of the degrees of freedom in R_1, then it degenerates into R_2 which is a ring with one pinned joint along it. The value of $q(R_2)$ becomes 87.61. As we progress to deteriorate the ring from R_1 to R_4, the value of $q(R_m)$ is decreasing from 97.29 to 68.22. This illustrates that the deterioration of a structural ring will reduce the quality of well-formedness of that ring.

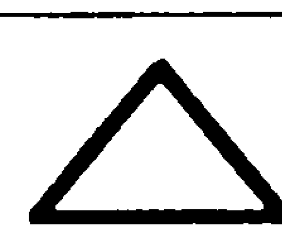
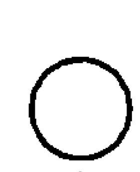
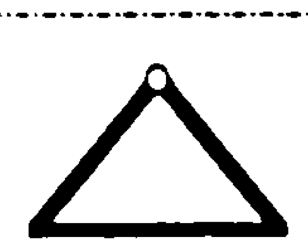
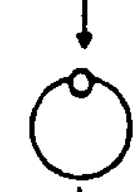
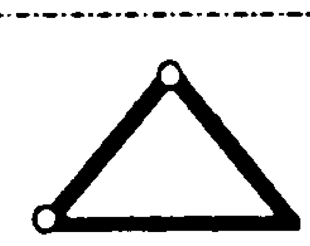
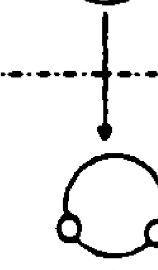
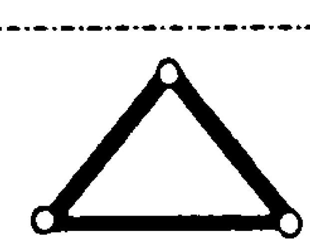
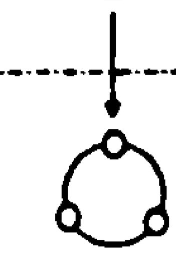
Ring R_m	Structure	Graphical Pattern	$q(R_m)$ (10^{10})
1			97.29
2			87.61
3			77.61
4			68.22

Fig. 4.6 Deterioration of a structural ring

We will see in Chapter 7 that the quality of the well-formedness of structural rings plays a significant role in the sense of vulnerability and robustness of a structure.

A structural ring with high value of well-formedness has the potential capability to resist loads in any arbitrary direction and is therefore robust. If at a certain level of hierarchy, a very badly formed ring is identified, the corresponding substructure is a potentially vulnerable part of the whole system.

4.6 Summary and Conclusions

In this chapter a measure of the well-formedness of structural rings has been developed. The well-formedness of a structural ring indicates its potential capability to resist loading in any arbitrary direction and to perform its desired function. In Chapter 3, it was shown that a structure is built up with many interconnected structural rings. A robust structure should therefore consist of a set of well formed structural rings. The well-formedness of a structural ring presented in this chapter provides a quantitative estimate of the vulnerability and the robustness of a structure.

The quality of the well-formedness of a structural ring is closely related to the well-formedness of the joints contained in the ring. The concept of the determinant of a joint has been introduced in this chapter. It has been found that the determinant of a joint is dependent on the degrees of freedom of the joint as well as the orientation and stiffness of the members framing in to the joint. Thus, the determinant of a joint also implies local physical or structural conditions for the resistance of load from any arbitrary direction.

In this Chapter we have examined the concepts of the eigenvalue and eigenvector of the stiffness matrix and the application to the analysis of structures. The principle stiffness coefficients and the principle displacement

Structural Clusters

5.1 Objectives

The objectives of this chapter are:

1. To introduce the role of clustering techniques in dealing with complex systems;
2. To present the concept of structural clusters;
3. To develop a measure to evaluate the structural quality of clusters;
4. To develop an algorithm of cluster formation, and;
5. To present an example.

5.2 Introduction

In Chapter 3, a structural ring has been defined as the basic element of a structure which is capable of resisting any arbitrary equilibrium set of applied forces and perform the desired function. A structure is made up of a set of structural rings. A complex structure, however, consists of a large number of structural rings which are highly interconnected. At this stage we are still faced with a great number of structural rings together with the complicated interactions between them. We would not be able to do the structural vulnerability analysis efficiently unless we classify the structural rings into manageable groups, which in some sense can be treated as whole elements.

Clustering techniques can be used to perform this data reduction. In this chapter we explore the use of clustering techniques to classify the structural rings of a structure into a number of groups. Each group contains a set of overlapping structural rings. We call them structural clusters.

We develop an algorithm of cluster formation, the major purpose of which is to group the structural rings of a structure into a number of interconnected structural clusters. These clusters form a new set of rings at the next higher level of definition. The number of structural rings as well as the interactions between the rings is reduced. This algorithm is then applied recursively to various levels of definition until the structure becomes a single cluster.

The process of cluster formation produces a hierarchical model in which a structure can be described at various levels of definition in terms of interconnected sets of structural rings. This provides a basis for the structural vulnerability analysis.

5.3 Clustering Analysis

Clustering analysis is concerned with techniques for the analysis of multivariate data to solve the following problem (Everitt,1974):

Given a sample of N objects or individuals, each of which is measured on each of p variables, a classification scheme is devised for grouping the objects into g classes. The number of classes and the characteristics of the classes are to be determined.

Clustering techniques have been widely used in diverse fields such as psychology, sociology, artificial intelligence, information retrieval (Everitt, 1974). In many such fields the research worker is faced with a great bulk of observations which are quite intractable unless classified into manageable groups, which in some sense can be treated as units. Clustering techniques can be used to perform this data reduction, reducing the information on the whole set of say N individuals to form about say g groups where necessarily g is very much smaller than N. In this way it may be possible to build up a hierarchy and to describe the observations at various levels of definition.

Clustering techniques also play an important role in the world of large-scale systems in many engineering fields, for example, engineering networks, environmental analysis, information technology (Elms, 1983) (Sangiovanni-Vincentelli, 1977). It is often required to partition such systems into many subsystems (clusters) such that elements in the same subsystem are strongly interconnected, whereas elements in the different subsystems are weakly interconnected. In some cases where the system has a simple layout, a fairly good cluster partition can be determined by inspection. For arbitrary systems, however, an algorithm must be used to systematically partition the associated system into a set of connected clusters.

In this thesis, we confine our attention to the clustering problem associated with structural systems.

Up to now we have a model - the OOGM, which represents the analysed problem - a structural system $S = (J, M)$. Very often, this 'whole' graph model is complex. It is likely to be composed of many interacting joint objects and member objects. The problem of handling structures composed of a large number of objects is important for practical application of safety and vulnerability analysis. It would be very helpful to engineers if the whole model could be reconstructed into a number of interrelated small models, thus allowing the analysis of the structure at different levels of description. We now shall consider what criterion to use as a basis for the formation of small models, or how these structural objects of a system are to be reclassified and reorganized.

Intuitively, it would seem that objects in a group share some common features or have more features in common than with objects outside the group.

There are a number of clustering algorithms which can be used to classify objects, such as DCMPOS (Owen, 1970). This algorithm was designed in such a way that it decomposes a graph into clusters which have the largest number of nodes for a given description of connectivity. Another method which was initially put forward by Alexander (1964) then developed by Elms (1983), simply divides the node set N in a graph $G=(N, L)$ into those subsets which are connected by as few connections of link set L as possible, thus leaving as many of connections as possible within the subsets.

It is not, however, intended here to cluster those joint objects or member objects of a structure rather we take structural rings as basic objects of clustering analysis. As discussed in Chapter 3, a structural ring is the basic object of a structural system which is capable of resisting an arbitrary equilibrium set of applied forces. A structural system can be described as a set of interconnected structural ring objects. Some of the rings are highly interconnected and some are less interconnected. Thus, the clustering procedure is to group ring objects into a number of subsystems in which the structural rings are as highly interconnected as possible within the subsystem and leave as few connections as possible between these subsystems. These subsystems of a structure are called structural clusters.

5.4 Structural Clusters

A *structural cluster* S^l_i at a level of definition l is defined as a group of overlapping structural rings in which the rings themselves are more densely connected to each other within the cluster than to other rings outside the cluster.

Having defined this, the following points should be noted:

(1). A structural cluster S^l_i at a level of definition l is a sub-set $S^l_i = (M^l_i, J^l_i)$ of $S=(M,J)$ in which M^l_i is a subset of M, and J^l_i is a subset of J.
(2). A structural cluster itself is a complete structural subsystem since it consists of interconnecting structural rings.
(3). A structural system itself is a structural cluster at the highest level of description.
(4). A primitive structural cluster is one which contains only one member object.

The fact that a structural cluster is a complete structural subsystem leads to the conclusion that the theories and algorithms we have developed in previous chapters can also apply to the analysis of structural clusters.

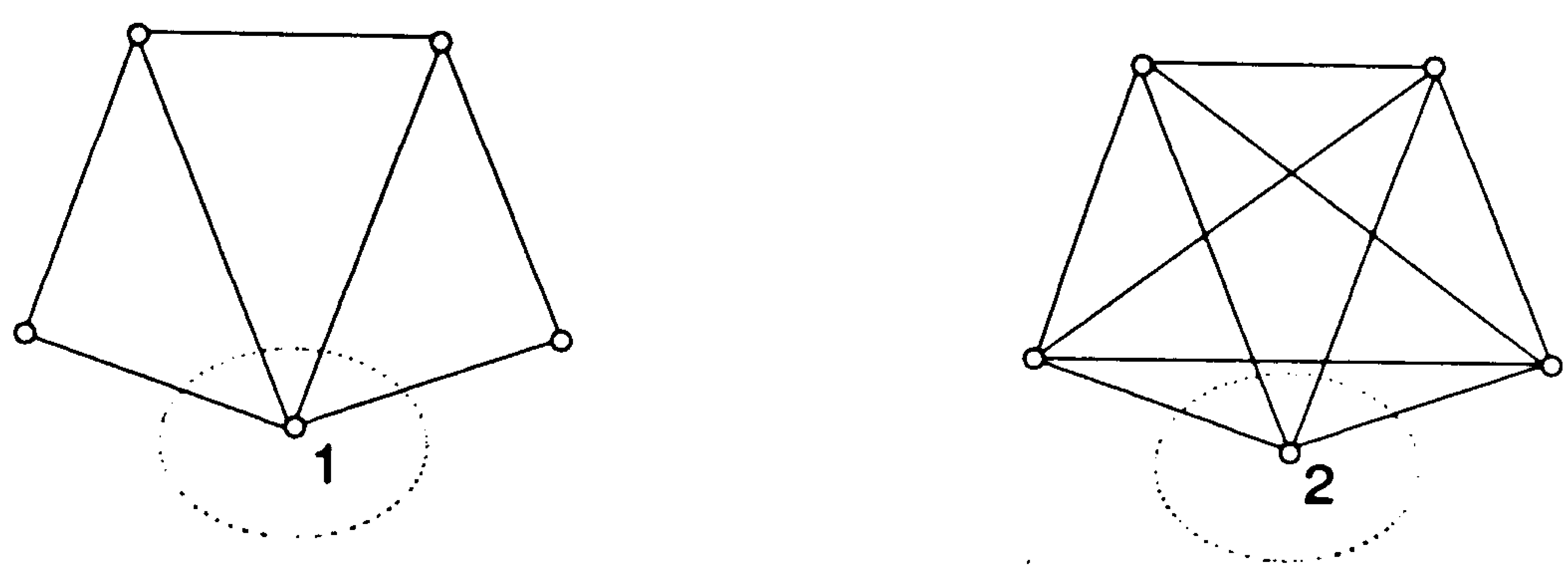
5.4.1 Connectivity of structural Clusters

One important characteristic of a structural cluster is that of connectivity.

The connectivity of a structural cluster implies

(i) the number of structural rings within the cluster;
(ii) the degree of overlap between them.

Therefore, a measure of the connectivity requires establishing a difference between the number of members adjacent to a joint and the number of structural rings associated with a joint. For example, in Fig.5.1(a) and (b), if we consider joint 1 in S^l_1 and joint 2 in S^l_2 separately, it seems that there is no immediate difference between the connectedness of joint 1 and joint 2 because there are four adjacent members framing in to both of the joints. But if we look for the number of structural rings associated with each joint, we can recognize the difference of the connectedness. Thus, as a measure of the connectivity, it is not sufficient simply to count the members attached to joints as the concept of connectivity implies that these joints must also be connected to one another to form interconnecting rings.



(a) S^{l}_{1} (b) S^{l}_{2}

Fig. 5.1 Connectedness of structural joints

The definition of a cluster requires that there is at least one structural ring associated with each joint in a cluster. For a given number of joints in a cluster, it seems logical that the more the number of structural rings in the cluster, the higher degree of connectedness the cluster will have.

To describe the degree of connectedness of a structural cluster, the concept of tightness is used here.

Tightness, is the degree of connectedness of a structural cluster S^{l}_{i} and equals to the ratio of the total number of structural rings to the total number of joints in the cluster.

$$T(S^{l}_{i}) = n_R / n \tag{5.1}$$

where n_R is the total number of structural rings in S'_i and n is the number of joint objects in S'_i.

The similar concept has been used by Elms(1983) to describe the connectivity of a graph, in which the tightness of a graph is defined as the ratio of the total number of 3-link loops to the total number of nodes in the graph. But here the tightness of a cluster has been defined in even broader sense. It includes not only 3-link length rings but all structural rings contained in a cluster.

For example, Fig.5.2 is shown a group of structural clusters and their corresponding tightness. We see that the greater the connections between joint objects, the 'tighter' is the cluster.

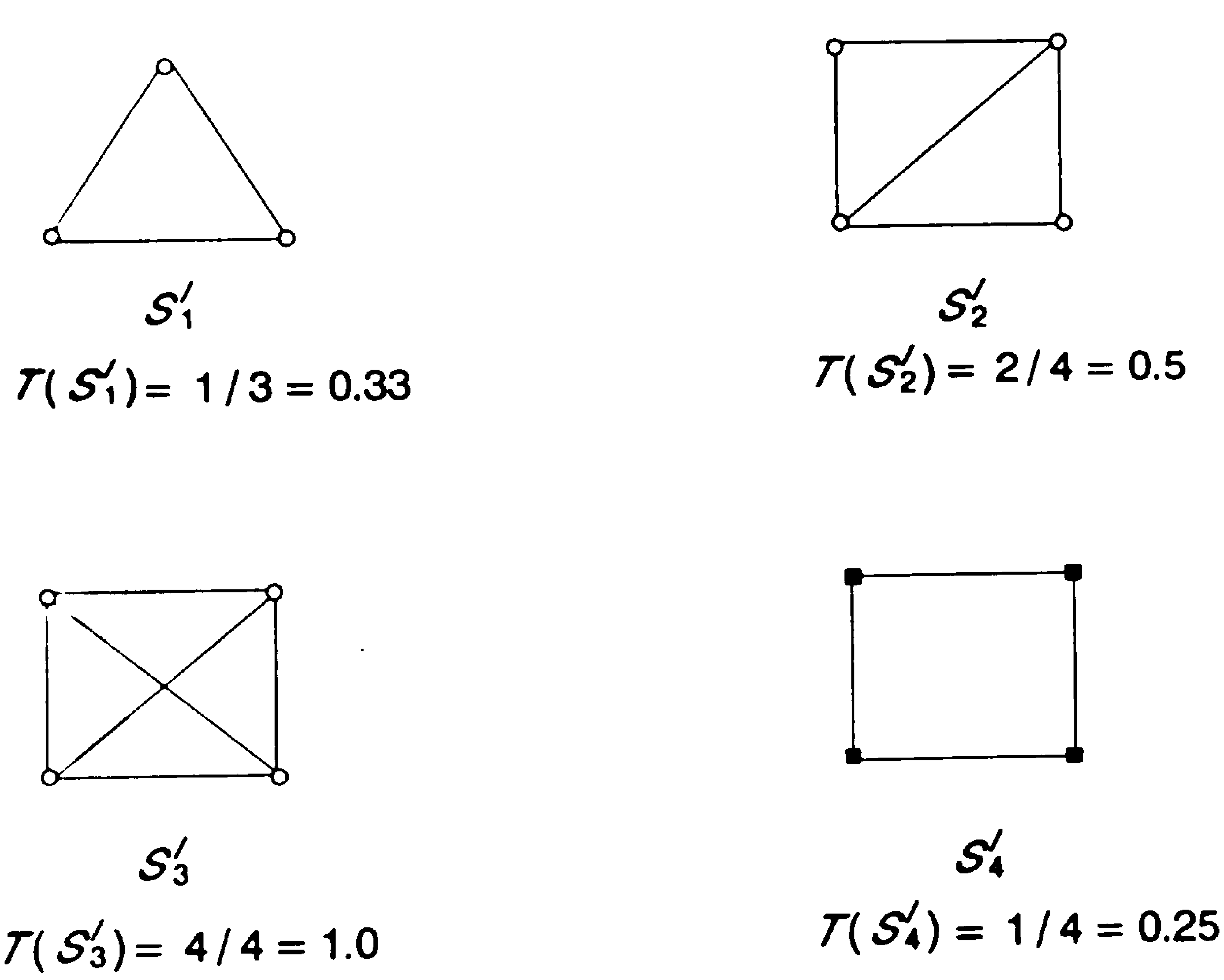


Fig. 5.2 Structural clusters and their tightness

For two structural clusters S'_1 and S'_2, denote the tightness of S'_1 and S'_2 as $T(S'_1)$, $T(S'_2)$, then we say that

(1). cluster S'_1 is tighter than cluster S'_2 if $T(S'_1) > T(S'_2)$;

(2). cluster S'_1 is as tight as cluster S'_2 if $T(S'_1) = T(S'_2)$

Fig.5.3 (a) and (b) have shown these two cases.

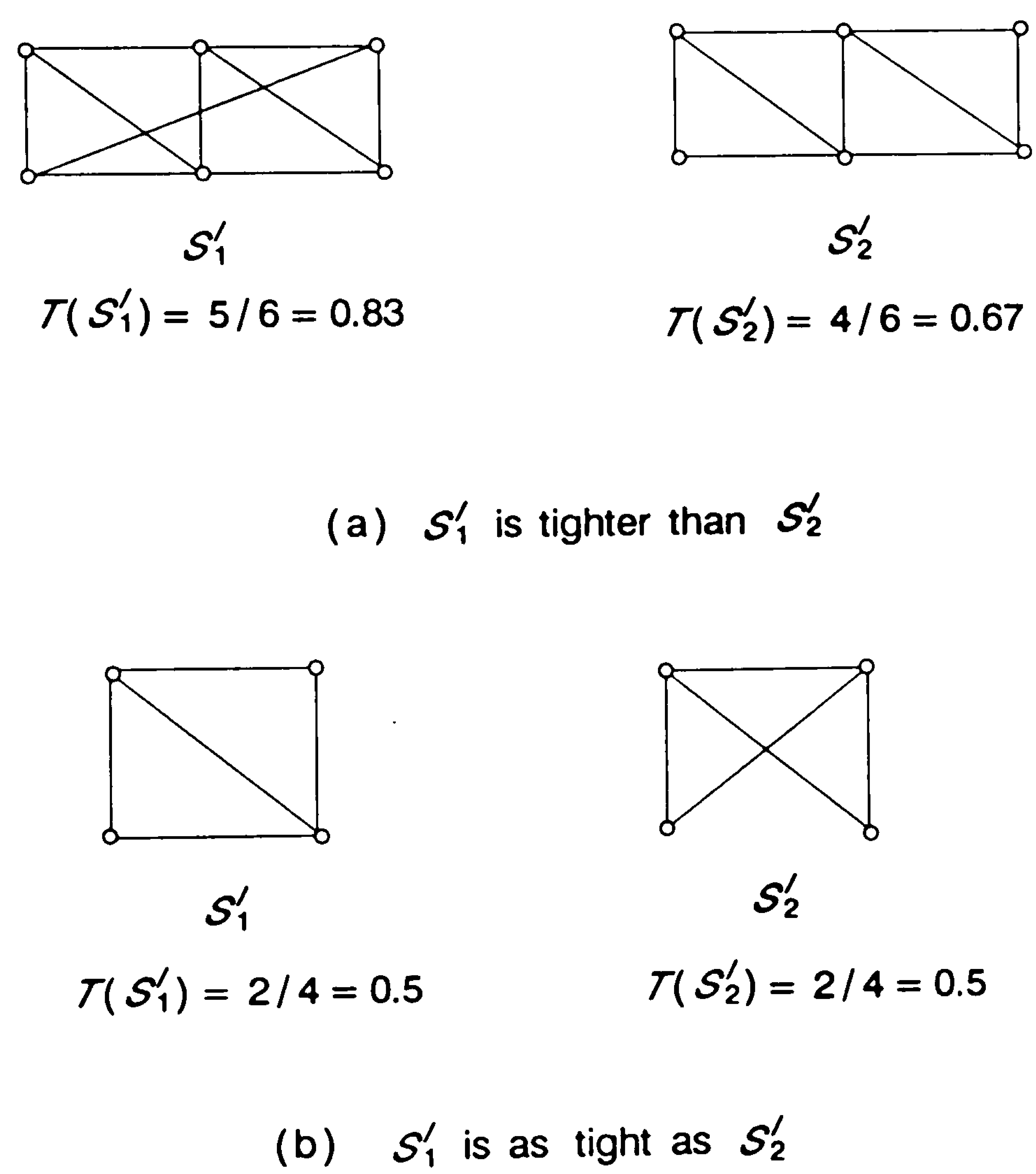


Fig. 5.3 Connectivity of structural clusters

5.4.2 Structural Tightness

The definition of the tightness of a cluster includes only the ratio of the number of structural rings to a given number of joints. As a measure of the quality of a cluster, however, the well-formedness of the structural rings contained in the cluster is also very important as far as the structural vulnerability concerned. If we consider the quality of well-formedness of each ring in a cluster, then the tightness becomes structural tightness which is defined as

Structural Tightness

$$Q(S^l_i) = \Sigma\, q(R_m) \,/\, n \quad (m = 1,\ldots,n_R) \tag{5.2}$$

where n is the number of joints in S^l_i and n_R is the total number of rings in S^l_i, and $q(R_m)$ is the quality of well-formedness of a ring R_m which is given by (4.21).

Thus, we can see that the structural tightness of a cluster is a measure of both the quantity and the quality of the connections within the cluster. It depends on

(i) the number of structural rings within the cluster;
(ii) the degree of overlap between the rings;
(ii) the quality of the well-formedness of the rings.

When a cluster S^l_i contains only one structural ring, R_m (5.2) then actually gives the value of the structural tightness of that ring R_m, denoted as

$$Q(S^l_i) = Q(R_m) = q(R_m) \,/ n \tag{5.3}$$

(5.3) implies that if two structural rings R_1 and R_2 have the same values of $q(R_i)$,

$i=1,2$, then the one with shorter path length, i.e. less number of joints, has the bigger value of structural tightness. Fig.5.4 has shown this case.

If the quality of well-formedness $q(R_1) = q(R_2)$

Then the structural tightness $Q(R_1) > Q(R2)$

Fig.5.4 Structural tightness of rings

The conclusion is that a structural ring with short path length tends to be robust. We will see later that the $Q(R_m)$ measure is very useful to identify the most robust structural ring at a given level of definition.

In clustering analysis, the ground or foundation of a structure is treated as a part of structure. We define the foundation or ground as a very special single cluster -- ground cluster, which is much more tightly and densely self connected and very well formed. A ground cluster is denoted as S_G. It is excluded from consideration when the structural tightness is calculated.

5.5 Principles of Cluster Formation

In this section we discuss the basic principles of cluster formation. Some requirements of a clustering technique are as follows:

(1). The given set of objects can be a sample taken from an even larger set of objects or else the set can be complete in itself;
(2). Interest may be centred on knowing the individual elements of a cluster, the elements in an over-all description of the cluster, or in both.
(3). An object may be allocated to only one cluster or to more than one. It is required to be an element of at least one cluster.

Generally speaking (Lance & Williams, 1966), a complete clustering algorithm may consist of three distinct process:

(1). a method for initiating a cluster;
(2). a method of allocating new objects to existing clusters;
(3). a method of determining whether further allocating may be regarded as unprofitable according to the criteria of clustering, so that objects remain unallocated as independent clusters.
(4). a method of reallocating some or all the objects to existing clusters when the main classifying process is completed; this is intended to readdress any misclassification produced by the first three process.

A clustering algorithm must include (1) and (2); but in any particular system we study either (3) or (4), or both, may be lacking.

The principles of cluster formation of a structure as considered in this thesis may be stated as follows

Given: A large complicated system, which consists of a set of joint objects connected by a set of member objects. The joint objects and member objects interconnect to each other to form a set of structural ring objects.

Find: A "set" of overlapping rings -- subsets of the original joint and member object set, such that: the rings are more densely connected to each other within the cluster than to other rings outside of the cluster. The judgement is based on the criterion of the tightness T or the structural tightness Q of the cluster, depending on the interest of the user.

The tightness T and the structural tightness Q can be used at different stage of analysis. The user may be only interested in the configuration or connectivity of a structural system by neglecting the physical and geometric details of the structure, in this case, the tightness T will be used in the judgement. When the well-formedness of the connections of a structural system are of interest, then the structural tightness Q will be used. Generally speaking, the different criterion will produce different results. The clustering algorithm in this chapter can be used with either of them.

5.6 Algorithm

In the process of cluster formation, structural clusters are built up with successively assigning overlapping structural rings to the forming clusters. The algorithm builds up a cluster, adding one structural ring object at a time. It keeps searching for two characteristics at each level l of description as follows:

(1). The set of ring objects R^l_C in the forming cluster at level l;

(2). The set of ring objects R^l_D which could possibly be added to R^l_C to further increase the forming cluster, and this is decided using Q.

The criterion for cluster formation will be that an overlapping structural ring will be added to the forming cluster if the value of Q (structural tightness) is increased. If there is no addition of a ring which can satisfy the above condition, the cluster is complete.

Basing on the criterion the process of cluster formation can be described as follows

(1). Supposing S^l_i is a forming cluster, denote the structural tightness of S^l_i as $Q(S^l_i)$.

(2). Adding an overlapping ring R_m into the forming cluster S^l_i, and the new value of structural tightness $Q^*(S^l_i)$ is then given by (5.2).

(3). The ring R_m will not belong to S^l_i unless

$$Q^*(S^l_i) > Q(S^l_i) \tag{5.4}$$

(4). If (5.4) is satisfied then the ring R_m should add into S^l_i. The forming cluster now becomes S^{l+1}_i and its value of structural tightness is $Q(S^{l+1}_i)$.

The ability to conduct an ordered search is very important to the development of the algorithm. This is necessary both to reduce search time and to improve the probability of finding clusters. Translated into the needs of an algorithm, it must be a device for placing the ring objects in an order for consideration that will enhance the probability of finding best-formed or tightest structural clusters quickly. The objective of this device is to approach that order by placing high on the list those rings which will tend to be nuclei around which clusters will form. To achieve this, structural rings are ranked according to the structural tightness of the rings $Q(R_m)$, given by (5.3).

