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Reliability Modeling and Prediction of Wireless Multi-Hop Networks with Correlated Shadowing

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Abstract—Shadowing losses on proximate wireless links have been experimentally proven to be highly correlated in various scenarios. However, most of the existing works on the reliability modeling of Wireless Multi-Hop Networks (WMHNs) assume independent link shadowing. Neglecting shadowing correlations could lead to inaccurate network simulation results and unreliable wireless system design. In this paper, we present a more realistic reliability model of WMHN that incorporates correlated link shadowing. In particular, we use the correlated shadowing model that was developed in our previous work. This model enables the efficient generation of spatially correlated shadowing. It has been proved to agree well with the literature in terms of statistical properties. The proposed network model allows us to predict the reliability metrics of WMHNs and study network designs that can lessen the effect of correlated shadowing. Numerical results indicate that it is important to use the correlated shadowing model when evaluating the reliability of densely deployed large-scale multi-hop networks. We also present some numerical results to show the influence of shadowing correlation on redundant node deployment. This paper makes a small but fundamental step towards reliable wireless multi-hop network design in the context of correlated link shadowing.

Keywords—wireless multi-hop networks; shadowing; spatial correlation; reliability modeling and prediction

I. INTRODUCTION

Recently, there has been increasing interest in wireless networks for industrial and medical applications [1], [2]. In these applications, data packets that fail to be received correctly may result in commercially significant damage, and even loss of life. Reliable transmissions assume special importance in these critical applications.

The reliability of a single wireless link may be inadequate due to attenuation, shadowing, fading, interference, noise and other inherent characteristics of the wireless channel. One common way to attain the high reliability levels demanded by emerging wireless applications is to employ a Wireless Multi-Hop Network (WMHN) to provide multiple redundant paths. In wireless multi-hop networks, including mesh and sensor networks, devices assist each other in transmitting packets through the network. A node can send and receive messages, as well as relay messages for its neighbors. Before the deployment of a WMHN, reliability modeling may be used to predict the performance of the network. These predictions can then be used to evaluate design feasibility, compare alternatives, trade off system parameters and track reliability improvement.

In recent decades, network reliability has been studied extensively in the context of wired networks [3]-[5]. Most of these papers model the network as a random graph and study the k-terminal reliability of the graph. The k-terminal reliability, defined as the probability that there exists a path that connects k nodes in the network, is the most general network reliability problem. Because determining k-terminal reliability is very time-consuming, most existing works focus on speeding up the calculations. Many algorithms, including path/cut-based [3], factoring theorem-based [4] and ordered binary decision diagram-based (OBDD-based) algorithms [5], have been presented to solve network reliability problems. Nevertheless, these evaluation methods are not generally applicable to wireless networks. The main reason for this is that these papers are based on the assumption that all failure events are mutually s-independent. However, in a wireless multi-hop network, proximate links may have correlated failures.

The connectivity problem of wireless multi-hop networks has attracted increasing attention [6]-[8], [10], [11]. Gupta and Kumar [6] presented a detailed analysis of wireless network connectivity based on the disk model. This model assumes that nodes that are within a certain distance of each other have a reception probability of one. Outside range, the successful transmission probability is zero. However, the disk model is idealistic and ignores the stochastic nature of the wireless channel. Bettstetter and Hartmann [7], and Hekmat and Mieghem [8], studied the connectivity of wireless *ad hoc* networks using a radio model that incorporates shadowing. The log-normal shadowing model considers link shadowing losses in dB to be independent and identically distributed (i.i.d.) Gaussian random variables with zero mean. Their results show that a higher variance of shadowing improves the network connectivity. Nevertheless, these papers do not consider the effect of shadowing correlations. Several empirical studies have shown that shadowing losses on proximate links are significantly correlated in various scenarios [9], [10]. Recently, Agrawal and Patwari [10] investigated spatially correlated link shadowing in multi-hop networks and developed the correlated network shadowing (NeSh) model. The same authors then applied the NeSh model to the connectivity analysis of some small multi-hop networks, e.g., 3 or 4 nodes arranged in a line [10], and 16 nodes arranged in grid structures [11]. Their

results show that the correlated shadowing suggests a significant negative effect on the connectivity of WMHNs.

