Orbit and gravity field
common versus sequential a alysis

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Gravity field mapping methology from GR E

and future gravity missions

Hotine Marussi 2013, Roma

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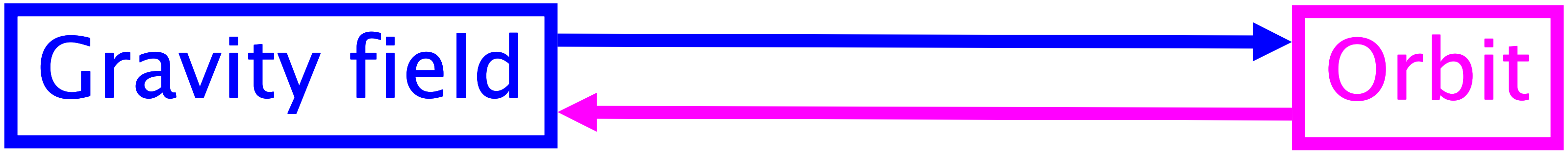
- Gravity field and Orbit
- Signal and Noise in monthly models (GRACE)
- Timewise analysis: the concept of Lumped Coefficients
- Resonance
- Discussion

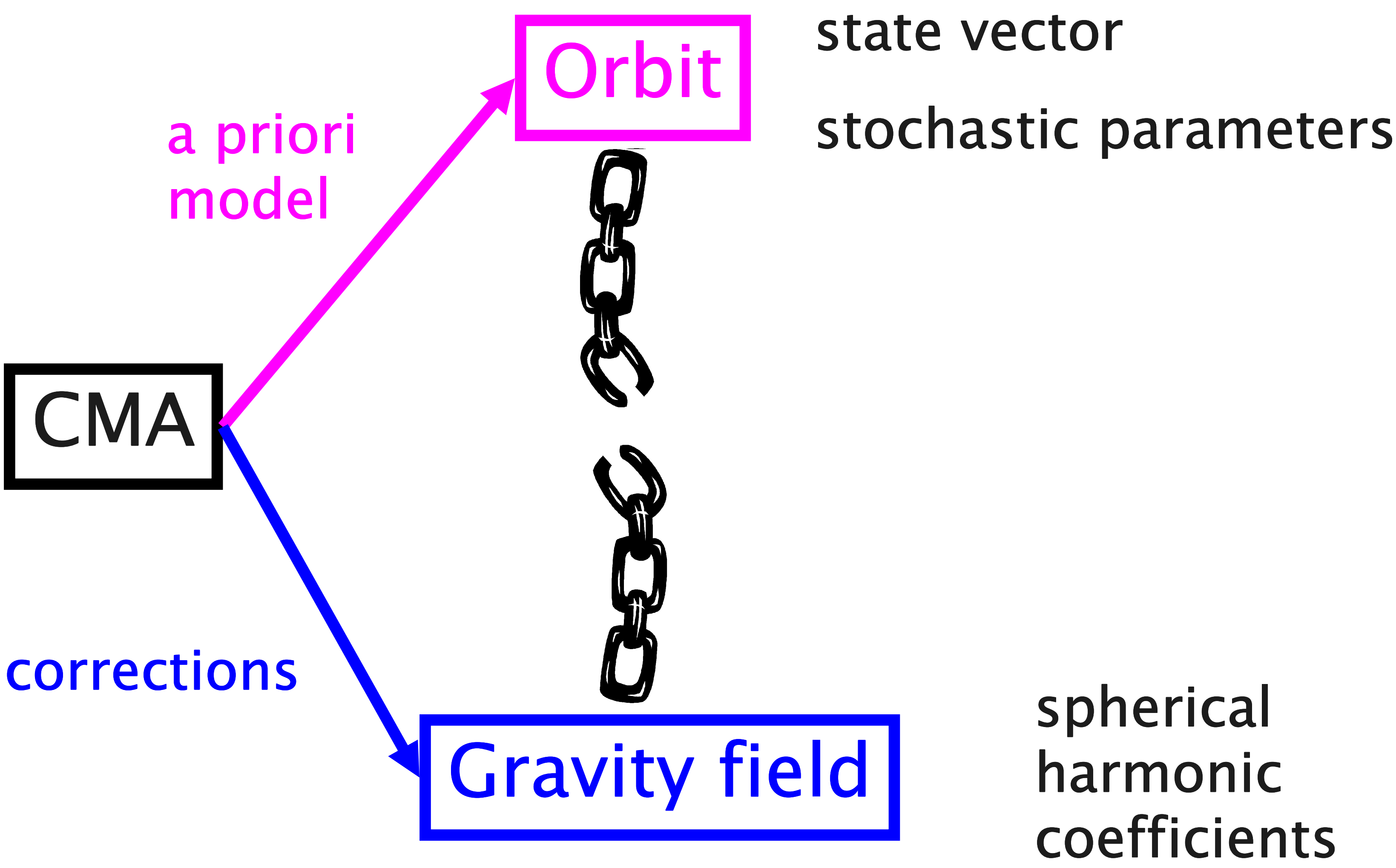
Gravity field and Orbit



Non-linear parameter
estimation problem

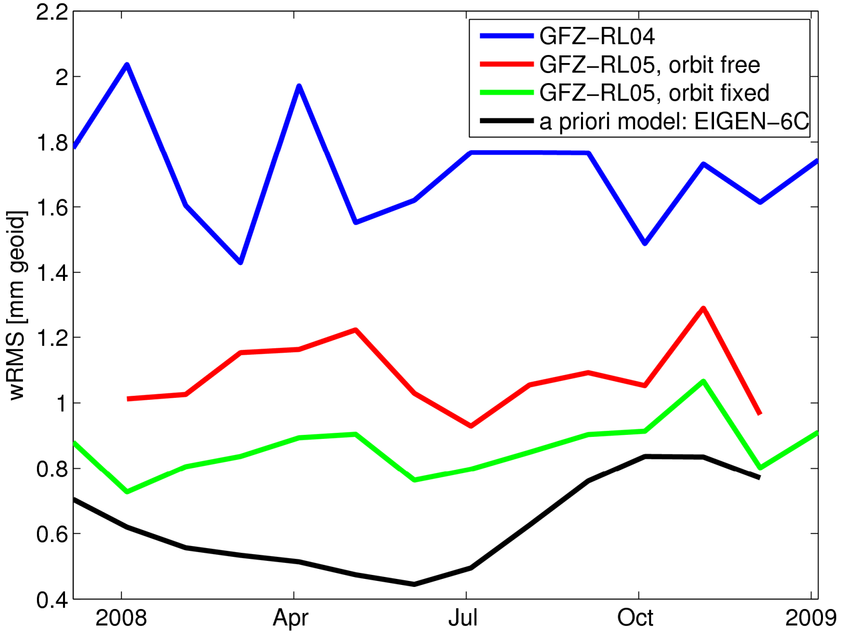
- A priori model (linearization)
- Observations
- Regularization (a priori knowledge via pseudo-observations)

A generalized orbit determination problem



Signal and Noise in monthly models (GRACE)

Noise



A priori model:

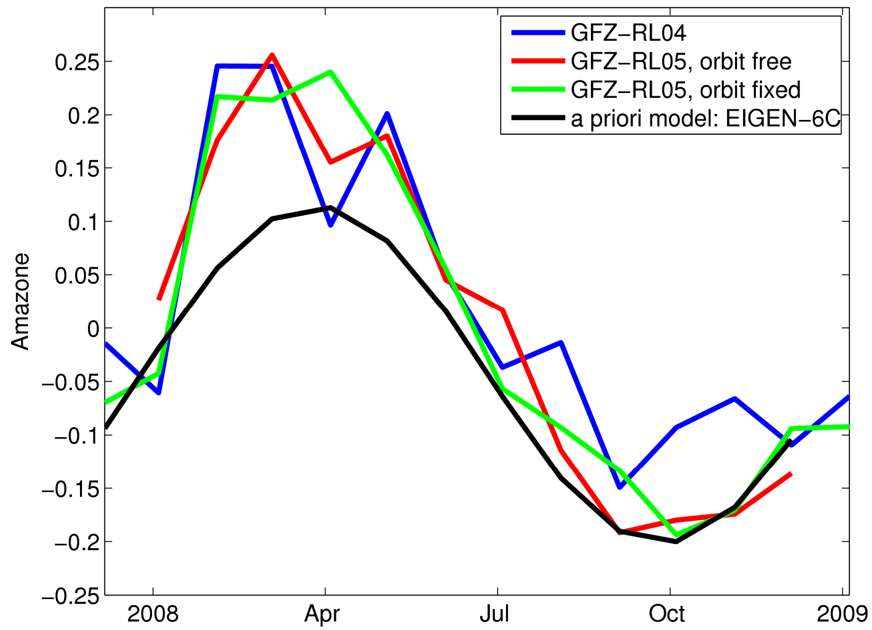
EIGEN–6C

(incl. time–var. d/o 50)

Stochastic accelerations:

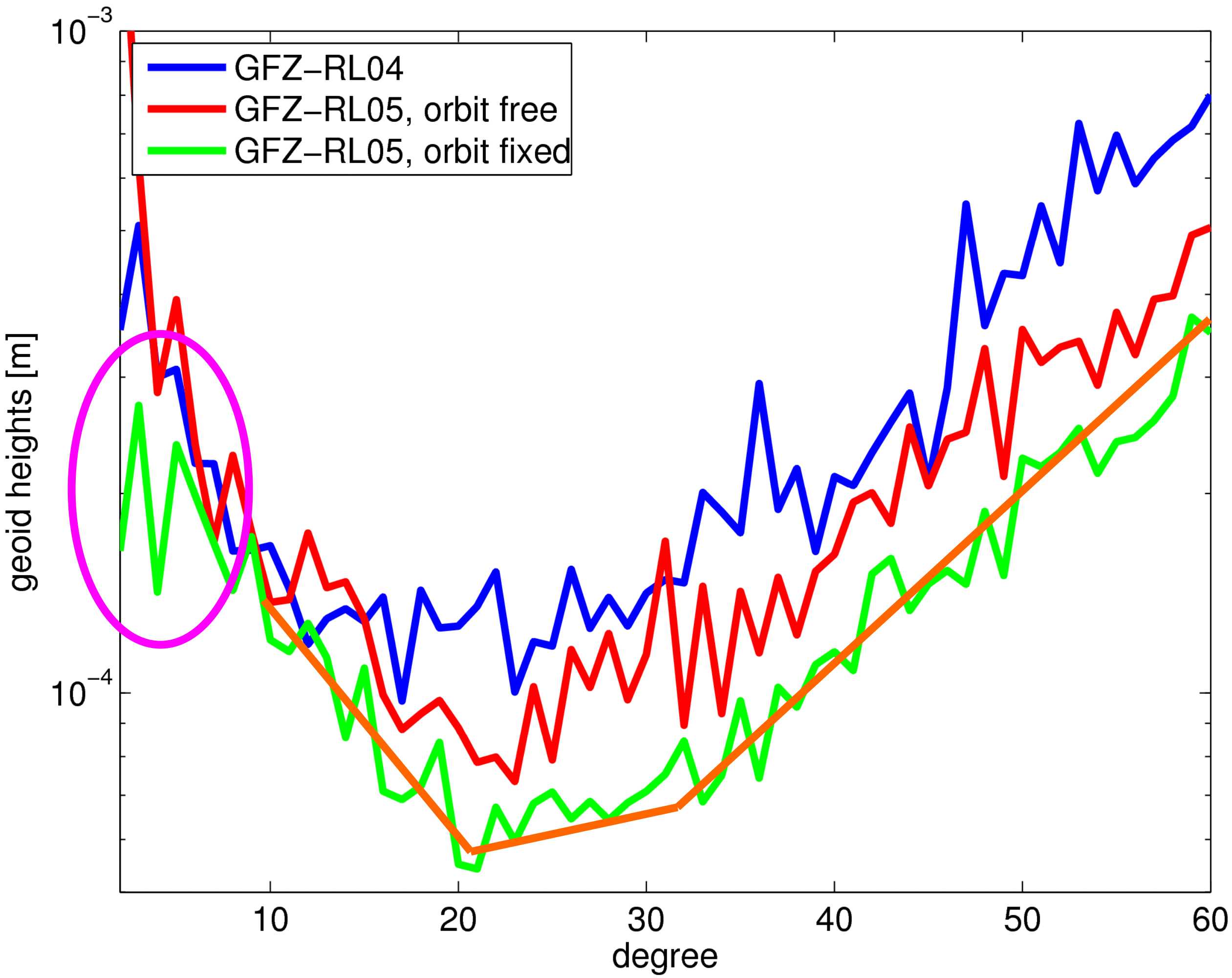
60 min, in R,S,W

Signal



Hotine–Marussi, 17th–21st June 2013, Roma

Signal and Noise in monthly models (GRACE)



Reference:

EIGEN-6C

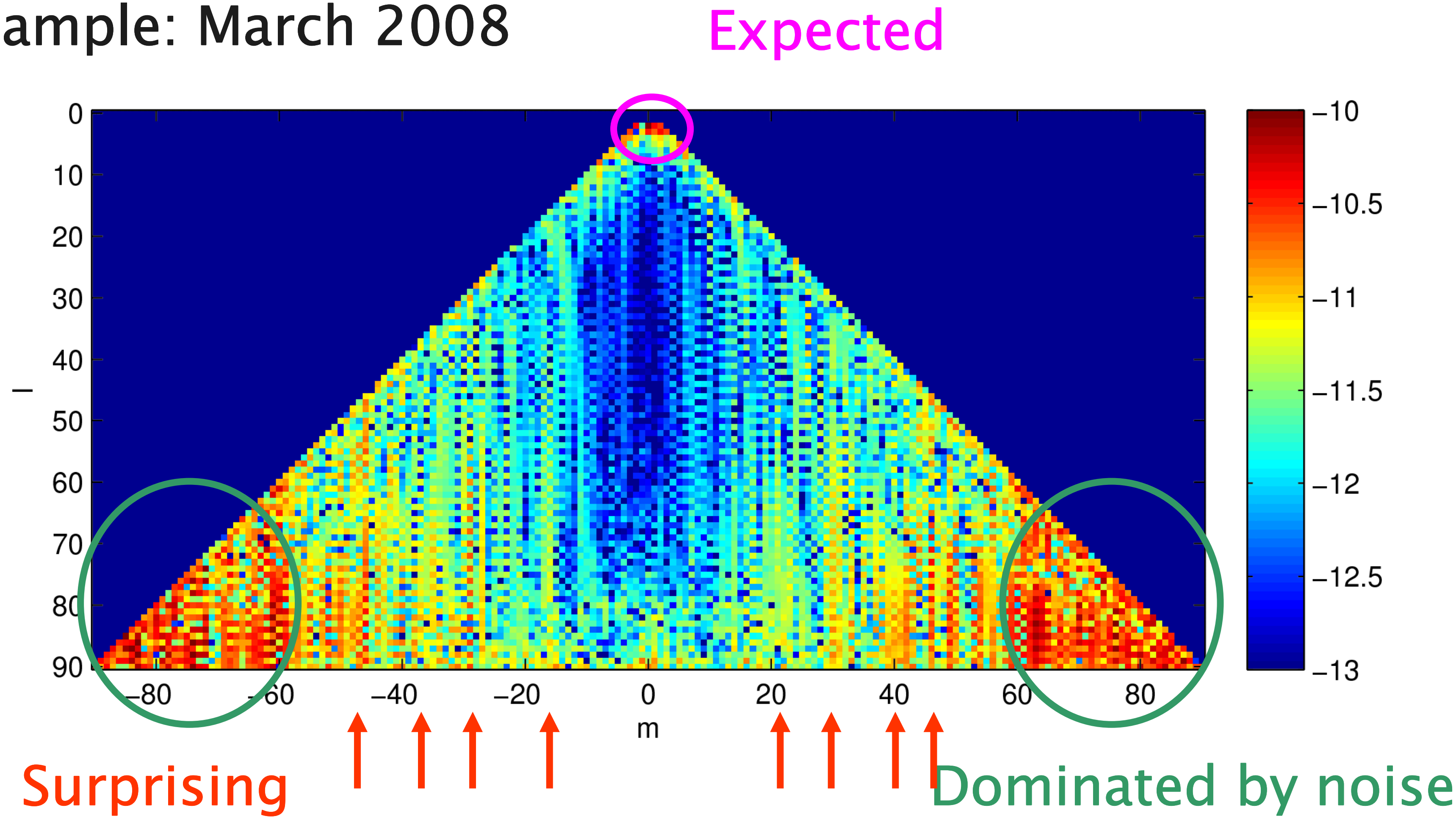
Expected

Surprising!

S,C−Coefficients

Difference: common estimation – orbit fixed

Example: March 2008



Direct – Spacewise – Timewise Analysis

- Direct approach:

  generalized orbit determination problem
  - arc specific parameters
  - model parameters
- Space wise approach:

  grid values are interpolated from observations

  => S,C–Analysis (integral formulas)
- Time wise approach:

  observations as timeseries along orbit

  Fourier–Analysis => Lumped Coefficients

  => Spherical Harmonic Coefficients

Timewise approach

- Potential along orbit
- Gravitational observations in satellite fixed frame
- Orbit perturbations relative to reference orbit (in satellite fixed frame)
- Inter-satellite observations
- Time derivatives

Gravity potential along orbit

$$V(r, I, u, \Lambda) =$$

$$\frac{GM}{r} \sum_{m=0}^{L} \sum_{k=-L}^{L} \sum_{l=\max(m,|k|)}^{L} \left(\frac{a_E}{r}\right)^l \bar{F}_{lmk}(I)$$

$$\left\{ \begin{array}{l} \bar{C}_{lm} \cos\psi_{km} + \bar{S}_{lm} \sin\psi_{km} \\ -\bar{S}_{lm} \cos\psi_{km} + \bar{C}_{lm} \sin\psi_{km} \end{array} \right\}_{l-m \text{ odd}}^{l-m \text{ even}}$$

$$= \sum_{m=0}^{L} \sum_{k=-L}^{L} A_{mk}^{V} \cos\psi_{km} + B_{mk}^{V} \sin\psi_{km}$$