The process of cluster formation has been implemented in a computer program. A schematic representation of the mechanics involved in the program is presented in Fig.5.5(a) and Fig.5.5(b). Even though this program is basically concerned with the formation of structural clusters, the various subroutines which constitute the program can be used in other applications. The program was developed in subroutine form so that it would be readily adaptable to a wide variety of computational schemes and problem types.

The following list contains a brief description of the computing system developed to implement the techniques and principles of cluster formation presented in previous sections. The program is composed of four subroutines. These subroutines and their primary functions are summarized below.

DATAIN: This is a standard input subroutine, which includes inputting of joints, string pattern of joints, end joints of each member, the physical and geometric details of each member, such as **E**, **A**, **I**, l.

RINGS: Identify all structural rings in a structure at a level of definition l. The principle of this subroutine is based on the algorithm developed in Section 3.7.

DET: Form the stiffness submatrix for each joint in a given ring. Compute determinant d_i, the quality of well-formedness q_i of each joint, given by (4.20); the quality of well-formedness $q(R_m)$, given by (4.21); and the structural tightness $Q(R_m)$ of the whole ring, given by (5.3).

These three subroutines, which are shown in Fig.5.5(a), are prerequisites for the subroutine of cluster formation. Since they show the source of information used by the subroutine **CLUSTER**, the procedure of which is shown in Fig.5.5(b).

The first main process in the subroutine **CLUSTER** is to initiate a forming cluster. The highest ranked ring not in a cluster is chosen.

The second main process in **CLUSTER** is a recursive process to form a complete cluster. Our strategy will be

(1) to find out an overlapping ring which is not in a cluster
(2) to add this ring to the forming cluster if it will increase the current value of Q of the forming cluster

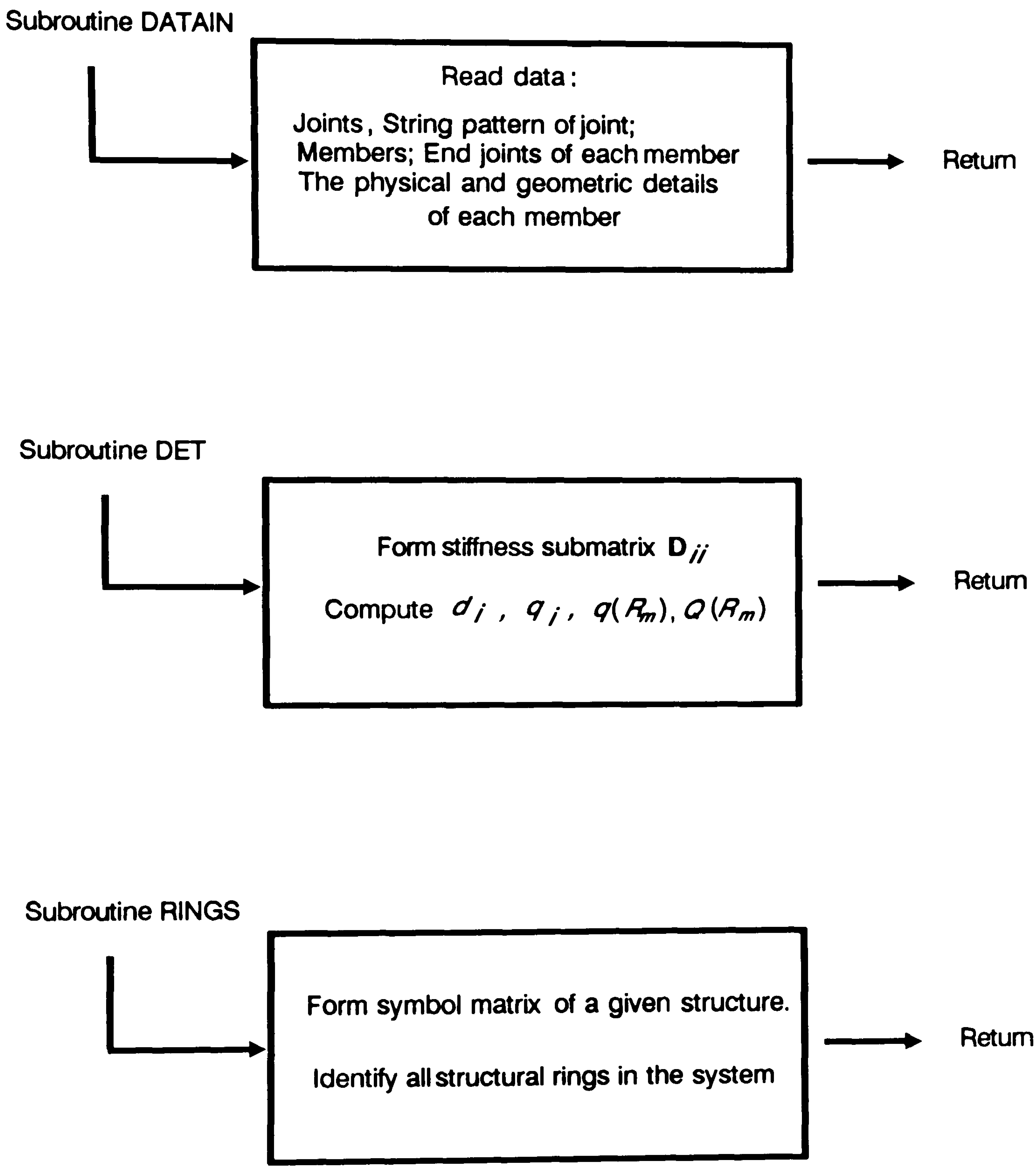


Fig. 5.5 (a) Subroutines

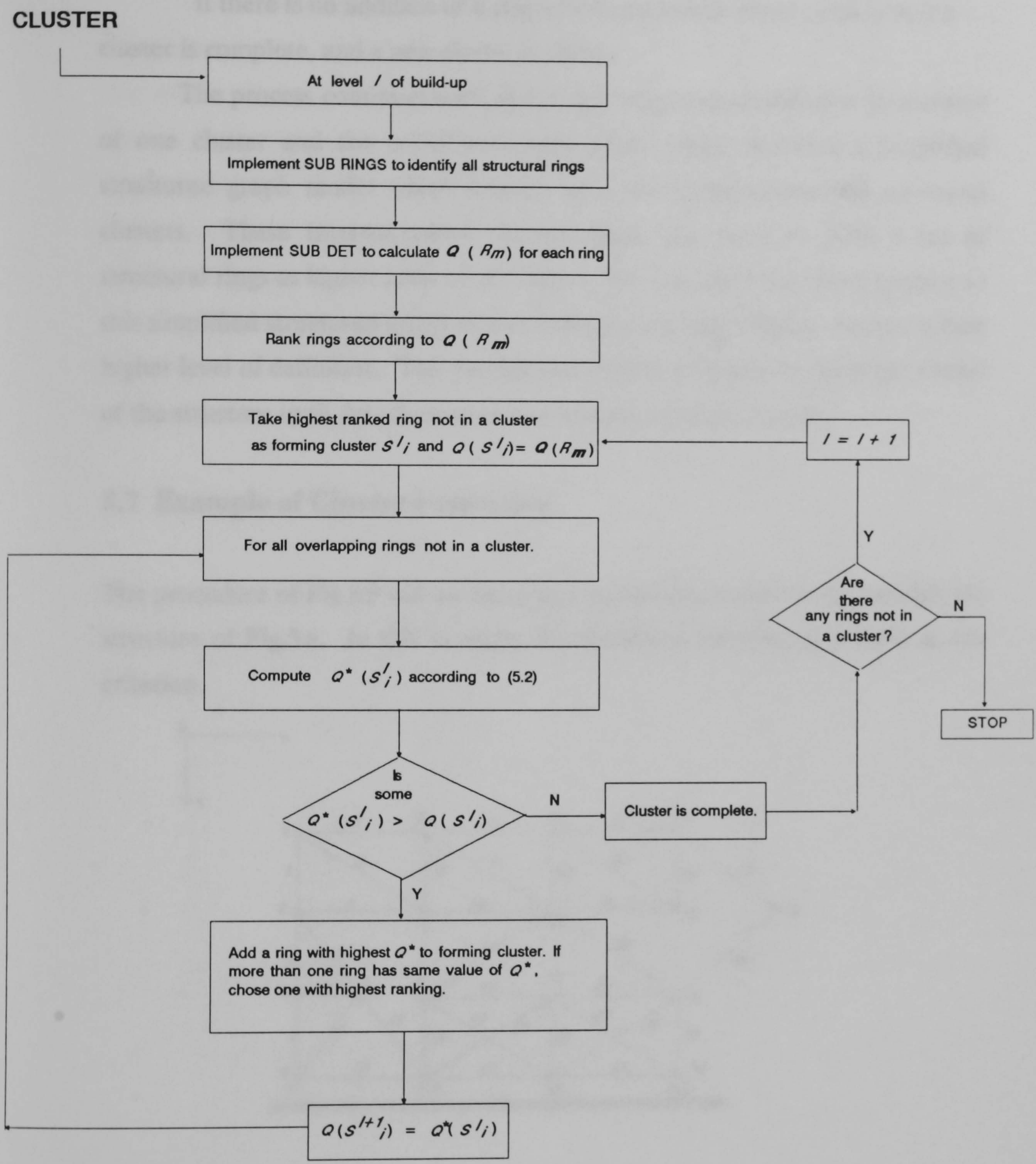


Fig. 5.5(b) Procedure for cluster formation

If there is no addition of a ring which can satisfy above condition, the cluster is complete, and a new cluster is started.

The process continues until at a stage every ring identified is an element of one cluster and the initial structural graph model becomes a simplified structured graph model which consists of a set of interconnected structural clusters. These interconnected clusters again are there to form a set of structural rings at higher level of definition. We can apply the above process to this simplified structured graph model to form even bigger union clusters at next higher level of definition. This process can recursively apply to the graph model of the structure until the whole structure becomes a single cluster.

5.7 Example of Cluster Formation

The procedure of Fig.5.5 will be illustrated by forming structural clusters for the structure of Fig.5.6. In this example, the structural tightness Q is used as the criterion.

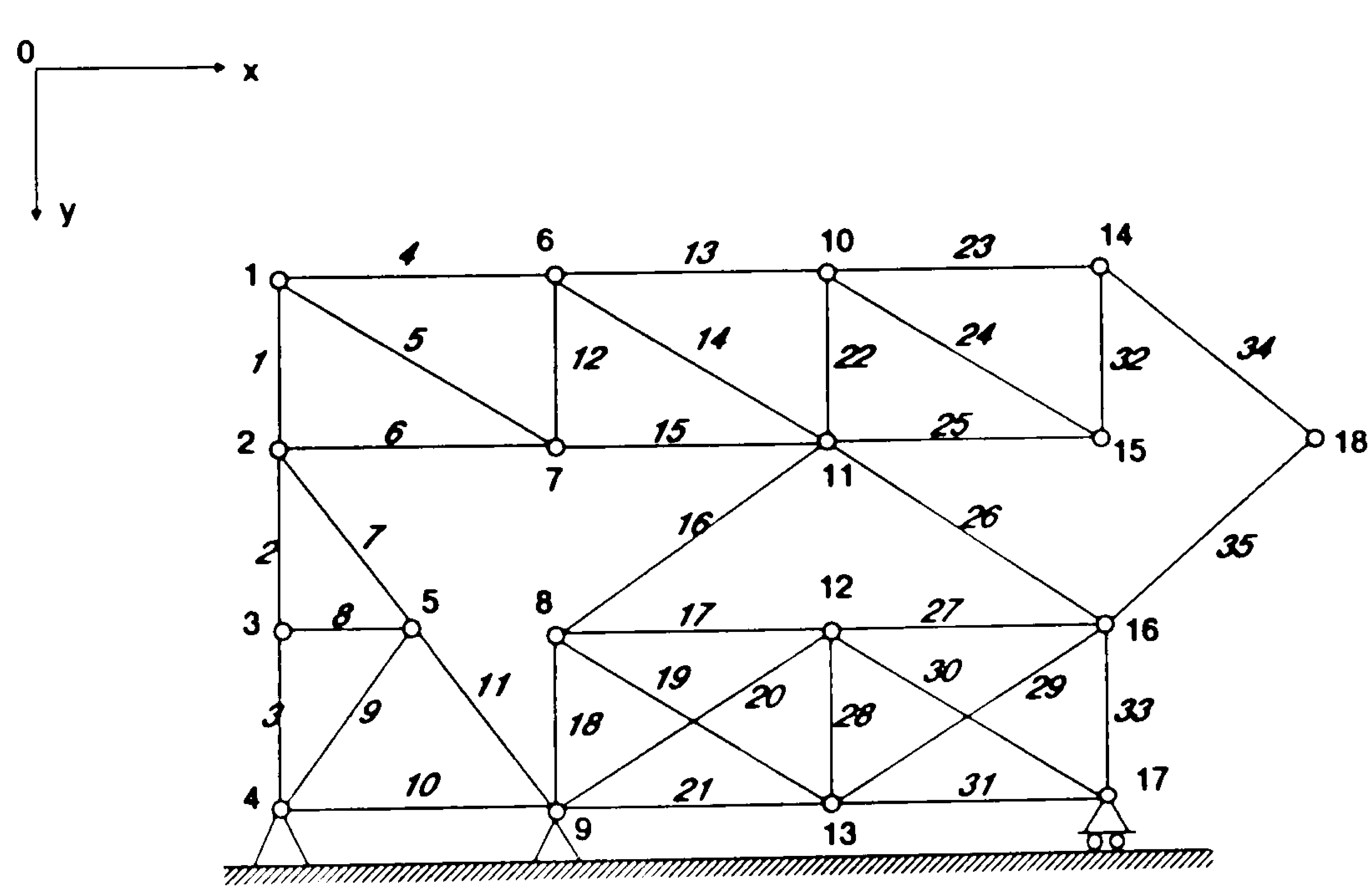


Fig. 5.6 A Structural System

The structural system of Fig.5.6 is a two dimensional pin-jointed frame. The members are denoted by italic numbers. The diagonal members cross at the centre of the frame without being connected there. All members of the frame have the same value of elastic modulus $E=2.0\times10^{8}kN/m^{2}$. By implementing the subroutine **DATAIN**, the detailed information about the sizes and properties of the structure can be obtained which are listed form Table 5.1 to Table 5.2.

TABLE 5.1

Joint co-ordinate table

Joint No.	X Co-od. (m)	Y Co-od. (m)	Joint No.	X Co-od. (m)	Y Co-od (m)
1	0.00	0.00	2	0.00	1.50
3	0.00	3.00	4	0.00	4.50
5	1.00	3.00	6	2.00	0.00
7	2.00	1.50	8	2.00	3.00
9	2.00	4.50	10	4.00	0.00
11	4.00	1.50	12	4.00	3.00
13	4.00	4.50	14	6.00	0.00
15	6.00	1.50	16	6.00	3.00
17	6.00	4.50	18	7.50	1.50

TABLE 5.2

Member end condition and properties table

Member No. m_k	Joint j_i	String Pattern	Joint j_j	String Pattern	Area $A(m^2)$
1	1	*p*	2	*p*	0.0016
2	2	*p*	3	*p*	0.0021
3	3	*p*	4	*p*	0.0021
4	1	*p*	6	*p*	0.0016
5	1	*p*	7	*p*	0.0016
6	2	*p*	7	*p*	0.0016
7	2	*p*	5	*p*	0.0028
8	3	*p*	5	*p*	0.0028
9	4	*p*	5	*p*	0.0028
10	4	*p*	9	*p*	0.0028
11	5	*p*	9	*p*	0.0028
12	6	*p*	7	*p*	0.0016
13	6	*p*	10	*p*	0.0016
14	6	*p*	11	*p*	0.0016
15	7	*p*	11	*p*	0.0016
16	8	*p*	11	*p*	0.0032
17	8	*p*	12	*p*	0.0032
18	8	*p*	9	*p*	0.0032
19	8	*p*	13	*p*	0.0032
20	9	*p*	12	*p*	0.0032
21	9	*p*	13	*p*	0.0032
22	10	*p*	11	*p*	0.0016
23	10	*p*	14	*p*	0.0016
24	10	*p*	15	*p*	0.0016
25	11	*p*	15	*p*	0.0016
26	11	*p*	16	*p*	0.0032
27	12	*p*	16	*p*	0.0032
28	12	*p*	13	*p*	0.0032
29	13	*p*	16	*p*	0.0032
30	12	*p*	17	*p*	0.0032
31	13	*p*	17	*p*	0.0032
32	14	*p*	15	*p*	0.0016
33	16	*p*	17	*p*	0.0032
34	14	*p*	18	*p*	0.0016
35	16	*p*	18	*p*	0.0016

Then we start the process of cluster formation and assume that this is the level *1* in the hierarchy.

Referring to Fig.5.5(b), the first main step in the process is to implement the subroutine **RINGS** to identify all structural rings at this level in the structure and use the subroutine **DET** to calculate the quality of well-formedness $q(R_m)$ for each ring and then rank them according to value of structural tightness $Q(R_m)$. The results are shown in Table 5.3.

It was mentioned in Section 5.4.2 that, the ground or foundation of the structure is considered as a single cluster very tightly and densely self-connected and very well formed. Thus, in the process of cluster formation, the ground cluster S_G comes up first, as shown in Fig.5.7(a)-(a).

The rest of the procedure of cluster formation is straight forward. To initiate the first cluster, we consult Table 5.3 and we see that rings R_7 and R_8 have the same rank. Choosing, arbitrarily, the first ring as the initial forming cluster, and its $Q(S^2{}_1)=Q(R_7)=10.13$.

By consulting Table 5.3, we find only ring R_8 is overlapping with the forming cluster $S^2{}_1$. Adding ring R_8 into $S^2{}_1$, and the value of structural tightness $Q^*(S^2{}_1)$, given by (5.2), is as

$$Q^*(S^2{}_1)=[q(R_7)+q(R_8)]/n=(30.93+30.93)/4=15.46>Q(S^2{}_1)=10.13$$

according to the criterion of cluster formation ring R_8 should add in to the forming cluster $S^2{}_1$. Now the forming cluster becomes $S^3{}_1$ and the value of structural tightness

$$Q(S^3{}_1)=Q^*(S^2{}_1)=15.46$$

Structural Rings and Their Ranks

Ring R_m	Joints	Structure	Graphical Pattern	String Pattern	$q(R_m)$ (10^{10})	$Q(R_m)$ (10^{10})	Rank
1	1-2-7-1	1 2 7		pspsps	5.91	1.97	4
2	1-6-7-1	1 6 7		pspsps	5.91	1.97	4
3	6-7-11-6	6 7 11		pspsps	5.91	1.97	4
4	6-10-11-6	6 10 11		pspsps	5.91	1.97	4
5	10-11-15-10	10 11 15		pspsps	5.91	1.97	4
6	10-14-15-10	10 14 15		pspsps	5.91	1.97	4
7	2-3-5-2	2 3 5		pspsps	30.93	10.13	1
8	3-4-5-3	3 5 4		pspsps	30.93	10.13	1
9	4-5-9-4	5 4 9		pspsps	20.25	6.75	3
10	8-9-13-8	8 9 13		pspsps	23.58	7.86	2
11	8-12-13-8	8 12 13		pspsps	23.58	7.86	2
12	8-9-12-8	8 12 9		pspsps	23.58	7.86	2
13	9-12-13-9	12 9 13		pspsps	23.58	7.86	2
14	12-13-17-12	12 13 17		pspsps	23.58	7.86	2
15	12-16-17-12	12 16 17		pspsps	23.58	7.86	2
16	12-13-16-12	12 16 13		pspsps	23.58	7.86	2
17	13-16-17-13	16 13 17		pspsps	23.58	7.86	2
18	8-11	8 11	#	cs	2.84 (10^{-5})	1.42 (10^{-5})	5
19	11-16	11 16	#	cs	2.84 (10^{-5})	1.42 (10^{-5})	5
20	14-18	14 18	#	cs	1.51 (10^{-5})	0.75 (10^{-5})	6
21	16-18	16 18	#	cs	1.51 (10^{-5})	0.75 (10^{-5})	6

TABLE 5.3

Then, similarly, ring R_9 is added into the forming cluster $S^3{}_1$, giving the value of structural tightness

$$Q^*(S^3{}_1) = [q(R_7) + q(R_8) + q(R_9)]/n = (30.93 + 30.93 + 20.25)/5 = 16.42 > Q(S^3{}_1)$$

At this stage no structural ring is overlapping with the forming cluster. The first cluster $S^4{}_1$ has thus been formed, and the value of structural tightness $Q(S^4{}_1) = Q^*(S^3{}_1) = 16.42$.

The whole process of forming clusters of the structure of Fig.5.6 is illustrated pictorially by Fig.5.7(a) to Fig.5.7(c). The $Q(S^l{}_i)$ values beside each diagram are there to indicate the structural tightness either the forming cluster currently being considered or the cluster just being formed. We can see how the structural tightness is increased when a structural cluster is building up.

The same procedures may be followed to form more clusters until all identified rings in Table 5.3 are an element in one cluster. At this stage the total set clusters together with their structural tightness valves are given in Table 5.4, which also indicates the order of cluster formation.

TABLE 5.4

Structural Clusters Table

Cluster $S^l{}_i$	Joint	Structural Tightness $Q(S^l{}_i)$ (10^{10})
1	2 3 5 4 9	16.42
2	8 9 12 13	23.85
3	12 13 16 17	23.85
4	1 2 6 7 11 10 15 14	4.43
5	8 11	1.42×10^{-5}
6	11 16	1.42×10^{-5}
7	14 18	0.76×10^{-5}
8	16 18	0.76×10^{-5}
G	Ground	∞