In our previous work [12], we improved the NeSh model in terms of both accuracy and efficiency. Our model enables the generation of spatially correlated shadowing for meshed links in multi-hop networks without a large computational overhead. It has been shown to agree well with the literature in terms of the statistical properties, i.e., for a single link, the resulting shadowing loss is a zero-mean Gaussian random variable with distance-dependent variance. Moreover, the correlation between link shadowing can be adjusted to match empirical results by properly setting some model parameters, e.g., de-correlation distance, and near field of a node.

In this paper, we apply for the first time the enhanced correlated shadowing model [12] to the reliability analysis of wireless multi-hop networks. In particular, we first model the WMHN as the combination of an undirected graph and the correlated channel model. We then define reliability metrics and propose corresponding evaluation methods for WMHNs with and without redundant relay nodes. The main contribution of this work is a method to evaluate the reliability metrics of wireless multi-hop networks considering spatially correlated shadowing. This, in turn, allows us to study network designs that can lessen the effect of correlated link shadowing.

The paper is organized as follows: in Section II, we describe the system models and assumptions made in this study. Section III describes the network reliability metric together with the corresponding evaluation method. In Section IV, we address the effect of the correlated shadowing model on network reliability prediction. We also present some numerical results to show the significance of redundant relay node deployment on the reliability improvement, and discuss wireless multi-hop network designs in the context of correlated shadowing. Finally, we conclude and discuss future research directions in Section V.

II. SYSTEM MODELS AND ASSUMPTIONS

A. Network Model

In wireless systems, link quality is affected by transmission and reception capabilities, and the characteristics of the wireless channel. When the transmitted signal propagates through the channel, part of the energy may be lost through absorption, reflection, diffraction and scattering. Link failure occurs when the received signal power drops below a certain threshold. In this work, we assume that all nodes have the same transmit power P_t and the same receiver sensitivity P_{thr}. We also assume a symmetrical wireless channel, i.e., for a particular wireless link $L_{i,j}$, the received power $P_{i,j}$ will be the same no matter the direction of transmission. We further define that the link $L_{i,j}$ fails if $P_{i,j} < P_{thr}$. Therefore a wireless multi-hop network with a number of nodes spread over a certain area can be described as an undirected graph $G = (V, E)$, where V indicates the set of all nodes, and E stands for the set of all successfully communicating wireless links.

B. Channel Model

A wireless channel is typically modeled as a combination of three components: path loss, shadowing and fading. The mean path loss is mostly determined by the distance between the transmitter and the receiver. Shadowing is caused by obstacles in the communication path and is defined as the fluctuation in the received power averaged over a few tens of wavelengths. Fading is caused by multi-path propagation and its statistical properties have been studied extensively in the literature [13]. Many methods, ranging from wideband systems to multi-antenna systems, have been developed for combating fading. Therefore, we assume that fading is managed through clever transceiver design. In this section, we describe the channel model as combination of two components: path loss and shadowing.

To describe the channel, we consider two nodes A and B that are located at a relative distance of d_{AB}. Node A transmits a signal with power P_t dBm. The mean received power at node B is given by an empirical formula as

$$\overline{P}_r(d_{AB}) = P_t - PL(d_0) - 10\alpha \log\left(\frac{d_{AB}}{d_0}\right) \qquad (1)$$

where $PL(d_0)$ is the path loss at a short reference distance d_0 from the transmitter antenna, and α is the path loss exponent. The path loss exponent depends on the environment. The value of α is 2 for free space, less than 2 for waveguide-like environment and larger when obstructions are present [13].