Inclination Functions

Lumped Coefficients

$$\psi_{km} = ku + m\Lambda$$

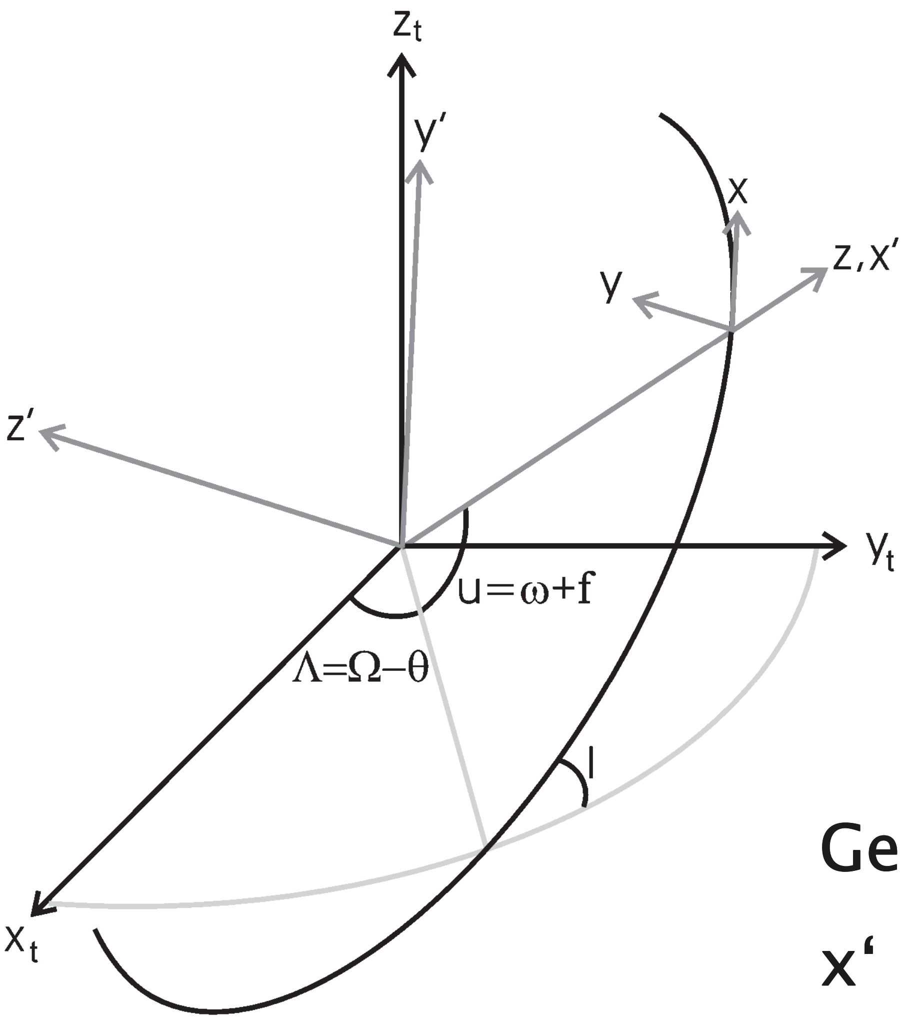
Lumped Coefficients: potential

$$A_{mk}^V = \sum_{l=\max(m,|k|)}^{L} \bar{H}_{lmk}^V \left\{ \begin{array}{c} \bar{C}_{lm} \\ -\bar{S}_{lm} \end{array} \right\}_{l-m \text{ odd}}^{l-m \text{ even}}$$

$$B_{mk}^V = \sum_{l=\max(m,|k|)}^{L} \bar{H}_{lmk}^V \left\{ \begin{array}{c} \bar{S}_{lm} \\ \bar{C}_{lm} \end{array} \right\}_{l-m \text{ odd}}^{l-m \text{ even}}$$

Transfer: $\bar{H}_{lmk}^V = \frac{GM}{r} \left(\frac{a_E}{r}\right)^l \bar{F}_{lmk}(I)$

Co–Rotating frame



Earth fixed frame:

x_t, y_t, z_t

Satellite fixed frame:

x = along–track

y = cross–track

z = radial

Geocentric, rotating frame:

x‘ || z, y‘ || x, z‘ || y

Potential => gravitational acceleration

Gradient in satellite fixed frame: $\nabla = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = \begin{pmatrix} \frac{1}{r\cos\phi'}\frac{\partial}{\partial\lambda'} \\ \frac{1}{r}\frac{\partial}{\partial\phi'} \\ \frac{\partial}{\partial r} \end{pmatrix}$

$\underline{\ddot{x}} = \nabla V$

Transfer:

$$\bar{H}^x_{lmk} = \frac{1}{r}\frac{GM}{r}\left(\frac{a_E}{r}\right)^l \bar{F}_{lmk}(I)k$$

$$\bar{H}^y_{lmk} = \frac{1}{r}\frac{GM}{r}\left(\frac{a_E}{r}\right)^l \bar{F}^\times_{lmk}(I)$$

$$\bar{H}^z_{lmk} = -\frac{l+1}{r}\frac{GM}{r}\left(\frac{a_E}{r}\right)^l \bar{F}_{lmk}(I)$$

Gravitational accelerations => orbit perturbations

Hill (1878)

Equations of motion:

$$\ddot{x} + 2n\dot{z} = \frac{\partial T}{\partial x}$$

$$\ddot{y} + n^2 y = \frac{\partial T}{\partial y}$$

$$\ddot{z} - 2n\dot{x} - 3n^2 z = \frac{\partial T}{\partial z}$$

Perturbing potential

x, y, z relative to circular reference orbit (n const.)