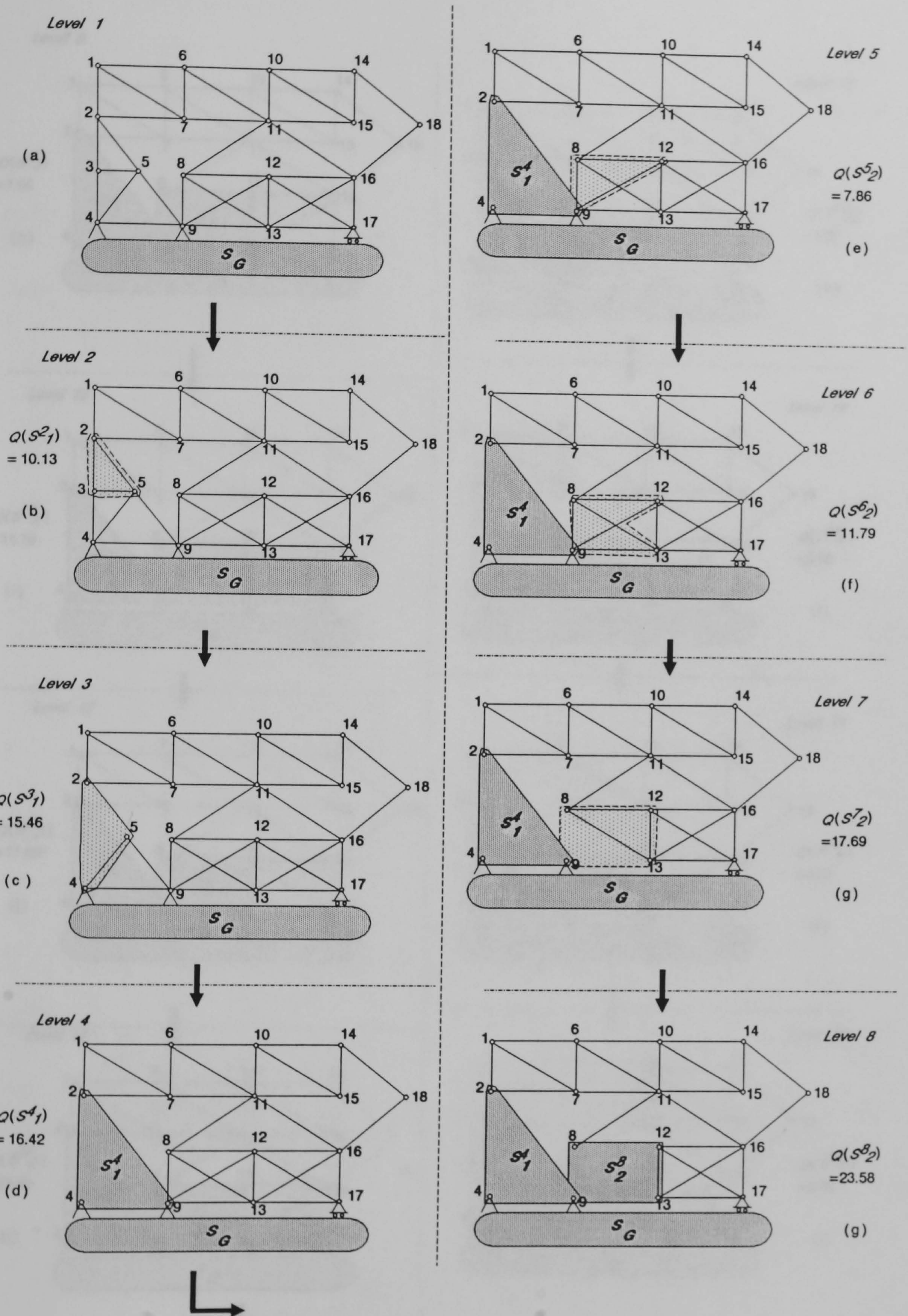


Fig. 5.7 (a) Cluster Formation

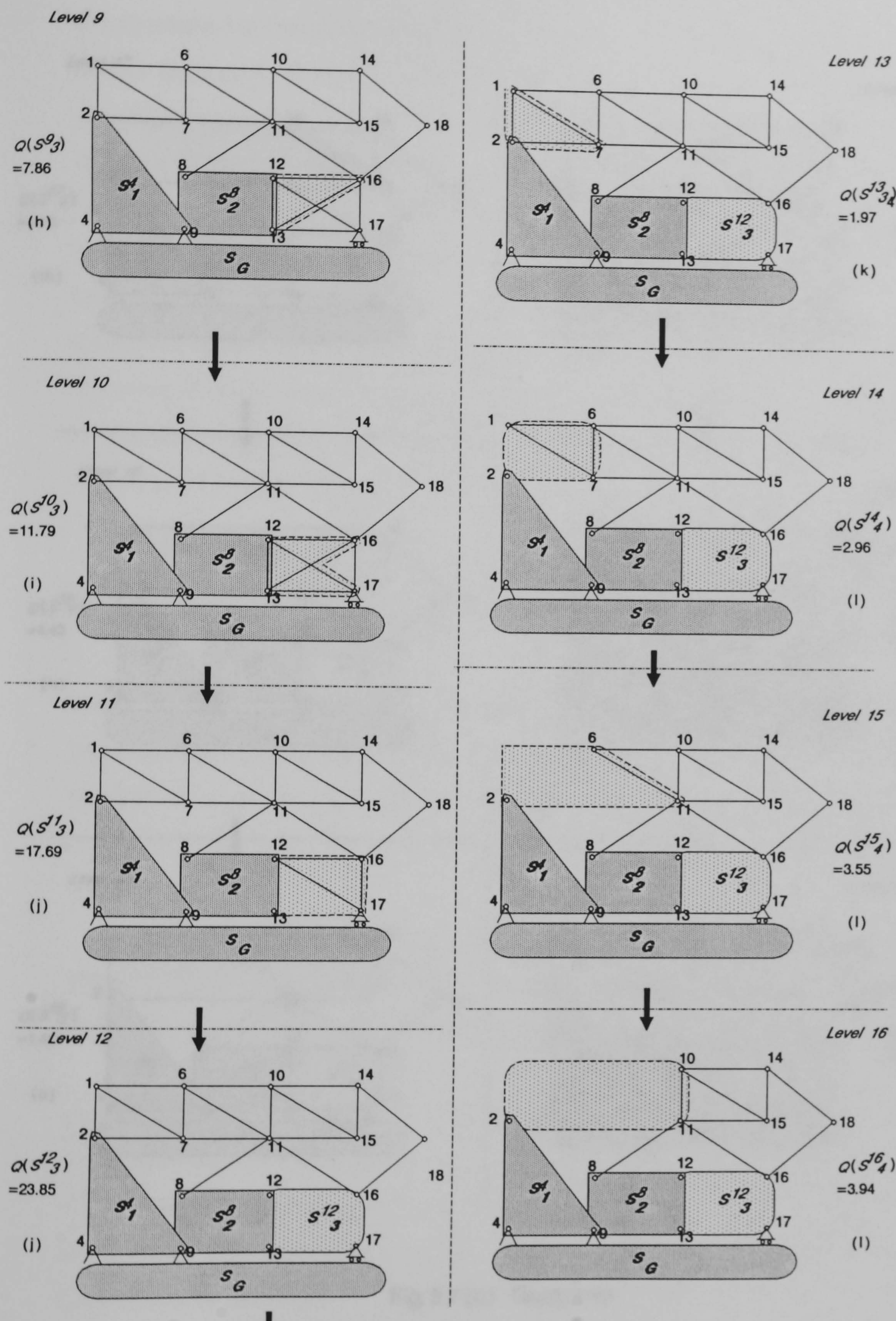


Fig. 5.7 (b) Continued

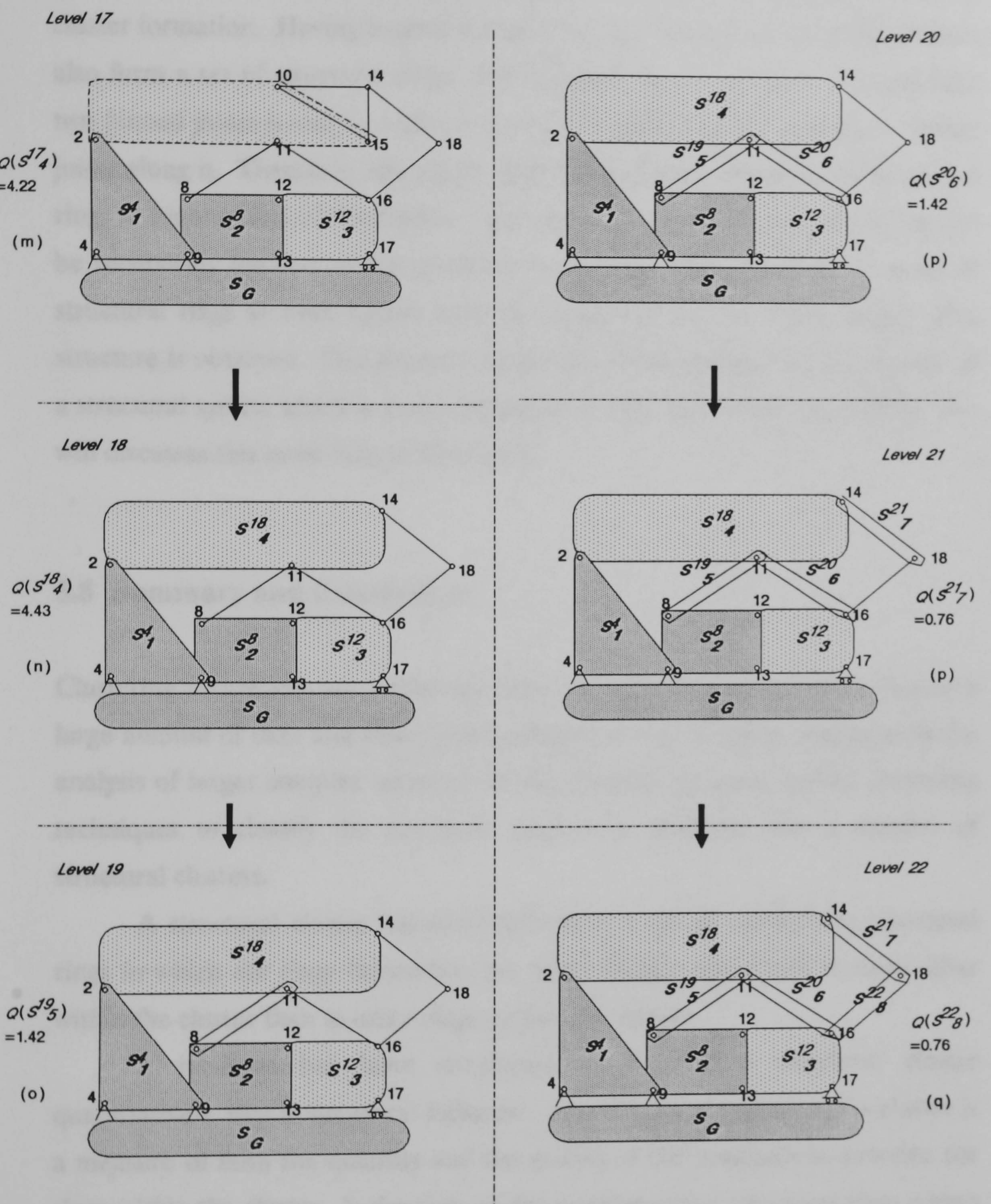


Fig. 5.7 (c) Continued

Comparing Fig.5.7(b) to Fig.5.6, we see that the original graph model of the structure has been transformed into a simpler model through the process of cluster formation. Having looked at Fig.5.7(c)-(q), these interconnected clusters also form a set of structural rings. For example, S_G, $S^8{}_2$, $S^{12}{}_3$, S_G is a ring with two pinned joints j_9 and j_{17}, whilst $S^8{}_2$, $S^{12}{}_3$, $S^{20}{}_6$, $S^{19}{}_5$ is a ring with three pinned joints along it. Therefore, this graph model still consists of a set interconnected rings at higher level of description. The same process of cluster formation can be recursively applied to this model to form even bigger clusters and a set of structural rings at even higher level of description until a single cluster of a structure is obtained. This process can produce a hierarchical representation of a structural system which is extremely useful for the structural vulnerability. We will discusses this more fully in Chapter 6.

5.8 Summary and Conclusions

Clustering techniques are concerned with the organization and reduction of a large amount of data and information, which are very useful in dealing with the analysis of larger complex systems. In this Chapter we have applied clustering techniques to classify the structural rings of a structure into a number of structural clusters.

A structural cluster has been defined as a set of overlapping structural rings in which the rings themselves are more densely connected to each other within the cluster than to other rings outside the cluster.

A measure has been developed to describe a structural cluster quantitatively, that is structural tightness. The structural tightness of a cluster is a measure of both the quantity and the quality of the connections between the rings within the cluster. It depends on the number of the structural rings within the cluster, the degree of overlap between them and the quality of well-formedness of the rings.

The process of cluster formation of a structure is, thus, based on the concept of a structural cluster and the measure of structural tightness. The criterion for cluster formation has been defined in such a way that an overlapping structural ring would be added to the forming cluster if the value of structural tightness is increased. By implementing the process of cluster formation, we can identify all structural clusters of a structure.

This process could apply recursively to a structure at each level of definition in which a number of structural clusters connect to form a set of structural rings until the whole structure becomes a single cluster. A structure then could be represented in a form of hierarchy which is very useful in the vulnerability analysis of the structure.

An example has been given in this chapter to illustrate the whole process of cluster formation.

Chapter 6

Hierarchical Representation of Structural Systems

6.1 Objectives

The objectives of this chapter are:

1. To discuss how a structure can be represented in a form of hierarchy;
2. To examine the systems concepts and their role in dealing with the analysis of complex systems;
3. To examine the different types of external connections between structural clusters at a level of definition, and;
4. To modify the clustering algorithm developed in Chapter 5;
5. To present an example.

6.2 Introduction

At the end of Chapter 5 the concept of the hierarchical representation of a structure as used in the structural vulnerability analysis was briefly mentioned. In this chapter we introduce the systems concepts and discuss how a structure can be represented in a form of hierarchy.

Hierarchy plays an important role in dealing with the analysis of complex systems. The organization of any complex systems is hierarchical (Alexander, 1964). It is argued that (Stone, 1989) it is desirable for a system to be structured hierarchically, since this gives a natural representation of a complex system and is an important factor in the ability of a knowledge based system to provide an appropriate degree of detail in response to analysis.

A structural system can also be described at various levels of definition. A structure is built up starting with a set of structural rings at the lowest level of description, grouping them into a number of small clusters at next higher level of description. These newly formed clusters connect to each other to form a set of new structural rings at that level of definition. The same process can recursively apply to the structural rings at various levels of definition, and such a process leads to the hierarchical representation of a structure in terms of a set of interconnected structural rings.

This chapter brings together the OOGM from Chapter 2, the structural ring model from Chapter 3, the quality of well-formedness of structural rings from Chapter 4 and the structural cluster formation from Chapter 5, together with the systems concepts to be introduced in this chapter, for forming the basis of a hierarchical representation of a structure. The aim is to transform a structure into a hierarchical model with various levels of description in terms of interconnected rings. This hierarchical model provides a basis for structural vulnerability analysis.

6.3 External Connectivity Between Structural Clusters

6.3.1 Structured Graphs

A graph $G=(N, L)$ is said to be structured at a level of definition l if any of its elements can be considered as the subgraphs of G, such elements are called macro-elements. The OOGM of a structural system $S=(J, M)$ may possibly be represented by its corresponding structured graph at a level of definition l, and the elements of which are structural clusters S^l_i of S.

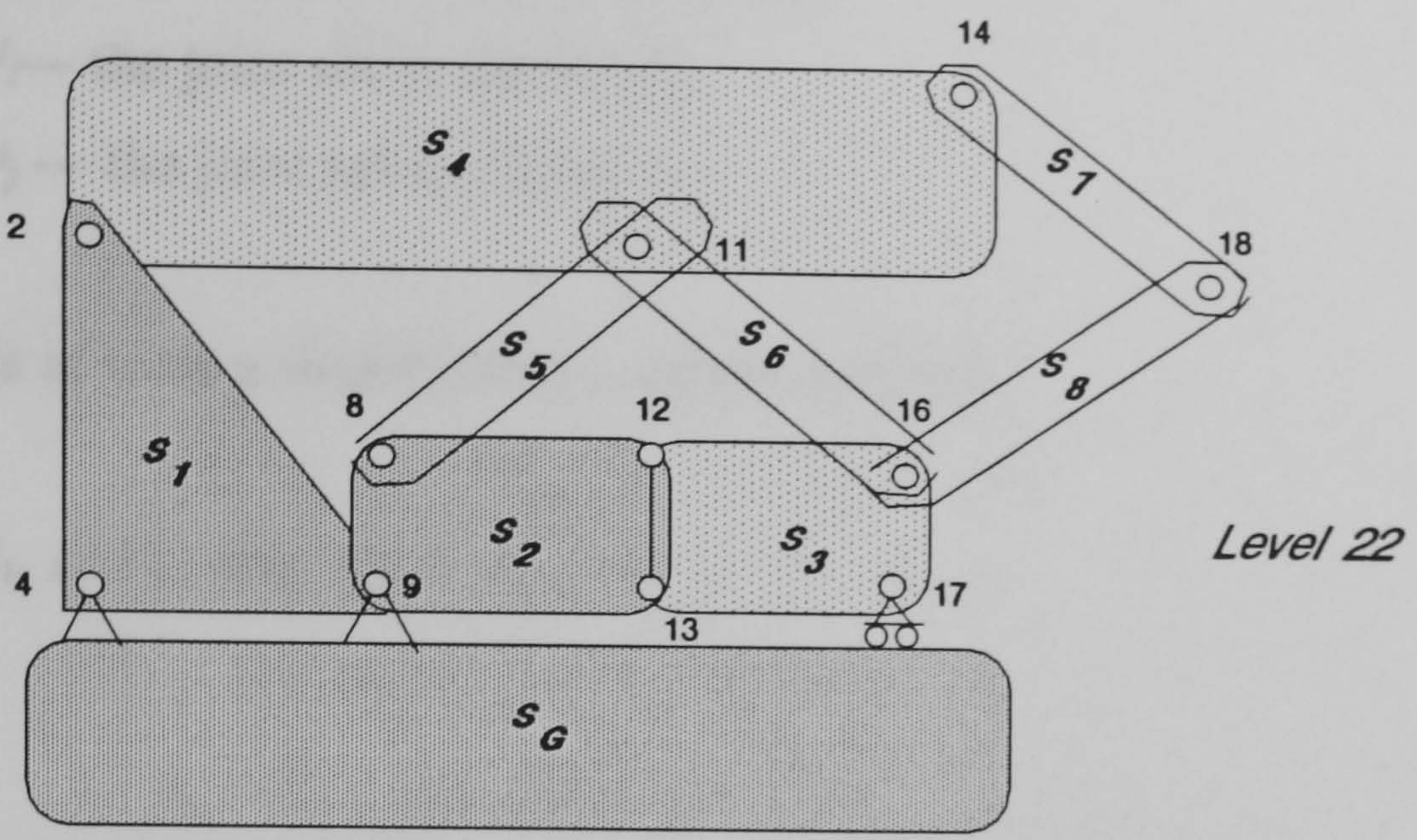


Fig. 6.1 A structured graph at a level of definition

Fig.6.1 is one of the structured graphs at level 22 in the process of cluster formation of the structure of Fig.5.6, and its elements are structural clusters at the corresponding level. It is obvious, from Fig.5.7(a) to Fig.5.7(b), that a structured graph at a higher level of description is a simpler model comparing to its original OOGM.

6.3.2 Connected Structural Clusters

Assume that S^l_i and S^l_j are two structural clusters at a level of definition l,

$$S^l_i=(M^l_i,J^l_i),\quad S^l_j=(M^l_j,J^l_j) \tag{6.1}$$

where

M^l_i--- the member set in cluster S^l_i

M^l_j--- the member set in cluster S^l_j

J^l_i--- the joint set in cluster S^l_i

J^l_j --- the joint set in cluster S^l_j

if there is at least a single joint j^l_k, which satisfies:

$$j^l_k \in J^l_i \text{ and } j^l_k \in J^l_j \tag{6.2}$$

then structural clusters S^l_i and S^l_j are said to be *connected*.

For example, in the structured graph of Fig.6.1, the structural clusters S_1 and S_2 are connected clusters.

6.3.3 Complex Joints

A *complex joint* is the intersection of any two connected structural clusters, that is,

$$j^l_C = S^l_i \cap S^l_j \tag{6.3}$$

According to the definition of connected clusters, a complex joint may be either

(1). a single joint, shown in Fig.6.2(a);
(2). a set of joints not directly connected but indirectly connected through the clusters which form the intersection, shown in Fig.6.2(b);
(3). a primitive cluster, i.e. a single member, shown in Fig.6.2(c). Note that the

member in this case must belong to both clusters;

(4). mixture of any above cases, shown in Fig.6.2(d).

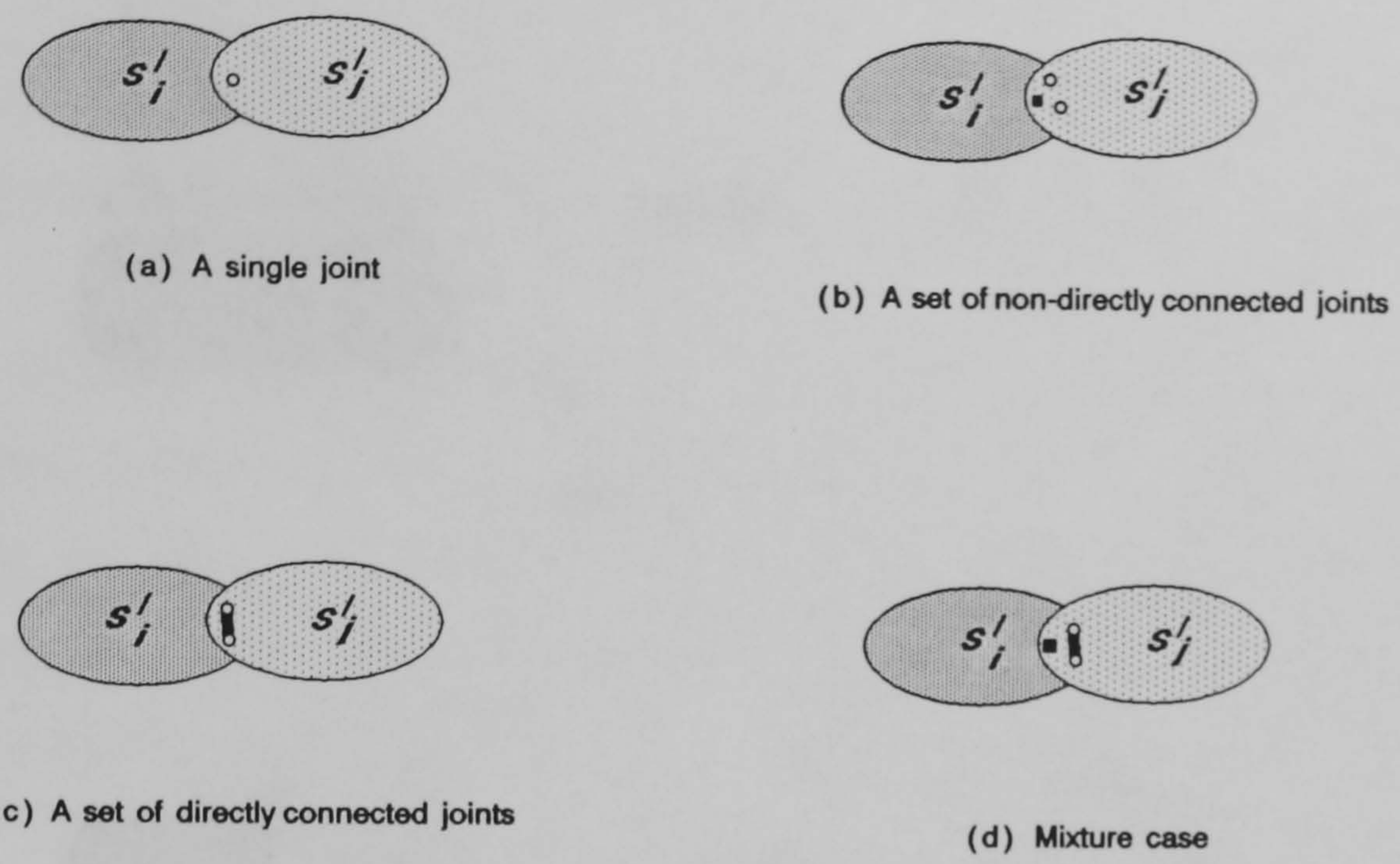


Fig. 6.2 Examples of complex joints

6.3.4 Structural Rings at a Level of Definition

For a given structured graph at level of definition l, there exist many structural loops and structural rings. The elements of these loops and rings are clusters and complex joints. At each level of description, only structural rings are of interest as far as building higher level clusters is concerned.

For example, in the structured graph of Fig.6.1, S_1, j_4, S_G, j_9, S_1 is a ring, shown in

Fig.6.3(a). Obviously, there are many other structural rings at this level.

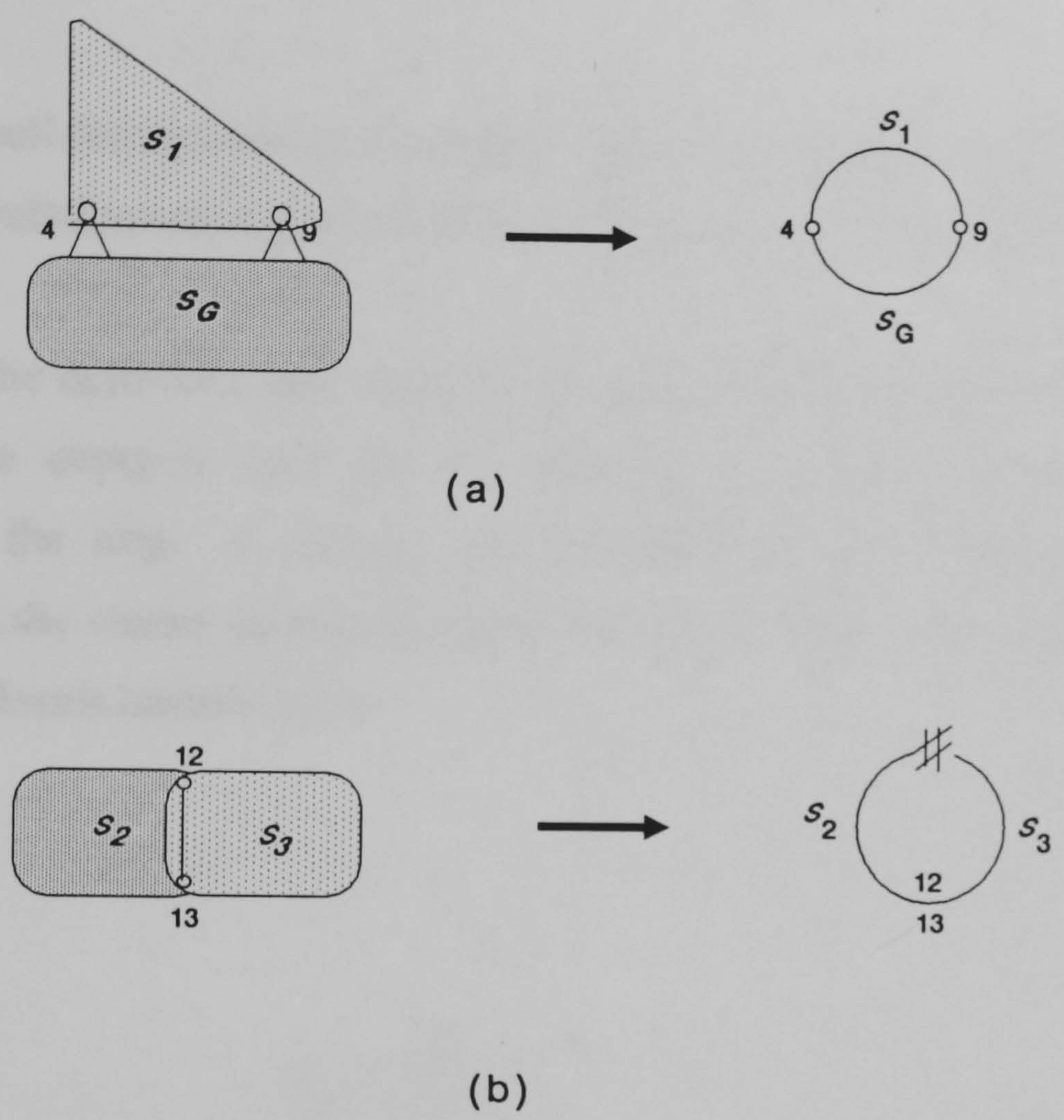


Fig. 6.3 Connected Clusters and Their Ring

It should be noted that if the complex joint between two connected clusters is a primitive cluster, there are at least two single joints belonging to both of the clusters, in this case, these two clusters can be considered as a continuous elements, the corresponding structural ring pattern is a ring with a cut along it. For instance, the complex joint between clusters S_2 and S_3 of Fig.6.1 is a primitive cluster, the structural ring of which can be denoted by a ring with a cut along it, shown in Fig.6.3(b).

The quality of well-formedness of a structural rings at a level of definition can similarly computed according to (4.20) and (4.21), that is, only those single joints contained in the ring are to be considered.

For example,

(a) the well-formedness of the ring of Fig.6.3(a) is equal to $q(R^l) = q_4 + q_9$;
(b) the well-formedness of the ring of Fig.6.3(b) is equal to $q(R^l) = q_{12} + q_{13}$.

The definition used here is that the well-formedness of a ring at a level of definition depends only on the external connections between the clusters forming the ring. A cluster only becomes an arc of the ring at that level although the cluster itself could possibly consist of a set of interconnected rings at lower levels of definition.

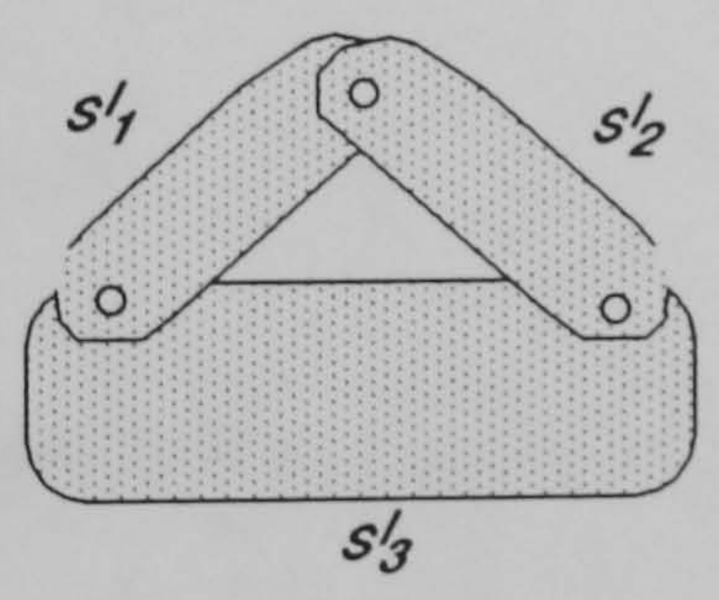


Fig 6.4 A structural ring at a level of definition

Take ring R^l at level of definition of Fig.6.4 as an example, assuming that three clusters S^l_1, S^l_2 and S^l_3 are all tightly self connected and very well formed. Three clusters are connected together by three pinned joints to form structural ring. From Chapter 3, we know that this ring is a just-stiff ring and only if one of the degrees of freedom is released adjacent to any one of the three joints the ring becomes a mechanism. In the design process of a structure, therefore, merely to increase the connectivity and stiffness of the clusters will not improve the quality of the well-formedness of structural rings. The structural vulnerability analysis is to examine not only the structural tightness of clusters making up a ring R^l at level of definition, but also, as a whole, the quality of the well-formedness of the ring. That is a very important concept in the sense of structural vulnerability. We will see in Chapter 7 that the robustness of a structure is dependent on the quality of the well-formedness of structural rings within a structure at various levels of definition.