Because of the shadowing effect, the received power will vary from its mean. Shadowing on a dB scale is commonly modeled by a zero-mean Gaussian random variable. Thus augmenting (1) to include contributions from shadowing gives

$$P_r(d_{AB}) = P_t - PL(d_0) - 10\alpha \log\left(\frac{d_{AB}}{d_0}\right) - X_\sigma \qquad (2)$$

where X_σ is a zero-mean Gaussian random variable with standard deviation σ.

In reality, link shadowing losses are spatially correlated and (2) fails to capture this. In this paper, we use a correlated shadowing model, adding an important element of realism to modeling capabilities. The correlated shadowing model is described in detail in our previous work [12]. It has been shown to agree with the empirically-observed link shadowing properties in WMHNs. In this section, we outline briefly its main components and properties for the sake of completeness.

The main assumption of this model is that the shadowing losses experienced on links in a wireless multi-hop network are a result of signals passing through an underlying shadowing map. We propose the correlated shadowing model as combination of the shadowing map and the link shadowing function. The map models the shadowing environment in which the wireless network operates. The function allows correlated link shadowing losses to be calculated from the underlying map. By connecting the link shadowing losses with the environment, we preserve the correlations that exist between wireless links in the real world.

As in prior literature [10], we assume that the underlying shadowing map is a stationary and isotropic Gaussian random field with zero-mean and exponentially-decaying spatial correlation. In this paper, we simulate a Gaussian random field $f(x)$ on a rectangular grid of size $n \times m$ as the shadowing map. Let Δd denote the spacing along the grid. The shadowing map will cover a simulation area of size $L \times W = n\Delta d \times m\Delta d$. In particular, we will generate a zero-mean Gaussian random process on each of the grid points $\{(i\Delta d, j\Delta d), i=0,\ldots,n-1, j=0,\ldots,m-1\}$ corresponding to a covariance function given by

$$\mathrm{cov}(f(s), f(t)) = \sigma^2_s \exp\left(-\frac{\|s-t\|}{\delta}\right) \quad (3)$$

where σ^2_s is the variance of the shadowing map, δ is the de-correlation distance, and $\|s-t\|$ is the Euclidian distance between s and t.

Although it is intuitively correct to approximate the link shadowing loss as the weighted sum of individual shadowing values along the communication path, the weighting coefficients need to be determined carefully. In the real world, obstacles that are close to the antenna have higher impacts on link shadowing. This is because the relative loss of diffracting or scattering over or around the object is more for the obstacles near the antenna. Therefore, the weighting coefficients must be distance-dependent to reduce the impact of obstacles in the middle of a link on the shadowing loss. We further abstract this empirical observation by assuming that the link shadowing is dominated by the shadowing values in the near field at both ends of the link. The following function is proposed for the shadowing loss X_{AB} of the link L_{AB} as

$$X_{AB} = \frac{1-\exp\left(-\dfrac{d_{AB}}{\delta}\right)}{\sqrt{2}\sqrt{1+\exp\left(-\dfrac{d_{AB}}{\delta}\right)}}(f(A)+f(B)) \quad (4)$$

where d_{AB} is the Euclidian distance between nodes A and B, and $f(A)$ and $f(B)$ represent the shadowing values in the near field of nodes A and B respectively.

Finally, the total received power at node B considering both path loss and correlated shadowing becomes

$$P_r(d_{AB}) = P_t - PL(d_0) - 10\alpha \log\left(\frac{d_{AB}}{d_0}\right) - X_{AB} \quad (5)$$

where X_{AB} is calculated by (4).

III.　Network Reliability

In this paper, we use simulations to generate graphs that represent different deployments of a wireless multi-hop network. The reliability of the network can then be studied through properties of the graphs. In this section, we first describe the simulation process of the graph. We then introduce

the network reliability metric together with the evaluation method.