Solvable analytically (exact)!

But only valid for circular orbits (approx.)

Transfer: orbit perturbations

$$\bar{H}^{dx}_{lmk} = \frac{(3n^2 + \dot{\psi}^2_{mk})k - 2n\dot{\psi}_{mk}(l+1)}{\dot{\psi}^2_{mk}(n^2 - \dot{\psi}^2_{mk})} \cdot \frac{GM}{a_E^2}\left(\frac{a_E}{r}\right)^{l+2} \bar{F}_{lmk}(I)$$

Transfer:

$$\bar{H}^{dy}_{lmk} = \frac{1}{n^2 - \dot{\psi}^2_{mk}} \frac{GM}{a_E^2}\left(\frac{a_E}{r}\right)^{l+2} \bar{F}^{\times}_{lmk}(I)$$

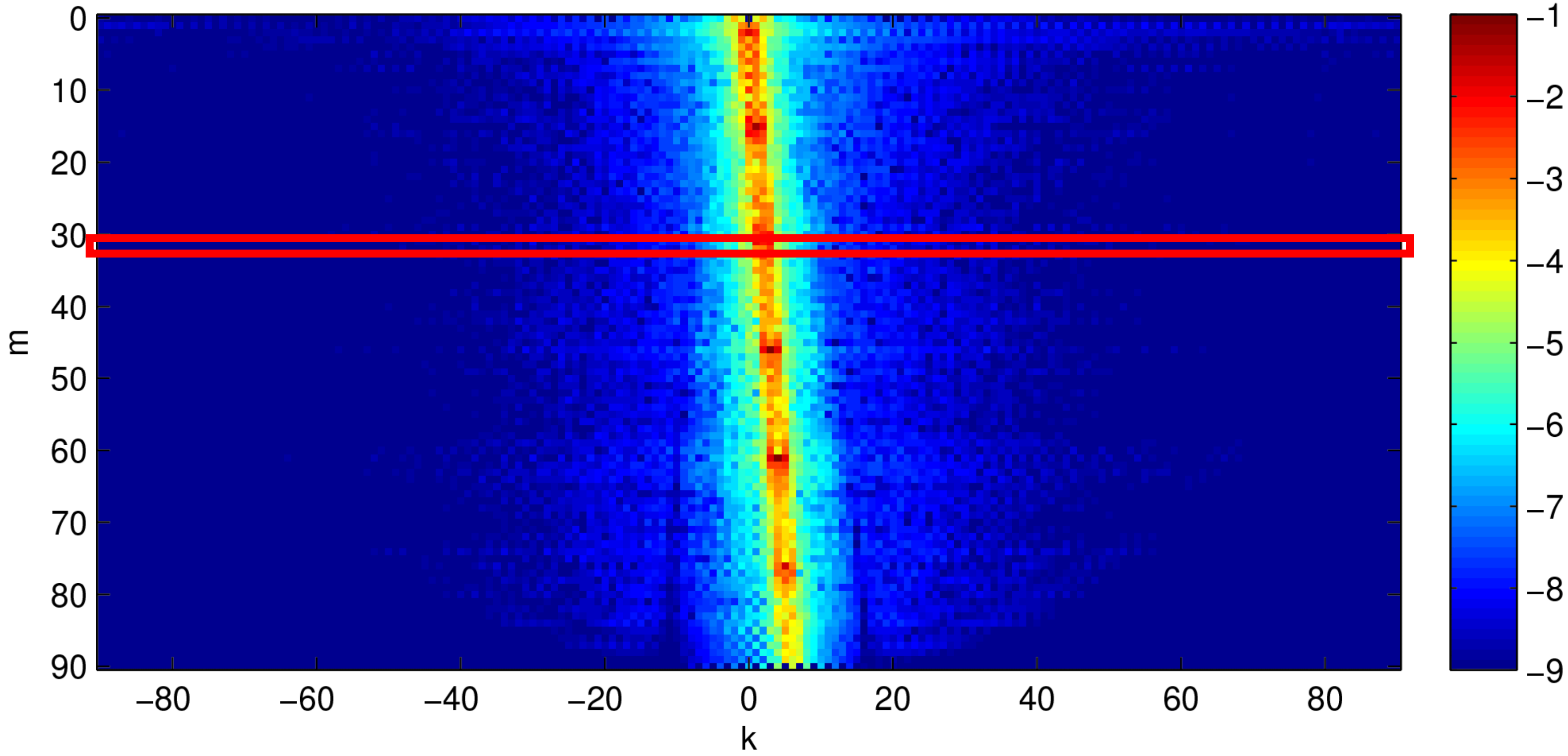
$$\bar{H}^{dz}_{lmk} = \frac{2nk - (l+1)\dot{\psi}_{mk}}{\dot{\psi}_{mk}(n^2 - \dot{\psi}^2_{mk})} \cdot \frac{GM}{a_E^2}\left(\frac{a_E}{r}\right)^{l+2} \bar{F}_{lmk}(I)$$