6.3.5 Relative Tightness

Assume that a structural ring at level of definition R^l consists of a number of clusters

$$R^l = \{S^l_i \mid i = 1,\ldots,k\}$$

then the relative tightness of a cluster S^l_i is defined as

$$\eta(S^l_i) = Q(S^l_i) \; / \; \Sigma\, Q(S^l_i) \quad (i = 1,\ldots,k) \tag{6.4}$$

where $Q(S^l_i)$ is the structural tightness of a cluster S^l_i and k is the total number of clusters in R^l.

For example, the relative tightness of cluster S^l_1 in the ring R^l of Fig.6.4 is equal to

$$\eta(S^l_1) = Q(S^l_1) \; / \; \Sigma\, Q(S^l_i) \qquad (i = 1,\ldots,3)$$

This relative measure $\eta(S^l_i)$ reflects the structural quality (connectivity and well-formedness) of a cluster with reference to other clusters in a ring.

From the structural vulnerability analysis point of view, the cluster with minimal value of relative tightness in a ring R^l is the most vulnerable part in that ring.

6.4 Systems Concepts

The systems view of the world is an holistic one where the whole is considered to be more than just sum of the parts(Comerford, 1989). The system has characteristics and exhibits behaviours which are due to its totality. This is in contrast to traditional scientific method which seeks to explain the structure and behaviour of the whole by examining the nature of its constituent and parts.

A system can be considered as a structure consisting of many levels of organisation. At each level there exists attributes and types of behaviour which do not exist at any other levels. Specific problems and phenomena emerge at each level of organization which are peculiar to that level. Consequently, there are multiple descriptions, models of the structure with each level having its own features and characteristics. Hence we may be able to understand and explain the system and behaviour of the system in many different ways all equally valid but of different levels of detail or complexity.

The organization of any complex systems is hierarchical (Alexander, 1964). The concept of hierarchy is one of a structure with discrete but interacting levels. It implies that a body or an organisation is classified in successively subordinate grades. Hierarchy can be used to represent the fact that systems can be ordered according to various criteria. The levels of hierarchy are subordinated to each other on defined criterion.

Viewing a system as a hierarchy is useful in describing its nature and explaining its behaviour. Hierarchical structures are generally characterised by repeated classification and neglect of details at successive transitions to high levels of the hierarchy.

Each entity in a hierarchy can be considered as either a part or a whole. It is a part with respect to entities above it in the hierarchy but a whole with respect to entities below it. Koestler (1968) referred to any such members of a hierarchy as a 'holon', which has already been mentioned in Chapter 2.

Hierarchy implies a framework that permits complex systems to be built from simpler ones. In turn, the existence of a hierarchy allows complex systems to be broken up into their component parts and subsystems. Hierarchy helps us to organize, to understand, to communicate, and to learn about complexity of systems.

6.5 Description of a Structure at Various Levels of Definition

In this section we apply the systems concepts discussed above to a structural system and the structure of Fig.6.5 will be used as an example.

At the lowest level of description the structure, $S=(M, J)$, is composed of many connected member objects and joint objects, where the member set $M=\{m_i \mid i=1,...,8\}$ and the joint set $J=\{j_k \mid k=1,...,7\}$. From Chapter 3 we know that these members and joints connect to each other to form a set of structural rings. Thus, at the next higher level of description--level 2, the structure is considered as a system consisting of a set of structural rings, one of which is

$$R^2=(S^2_1, j^2_2, S^2_2, j^2_3\ S^2_3, j^2_5, S^2_4, j^2_6)$$

where S^2_i $(i=1,...,4)$ are all primitive clusters, shown in Fig.6.5(b).

If the ring R^2 is replaced by a single structural cluster then Fig.6.5(b) becomes a new structured graph at level 3, shown in Fig.6.5(c).

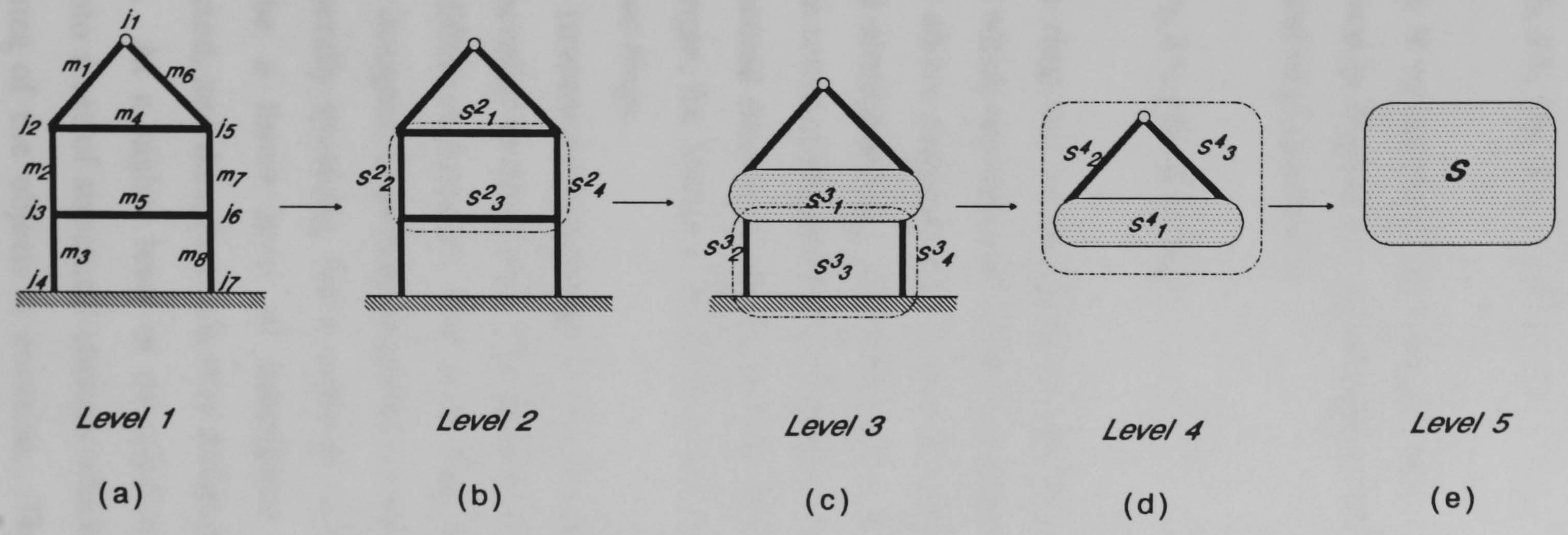


Fig6.5 Cluster formation

Having looked at this structured graph model of Fig.6.5(c), we see that this newly-formed cluster, together with other primitives clusters, forms two structural rings. Denote one of them as

$$R^3 = (S^3{}_1, j^3{}_3, S^3{}_2, j^3{}_4, \; S^3{}_3, j^3{}_7, S^3{}_4, j^3{}_6)$$

and replace it by another single cluster and we get a new structured graph at level 4, shown in Fig.5.6(d). At this stage the structured graph consists of only one structural ring, denoted as

$$R^4 = (S^4{}_1, j^4{}_1, S^4{}_2, j^4{}_2, \; S^4{}_3, j^4{}_5)$$

This ring can be replaced by a single cluster at the highest level of description which represents the whole structure.

The above example has illustrated that, with the concepts of structural cluster and structural ring, a structure can be described at various levels of definition in terms of interconnected structural ring. At each level there exists a set of connected structural rings which do not exist at any other levels. In the above example, for instance, at level 4 there exist one ring whereas at level 3 there are two rings.

The structural rings at each level have their own structural characteristics which are peculiar to that level. The quality of well-formedness of the ring R^4 is obviously different from R^3, and each ring at a level of definition within a structure is designed to perform a particular desired function.

Generally speaking, for a complex structure, the number of structural rings at the a lower level of description is large and they are highly interconnected, and therefore it is very difficult to recognise the organisation of the system. At a higher level of description, the structural ring objects are classified into a set of structural clusters which indicates more general features and attributes of the objects it contains. The detail information about ring objects in a cluster is hidden inside the cluster. These connected clusters form a set of new structural rings. But at this stage the number of rings and the interaction between them become much smaller and simpler compared with those at lower levels of description.

With the concepts of hierarchy, a structural cluster at level of definition itself is a complete system which consists of a set of rings at lower levels of definition, meanwhile it is a part of a structural ring at higher level of definition. Therefore, how we define a cluster really depends on at which level we look at it. There is no immediate difference between a huge structural cluster and a single member if they are observed at different levels of description. At very high level of definition a complex structural cluster can still be treated as a single object.

At the lowest levels of description all structural clusters are primitive clusters i.e. single members. At the highest level of the hierarchy there is only one cluster which represents the whole structure being studied.

Summarising the discussion so far, some conclusions can be drawn:

(1) A structural system can be represented by successively subordinate structural rings at each level of definition.

(2) A structural ring R^l at the level of definition l represents a substructural system.

(3) Given a structural ring R^l in the hierarchy, any of its arcs can be regarded as the condensation of a set of structural rings at lower levels of definition and itself can be an (or part of) arc of a structural ring at a higher level of definition.

(4) Structural rings at lower levels of the hierarchy are a more detailed description of a structure than those at higher levels of definition.

6.6 Hierarchy Formation

In Chapter 4, it was stated that the quality of the well-formedness of a structural ring is a measure of its ability to resist damage from any arbitrary direction and to perform its desired function. A well formed structural ring is more robust than a badly formed ring. From the Section 6.5 it has been shown that a structure can be described at various levels of definition in terms of interconnected structural rings. The robustness of a structural system is, thus, dependant on the quality of the well-formedness of rings at various levels of definition within a structure..

For a ring the robustness is the same as the quality of well-formedness and for a structure it is the structural ring with worst quality of well-formedness over all levels of definition i.e. there is one level of definition which the structural ring is the weakest.

In order to find the measure of robustness for a structure, it is desirable that a clustering algorithm should be designed in such a way that it can identify the structural rings at various levels of definition. The purpose of this clustering algorithm is to find a ring at a given level of definition which has the best quality of the well-formedness.

Basing on the cluster algorithm developed in Chapter 5, some modification about that algorithm will be made in this section as follows:

At a given level of definition identify all structural rings. Calculate the structural tightness $Q(R^l_m)$ for each ring according to (5.3). Choose the ring with the maximal value of $Q(R^l_m)$. Replace it by a single cluster S^{l+1}_j. Go to next hierarchical level of definition $l=l+1$. Then the structural quality of this cluster is defined as

$$Q(S^{l+1}_j)=Q(R^l_m) \tag{6.5}$$

Repeat the process until the whole structure becomes a single union cluster.

This new clustering algorithm is described as follows

(1) At level l of build-up.
(2) Implement **SUB RINGS** to identify all structural rings. If no structural ring can be found then go to step (9), otherwise
(3) Implement **SUB DET** to calculate $Q(R^l_m)$ for each ring.
(4) Rank rings according to $Q(R^l_m)$.
(5) Replace highest ranked ring by a single cluster.
(6) Let $S^{l+1}_j = \{R^l_m\}$ and $Q(S^{l+1}_j) = Q(R^l_m)$
(7) Go to next hierarchical level of definition $l = l + 1$.
(8) Go to step (1).
(9) Stop.

The process of cluster formation produces a hierarchical model of a structure which is particularly useful in the identification of failure scenarios. The structure to be analyzed is modelled as a hierarchical set of structural rings. This hierarchical model of a structure $S = (M, J)$ can therefore be described as

$$S = \{ R^l \} \tag{6.6}$$

where R^l, $l = 1,2,...,q_s$ is a structural ring at the level of definition l and there are q_s levels in the hierarchy.

A structural ring R^l represents a substructure of S. It consists of a number of joints and arcs. Each arc itself may be self contained sub-structure of R^l, i.e. structural cluster, denoted by S^l_i. The structural ring R^l thus can be represented by .

$$R^l = \{\, j^l_i, S^l_i \mid i=1,\dots,n \} \tag{6.7}$$

where j^l_i is a joint and n is the total number of joints in the ring R^l.

The structural cluster S^l_i again consists of a sequence of structural rings at lower levels of definition. The cluster S^l_i may also be described as a hierarchical set of structural rings

$$S^l_i = \{ R^{q_i} \} \quad (q_i < l) \tag{6.8}$$

and

$$R^{q_i} = \{\, j^{q_i}_x , S^{q_i}_x \mid x=1,\dots, n_i) \tag{6.9}$$

where n_i is the total number of joints in the ring R^{q_i} and q_i is the next lowest level of definition of S^l_i.

The structure of Fig.6.5 can then be represented in a form of hierarchy, shown in Fig.6.6

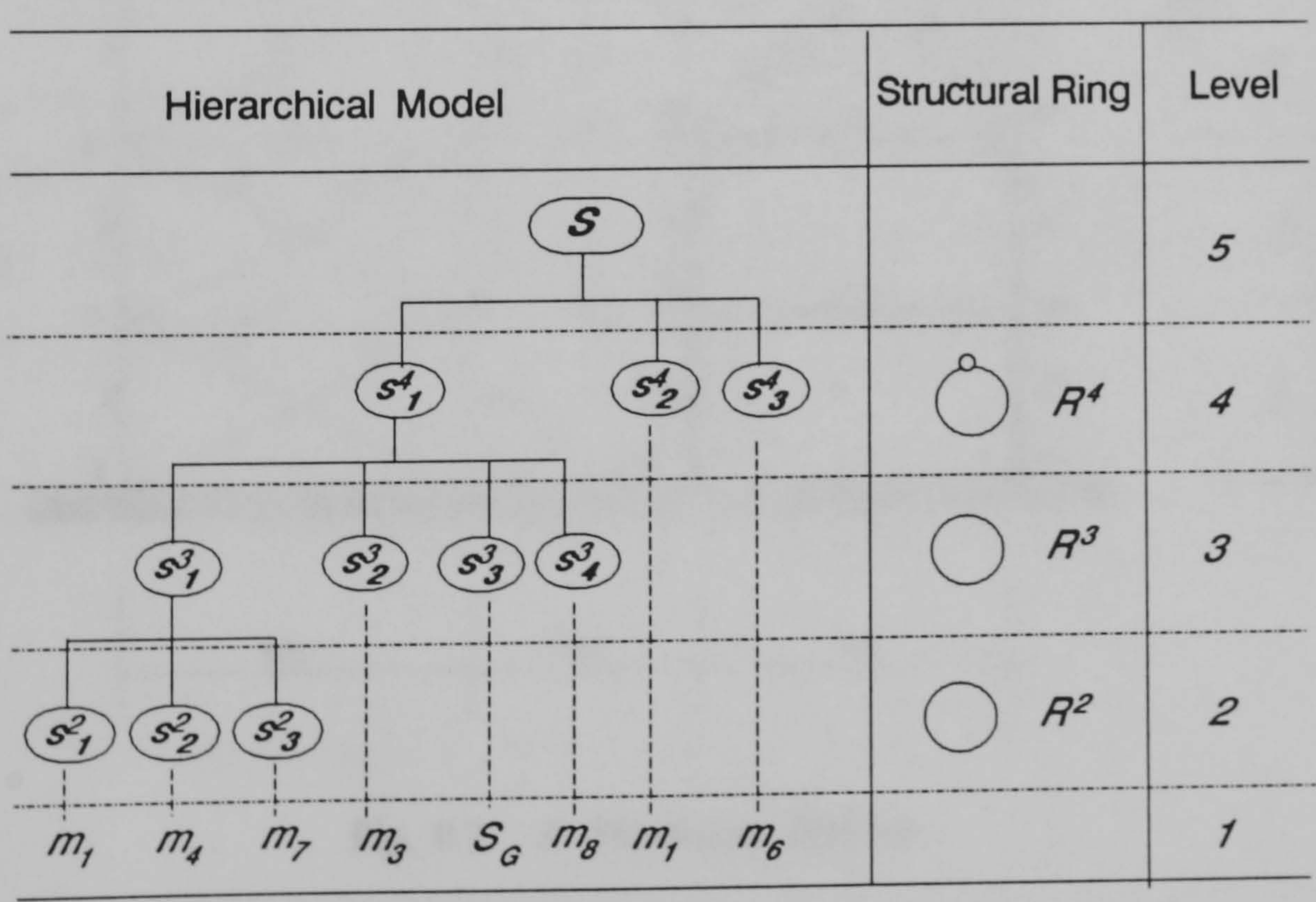


Fig. 6.6 Hierarchical Model of A Structure

6.7 Example

In this section we present another example to illustrate the clustering algorithm developed in this chapter by forming structural clusters for the structure of Fig.6.7. This has already been used to illustrate the process of structural ring identification in Section 3.7.

The structure of Fig.6.7 is a two dimensional framed structure with pinned and fixed joints as shown. The members are denoted by italic numbers. By implementing the subroutine **DATAIN**, the detailed information about the sizes and properties of the structure can be obtained which are listed form Table 6.1 to Table 6.3.

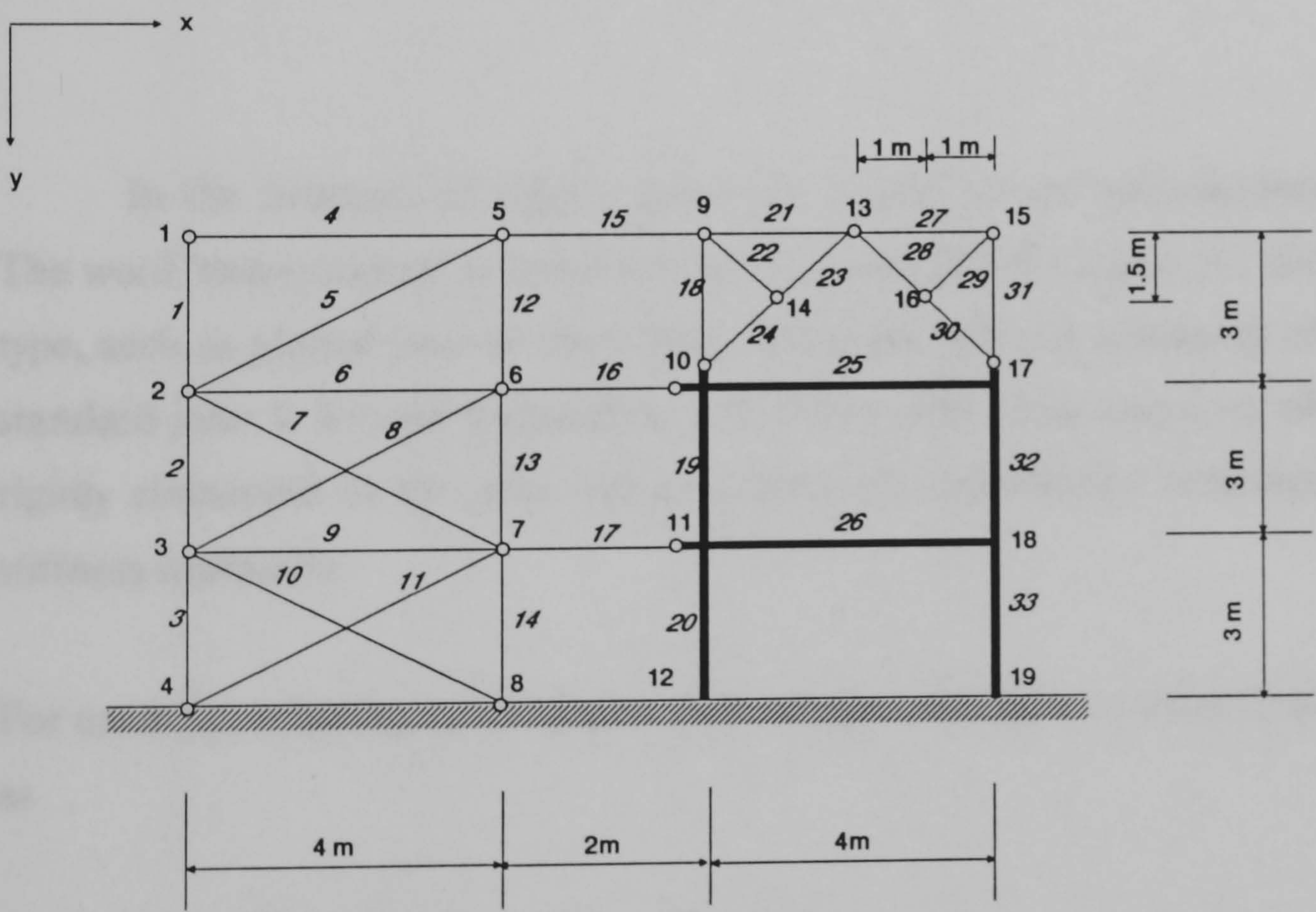


Fig. 6.7 A Structural System

TABLE 6.1

Joint co-ordinate table

Joint No.	X Co-od. (m)	Y Co-od. (m)	Joint No.	X Co-od. (m)	Y Co-od (m)
1	0.00	0.00	2	0.00	3.00
3	0.00	6.00	4	0.00	9.00
5	4.00	0.00	6	4.00	3.00
7	4.00	6.00	8	4.00	9.00
9	6.00	0.00	10	6.00	3.00
11	6.00	6.00	12	6.00	9.00
13	8.00	0.00	14	7.00	1.50
15	10.00	0.00	16	9.00	1.50
17	10.00	3.00	18	10.00	6.00
19	10.00	9.00			

In the structure of Fig.6.7, joints 10, 11 and 17 are non-standard joints. The word 'non-standard' is intended here to mean that the joints are not of one type, such as pinned joint or fixed joint. When the stiffness submatrix of a non-standard joint is formed it should be noted that only those members which are rigidly connected to the joint will contribute the calculated θ rotations to the stiffness submatrix.

For example, referring to Chapter 4, the stiffness submatrix of joint 11 is formed as

$$\mathbf{D}_{11} = [\, \mathbf{k}_{11}^{17} + \mathbf{k}_{11}^{19} + \mathbf{k}_{11}^{20} + \mathbf{k}_{11}^{26} \,]$$

whereas $\mathbf{k}_{11}^{19}$, $\mathbf{k}_{11}^{20}$, $\mathbf{k}_{11}^{26}$ are all 6x6 matrices but $\mathbf{k}_{11}^{17}$ is a 4x4 matrix in which the items related to the rotation displacement θ are zero.

TABLE 6.2

Member end condition and joint table

Member No.	Joint A	String Pattern	Joint B	String Pattern
1	1	*p*	2	*p*
2	2	*p*	3	*p*
3	3	*p*	4	*p*
4	1	*p*	5	*p*
5	2	*p*	5	*p*
6	2	*p*	6	*p*
7	2	*p*	7	*p*
8	3	*p*	6	*p*
9	3	*p*	7	*p*
10	3	*p*	8	*p*
11	4	*p*	7	*p*
12	5	*p*	6	*p*
13	6	*p*	7	*p*
14	7	*p*	8	*p*
15	5	*p*	9	*p*
16	6	*p*	10	*p*
17	7	*p*	11	*p*
18	9	*p*	10	*p*
19	10	*f*	11	*f*
20	11	*f*	12	*f*
21	9	*p*	13	*p*
22	9	*p*	14	*p*
23	14	*p*	13	*p*
24	10	*p*	14	*p*
25	10	*f*	17	*f*
26	11	*f*	18	*f*
27	13	*p*	15	*p*
28	13	*p*	16	*p*
29	16	*p*	15	*p*
30	16	*p*	17	*p*
31	15	*p*	17	*p*
32	17	*f*	18	*f*
33	18	*f*	19	*f*

TABLE 6.3

Member properties table

Member No.	E. Value (KN/m^2)	Area(A) (m^2)	Inertia(I) (m^3)
1	2.1×10^8	3.23×10^{-3}	3.749×10^{-5}
2	2.1×10^8	4.2×10^{-3}	4.856×10^{-5}
3	2.1×10^8	4.2×10^{-3}	4.856×10^{-5}
4	2.1×10^8	3.23×10^{-3}	3.749×10^{-5}
5	2.1×10^8	3.23×10^{-3}	3.749×10^{-5}
6	2.1×10^8	4.2×10^{-3}	4.856×10^{-5}
7	2.1×10^8	4.2×10^{-3}	4.856×10^{-5}
8	2.1×10^8	4.2×10^{-3}	4.856×10^{-5}
9	2.1×10^8	4.2×10^{-3}	4.856×10^{-5}
10	2.1×10^8	4.2×10^{-3}	4.856×10^{-5}
11	2.1×10^8	4.2×10^{-3}	4.856×10^{-5}
12	2.1×10^8	3.23×10^{-3}	3.749×10^{-5}
13	2.1×10^8	4.2×10^{-3}	4.856×10^{-5}
14	2.1×10^8	4.2×10^{-3}	4.856×10^{-5}
15	2.1×10^8	3.23×10^{-3}	3.749×10^{-5}
16	2.1×10^8	3.23×10^{-3}	3.749×10^{-5}
17	2.1×10^8	3.23×10^{-3}	3.749×10^{-5}
18	2.1×10^8	3.23×10^{-3}	3.749×10^{-5}
19	2.1×10^8	4.74×10^{-3}	5.544×10^{-5}
20	2.1×10^8	4.74×10^{-3}	5.544×10^{-5}
21	2.1×10^8	3.23×10^{-3}	3.749×10^{-5}
22	2.1×10^8	3.23×10^{-3}	3.749×10^{-5}
23	2.1×10^8	3.23×10^{-3}	3.749×10^{-5}
24	2.1×10^8	3.23×10^{-3}	3.749×10^{-5}
25	2.1×10^8	6.83×10^{-3}	11.686×10^{-5}
26	2.1×10^8	6.83×10^{-3}	11.686×10^{-5}
27	2.1×10^8	3.23×10^{-3}	3.749×10^{-5}
28	2.1×10^8	3.23×10^{-3}	3.749×10^{-5}
29	2.1×10^8	3.23×10^{-3}	3.749×10^{-5}
30	2.1×10^8	3.23×10^{-3}	3.749×10^{-5}
31	2.1×10^8	3.23×10^{-3}	3.749×10^{-5}
32	2.1×10^8	4.74×10^{-3}	5.544×10^{-5}
33	2.1×10^8	4.74×10^{-3}	5.544×10^{-5}

Then we start the process of cluster formation and assume that this is the level *1* in the hierarchy.

As before the ground or foundation of the structure is considered as a single cluster very tightly and densely self-connected and very well formed. Thus, in the process of cluster formation, the ground cluster S_G comes up first, as shown in Fig.6.8(a)-(a).

Implement the subroutine **RINGS** to identify all primitive rings in the system and use the subroutine **DET** to calculate the quality of well-formedness $q(R_m)$ for each ring and then rank them according to value structural tightness $Q(R_m)$. The results are shown in Table 6.4.

The rest of the procedure of cluster formation is straight forward. To initiate the first cluster, we consult Table 6.4 and choose the highest ranked ring R_{14}, and its $Q(R_{14})=4.85\times10^5$. Replace it by a single cluster $S^2{}_1$ and a new structured graph is generated, shown in Fig.6.8(a)-(b). Repeat the above process to identify a structural ring with the best quality of the well-formedness at this level. Again replace it by a single cluster, and the structured graph at level 3 is shown in Fig.6.8(a)-(c).