A.　Generating A Graph

Our simulator takes the following steps to generate a graph:

1. Specify transmit power P_t, receiver sensitivity P_{thr}, and path loss exponent α;

2. Set the parameters of the underlying shadowing map, which include: dimensions of the simulated region L and W, spatial resolution Δd, shadowing standard deviation σ_s, and de-correlation distance δ;

3. Generate a shadowing map with the covariance given in (3) using the Circulant Embedding method [14];

4. Set node locations, establish links and calculate the path loss on each link using (1);

5. Calculate the link shadowing losses from the pre-generated shadowing map using (4);

6. Calculate the total received power in (5) and determine whether a link has failed by checking if $P_r < P_{thr}$;

7. Delete the failed links from the original set of links.

Some guidelines on parameter settings are given in [12].

B.　K-Terminal Reliability

Consider the multi-hop network illustrated in Fig. 1. This network can be described as an undirected graph G that includes 3 source nodes (N2, N3, N4), one sink node (N1), and one redundant relay node (R). Under regular network operation, packets are generated by each source node and transferred to the sink node. However, the redundant node only relays messages for its neighbors, and does not provide any useful information itself. If any source node is disconnected from the sink node, the network has failed. In other words, the network is only considered reliable (i.e., operating successfully) if all the source nodes and the sink node are connected, i.e., all source and sink nodes are critical, and can successfully exchange information when required. Redundant nodes are not considered critical.

This is related to application scenarios in industrial monitoring and control, where wireless sensors (source nodes) monitor some critical physical parameters of the system (e.g., temperature, pressure) and periodically send the data back to the central control room (sink node). The control system then makes decisions according to the received data, e.g., the emergency shutdown of a machine or plant to avoid explosion.
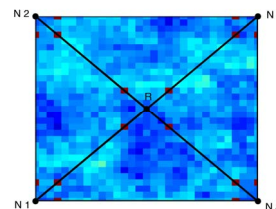

Fig. 1. A wireless multi-hop network in an underlying environment.

In this case, the reliability measure of interest is the probability that there exists at least one operational path that connects all the critical nodes in the network. This can be seen as the *k*-terminal reliability, where *k* is equal to the number of critical nodes. When there is no redundant relay node in the network, it becomes the all-terminal reliability.

C. Evaluation Methods

We evaluate the network reliability metric by simulation, as in [11]. In particular, we generate *N* graphs to represent different deployments of a wireless multi-hop network by following the simulation process described in Section III-A. Each graph is then checked for connectedness between the *k* critical nodes. Finally we count the number of graphs where the *k* critical nodes are connected. The *k*-terminal reliability, which is a success probability for operation in a random environment, can be estimated by [15]

$$P_{success} = \frac{1}{N}(graph\_count) \qquad (6)$$

As *N* approaches infinity, $P_{success}$ approaches true reliability.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, a range of numerical results are presented using the proposed network model. *k*-terminal reliability is studied with different parameter settings relevant to correlated shadowing. We include the result of using the independent shadowing model for comparison. We also present some results to show the influence of shadowing correlation on redundant node deployment.

For each data point on the graph, we take the mean value for the reliability of 10 independently performed experiments. The error bars indicate the 95% confidence interval. The size of each experiment is *N*=1000, and the reliability is calculated using (6). In each experiment, we vary the transmit power of the node. This allows us to study the effects of increasing link reliability. We have chosen some node parameters according to the datasheet of TelosB [16], where the radio operates at 2.4GHz, the transmit power P_t can range from -24dBm to 0dBm, and the receiver sensitivity P_{thr} is -90dBm. For the underlying environment, we set the path loss exponent $\alpha = 3$, and the de-correlation distance $\delta = 2m$. These are valid assumptions for an indoor environment, where an obstruction may have the size of 2m×2m. Because we approximate the shadowing value by returning the value of the nearest grid point, the spatial resolution must be much less than the distance up to which the shadowing value remains approximately constant. The empirically accepted bound is a few tens of wavelengths. Therefore we set spatial resolution $\Delta d = 1m$. For the near field of node, defined as $a\delta$ from the node location in [12], *a* is chosen to be 0.5, 1, and 1.5 corresponding to link distance of 10, 16, and 30m to match the empirically observed link correlation coefficients [12].