0–RESONANCE

N–RESONANCE

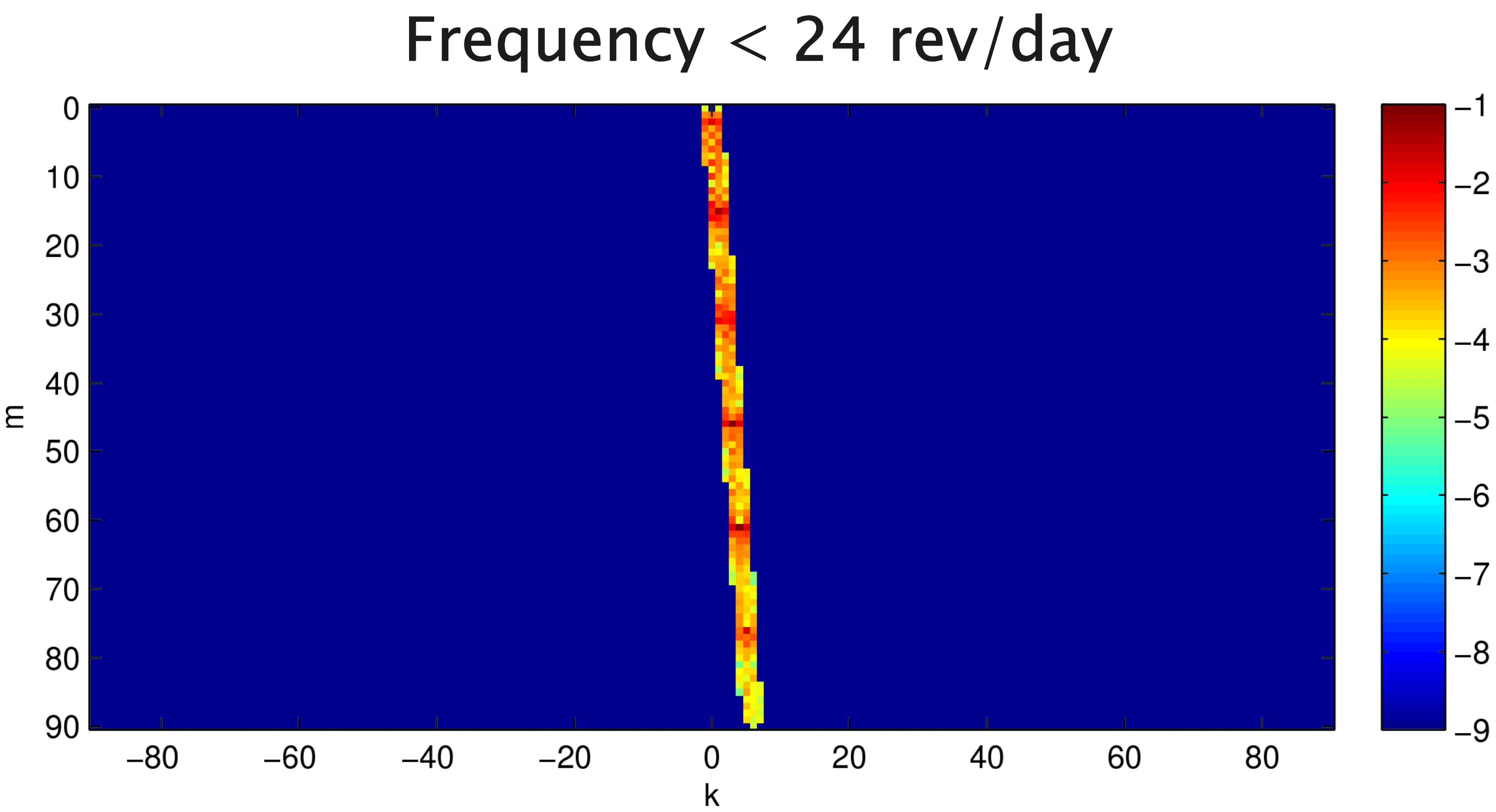
Lumped Coef.: Along-track orbit perturbations