The whole process of the cluster formation is illustrated pictorially by Fig.6.8(a) to Fig.6.8(c). The Q values beside each diagram are there to indicate the structural tightness of either the forming cluster currently being considered or the cluster just being formed. We can see how the structural tightness is increased as a structural cluster builds up

Comparing to the clustering algorithm of Chapter 5, we can see that this modified algorithm is simpler. It can also tell us which structural ring has the best quality of the well-formedness at each level of hierarchy. We will see in the next chapter that the robustness of a structure depends on the quality of the well-formedness of structural rings at various hierarchical levels of the structure.

Structural Rings and Their Ranks

Ring R_m	Joints	Structure	Graphical Pattern	String Pattern	$q(R_m)$ (x 10^{10})	$Q(R_m)$ (x 10^{10})	Rank
1	1-2-5-1			pspsps	6.62	2.21	7
2	2-5-6-2			pspsps	8.02	2.67	6
3	2-3-6-2			pspsps	11.20	3.73	5
4	2-3-7-2			pspsps	11.20	3.73	5
5	2-6-7-2			pspsps	11.20	3.73	5
6	3-6-7-3			pspsps	11.20	3.73	5
7	3-4-7-3			pspsps	11.20	3.73	5
8	3-7-8-3			pspsps	11.20	3.73	5
9	9-10-14-9			pspsps	17.29	5.77	4
10	9-13-14-9			pspsps	29.73	9.91	3
11	13-15-16-13			pspsps	29.73	9.91	3
12	15-16-17-15			pspsps	17.29	5.77	4
13	12-11-18-19			s	9.70(10^5)	2.43(10^5)	2
14	10-11-18-17-10			s	19.40(10^5)	4.85 (10^5)	1
15	5-9-5		#	cs	3.39(10^{-5})	1.70(10^{-5})	8
16	6-10-6		#	cs	3.39(10^{-5})	1.70(10^{-5})	8
17	7-11-7		#	cs	3.39(10^{-5})	1.70(10^{-5})	8

TABLE 6.4

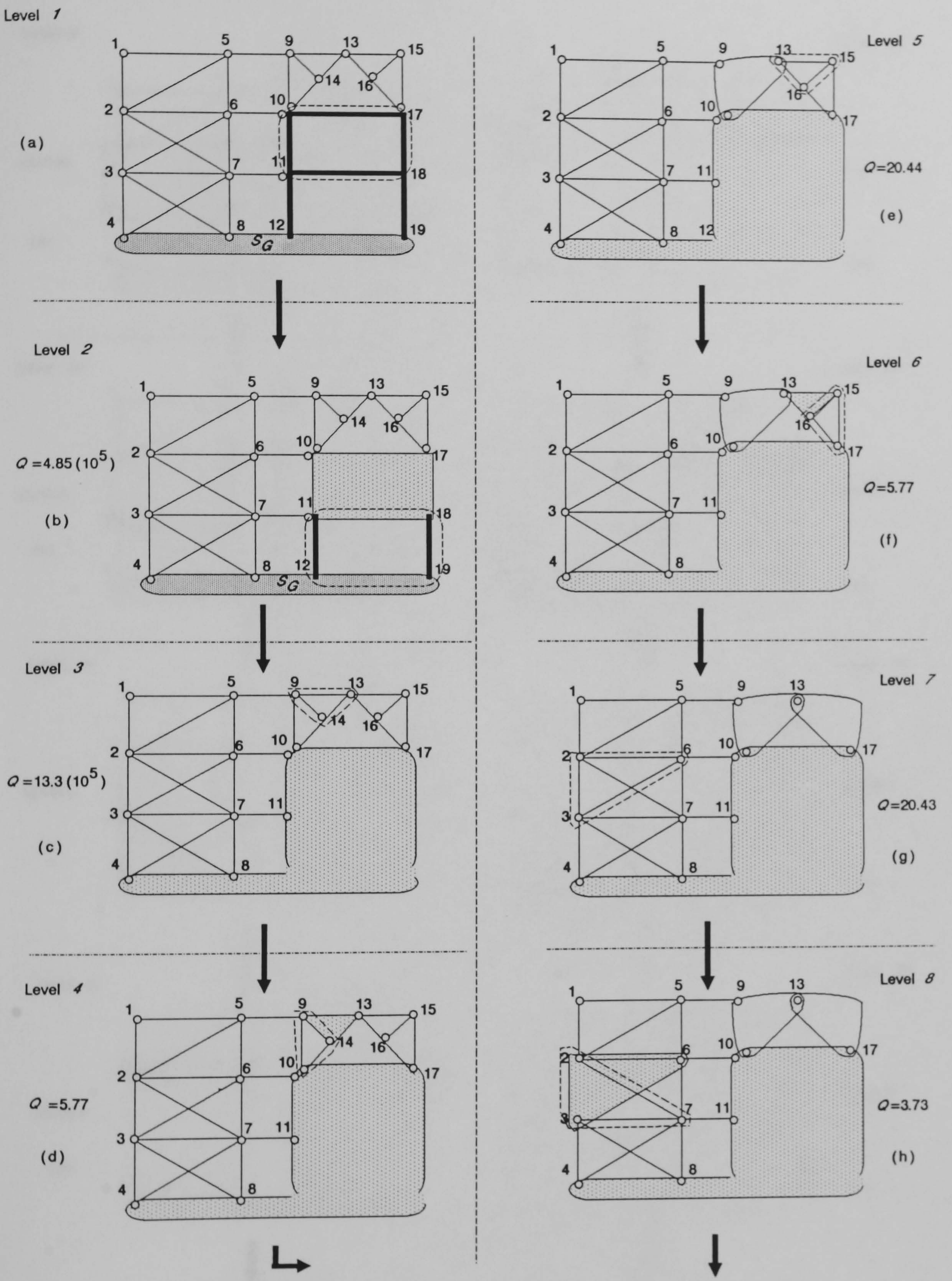


Fig. 6.8 (a) Cluster Formation

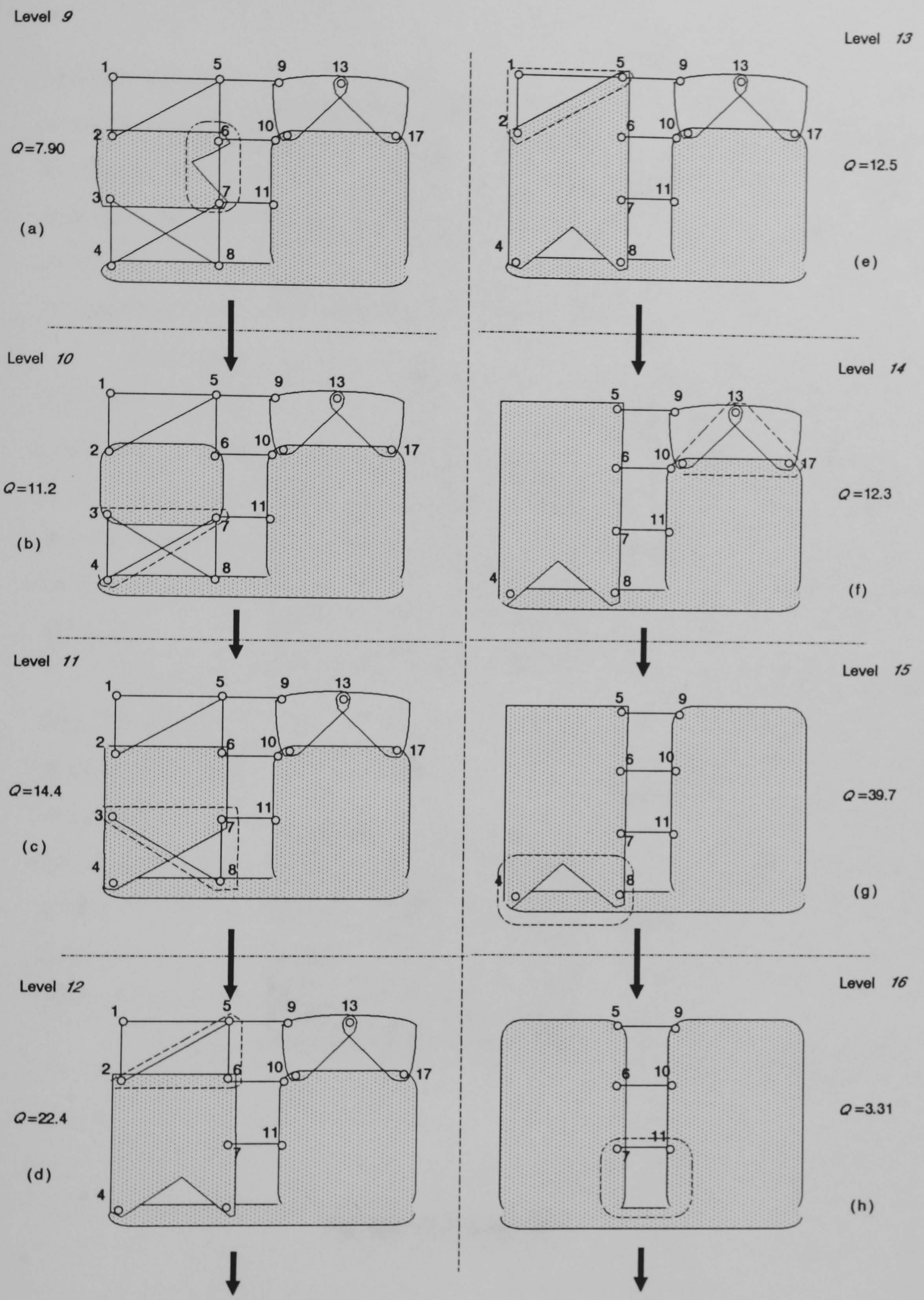


Fig. 6.8 (b) (Continued)

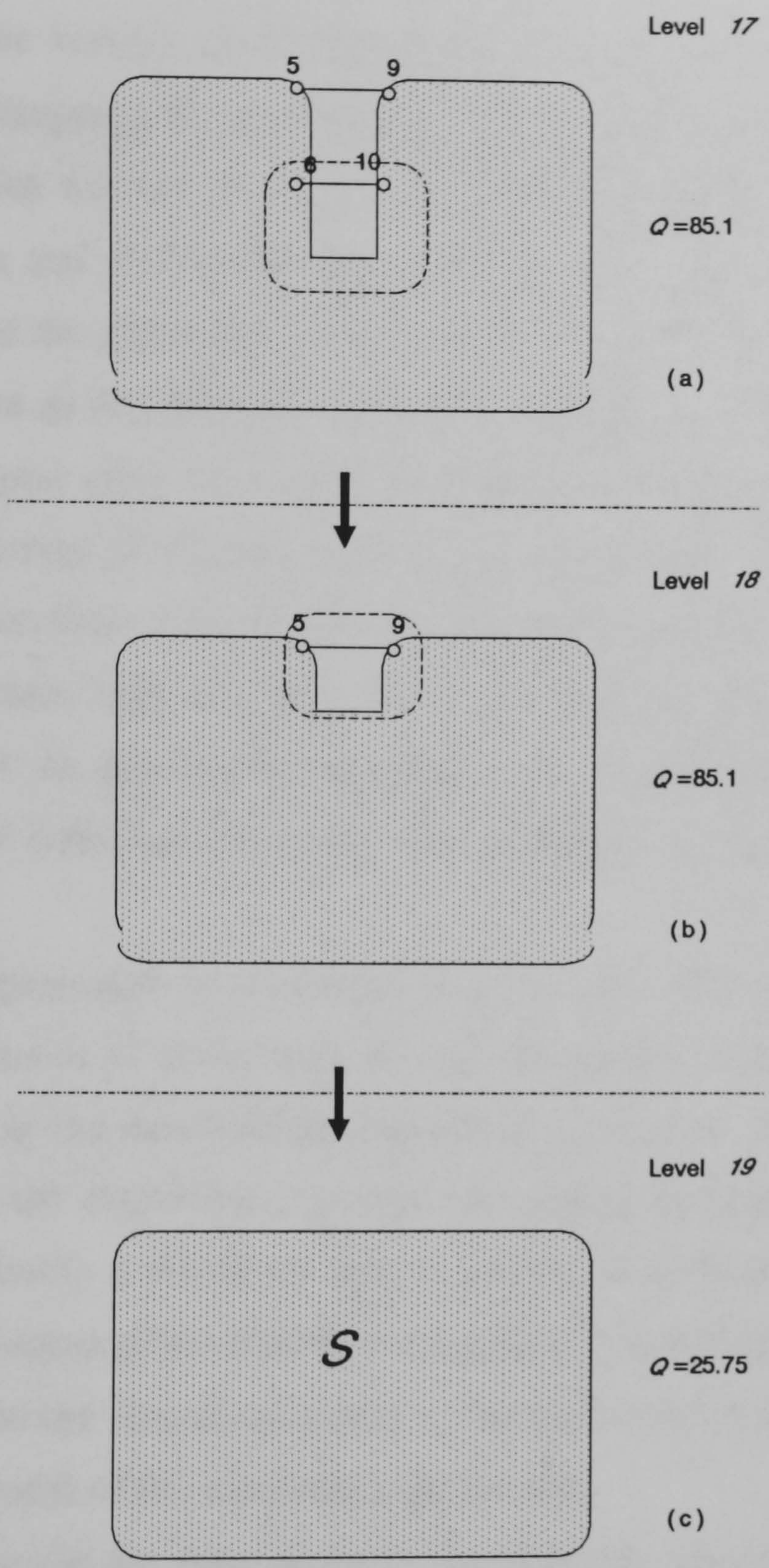


Fig. 6.8 (c) (Continued)

6.8 Summary and Conclusions

Systems can be viewed as structures made up of many levels with associated concepts and language for description at each level. Hierarchies are multi-level structures which can be used for representing different levels of definition of concepts, data and structures. Hierarchical structures allow multiple levels of modelling, and therefore representations of concepts and data to be used which are appropriate to the inference and problems of concern.

A complex structure is a system which can be represented in a form of hierarchy in terms of interconnected structural rings. The structural rings at each level have their own structural characteristics which are peculiar to that level. A structural ring at a given level of definition represents a substructural system. Any of its arcs can be regarded as the condensation of a set of rings at lower levels of definition and itself may be an arc of a ring at a higher level of definition.

The exploration of structural rings at each level of hierarchy, together with the properties of these rings such as the quality of well-formedness, is one of main tasks in the vulnerability analysis of structures. Some modification has been made to the clustering algorithm developed in Chapter 5, the purpose of which is to identify a structural ring at a level of definition which has the best quality of well-formedness and then to replace it by a single cluster. Repeat the same process to the structured graph at the each level of definition and finally a hierarchical model of the structure is generated.

An example has been given to illustrate this new clustering algorithm.

Chapter 7

Vulnerability Analysis of Structural Systems

7.1 Objectives

The objectives of this chapter are:

1. To develop the theory of structural vulnerability;
2. To present an analytical method to identify the failure scenarios for a structural ring;
3. To examine the concept of a deteriorating event and to define a measure of the damage demand for a failure scenario;
4. To explore the concept of the 'robustness' of a structure.

7.2 Introduction

The vulnerability of a structure indicates the ease with which it is capable of being damaged or deteriorated. Alternatively the robustness of a structure is concerned with the strength or toughness of the constitution of the structure; or the physical strength to resist damage or to experience particular deterioration without dissatisfying functional requirements.

It is desirable that a structural system is robust. Robustness implies strength and sturdiness in all possible limiting states. It is useful if a structural engineer can identify how a structural system is vulnerable. Vulnerability implies susceptibility to damage or failure.

We present a theory in this chapter the purpose of which is to identify the most vulnerable parts of a structural system so that they may be suitably protected and monitored. The emphasis of this method is not the usual one of analysing a structure under some given loading condition. Rather it is to examine the quality of the well-formedness of the various structural rings in a structure and to identify those most vulnerable or critical rings the deterioration of which could directly cause the failure of the structure.

In previous chapters it has been shown that a structure can be described at various levels of definition in terms of interconnected sets of structural rings. This provides a basis for the structural vulnerability analysis

An analytical method is developed to identify and enumerate all possible failure scenarios for a structural ring. This analytical process can be recursively applied to structural rings at various levels of definition. The results are concerned with the identification of potentially interesting failure scenarios together with the actions which may cause them, such as the minimal failure scenario and the maximal failure scenario.

Finally the concept of the "robustness" of a structure is examined.

7.3 Key Concepts Used in the Theory of Structural Vulnerability

We start in this section with some concepts and definitions

Deteriorating event: In structural vulnerability analysis, it is assumed that any damage or faults which occur in a structural ring are triggered by deteriorating events. *A deteriorating event* is the result of actions which would cause the loss, by a structural ring, of the capacity to transmit a degree of freedom. The action here is defined in very general sense. It may be natural (e.g. wind or earthquake) or human (e.g. sabotage).

Limit state boundary: The response of a structural ring to a given deteriorating event is either acceptable or unacceptable. The boundary between acceptable and unacceptable is the limit state boundary.

Failure scenario: *A failure scenario* is a sequence of deteriorating events which transforms a structural ring into a mechanism.

Event sequence diagram (ESD): A structural ring may have a set of possible failure scenarios. Each failure scenario will involve a sequence of deteriorating events in a tree-like structure of temporally linked events. This is called the *event sequence diagram* (**ESD**) of a failure scenario.

Damage demand: The damage demand is a measure of the effort which is required to make the occurrence of a specific deteriorating event. The damage demand of a failure scenario is therefore equal to the sum of the damage demands of all deteriorating events in that scenario.

Minimal failure scenario: The minimal failure scenario of a structural ring at level of definition l is the one in which the damage demand required to transform the structural ring into a mechanism is a minimum.

Separateness : The separateness of a structural ring at a level of definition is a description of the consequence of a failure scenario. It is the number of structural clusters structurally disconnected from a reference cluster contained in that ring.

Reference cluster: A reference cluster at a level of definition may be any cluster chosen for its importance or because it has the highest value of structural tightness. On earth the reference cluster would normally be the ground cluster S_G or a cluster which contains S_G.

Effective consequence : The effective consequence of a failure scenario at level of definition is measured by the ratio of the separateness of a structural ring caused by that failure scenario to the total required damage demands.

Maximal failure scenario: The maximal failure scenario of a structural ring at level of definition is one in which the effective consequence is maximal. The maximal failure scenario of whole structure is that for which the effective consequence over all levels of definition is maximal.

Structural vulnerability analysis: Structural vulnerability analysis is concerned with the identification of

(i) the minimal failure scenario;
(ii) the maximal failure scenario;
(iii) any particular interesting failure scenarios with respect to a given reference cluster.

Robustness: The robustness of a ring is measured by the size of the damage demand. The most robust ring is the one with maximal damage demand. For a ring the robustness is the same as the damage demand and for a structure it is the minimal damage demand over all levels of definition, i.e. there is one level of definition which is the weakest.

7.4 Failure Scenarios

7.4.1 Description of a Structural Ring at Level of Definition

Let us confine our attention initially to a structural ring R^l at a level of definition l. It consists of a number of arcs and joints. Each joint will have 3 or less degrees of freedom which can be transmitted to the arcs, which themselves may be self contained substructures of R^l - we call them structural clusters.

In Chapter 2, a degree of freedom was defined as the capacity of a structural joint j_i to permit the transmission of movements in a defined coordinate direction (separate and independent of other movements).

Here we characterise a ring by a string pattern representing the degrees of freedom (DOF) carried at each joint.

A DOF set at a joint i for a ring R^l is

$$D^l_i = \{ d^l_{i,j} \mid j=u,v,\theta\} \tag{7.1}$$

i.e a set of $d^l_{i,j}$ such that $j=u$ or v or θ.

where

(a) $d^l_{i,u}=1$, if a translation force can be transmitted, otherwise $d^l_{i,u}=0$.
(b) $d^l_{i,v}=1$, if a translation force can be transmitted, otherwise $d^l_{i,v}=0$.
(c) $d^l_{i,\theta}=1$ if a rotation force can be transmitted, otherwise $d^l_{i,\theta}=0$.

Thus,

a fixed joint at i is $D^l_i=\{1,1,1\}$
a pinned joint at i is $D^l_i=\{1,1,0\}$
a joint with rigidly connected member to u axis roller at i is $D^l_i=\{0,1,1\}$

An arc i in a ring R^l at a level of definition l is a structural cluster S^l_i. It is a continuous element along which the degrees of freedom can be transmitted from one of its end joints to another.

The capacity of a structural cluster S^l_i for a ring R^l to transmit the degrees of freedom is described as

$$S^l_i = \{ s^l_{i,j} \mid j=u,v,\theta \} \tag{7.2}$$

where

(a) $s^l_{i,u}=1$ if there is the capacity for a cluster to transmit a translation force from one end joint to another, otherwise $s^l_{i,u}=0$.
(b) $s^l_{i,v}=1$ if there is the capacity for a cluster to transmit a translation force from one end joint to another, otherwise $s^l_{i,v}=0$.
(c) $s^l_{i,\theta}=1$ there is the capacity for a cluster to transmit a rotation force from one end joint to another, otherwise $s^l_{i,\theta}=0$.

In Chapter 6 it was shown that a structural ring R^l at a level of definition l can be represented as

$$R^l=\{ j^l_i , S^l_i \mid i=1,2,...,n\} \tag{7.3}$$

Substituting (7.1) and (7.2) into (7.3) we have

$$\begin{aligned} R^l &=\{ D^l_i , S^l_i \mid i=1,2,...,n\} \\ &=\{ d^l_{i,j} \;,\; s^l_{i,j} \mid j=u,v,\theta \,;\, i=1,2,...,n\} \end{aligned} \tag{7.4}$$

(7.4) thus can be used to describe a structural ring R^l in terms of the degrees of freedom being transmitted along the ring.

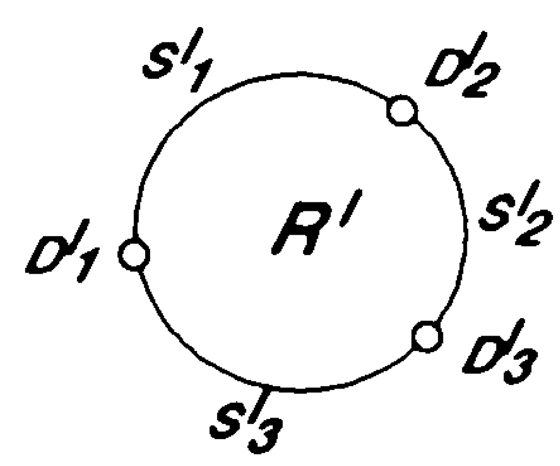


Fig. 7.1 A structural ring

For example, a structural ring of Fig.7.1 would be

$$R^l = \{ D^l_1, S^l_1, D^l_2, S^l_2, D^l_3, S^l_3 \}$$

where

$D^l_1 = (1,1,0)$, $S^l_1 = (1,1,1)$,
$D^l_2 = (1,1,0)$, $S^l_2 = (1,1,1)$,
$D^l_3 = (1,1,0)$, $S^l_3 = (1,1,1)$,

Now a fully fixed ring could theoretically consists of an infinite number of joints, but in practice n would generally be finite and this will be assumed in what follows. For a fully fixed circular ring all that will be required is that a finite number of joints are chosen in the model. As discussed in Chapter 3 we will decide that the number of adjacent fixed joints is 4, so that a ring description contains a minimum of 4 clusters and 4 joints.

Thus a fully fixed ring could be

$$R^l = \{ D^l_1, S^l_1, D^l_2, S^l_2, D^l_3, S^l_3, D^l_4, S^l_4 \}$$

where

$$D^l_1 = (1,1,1), S^l_1 = (1,1,1),$$
$$D^l_2 = (1,1,1), S^l_2 = (1,1,1),$$
$$D^l_3 = (1,1,1), S^l_3 = (1,1,1),$$
$$D^l_4 = (1,1,1), S^l_4 = (1,1,1),$$

shown in Fig.7.2(a).

(a) A fully fixed ring

(b) A ring with a cut

Fig. 7.2

A ring with a cut would be

$$R^l = \{ D^l_1, S^l_1, D^l_2, S^l_2, D^l_3, S^l_3, D^l_4, S^l_4 \}$$

where

$$D^l_1 = (1,1,1), S^l_1 = (1,1,1),$$
$$D^l_2 = (1,1,1), S^l_2 = (1,1,1),$$
$$D^l_3 = (1,1,1), S^l_3 = (1,1,1),$$
$$D^l_4 = (0,0,0), S^l_4 = (1,1,1),$$

shown in Fig.7.2(b).

Any structural loop L^l with n joints may similarly be described as $L^l = \{D^l_i, S^l_i\}$ which could possibly be matched with a ring R^l in the DHSR. If $L^l = \{D^l_i, S^l_i\}$ are not identical for some $R^l = \{D^l_i, S^l_i\}$ in the DHSR then L^l and R^l do not match and L^l is only a structural loop and not a structural ring.

7.4.2 Deteriorating Events and Failure Scenarios

In Chapter 3 we have developed the DHSR-the hierarchical deterioration of structural rings, showing all of the possible ways in which a structural ring deteriorates into a mechanism.

Generally in the DHSR for any ring R^l, each arc of the ring is in effect a structural cluster joined to the next cluster by a joint which is either fixed, pinned, on rollers, or is a break or cut, or a pin and roller combination. The structural clusters may be just stiff or statically indeterminate.

A path through the DHSR is an ordered subset of DHSR and this is a *scenario* $F_h(R^l)$ of a ring R^l.

$$F_h(R^l) = \{ R^l_k \mid k=1,\ldots,m_h\} \tag{7.5}$$

where each R^l_k is a deteriorated ring of R^l and there are m_h such rings in the scenario $F_h(R^l)$ and we define $R^l_1 = R^l$.

In a scenario the R^l_k are ordered in the sense that R^l_{k+1} is more deteriorated than R^l_k.

A *failure scenario* is one in which the final element is a mechanism.

In the structural vulnerability analysis, it is assumed that any damage or faults which occur to a structural ring are triggered by deteriorating events. *A deteriorating event* is the result of a sequence of actions which would cause the loss, by a structural ring, of the capacity to transmit a degree of freedom.
The action here is defined in very general sense. It may be natural (eg wind or earthquake) or human (eg sabotage). It is not intended at this stage to consider the details of those actions. That must be considered in further research. From the point of view of this research we are more interested in the consequence of actions which result in the loss, by a structural ring, of the capacity to transmit a degree of freedom.

In a failure scenario $F_h(R^l)=\{ R^l_k \mid k=1,...,m_h\}$, it is also assumed that from R^l_k to R^l_{k+1} only one deteriorating event occurs and it is the k^{th} event in a scenario $F_h(R^l)$. A failure scenario then could be considered as a sequence of deteriorating events which transforms a structural ring into a mechanism. In a failure scenario $F_h(R^l)=\{ R^l_k \mid k=1,...,m_h\}$ there are total m_h-1 deteriorating events.