A. Effects of the Correlated Shadowing Model

We first consider a WMHN of 25 nodes deployed regularly in a 5 by 5 grid. The nodes are spaced at 10m. Fig. 2 plots the network reliability against varying transmit power for different de-correlation distances. As δ goes to zero, i.e., no correlation in the shadowing environment, the correlated model is equivalent to the independent model. We also observe that for the same transmit power, the all-terminal reliability decreases as δ increases. This is because a larger obstruction in the environment is likely to cause more links to fail simultaneously, resulting in lower network reliability. Fig. 3 shows the all-terminal reliability of the network in environments of the same path loss exponent but different level of shadowing. For the same transmit power, the network reliability decreases with increasing shadowing variance. This is because larger link shadowing increases the failure probability of the link, and thus the overall failure probability of the network.

We then fix the shadowing environment, and vary the distance between two nodes. Fig. 4 illustrates the all-terminal reliability of network of varying link distances. For the same
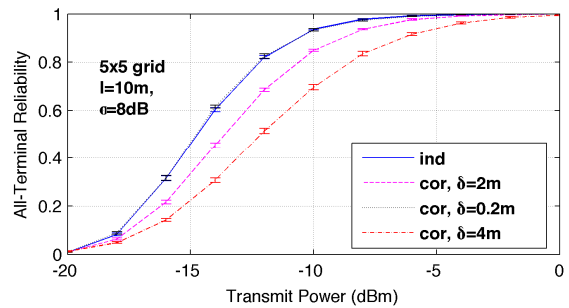

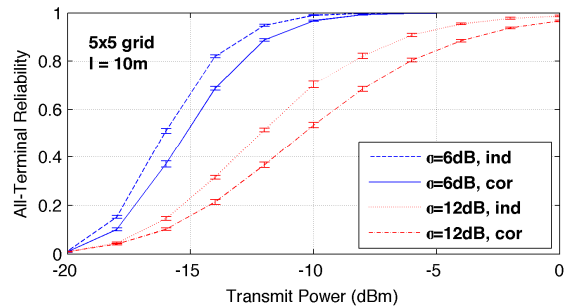Fig. 2. Effects of de-correlation distance on all-terminal reliability.

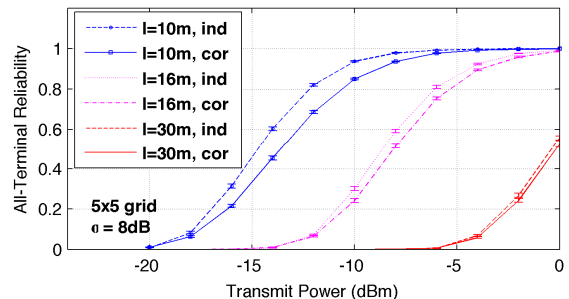Fig. 3. Effects of shadowing variance on all-terminal reliability.

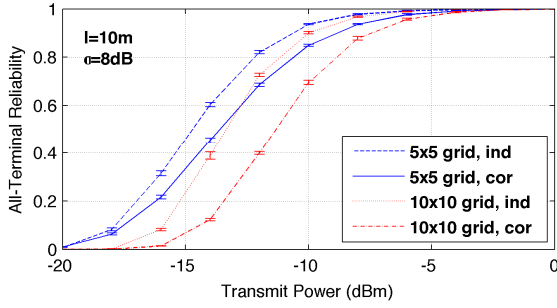Fig. 4. Effects of inter-node distance on all-terminal reliability.

Fig. 5. Effects of network size on all-terminal reliability.

Fig. 7. K-terminal reliability after introducing the redundant node.

Fig. 8. Percentage overestimation of k-terminal reliability improvement.

transmit power, the network reliability decreases as the inter-node distance increases. Also when links are closer, their shadowing correlations become stronger. The negative effect of correlated shadowing on network reliability is magnified.

Finally, we study the effect of different network sizes on all-terminal reliability. Because of the efficient modelling technique, we are able to extend the reliability analysis to a network size of 100 nodes or more. As shown in Fig. 5, it is harder to achieve all-terminal reliability for a larger sized network even if all other parameters are kept the same.