Difference: common estimation - orbit fixed



S_{lm}, C_{lm} depend on A_{mk}, B_{mk} of same order m

Lumped Coef.: Along-track orbit perturbations

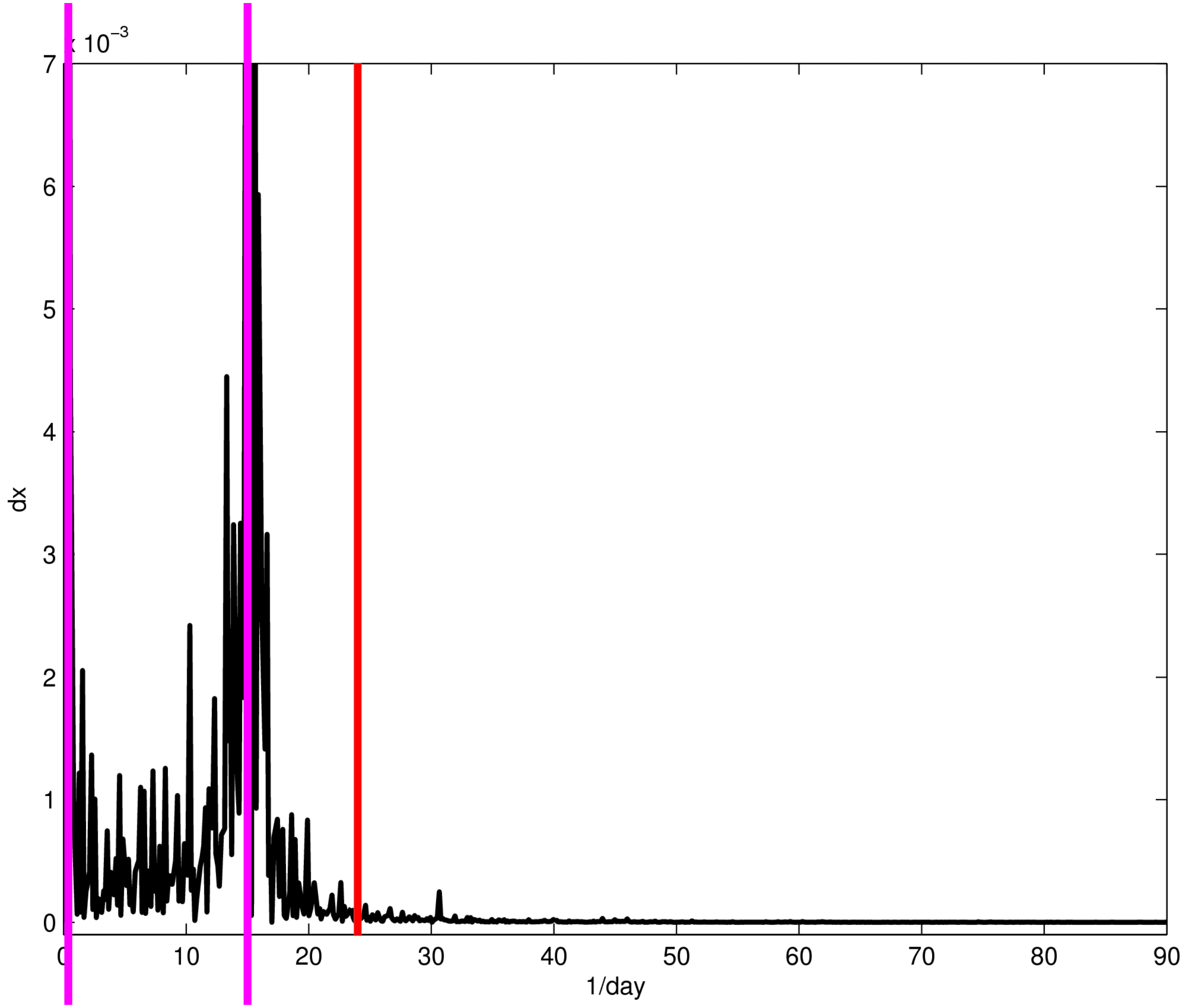


$$\dot{\psi}_{km} = k\dot{u} + m\dot{\Lambda}$$

Amplitude Spectrum (Lumped Coef. dx)

Frequency of stoch. accelerations

Resonance of Transfer



Discussion

- Stochastic orbit parameters increase consistency between a priori and estimated gravity field.
- Aggravated when correlations are broken.
- Whole S,C-spectrum is affected by only few low frequent stochastic accelerations.
- Can be explained via lumped coefficients by timewise analysis.
- Could probably be useful to regularize lumped coefficients (instead of S, C).
- Is complicated by resonance effects in case of orbit perturbations and derivatives.