A deteriorating event may occur adjacent to a joint or inside a cluster causing it to become two separate parts connected by a joint. If a ring deteriorates until it becomes a just-stiff ring then one more deteriorating event will cause it to become a mechanism. Note that if a just-stiff structural ring is identified then the failure scenario will contain only one deteriorating event.

The first deteriorating event in a failure scenario is called the *starting event*. The last deteriorating event is called the *terminating event*. When describing a failure scenario, it is usually necessary to identify the starting event and the terminating event.

Each failure scenario will involve a sequence of deteriorating events in a tree-like structure of temporily linked events. This is called the *event sequence diagram* (ESD) of a failure scenario. For a cluster consisting of a connected set of rings the failure event sequence diagram would represent the conjunction of the set of possible scenarios and the setting up and interpretation of such a diagram is the subject of the vulnerability analysis.

A set of all possible failure scenarios for a single structural ring R^l thus can be found in the DHSR, denoted by

$$F(R^l) = \{ F_h(R^l) \mid h = 1,\ldots,p\} \qquad (7.6)$$

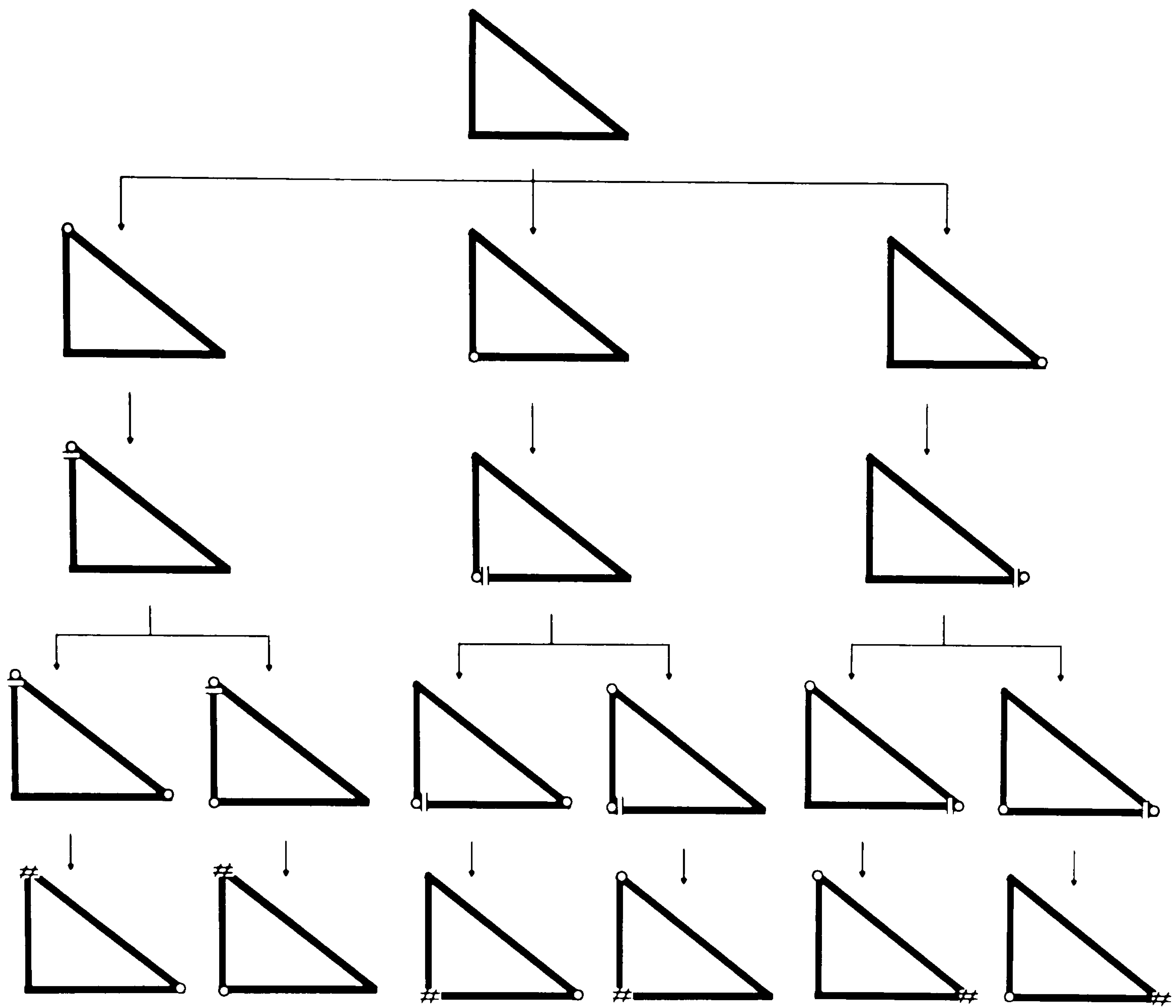
where p is the total number of failure scenarios for a ring R^l.

One important point should be noted that each path in the DHSR shows only an abstract pattern of a failure scenario. For a structural ring to be analysed, the actual failure scenario would depends on the locations on which the deteriorating events occur.

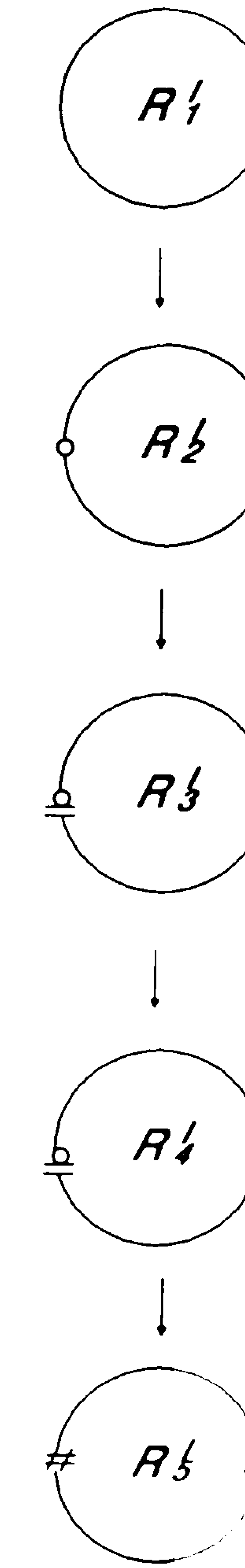
For example, the triangle frame of Fig.7.3(a) is a fully fixed structural ring R^l. Matching it in to the DHSR a failure scenario pattern of R^l, shown in Fig7.3(b), may be described as

$$F_h(R^l) = \{ R^l_k \mid k = 1,\ldots,5\}$$

Based on this pattern we can enumerate six possible failure scenarios in which the final failure modes of the structure are exact the same, shown in Fig.7.3(a). A set of all possible failure scenarios for the structure can be found using the similar approach.



(a) Failure scenarios of a structure



(b) A failure scenario pattern

Fig. 7.3

Therefore the identification of a set of all possible failure scenarios for a structural ring R^l to be analysed would consists of three phases.

(i) Match the ring $R^l = \{ D^l_i, S^l_i \mid i=1,..., n)$ in the DHSR;

(ii) For a particular failure scenario pattern in the DHSR, enumerate all possible failure scenarios with respect to that pattern;

(iii) Go through all failure scenario patterns in the DHSR, and find out a set of all possible failure scenarios $F(R^l) = \{F_h(R^l) \mid h=1,..., p\}$ for a ring R^l.

We will study this more fully in the following sections

7.4.3 Description of Failure Scenarios

It was shown in Section 7.4.2 that a failure scenario consists of a sequence of deteriorating events. Each deteriorating event causes the loss of the capacity to transmit a degree of freedom. Hence, with (7.3) and (7.6), a failure scenario for a ring R^l becomes

$$
\begin{aligned}
F_h(R^l) &= \{R^l_k \mid k=1,..., m_h\} \\
&= \{ D^l_{i,k}, S^l_{i,k} \mid k=1,...,m_h \,;\, i=1,..., n\} \\
&= \{d^l_{i,j,k} \,,\, s^l_{i,j,k} \mid k=1,..., m_h \,;\, i=1,2,...,n \,;\, j=u,v,\theta; \}
\end{aligned}
\tag{7.7}
$$

where n is the total number of joints contained in the ring R^l and m_h is the total number of the deteriorated rings in the failure scenario $F_h(R^l)$.

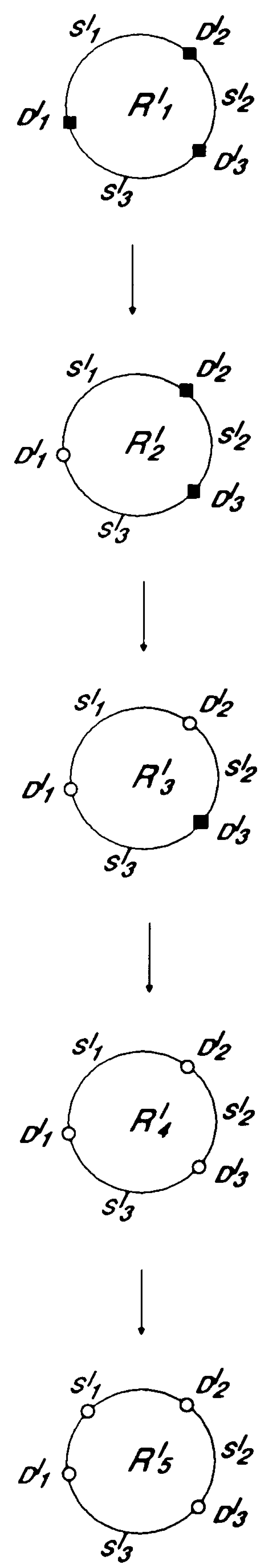


Fig. 7.4 A failure scenario

For example a failure scenario for the ring R^l of Fig.7.4 might be described as

$$R^l = \{ D^l_1, S^l_1, D^l_2, S^l_2, D^l_3, S^l_3 \}$$

and the failure scenario

$$\begin{aligned} F_h(R^l) &= \{ R^l_1, R^l_2, R^l_3, R^l_4, R^l_5 \} \\ &= \{ D^l_{i,k}, S^l_{i,k} \mid k=1,...,5 ; i=1,...,3 \} \\ &= \{ d^l_{i,j,k} \; , \; s^l_{i,j,k} \mid k=1,...,5 ; i=1,...,3 ; j=u,v,\theta \} \end{aligned}$$

$$\begin{aligned} = \{&[(1,1,1,), (1,1,1), (1,1,1), (1,1,1),(1,1,1), (1,1,1)], \\ &[(1,1,0,), (1,1,1), (1,1,1), (1,1,1),(1,1,1), (1,1,1)], \\ &[(1,1,0,), (1,1,1), (1,1,0), (1,1,1),(1,1,1), (1,1,1)], \\ &[(1,1,0,), (1,1,1), (1,1,0), (1,1,1),(1,1,0), (1,1,1)], \\ &[(1,1,0,), (1,1,0), (1,1,0), (1,1,1),(1,1,0), (1,1,1)],\} \end{aligned}$$

where $m_h = 5$ and $n = 3$.

For even more generality, we define that

(i) a deteriorating event occurring adjacent to a joint in a failure scenario $F_h(R^l)$ for a ring R^l as

$$f^l_{i,j,k} = d^l_{i,j,k} - d^l_{i,j,k+1} \qquad (7.8)$$

$f^l_{i,j,k}$ implies that this is the k^{th} deteriorating event in a failure scenario to cause the loss of a degree of freedom $d^l_{i,j}$ adjacent to a joint i.

thus

$$\begin{aligned} f^l_{i,j,k} &> 0 \quad \text{when an event occurs;} \\ f^l_{i,j,k} &= 0 \quad \text{otherwise.} \end{aligned} \qquad (7.9)$$

(ii) similarly, a deteriorating event occurring within a cluster in a failure scenario $F_h(R^l)$ for a ring R^l as

$$g^l_{i,j,k} = s^l_{i,j,k} - s^l_{i,j,k+1} \tag{7.10}$$

$g^l_{i,j,k}$ indicates this is the k^{th} deteriorating event in a failure scenario to cause the loss of the capacity $s^l_{i,j}$ of a cluster S^l_i to transmit a degree of freedom.

thus,

$$g^l_{i,j,k} > 0 \quad \text{when an event occurs;} \tag{7.11}$$
$$g^l_{i,j,k} = 0 \quad \text{otherwise.}$$

Therefore for a failure scenario there are a series of deteriorating events represented by

$$F_h(R^l) = \{ f^l_{i,j,k} \,,\; g^l_{i,j,k} \mid k = 1,\ldots,m_h\text{-}1 \,;\, i = 1,2,\ldots,n \,;\, j = u,v,\theta \} \tag{7.12}$$

With (7.8) to (7.12) the failure scenario of Fig.7.4 could simply be expressed as

$$\begin{aligned} F_h(R^l) &= \{ f^l_{i,j,k} \,,\; g^l_{i,j,k} \mid k = 1,\ldots,m_h\text{-}1 \,;\; i = 1,2,\ldots,n \,;\, j = u,v,\theta \} \\ &= \{ f^l_{1,\theta,1} \,,\; f^l_{2,\theta,2} \,, f^l_{3,\theta,3} \,,\; g^l_{1,\theta,4} \} \end{aligned}$$

in which the first deteriorating event is $f^l_{1,\theta,1}$ and the terminating event $g^l_{1,\theta,4}$ and there are a total of four deteriorating events in this failure scenario.

7.5 Damage Demand

In the DHSR each structural ring can be destroyed in a number of ways. Each way is a failure scenario which consists of a sequence of deteriorating events. A deteriorating event is the result of actions which causes the loss, by a structural ring, of the capacity to transmit a degree of freedom either adjacent to a joint or within a cluster contained in the ring. Thus for a given deteriorating event a certain amount effort is required in order to achieve the desired result. This effort is called the *damage demand* with respect to a given deteriorating event.

The damage demand for a deteriorating event $f^l_{i,j,k}$ or $g^l_{i,j,k}$ is defined as $e(f^l_{i,j,k})$ and $e(g^l_{i,j,k})$ respectively, where

(i) $e(f^l_{i,j,k}) > 0$ when $f^l_{i,j,k} > 0$, (7.13)
$e(f^l_{i,j,k}) = 0$ otherwise.

(ii) $e(g^l_{i,j,k}) > 0$ when $g^l_{i,j,k} > 0$, (7.14)
$e(g^l_{i,j,k}) = 0$ otherwise.

Thus the total damage demand for a failure scenario $F_h(R^l)$ is

$$E[F_h(R^l)] = \Sigma_k\Sigma_i\Sigma_j\, e(f^l_{i,j,k}) + \Sigma_k\Sigma_i\Sigma_j\, e(g^l_{i,j,k}) \tag{7.15}$$
$$(k = 1,\dots, m_h\text{-}1\ ;\ i = 1,2,\dots,n\ ;\ j = u,v,\theta)$$

The question now arises that what is the measure of the damage demand $e(f^l_{i,j,k})$ or $e(g^l_{i,j,k})$ for a deteriorating event.

Let us confine our attention initially in this section to a failure scenario for a ring R^l given by (7.4), with a failure scenario given by (7.12) in which S^l_i $(i = 1,\dots, n)$ are all primitive clusters.

Thus the damage demand for a failure scenario, $F_h(R^l)$, is given by (7.15)

Firstly let us consider the calculation of $e(f'_{i,j,k})$.

In Chapter 4, it was proved that the well-formedness of a joint in a structural ring is dependant on

(i) the orientation and stiffness of the members framing into the joint ;
(ii) the stiffness of the joint(pinned or fixed).

The quality of the well-formedness of a joint q_i can be described quantitatively by its principal stiffness coefficients i.e the products of the eigenvalues of the stiffness submatrix of the joint. We also know that (i) the principal displacement axes of a joint are linearly independent; (ii) the q_i measure is independent of the co-ordinate system.

A principle stiffness coefficient represents the capability of a joint to resist applied forces along the corresponding principle axis. The bigger value a principle stiffness coefficient has, the more damage demand is required to make the occurrence of the deteriorating event--that is, the loss of the corresponding degree of freedom.

Any deteriorating event occurring adjacent to a joint would definitely cause damage to the quality of well-formedness of that joint. It seems logical that the effort which is required to achieve a particular deteriorating event should be proportional to the damage to the quality of the well-formedness of that joint.

We need a measure of damage demand which will allow the identification of a minimal failure scenario. Thus the measure need only be proportional to the actural demand in a real structure. The damage demand with respect to a deteriorating event occurring adjacent to a joint can therefore be measured, for the purpose of vulnerability analysis, by the principal stiffness coefficients of that joint, which is defined as

$$e(f^l_{i,j,k}) = (w^l_{i,j,k} \times \lambda^l_{i,j,k} \times d^l_{i,j,k}) - (w^l_{i,j,k+1} \times \lambda^l_{i,j,k+1} \times d^l_{i,j,k+1}) \tag{7.16}$$

where $w^l_{i,j,k}$ is a constant and $\lambda^l_{i,j,k}$ is the stiffness coefficient of a joint i before the k^{th} event and $\lambda^l_{i,j,k+1}$ is the stiffness coefficient of a joint i after the k^{th} event.

Then we consider the calculation of $e(g^l_{i,j,k})$ when S^l_i is a primitive cluster.

S^l_i is a primitive cluster when it contains only one single member. It is assumed here that a deteriorating event occurs in the middle of a structural member which can be considered as two smaller parts connecting by a fixed joint in the middle. Thus the damage demand $e(g^l_{i,j,k})$ cluster could be calculated similarly by

$$e(g^l_{i,j,k}) = (w^l_{i,j,k} \times \lambda(s^l_{i,j,k}) \times d^l_{i,j,k}) - (w^l_{i,j,k+1} \times \lambda(s^l_{i,j,k+1}) \times d^l_{i,j,k+1}) \tag{7.17}$$

where $w^l_{i,j,k}$ is a constant and $\lambda(s^l_{i,j,k})$ is the stiffness coefficient of the central joint of the cluster S^l_i before the k^{th} event and $\lambda(s^l_{i,j,k+1})$ is the stiffness coefficient after the k^{th} event.

A structure of Fig.7.5 will be used as an example to illustrate the procedure of calculating of the damage demand. It is a triangular fixed frame and all members have the same values of $A=0.00683m^2$, $I=1.169\times10^{-4}$ and $E=2.1\times10^8$. The corresponding ring R^l is represented as

$$R^l = \{ D^l_1, S^l_1, D^l_2, S^l_2, D^l_3, S^l_3 \}$$

where S^l_1, S^l_2 and S^l_3 are all primitive clusters and the length of the three members are equal to 5m, 3m and 4m respectively.

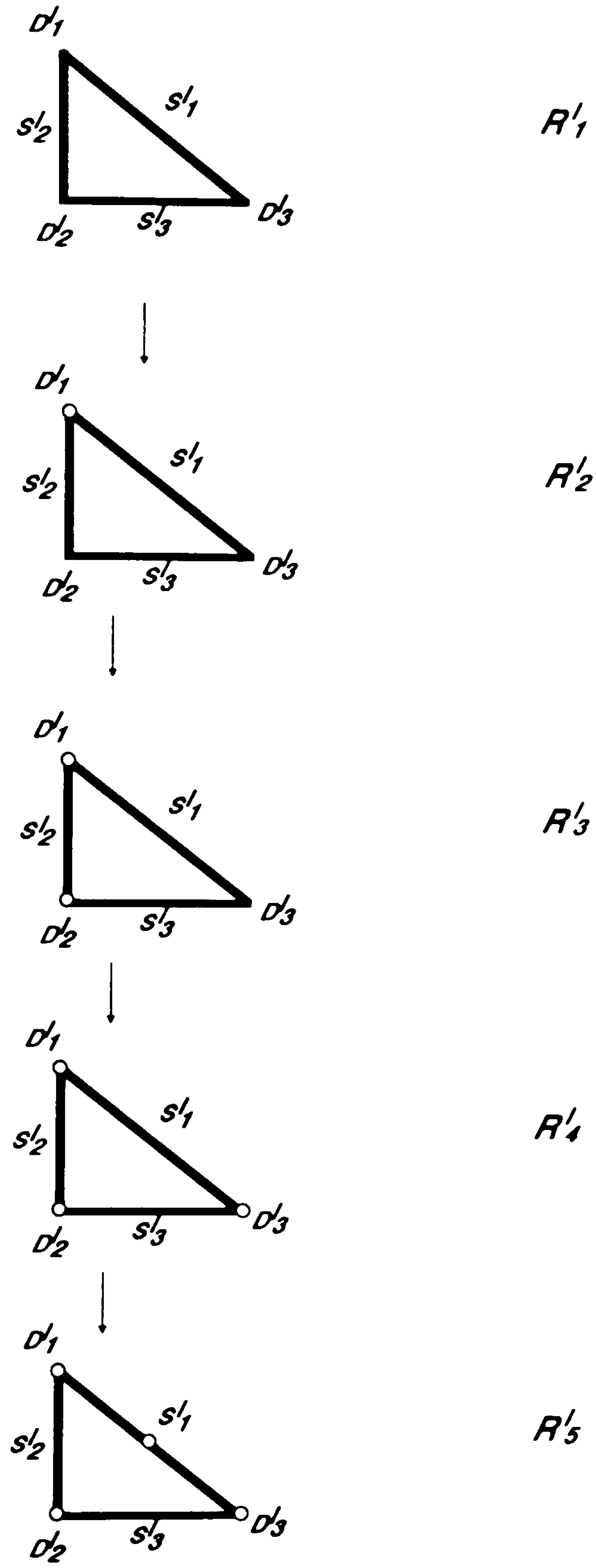


Fig. 7.5 Damage demand for a failure scenario

A failure scenario of R^l of Fig.7.5 according to (7.16) can be described as

$$F_h(R^l) = \{f^l_{i,j,k}, g^l_{i,j,k} \mid k=1,...,m_h\text{-}1 \,;\, i=1,2,...,n \,;\, j=u,v,\theta\}$$
$$= \{ f^l_{1,\theta,1},\ f^l_{2,\theta,2}, f^l_{3,\theta,3},\ g^l_{1,\theta,4} \}$$

By implementing the subroutine **DET** developed in Chapter 5, the eigenvalues of all joints in the ring R^l_k can be obtained and the results are shown in Table 7.1.

TABLE 7.1

Stiffness coefficients table

(10^{-5})

R^l_k	$\lambda^l_{1,j,k}$			$\lambda^l_{2,j,k}$			$\lambda^l_{3,j,k}$			$\lambda(s^l_{1,j,k})$		
	u	v	θ	u	v	θ	u	v	θ	u	v	θ
$k=1$	6.26	1.56	0.48	4.82	3.70	0.56	5.81	0.76	0.38	11.47	0.59	0.09
$k=2$	6.26	1.56	0.00	4.82	3.70	0.56	5.81	0.76	0.38	11.47	0.59	0.09
$k=3$	6.26	1.56	0.00	4.82	3.70	0.00	5.81	0.76	0.38	11.47	0.59	0.09
$k=4$	6.26	1.56	0.00	4.82	3.70	0.00	5.81	0.76	0.00	11.47	0.59	0.09
$k=5$	6.26	1.56	0.00	4.82	3.70	0.00	5.81	0.76	0.00	11.47	0.59	0.00

Now the damage demand for each individual deteriorating event can be obtained basing on (7.16) and (7.17). Here for the sake of simplicity, it is assumed that the constant $w^l_{i,j,k}(k=1,...,m_h\text{-}1)$ in (7.16) and (7.17) is equal to 10^{-5}. We have

$$e(f^l_{1,\theta,1}) = (w^l_{1,\theta,1} \times \lambda^l_{1,\theta,1} \times d^l_{1,\theta,1}) - (w^l_{1,\theta,2} \times \lambda^l_{1,\theta,2} \times d^l_{1,\theta,2})$$

Similarly

$$e(f^l{}_{2,\theta,2}) = (w^l{}_{2,\theta,2} \times \lambda^l{}_{2,\theta,2} \times d^l{}_{2,\theta,2}) - (w^l{}_{2,\theta,3} \times \lambda^l{}_{2,\theta,3} \times d^l{}_{2,\theta,3}) = 0.56$$
$$e(f^l{}_{3,\theta,3}) = (w^l{}_{3,\theta,3} \times \lambda^l{}_{3,\theta,3} \times d^l{}_{3,\theta,3}) - (w^l{}_{3,\theta,4} \times \lambda^l{}_{3,\theta,4} \times d^l{}_{3,\theta,4}) = 0.38$$
$$e(g^l{}_{1,\theta,4}) = (w^l{}_{1,\theta,4} \times \lambda(s^l{}_{1,\theta,4}) \times d^l{}_{1,\theta j,4}) - (w^l{}_{1,\theta,5} \times \lambda(s^l{}_{1,\theta,5}) \times d^l{}_{1,\theta,5}) = 0.09$$

Thus, the total damage demand for a failure scenario $F_h(R^l)$ can be obtained from (7.15)

$$E[F_h(R^l)] = \Sigma_k\Sigma_i\Sigma_j\, e(f^l{}_{i,j,k}) + \Sigma_k\Sigma_i\Sigma_j\, e(g^l{}_{i,j,k})$$
$$(k=1,...,4\ ;\ i=1,2,3\ ;\ j=u,v,\theta)$$
$$= e(f^l{}_{1,\theta,1}) + e(f^l{}_{2,\theta,2}) + e(f^l{}_{3,\theta,3}) + e(g^l{}_{1,\theta,4})$$
$$= 1.51$$

Generally for a structural ring R^l there exists a set of possible failure scenarios $F(R^l) = \{F_h(R^l)\ |\ h=1,..,p\}$. A computer program can be developed to find out the damage demand of $E[F_h(R^l)]$ for all failure scenarios. We are more interested in the failure scenario of a ring R^l in which the least damage demand is required to deteriorate a structural ring into a mechanism as far as the structural vulnerability analysis concerned. This is called the *minimal failure scenario*, defined as

$$E_{min}[F(R^l)] = min\ \{\ E[F_h(R^l)]\ |\ h=1,2,...,p\} \tag{7.18}$$

where p is the total number of failure scenarios for the ring R^l.

7.6 Deterioration of a Structural Ring at Level of Definition

So far we have developed a method to calculate the damage demand for a

So far we have developed a method to calculate the damage demand for a failure scenario of a ring $R^l = \{ D^l_i, S^l_i \mid i=1,2,...,n\}$ in which S^l_i, $i=1,2,...,n$ are all primitive clusters. In this section we consider a ring R^l in which some of clusters are not primitive clusters.

It was shown in Chapter 6 that the process of cluster formation produces a hierarchical model of a structure which is particularly useful in the identification of failure scenarios. The structure being analyzed is modelled as a hierarchical set of structural rings. This hierarchical model of a structure $S=(M,J)$ has been defined as

$$S = \{ R^l \} \tag{7.19}$$

where R^l, $l=1,2,...,q_s$ is a structural ring at the level of definition l and there are q_s levels in the hierarchy.