A common observation in all these results is that the difference between reliability using the independent model and the correlated model is not constant. For the higher transmit power, the absolute difference becomes smaller. This is because when the link has been designed with a large margin, it is more robust to shadowing. The probability of link failures due to shadowing, either correlated or independent, is small. Therefore, in this case, the correlated and independent model will produce similar results.

However, in practice, the nodes do not transmit at maximum power in order to minimize interference to other nodes, and to conserve energy to prolong network life. Therefore it is of critical importance to use the correlated shadowing model to ensure an accurate prediction of the network reliability, especially in densely deployed networks with a large number of nodes.

B. Influence on Redundant Node Deployment

In this section, we consider a simple 4-node multi-hop network with a redundant relay node. As shown in Fig. 6, the redundant node can be deployed at two different locations: one in the center of the network; the other next to the source node. The 4-terminal reliability results for networks without a
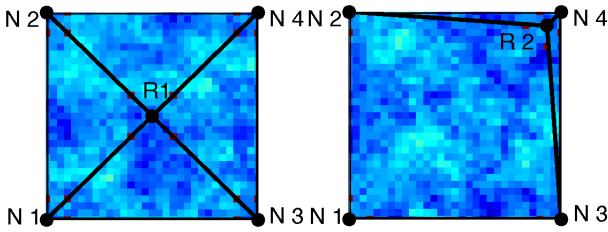

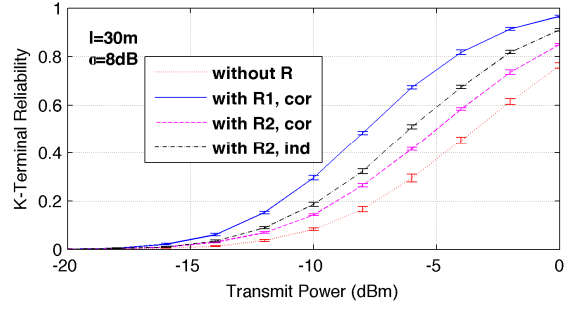Fig. 6. Networks with two different redundant node deployments.
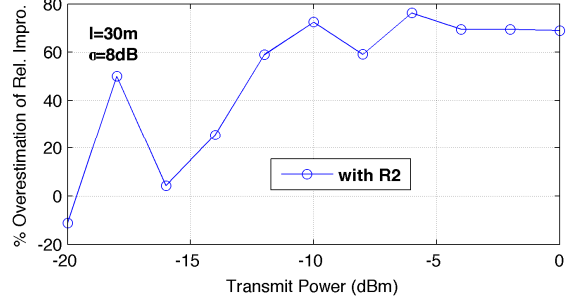
redundant node, with redundant node R1, and with redundant node R2 are given in Fig. 7. It can be seen that the deployment of R1 results in a larger improvement in network reliability. R1 introduces more "effective" redundant paths. Because R2 is too close to one of the original nodes, if that node is shadowed, there is a large chance that R2 is also shadowed. Nevertheless, an independent model fails to capture this. Fig. 8 illustrates the percentage-overestimation of reliability improvement after the placement of R2. The result indicates that not considering shadowing correlations the deployment of a network may not have the desired level of reliability.

For a more complex network, finding a good position for relay node in the context of correlated shadowing might become quite an interesting problem for future research.

V. CONCLUSIONS

This paper presents a mathematical model of the wireless multi-hop network, specifically, the combination of an undirected graph and the correlated channel model. The proposed network model allows us to predict the reliability metrics of WMHNs and study network designs that can lessen the effect of the correlated shadowing. Numerical results indicate that it is important to use the correlated shadowing model when evaluating the reliability of densely deployed large-scale multi-hop networks. We also present some numerical results to show the influence of correlated shadowing on redundant node deployment. Future work will incorporate higher layer protocols into the existing model and evaluate the protocol performance. Another interesting direction to take the research further is to study optimal relay node placement in a more complex network.

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