A structural ring R^l represents a substructure of S. It consists of a number of joints and arcs. Each arc itself may be a self contained sub-structure of R^l, i.e. structural cluster, denoted by S^l_i The structural ring R^l thus can be represented by (7.4) in terms of the degrees of freedom being transmitted along the ring.

$$R^l = \{ D^l_i, S^l_i \mid i=1,...,n\}$$

where n is the total number of joints in the ring R^l.

The structural cluster S^l_i again consists of a sequence of structural rings at lower levels of definition. The cluster S^l_i may also be described as a hierarchical set of structural rings

$$S^l_i = \{ R^{q_i} \} \; (q_i < l) \tag{7.20}$$

and $$R^{q_i} = \{ D^{q_i}_x, S^{q_i}_x \mid x=1,..., n_i) \tag{7.21}$$

level of definition of S^l_i.

At the lowest levels of the hierarchy all structural clusters are primitive clusters i.e. single members. At the highest level of the hierarchy there is only one cluster which represents the whole structure being studied.

Therefore in order to deteriorate a structural ring R^l at a level of definition l we could deteriorate either the joints D^l_i or the clusters S^l_i contained in that ring. Generally for a cluster S^l_i it consists of a set of structural rings at lower levels of definition. We deteriorate each of those rings in turn by using the same approach so that the cluster S^l_i is effectively damaged by the loss of the integrity of the rings making up that clusters, which in turn causes directly the failure of the ring R^l.

An algorithm can be developed to deal with the deterioration of a ring R^l. This algorithm can be applied recursively to the structural rings at the various levels of hierarchy of a structure. This will help us to identify different failure scenarios as well as the damage demands for those failure scenarios.

This algorithm is as follows

Step 1 For a structural ring at a level of definition l, according to .(7.4)

$$R^l = \{ D^l_i , S^l_i \mid i=1,...,n\}$$
$$= \{ d^l_{i,j} \;,\; s^l_{i,j} \mid i=1,2,...,n \;; j=u,v,\theta\}$$

Step 2 Match it in the DHSR.

Step 3 Find out a set of all possible failure scenarios for R^l

$$F(R^l) = \{F_h(R^l) \mid h=1,..., p\}$$

where p is the total number of possible failure scenarios and where

$$F_h(R^l) = \{R^l_k \mid k=1,\dots,m_h\}$$
$$= \{f^l_{i,j,k}, g^l_{i,j,k} \mid k=1,\dots,m_h\text{-}1 \,;\, i=1,2,\dots,n \,;\, j=u,v,\theta\}$$

where m_h is the total number of deteriorated rings in $F_h(R^l)$, and n is the total number of joints in R^l.

Step 4 Calculate the damage demand for each failure scenario $F_h(R^l)$, basing on (7.15) and (7.17) as

$$E[F_h(R^l)] = \Sigma_k\Sigma_i\Sigma_j\, e(f^l_{i,j,k}) + \Sigma_k\Sigma_i\Sigma_j\, e(g^l_{i,j,k})$$
$$(k=1,\dots, m_h\text{-}1 \,;\, i=1,2,\dots,n;\, j=u,v,\theta\}$$

where

$$e(f^l_{i,j,k}) = (w^l_{i,j,k} \times \lambda^l_{i,j,k} \times d^l_{i,j,k}) - (w^l_{i,j,k+1} \times \lambda^l_{i,j,k+1} \times d^l_{i,j,k+1})$$

where $w^l_{i,j,k}$ is a constant and $\lambda^l_{i,j,k}$ is the stiffness coefficient of a joint i before the k^{th} event and $\lambda^l_{i,j,k+1}$ is the stiffness coefficient of a joint i after the k^{th} event.

and if cluster S^l_i is a primitive cluster then

$$e(g^l_{i,j,k}) = (w^l_{i,j,k} \times \lambda(s^l_{i,j,k}) \times d^l_{i,j,k}) - (w^l_{i,j,k+1} \times \lambda(s^l_{i,j,k+1}) \times d^l_{i,j,k+1})$$

where $w^l_{i,j,k}$ is a constant and $\lambda(s^l_{i,j,k})$ is the stiffness coefficient of the central joint of the cluster S^l_i before the k^{th} event and $\lambda(s^l_{i,j,k+1})$ is the stiffness coefficient after the k^{th} event.

$(z=1,\ldots, m_r\text{-}1\ ;\ x=1,2,\ldots,n_i\ ;\ y=u,v,\theta\}$

(iii) thus the damage demand for a deteriorating event $e(g^{l}_{i,j,k})$ is equal to

$$e(g^{l}_{i,j,k}) = \min \{E[F_r(R^{q_i})] \mid r=1,\ldots,m_r\text{-}1\}$$

where $F_r(R^{q_i})$ is a failure scenario which will cause the occurrence of a event $g^{l}_{i,j,k}$.

Step 7 For each non-primitive cluster S^{l}_{i} in the ring R^l repeat *Step 5* to *Step 6* and calculate $e(g^{l}_{i,j,k})$.

Step 8 Bring back the result of $e(g^{l}_{i,j,k})$ to the damage demand $E[F_h(R^l)]$ which has not been decided in *Step 4*.

Step 9 Identify the minimal failure scenario of R^l

$$E_{min}[F(R^l)] = min\ \{\ E[F_h(R^l)] \mid h=1,2,\ldots,p\}$$

The procedure of this algorithm will be illustrated by deteriorating the structure of Fig.7.6(a). Assuming that the bending and axial rigidities of the members are as follows: for members m_1-m_6, EI$=1.164\times10^4$kNm and AE$=9.95\times10^5$kN; for members m_7-m_{12}, EI$=0.787\times10^4$kNm and AE$=6.783\times10^5$kN. The process of cluster formation has been implemented according to the algorithm developed in Chapter 6 in which a ring with the best quality of the well-formedness at each level of definition is identified and replaced by a single cluster, shown in Fig.7.6.

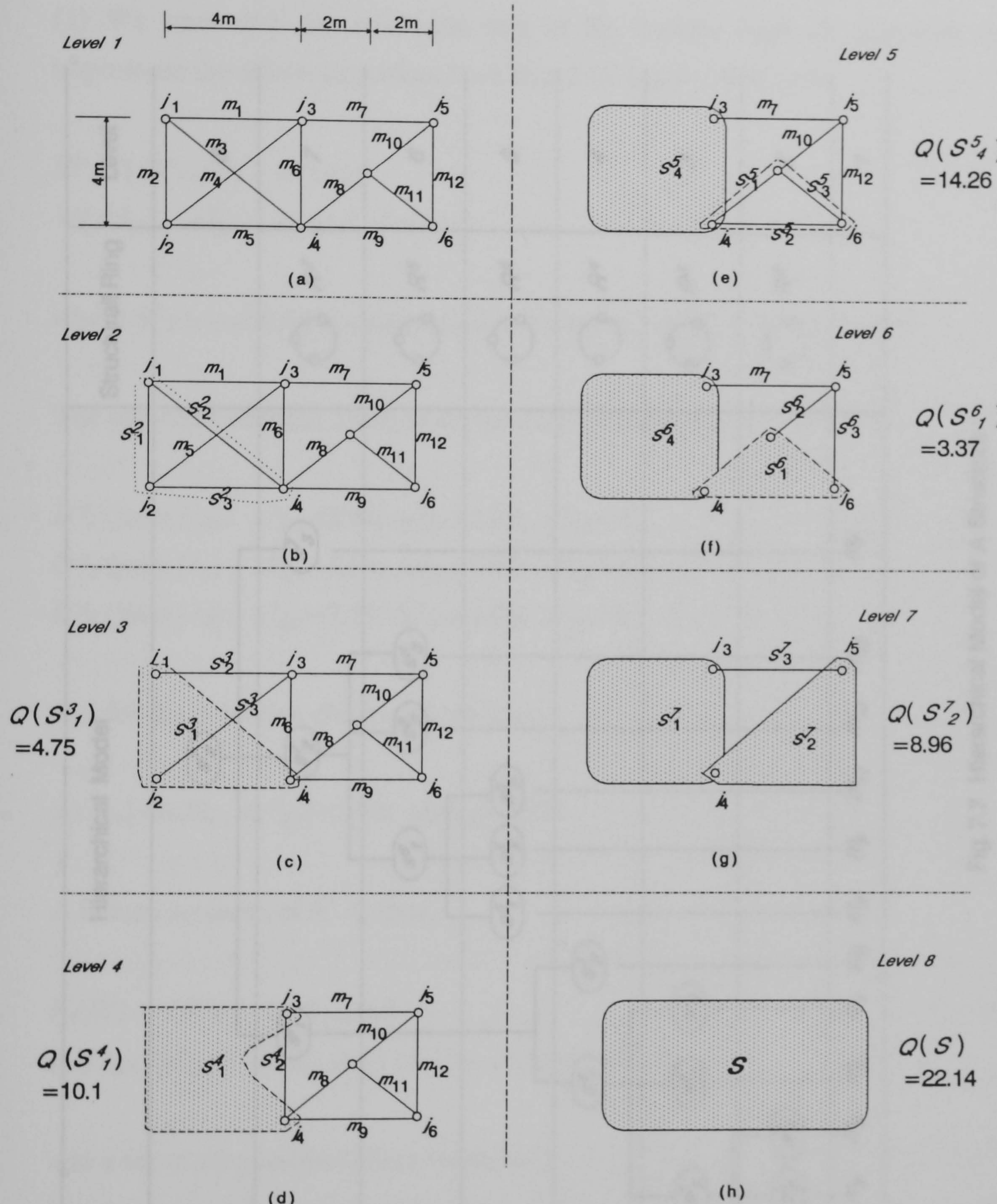


Fig. 7.6 Cluster formation

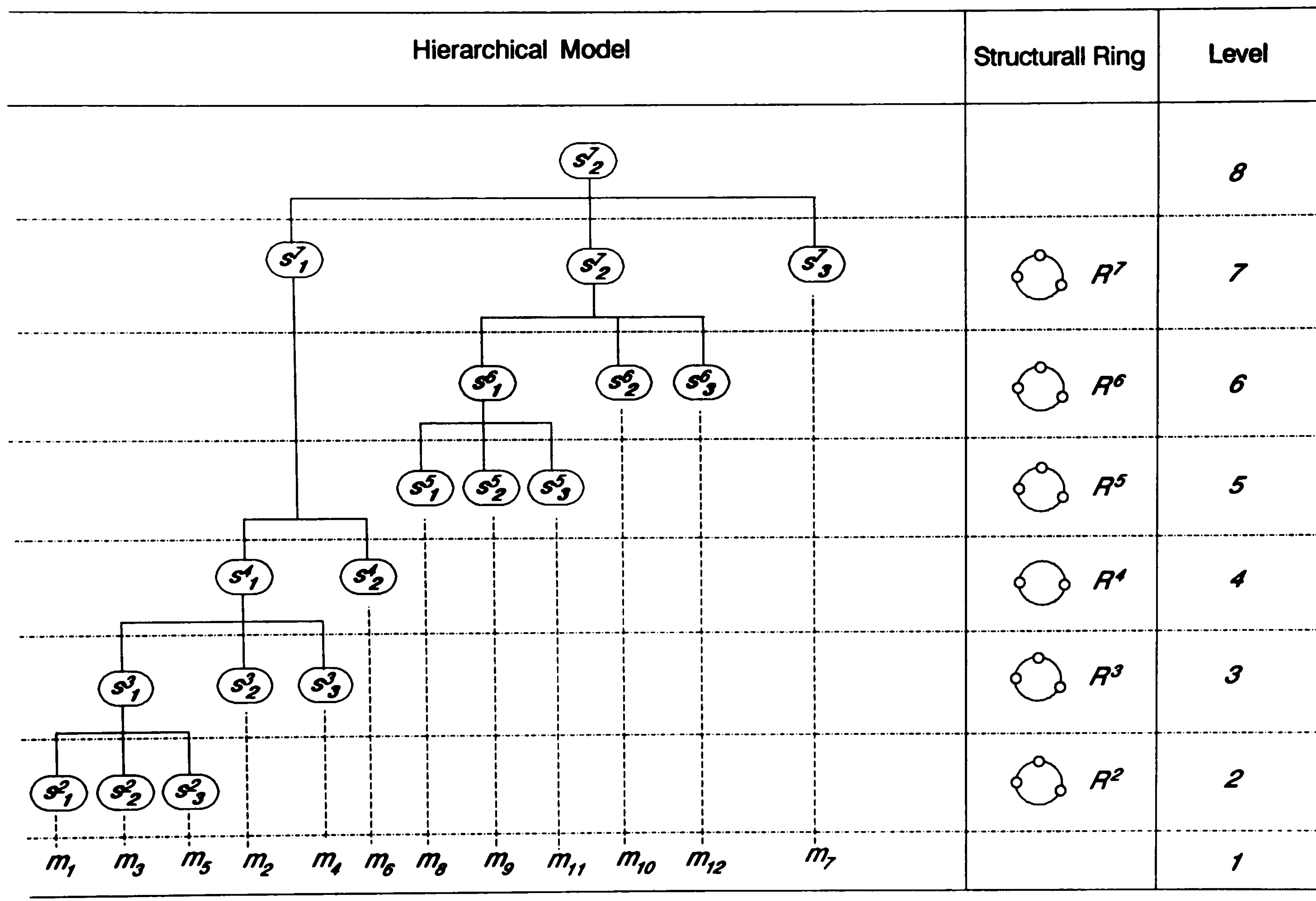


Fig. 7.7 Hierarchical Model of A Structure

It is obvious that the structure is built up with structural rings at various levels of definition and its hierarchical model is shown in Fig.7.7.

(1) We start with the structural ring at the highest level of definition and implement the above algorithm from *Step 1* to *Step 4*. We have

$$R^7 = \{ D^7_i , S^7_i \mid i=1,2,3\}$$
$$=\{ d^7_{i,j} \ , \ s^7_{i,j} \mid i=1,2,3 \,; j=u,v,\theta\}$$

where S^7_3 is a primitive cluster, and referring to Fig.7.7, it is member m_7

The principal stiffness coefficients for each joint D^7_i are as

D^7_1 (joint j^7_3) : $\lambda^7_{1,u}=5.94$; $\lambda^7_{1,v}=3.56$; $\lambda^7_{1,\theta}=0$
D^7_2 (joint j^7_6) : $\lambda^7_{2,u}=4.64$; $\lambda^8_{2,v}=2.03$; $\lambda^7_{2,\theta}=0$
D^7_3 (joint j^7_4) : $\lambda^7_{3,u}=7.25$; $\lambda^7_{3,v}=4.96$; $\lambda^7_{3,\theta}=0$

and for the primitive cluster S^7_3 we have

$\lambda(s^7_{3,u})=6.78$; $\lambda(s^7_{3,v})=0.24$; $\lambda(s^7_{3,\theta})=0.06$

A failure scenario of R^7 is, thus,

$$F_h(R^7) = \{R^7_k \mid k=1,\dots,m_h\}$$
$$=\{f^7_{i,j,k} , g^7_{i,j,k} \mid k=1,\dots,m_h\text{-}1 \,; i=1,2,\dots,n \,; j=u,v,\theta\}$$

and a set of all possible failure scenario is

$$F(R^7)=\{F_h(R^7) \mid h=1,\dots, p\}$$

Note that the ring R^7 is a just-stiff ring, any of its failure scenarios contains only one deteriorating event. Match R^7 in the **DHSR** and find out a set of failure scenarios, given by (7.15) to (7.17).

Failure scenario $F_h(R^7)$	$e(f^7_{i,j,k})$ or $e(g^7_{i,j,k})$	Damage demand E $[F_h(R^7)]$
$F_1(R^7)=f^7_{1,u,1}$	$e(f^7_{1,u,1})=5.94$	$E[F_1(R^7)]=5.94$
$F_2(R^7)=f^7_{1,v,1}$	$e(f^7_{1,v,1})=3.56$	$E[F_2(R^7)]=3.56$
$F_3(R^7)=f^8_{2,u,1}$	$e(f^7_{2,u,1})=4.64$	$E[F_3(R^7)]=4.64$
$F_4(R^7)=f^7_{2,v,1}$	$e(f^7_{2,v,1})=2.03$	$E[F_4(R^7)]=2.03$
$F_5(R^7)=f^7_{3,u,1}$	$e(f^7_{3,u,1})=7.25$	$E[F_5(R^7)]=7.25$
$F_6(R^7)=f^7_{3,\chi,1}$	$e(f^7_{3,v,1})=4.96$	$E[F_6(R^7)]=4.96$
$F_7(R^7)=g^7_{1,u,1}$	$e(g^7_{1,u,1})=?$	$E[F_7(R^7)]=?$
$F_8(R^7)=g^7_{1,u,1}$	$e(g^7_{1,v,1})=?$	$E[F_8(R^7)]=?$
$F_9(R^7)=g^7_{1,\theta,1}$	$e(g^7_{1,\theta,1})=?$	$E[F_9(R^7)]=?$
$F_{10}(R^7)=g^7_{2,u,1}$	$e(g^7_{2,u,1})=?$	$E[F_{10}(R^7)]=?$
$F_{11}(R^7)=g^7_{2,u,1}$	$e(g^7_{2,v,1})=?$	$E[F_{11}(R^7)]=?$
$F_{12}(R^7)=g^7_{2,\theta,1}$	$e(g^7_{2,\theta,1})=?$	$E[F_{12}(R^7)]=?$
$F_{13}(R^7)=g^7_{3,u,1}$	$e(g^7_{3,u,1})=6.78$	$E[F_{13}(R^7)]=6.78$
$F_{14}(R^7)=g^7_{3,u,1}$	$e(g^7_{3,v,1})=0.24$	$E[F_{14}(R^7)]=0.24$
$F_{15}(R^7)=g^7_{3,\theta,1}$	$e(g^7_{3,\theta,1})=0.06$	$E[F_{15}(R^7)]=0.06$

Since clusters S^7_1 and S^7_2 are not primitive clusters we cannot calculate $e(g^7_{1,j,1})$ or $e(g^7_{2,j,1})$ $(j=u,v,\theta)$ at this stage. Now we go to *Step 5* and look for the next lowest level structural rings making up clusters S^7_1 and S^7_2.

(2) We choose the cluster S^7_2 since it has a smaller value of the structural tightness $Q(S^7_2)$. Referring to Fig.7.6 and Fig.7.7 we see that the next lowest level ring of S^7_2 is the ring R^6

$S^7_2 = \{ R^6 \}$

$R^6 = \{ D^6_i , S^6_i \mid i=1,2,3\}$

$= \{ d^6_{i,j} \;,\; s^6_{i,j} \mid i=1,2,3 \,; j=u,v,\theta\}$

where S^6_2 and S^6_3 are two primitive clusters and referring to Fig.7.7, they are members m_{10} and m_{12}.

The principal stiffness coefficients for each joint D^6_i are as

D^6_1 (joint j^6_5) : $\lambda^6_{1,u}=5.80$; $\lambda^6_{1,v}=2.34$; $\lambda^6_{1,\theta}=0$

D^6_2 (joint j^6_6) : $\lambda^6_{2,u}=3.99$; $\lambda^6_{2,v}=0.98$; $\lambda^6_{2,\theta}=0$

D^6_3 (joint j^6_7) : $\lambda^6_{3,u}=4.64$; $\lambda^6_{3,v}=2.03$; $\lambda^6_{3,\theta}=0$

and for the primitive clusters S^6_2 and S^6_3 we have

$\lambda(s^6_{2,u})=10.85$; $\lambda(s^6_{2,v})=0.38$; $\lambda(s^6_{2,\theta})=0.24$

$\lambda(s^6_{3,u})=9.04$; $\lambda(s^6_{3,v})=0.31$; $\lambda(s^6_{3,\theta})=0.14$

A failure scenario of R^6 is

$F_h(R^6) = \{R^6_k \mid k=1,\dots,m_h\}$

$= \{f^6_{i,j,k} \,, g^6_{i,j,k} \mid k=1,\dots,m_h\text{-}1 \,; i=1,2,\dots,n \,; j=u,v,\theta\}$

and a set of all possible failure scenario

$F(R^6) = \{F_h(R^6) \mid h=1,\dots,p\}$

Note that the ring R^6 is again a just-stiff ring, any of its failure scenarios contains only one deteriorating event. Similarly we have

Failure scenario $F_h(R^6)$	$e(f^6_{i,j,k})$ or $e(g^6_{i,j,k})$	Damage demand $E[F_h(R^6)]$
$F_1(R^6)=f^6_{1,u,1}$	$e(f^6_{1,u,1})=5.80$	$E[F_1(R^6)]=5.80$
$F_2(R^6)=f^6_{1,v,1}$	$e(f^6_{1,v,1})=2.34$	$E[F_2(R^6)]=2.34$
$F_3(R^6)=f^6_{2,u,1}$	$e(f^6_{2,u,1})=3.99$	$E[F_3(R^6)]=3.99$
$F_4(R^6)=f^6_{2,v,1}$	$e(f^6_{2,v,1})=0.98$	$E[F_4(R^6)]=0.98$
$F_5(R^6)=f^6_{3,u,1}$	$e(f^6_{3,u,1})=4.64$	$E[F_5(R^6)]=4.64$
$F_6(R^6)=f^6_{3,\chi,1}$	$e(f^6_{3,v,1})=2.03$	$E[F_6(R^6)]=2.03$
$F_7(R^6)=g^6_{1,u,1}$	$e(g^6_{1,u,1})=?$	$E[F_7(R^6)]=?$
$F_8(R^6)=g^6_{1,v,1}$	$e(g^6_{1,v,1})=?$	$E[F_8(R^6)]=?$
$F_9(R^6)=g^6_{1,\theta,1}$	$e(g^6_{1,\theta,1})=?$	$E[F_9(R^6)]=?$
$F_{10}(R^6)=g^6_{2,u,1}$	$e(g^6_{2,u,1})=10.85$	$E[F_{10}(R^6)]=10.85$
$F_{11}(R^6)=g^6_{2,u,1}$	$e(g^6_{2,v,1})=0.38$	$E[F_{11}(R^6)]=0.38$
$F_{12}(R^6)=g^6_{2,\theta,1}$	$e(g^6_{2,\theta,1})=0.24$	$E[F_{12}(R^6)]=0.24$
$F_{13}(R^6)=g^6_{3,u,1}$	$e(g^6_{3,u,1})=9.04$	$E[F_{13}(R^6)]=9.04$
$F_{14}(R^6)=g^6_{3,u,1}$	$e(g^6_{3,v,1})=0.31$	$E[F_{14}(R^6)]=0.31$
$F_{15}(R^6)=g^6_{3,\theta,1}$	$e(g^6_{3,\theta,1})=0.14$	$E[F_{15}(R^6)]=0.14$

At this stage only cluster S^6_1 are not primitive cluster and we go to next step.

(3) We now look for the cluster S^6_1 and we see that

$$S^6_1 = \{ R^5 \}$$
$$R^5 = \{ D^5_i , S^5_i \mid i=1,2,3\}$$
$$=\{ d^5_{i,j} \ , \ s^5_{i,j} \mid i=1,2,3\,; j=u,v,\theta\}$$

where S^5_1, S^5_2 and S^5_3 are all primitive clusters and referring to Fig.7.7, they are members m_8, m_9 and m_{11}.

The principal stiffness coefficients for each joint D^5_i is as

$D^5{}_1$ (joint $j^5{}_4$) : $\lambda^5{}_{1,u}=3.99$; $\lambda^5{}_{1,v}=0.41$; $\lambda^5{}_{1,\theta}=0$

$D^5{}_2$ (joint $j^5{}_5$) : $\lambda^5{}_{2,u}=3.47$; $\lambda^5{}_{2,v}=1.95$; $\lambda^5{}_{2,\theta}=0$

$D^5{}_3$ (joint $j^5{}_7$) : $\lambda^5{}_{3,u}=3.99$; $\lambda^5{}_{3,v}=0.41$; $\lambda^5{}_{3,\theta}=0$

and for three primitive clusters we have

$\lambda(s^5{}_{1,u})=10.85$; $\lambda(s^5{}_{1,v})=0.38$; $\lambda(s^5{}_{1,\theta})=0.24$

$\lambda(s^5{}_{2,u})=6.78$; $\lambda(s^5{}_{2,v})=0.24$; $\lambda(s^5{}_{2,\theta})=0.06$

$\lambda(s^5{}_{3,u})=10.85$; $\lambda(s^5{}_{3,v})=0.38$; $\lambda(s^5{}_{3,\theta})=0.24$

Repeat the above process for ring R^5 and we have

Failure scenario$F_h(R^5)$	$e(f^5{}_{i,j,k})$ or $e(g^5{}_{i,j,k})$	Damage demand $E[F_h(R^5)]$
$F_1(R^5)=f^5{}_{1,u,1}$	$e(f^5{}_{1,u,1})=3.99$	$E[F_1(R^5)]=3.99$
$F_2(R^5)=f^5{}_{1,v,1}$	$e(f^5{}_{1,v,1})=0.41$	$E[F_2(R^5)]=0.41$
$F_3(R^5)=f^5{}_{2,u,1}$	$e(f^5{}_{2,u,1})=3.47$	$E[F_3(R^5)]=3.47$
$F_4(R^5)=f^5{}_{2,v,1}$	$e(f^5{}_{2,v,1})=1.95$	$E[F_4(R^5)]=1.95$
$F_5(R^5)=f^5{}_{3,u,1}$	$e(f^5{}_{3,v,1})=3.99$	$E[F_5(R^5)]=3.99$
$F_6(R^5)=f^5{}_{3,\chi,1}$	$e(f^5{}_{3,\theta,1})=0.41$	$E[F_6(R^5)]=0.41$
$F_7(R^5)=g^5{}_{1,u,1}$	$e(g^5{}_{1,u,1})=10.85$	$E[F_7(R^5)]=10.85$
$F_8(R^5)=g^5{}_{1,u,1}$	$e(g^5{}_{1,v,1})=0.38$	$E[F_8(R^5)]=0.38$
$F_9(R^5)=g^5{}_{1,\theta,1}$	$e(g^5{}_{1,\theta,1})=0.24$	$E[F_9(R^5)]=0.24$
$F_{10}(R^5)=g^5{}_{2,u,1}$	$e(g^5{}_{2,u,1})=6.78$	$E[F_{10}(R^5)]=6.78$
$F_{11}(R^5)=g^5{}_{2,v,1}$	$e(g^5{}_{2,v,1})=0.24$	$E[F_{11}(R^5)]=0.24$
$F_{12}(R^5)=g^5{}_{2,\theta,1}$	$e(g^5{}_{2,\theta,1})=0.06$	$E[F_{12}(R^5)]=0.06$
$F_{13}(R^5)=g^5{}_{3,u,1}$	$e(g^5{}_{3,u,1})=10.85$	$E[F_{13}(R^5)]=10.85$
$F_{14}(R^5)=g^5{}_{3,u,1}$	$e(g^5{}_{3,v,1})=0.38$	$E[F_{14}(R^5)]=0.38$
$F_{15}(R^5)=g^5{}_{3,\theta,1}$	$e(g^5{}_{3,\theta,1})=0.24$	$E[F_{15}(R^5)]=0.24$

(4) The results

(i) The minimal failure scenario for the ring R^5, from (3), is equal to

$$E_{min}[F(R^5)] = min\ \{ E[F_h(R^5)] \mid h=1,2,...,15\}$$
$$=E[F_{12}(R^5)]=0.09$$

where the corresponding failure scenario is $F_{12}(R^5)=g^5{}_{2,\theta,1}$, that is to form a pin within the cluster $S^5{}_2$ (member m_9). This is the easiest way to fail the ring R^5.

(ii) Bring the results from (3) back to (2) we have

$e(g^6{}_{1,u,1})=E[F_{13}(R^5)]=10.85$ thus, $E[F_7(R^6)]=10.85$

$e(g^6{}_{1,v,1})=E[F_{14}(R^5)]=0.38$ thus, $E[F_8(R^6)]=0.38$

$e(g^6{}_{1,\theta,1})=E[F_{15}(R^5)]=0.24$ thus, $E[F_9(R^6)]=0.24$

and the minimal failure scenario for ring R^6 is

$$E_{min}[F(R^6)] = min\ \{ E[F_h(R^6)] \mid h=1,2,...,15\}$$
$$=F_{15}(R^6)=E[F_{15}(R^6)]=0.14$$

where the corresponding failure scenario is $F_{12}(R^6)=g^6{}_{3,\theta,1}$, that is to form a pin within the cluster $S^6{}_3$ (member m_{12}). This is the easiest way to fail the ring R^6.

(iii) Similarly bring the results from (2) back to (1) those $e(g^l{}_{i,j,k})$ which has not been decided.

$$e(g^7{}_{2,u,1})=min\ \{E[F_3(R^6); E[F_5(R^6); E[F_7(R^6);\}$$
$$=min\{ 3.99; 4.64; 10.85\}=3.99$$
$$e(g^7{}_{2,v,1})=min\ \{E[F_2(R^6); E[F_4(R^6); E[F_8(R^6);\}$$

$=min\{ 2.34; 0.98; 0.38\} = 0.38$

$e(g^7_{2,\theta,1}) = min\ \{E[F_9(R^6); E[F_{12}(R^6); E[F_{15}(R^6);\}$

$=min\{ 0.24; 0.24; 0.14\} = 0.14$

Referring to the results in (1) we see so far the minimal value of damage demand is 0.09.

Cluster S^8_1 can be analysed using the same procedure to decide $e(g^8_{1,u,1})$, $e(g^8_{1,v,1})$ and $e(g^8_{1,\theta,1})$. We have found that none of these three values exceeds 0.09, and therefore the minimal failure scenario for the ring R^7 is

$$E_{min}[F(R^7)] = min\ \{ E[F_h(R^7)]\ |\ h = 1,2,...,15\}$$
$$= F_{15}(R^7) = E[F_{15}(R^7)] = 0.09$$

where the corresponding failure scenario is $F_{15}(R^7) = g^7_{3,\theta,1}$, that is to form a pin within the cluster S^7_3 (member m_9). This is the easiest way to fail the ring R^7.

Some conclusions can be drawn from the above example

(1) For a structural ring R^l at a level of definition l there are a number of ways to destroy the ring. Each way is a failure scenario.

(2) A failure scenario for a ring R^l consists of a sequence of deteriorating events occurring either adjacent to the joints or within the clusters making up the ring.

(3) The damage demand for a deteriorating event is dependant on the location on which it occurs.

(4) The damage demand required for a failure scenario of the ring R^l is dependant on the quality of the well-formedness of R^l.

(5) Among all of the possible failure scenarios of a ring R^l the above algorithm can identify the minimal failure. The minimal failure scenario indicates the easiest way to transform the ring into a mechanism.

7.7 Structural Vulnerability Analysis

Structural systems, like other engineering systems, are subjected to random damage or deteriorations, such as technical deficiencies, human errors, unexpected excessive loads, environmental influence (earthquake, heavy storm), etc. The damage may occur in the members or in the joints of a structure. The failure of these elements or units cause other units to fail which in turn lead to large structural clusters failing, ultimately causing the complete breakdown of the whole structure. A breakdown in a structures has a severe effect and risk on the users' activity and life. Although the damage involved might be measured by its cost, it is usually agreed that such events are unacceptable in a well developed community. It is, thus, desirable that a structural system is robust. It is useful if a structural engineer can identify how a structural system is vulnerable. The structural vulnerability implies the susceptibility of being damaged or deteriorated.

It has been mentioned in Section 1.2 that the emphasis of structural vulnerability analysis described in this thesis is not the usual one of analysing a structure under some given loading condition, rather it is to examine the quality of the well-formedness of the structural rings at various levels of definition within a structure. The algorithm described in this chapter does not relate to the real actions which occur in a structure to the losses of the degrees of freedom (DOF) in the structure. The point is that, at this stage, we are only considering the geometric stability of a structural system - the effects of loads (for example when a DOF is lost there will be a redistribution of stresses within the structure) are not being considered here. The mapping between actions and losses of DOF is for future work.

Up to now we have demonstrated that a structure is built up with a hierarchical set of structural rings. Each structural ring at a level of definition has a certain quality of well-formedness which implies its ability to resist damage or loading from any arbitrary direction. The structural vulnerability, therefore, may be measured by the quality of well-formedness of alternative structural rings which exist at different levels of hierarchy to perform a particular function.

The main purpose of the structural vulnerability analysis is thus the identification of the most vulnerable or critical rings together with the failure scenarios which might cause failure.

The algorithm developed in the previous sections then can perform this task. It can be applied recursively to the structural rings at various levels of the hierarchy and to identify all possibly failure scenarios for a structure.

Generally, however, for a complex structural system consisting of many structural rings at various levels of definition, the description of all possible failure scenarios is clearly a complex exercise. The use of a computer is thus essential and a computer program can be developed to implement the analysis.

Comerford (1989) has utilised the logic programming language PROLOG and developed the **SIPIT** system which is particularly powerful to perform pattern matching and symbol manipulation. A similar computing system can be developed using PROLOG to perform structural ring pattern matching and failure scenario identification. It was not intended, however, to develop such a computer programming in this work. That must be done in further development of this research project, but rather to provide the basic principles for structural vulnerability analysis.

Among all of the possible failure scenarios for a structural system a number of failure scenarios are of particular interest as far as the structural vulnerability in concerned, for example

(1) *The minimal failure scenario*:

The minimal failure scenario of a structural ring at level of definition l is the one in which the least damage demand is required to transform the structural ring into a mechanism. From the structural vulnerability point of view, if a structural ring can be destroyed by using a very little damage demand then this ring is a very vulnerable structure. The example in Section 7.6 has illustrated the minimal failures of the structural rings at various levels of definition.

(2) *The maximal failure scenario*:

The consequence of a failure scenario is to cause a structural ring to lose its integrity and the clusters contained in the ring to structurally disconnect from each other. This consequence can be described by the separateness of a structural ring.

Assuming that a structural ring R^l at a level of definition l consists of a number of clusters S^l_i $(i=1,...,n)$ and $Q(S^l_i)$ is the corresponding structural tightness. A cluster S^l_r $(1 \leq r \leq n)$ is chosen as a reference cluster. A failure scenario causes some of the clusters in the ring R^l to structurally disconnect from the reference cluster S^l_r. Then the *separateness of a structural ring* R^l with respect to that failure scenario is defined as

$$\gamma[\mathrm{F}_h(R^l)] = \Sigma Q(S^l_i) \,/\, \Sigma Q(S^l_j) \tag{7.19}$$

where $\Sigma Q(S^l_i)$ is the sum of the structural tightness of all clusters which structurally disconnect form the reference cluster S^l_r, and $\Sigma Q(S^l_j)$ is the sum of the structural tightness of all clusters which still structurally connects with the reference cluster S^l_r.

A reference cluster may be any cluster chosen for its importance or because it has the highest value of structural tightness. On earth the reference cluster would normally be the ground cluster S_G or any union cluster contained S_G. A reference cluster is denoted as S^l_r.

For example, consider a ring R^7 of Fig.7.6 and cluster S^7_1 is chosen as a reference cluster. If a failure scenario $F_h(R^7)$ causes clusters S^7_2 and S^7_3 disconnect form S^7_1 then the separateness of ring R^7 with respect to that scenario is

$$\gamma[F_h(R^7)] = [Q(S^7_2)+Q(S^7_3)]/\, Q(S^7_1) =$$
$$=(8.96+\ 1.68\times10^{-5})/14.26=0.62$$

The *effective consequence of a failure scenario* at a level of definition is thus defined as the ratio of the separateness to the total damage demand for that scenario.

$$\xi[F_h(R^l)] = \gamma[F_h(R^l)] \,/\, E[F_h(R^l)] \tag{7.20}$$

The *maximal failure scenario* is therefore one in which the least damage demand is required to cause the maximal number of clusters to structurally disconnect from a reference cluster at a given level of definition. The judgement is the value of the effectiveness of a failure scenario $\xi[F_h(R^l)]$.

In the above example the maximal failure scenario for ring R^7 is

$$\xi_{max}[\ _h\ ^7 \qquad _h\ ^7 \qquad _{min}\ \xi_h\ [F\ (R\ )] = \gamma[F\ (R\ )] \,/\, E\quad [F\ (R\ )]$$
$$=0.62/0.09=6.98$$

(3) *Any particular interesting failure scenarios*

Note that in the design of an engineering structure there are many complex factors to take into account. Thus the designer may well need to consider other possible failure scenarios for particular practical reasons or through observation, judgement, or physical analysis of the structure.

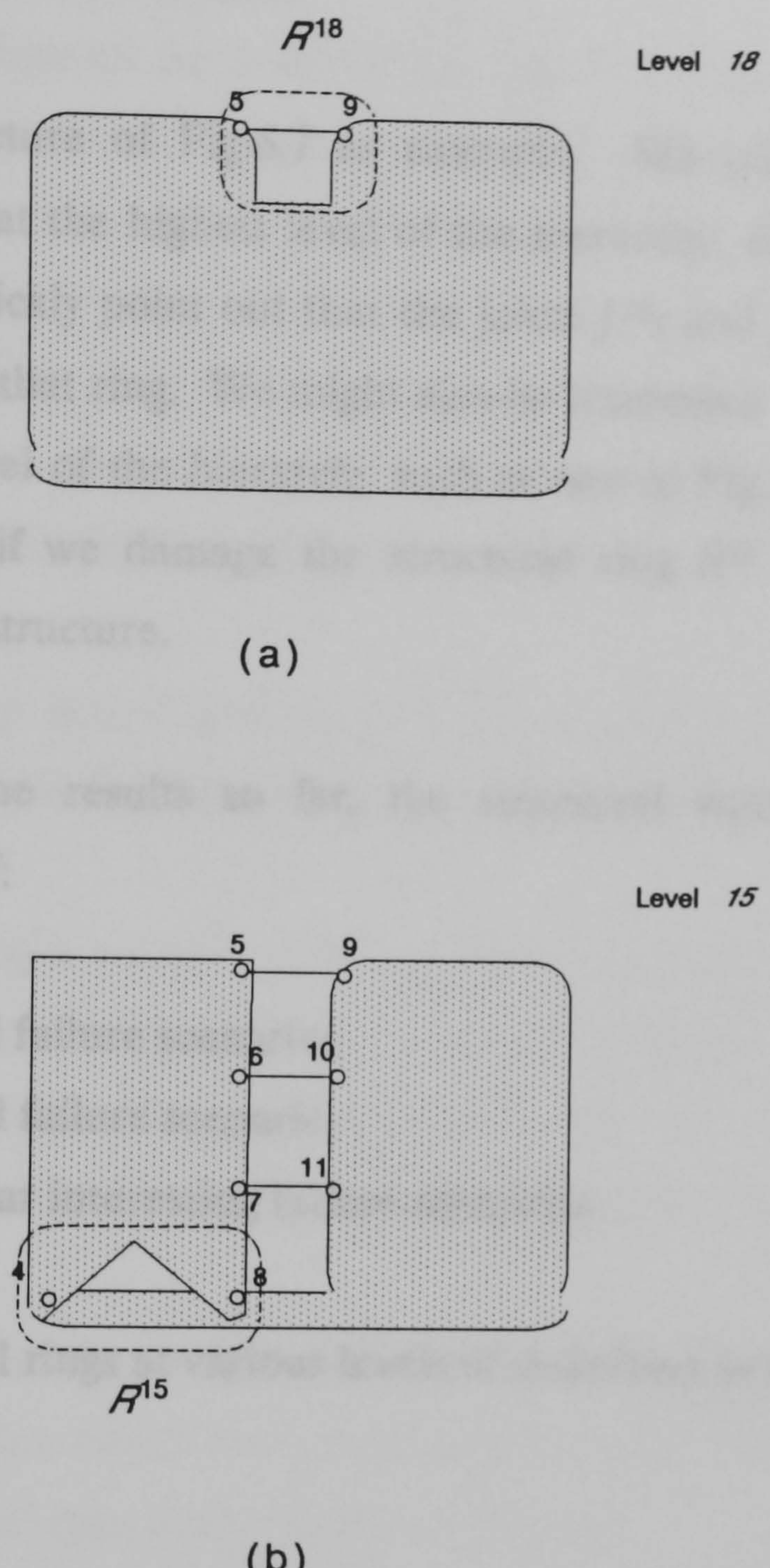


Fig. 7.8 Structured graphs

The decision will be made by engineers to pick up a structural ring at a given level of definition together with potentially interesting failure scenarios which may cause them failure.

Take the structure of Fig.6.7 as example. We might be interested in the structural ring at the highest level of the hierarchy, shown in Fig.7.8(a). From that we can quickly point out that the joints $j^{18}{}_5$ and $j^{18}{}_9$ are two critical joints with respect to that ring. We might also be interested in the structured graph at a particular level of the hierarchy such as one in Fig.7.8(b). We can see from Fig7.8(b) that if we damage the structural ring R^{15} we would cause severe damage to the structure.

Summarising the results so far, the structural vulnerability analysis is the identification of:

(1). the minimal failure scenario;
(2) the maximal failure scenario;
(3) any particular interesting failure scenarios.

for the structural rings at various levels of definition in the structure.

7.8 Robustness of a Structure

The robustness of a structure is related to the structure's strength or toughness of constitution; or the physical strength to resist damage or to experience particular deterioration without dissatisfying functional requirements. The structural vulnerability analysis can help us to identify how a structure is robust.

We will define a measure of the robustness of a structural ring to be the size of the damage demand. The most robust ring is the one with maximal damage demand.

The damage demand for a structural ring is dependant on the quality of the well-formedness of that ring. A well formed ring needs more damage demands to deteriorate it into a mechanism than a badly formed ring.

For a structural ring the robustness is the same as the damage demand and for a structure it is the minimal damage demand over all levels of definition i.e. there is one level of definition which is the weakest.

A structural ring is strong or sturdy with respect to a given deteriorating event if it is capable of resisting the actions that are attempting to cause that event.

A structural ring is robust with respect to a particular failure scenario if it is strong with respect to all or some of the deteriorating events within that failure scenario. A structural ring with a good quality of the well-formedness tends to be robust with respect to a set of possible failure scenarios.

A structure is therefore robust if it consists of a set of structural rings at various levels of definition which have good quality of well-formedness and are robust with respect to all conceivable failure scenarios.

A robust structure is therefore one which is strong against a variety of deteriorating events rather than a limited set of deteriorating events.

7.9 Summary and Conclusions

The structural vulnerability analysis is mainly concerned with the identification of the minimal failure scenario; the maximal failure scenario; and any particular interesting failure scenarios for the structural rings at various levels of the hierarchy of a structure. Through the structural vulnerability analysis we are able to identify the most vulnerable rings in a structure.

An analytical method has been developed in this chapter . It is designed in such a way that it is to examine the quality of the well-formedness of a ring at a level of definition and to identify all possible failure scenarios for the ring. This process can be recursively applied to structural rings at various levels of definition.

The concepts of a deteriorating event and damage have been presented in this chapter. The damage demand for a failure scenario provides a quantitative estimate of the robustness of a structural ring.

Finally the concept of the robustness of a structure has been examined.

Chapter 8

Conclusions and Recommendations

8.1 Objectives

The objectives of this chapter are:

1. To summarize the conclusions which can be drawn from this research.
2. To identify issues raised in this research which deserve further study.
3. To suggest applications for the techniques and methodology developed in this research.

8.2 Conclusions

1. A foundation of a theory of structural vulnerability has been laid. It is anticipated that structural vulnerability analysis will be able to play an important role in structural engineering, in particular, in the areas of structural reliability and safety assessments.

2. It has been shown that the purpose of structural vulnerability analysis, which is to enable the identification of the most vulnerable parts of a structural system, can be achieved.

3. Methods to identify (i) the minimal failure scenario; (ii) the maximal failure scenario; (ii) any particular interesting failure scenarios; for the structural rings of a structure at various hierarchical levels of

definition have been derived.

4. In the theory of structural vulnerability, it is assumed that any damage or faults which occur in a structural ring are triggered by deteriorating events. A deteriorating event is the result of actions which would cause the loss, by a structural ring, of the capacity to transmit a degree of freedom. A failure scenario consists of a sequence of deteriorating events which transforms a structural rings into a mechanism.

5. A measure of the effort which is required to make the occurrence of a specific deteriorating event or damage demand, has been derived. The damage demand of a failure scenario for a structural ring at a level of definition is dependant on the quality of well-formedness of that ring. The measure is based on the idea that a well formed structural ring needs more damage demand in order to deteriorate it into a mechanism than a badly formed ring.

6. The robustness of a structural ring is measured by the size of the damage demand. The most robust ring is the one with maximal damage demand.

7. For a structural ring at a level of definition the robustness is the same as the damage demand and for a structure it is the minimal damage demand over all levels of definition i.e. there is one level of definition which is the weakest.

8. The well-formedness of a structural ring is a measure of its ability to resist damage or loading from any arbitrary direction.

9. The quality of the well-formedness of a structural ring is dependant on (i) the orientation and stiffness of the members framing into the joints within the ring; (ii) the stiffness of the joints (whether pinned or fixed).

10. The quality of the well-formedness of a structural ring provides a quantitative estimate of the robustness of that ring. A well formed structural ring is more robust than a badly formed ring.

11. The quality of the well-formedness of a structural ring has been quantitatively defined as the sum of the determinants of the joints contained in the ring.

12. The determinant of the stiffness submatrix associated with a joint is a measure of the ability of the joint to resist damage and loading from any arbitrary direction. The determinant of a joint is equal to the products of the eigenvalues of the stiffness submatrix associated with that joint.

13. The deterioration hierarchy of structural rings, DHSR, is a very important part in the theory of structural vulnerability. The DHSR shows all possible ways in which a fully fixed ring can deteriorate into a mechanism.

14. A path through the DHSR is a failure scenario that indicates a particular way in which a structural ring deteriorates into a mechanism. If we can model a structure as a ring and match it in the DHSR then we can find out a set of all possible failure scenarios to fail the structure.

15. A structural ring is an over-stiff or just-stiff structure which can transmit forces around a closed loop. A structural ring represents a substructural system of a structure which is capable of resisting an arbitrary equilibrium set of applied forces and performs a desired function.

16. A structural system can be modelled as the objected oriented graph model--OOGM. The OOGM of a structure is a description of (i) the interactions between joint objects and members; (ii) the specific characteristics in terms of features, behaviours and constraints of that structure.

17. Associated matrices and symbol matrices are two useful tools which can be used to identify various structural rings within an OOGM.

18. Clustering techniques are useful in dealing with the analysis of a complex structural system.

19. The concept of structural cluster has been developed and applied in this research. A structural cluster consists of a set of structural rings which are more tightly interconnected to each other within the cluster than other rings outside the cluster.

20. The structural quality of a cluster can be quantitatively described by structural tightness which depends on the number of structural rings within it, the degree of overlap between them and the well-formedness of the rings.

21. The cluster algorithm developed in this thesis can be used to form structural clusters at various levels of definition and to transform a

structure | structure into a form of hierarchy.

22. For a structure, at each level of hierarchy, there exists a set of connected structural rings. The elements of these rings are clusters and complex joints.

23. A structure can be described at various level of definition in terms of sets of interconnected structural rings. That provides a basis for the structural vulnerability analysis.

24. A structural ring at a level of definition represents a substructural system which has its particular characteristic which does not exist at other levels in terms of its well-formedness and connectedness.

25. Given a structural ring at a level of hierarchical definition, any of its arcs can be regarded as the of a set of structural rings at lower levels of definition and itself can be (or be part of) an arc of a structural ring at a higher level of definition.

26. Structural rings at lower levels of hierarchy are a more detailed description of a structure than those at higher levels of definition.

8.3 Issues for Further Research

In Chapter 7, we have defined a deteriorating event as the results of actions which would cause the loss of the capacity to transmit a degree of freedom. The action has been described in very general sense such that it is either natural (eg. wind or earthquake) or human (eg sabotage). It has also been assumed that any damage or faults which occur to a structural ring are triggered by deteriorating events.

In practice, however, faults or damage which occur to a structure are caused by more complex factors. The members and joints of a structure could be damaged due to original defect, fatigue, accidental loading, etc. The failure modes could be such that, forming a plastic hinge in a member, buckling of a whole member, local shear failure, and brittle fracture or fatigue fracture.

The relationship between the real damage and a deteriorating event has not been fully studied. That deserves further study.

It was mentioned in Chapter 7 that for a complex structure the identification of all possible failure scenarios is a complex exercise. The use of a computer is essential and it is desirable to develop a computer program for implementing the process of structural vulnerability analysis. But in this thesis this computer program has not yet been developed.

Comerford (1989) has utilised the logic programming language PROLOG and developed the SIPIT system which is particularly powerful to perform pattern matching and symbol manipulation.

So the next step is to develop a similar computing system for implementing the process of structural vulnerability analysis by using a appropriate language such as PROLOG or C.

In this thesis we have confined ourselves to the two dimensional structures. From practical point of view, the analysis of the vulnerability for three dimensional structures are even more important. Clearly the structural paths or loops in three dimensional structures are different from those in two dimensional structures. But the principles and techniques developed in this thesis can extend to the vulnerability analysis of three dimensional structures.

8.4 Possible Applications of the Developed Techniques

There are many possible areas for application of the ideas, techniques and principles developed in this research. One of them is to apply the principles of the structural vulnerability analysis to the analysis of structural system reliability.

In structural system reliability theory, two fundamental types of systems, namely series systems and parallel systems (Ditlevsen & Bjerager, 1986) have attracted most attention. A structure is modelled as a series system if it is in a state of failure whenever any of its component fails. A structure is a parallel system if it will only fail when all components in that system fail. Then the predicating structural system reliability has been formulated as determining the system reliability from the component reliability (Moses, 1990).

In practice, however, there are few cases that a complex structure can be modelled by the idealized series system, or parallel system. The research in this thesis has shown that a complex structure is actually built up with structural rings at various levels of definition. The structural safety assessments will mainly depend on the quality of the well-formedness of the rings at various levels of definition making up the structure.

It is argued here that for a large-scale structures, the system reliability should be divided into two phases

(1) Structural vulnerability analysis, which means the identification of the most vulnerable, or critical structural rings in a structure together with failure scenarios which cause failure.

(2) Probabilistic calculation to assess the failure probability or safety index for each individual failure scenario and subsequently combining these into a single system reliability assessment.

The research in this thesis is concerned with the first phase i.e. the structural vulnerability analysis. The second part of the system reliability analysis i.e probabilistic calculation therefore can be done by using the probabilistic theory such as the interval probability theory developed by Cui and Blockley (1990).

Another important application of the structural vulnerability analysis is to monitor and test a structure. Once the most critical parts of a structural system are identified they can be suitably protected and monitored by sensing the conditions on those critical elements, such as (i) loading (e.g. pressure, forces); (ii) atmospheric conditions (e.g. temperature, moisture, corrosive conditions); (iii) structure behaviour (e.g. strain and acceleration); and (iv) material condition (e.g. thickness losses).

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APPENDIX A

Eigenvalues and Eigenvectors

In this appendix we will briefly review the concepts of eigenvalue and eigenvector and some of their properties (Williams, G. 1976) (Anton, 1984).

Definition. Let C be and $n \times n$ matrix, then a non-zero vector $\mathbf{x}$ in $\mathbf{R}^n$ ($\mathbf{R}^n$ is defined to be the set of all ordered collections of n real numbers) is said to be an *eigenvector* of C if there exists a scalar λ such that

$$C\mathbf{x} = \lambda\mathbf{x} \tag{A.1}$$

λ is called the *eigenvalue* of C corresponding to the eigenvector v. The set of all eigenvalues is called the *spectrum* of C.

To find the eigenvalues of an $n \times n$ matrix C we rewrite (A.1) as

$$(C - \lambda I_n)\mathbf{x} = \mathbf{0} \tag{A.2}$$

where I_n is $n \times n$ unit matrix.

For λ to be an eigenvalue, there must be a nonzero solution of this equation. The (A.2) will have a nonzero solution if and only if the determinant of the matrix $(C - \lambda I_n)$ is equal to zero.

$$det(C - \lambda I_n) = 0 \tag{A.3}$$

This is called the *characteristic equation* of C; the scalars satisfying this equation are the eigenvalues of C. When expanded, the determinant $det(C - \lambda I_n)$ is a polynomial in λ called the *characteristic polynomial* of C.

Theorem 1. If C is an $n \times n$ matrix, then the following are equivalent.

(a) λ is an eigenvalue of C.
(b) The system equation $(C-\lambda I_n)\mathbf{x}=\mathbf{0}$ has nontrivial solutions.
(c) There is a nonzero vector $\mathbf{x}$ in $\mathbf{R}^n$ such that $C\mathbf{x}=\lambda\mathbf{x}$.
(d) λ is a real solution of the characteristic equation $det(C-\lambda I_n)= 0$.

Definition. A square matrix C is called *orthogonally diagonalizable* if there is an orthogonal matrix P such that $H=P^{-1}CP(=P^tCP)$ is diagonal; the matrix P is said to orthogonally diagonalize C, where H is the diagonal matrix having the eigenvalues λ_1, λ_2, ..., λ_n of C on the main diagonal.

Theorem 2. If C is an $n \times n$ matrix, then the following are equivalent.

(a) C is orthogonally diagonalizable.
(b) C has an orthogornal set of n linearly independent eigenvectors.
(c) C is symmetric.

Theorem 3.

(a) The characteristic equation of a symmetric matrix C has only real roots.
(b) If an eigenvalue λ of a symmetric matrix C is repeated k times as a root of the characteristic equation, then the eigenspace corresponding to λ is k-dimensional.

If C is a symmetric matrix, according to **Theorems 2** and **3**, the determinant of matrix C would be

$$det(C) = det(PHP^{-1}) = det(H) = \lambda_1 \times \lambda_2 \times, \dots, \times \lambda_n \tag{A.4}$$

and we also know that the sum of all eigenvalues is a constant

$$\text{Constant} = \lambda_1 + \lambda_2 +, \dots, + \lambda_n \tag{A.5}$$

where λ_i $(i=1,\dots,n)$ is the eigenvalue of C.