PROCEEDINGS OF SCIENCE

Isospin breaking in pion–deuteron scattering and the pion–nucleon scattering lengths

Martin Hoferichter[*], [ab] **Vadim Baru**, [cd] **Christoph Hanhart**, [e] **Bastian Kubis**, [a] **Andreas Nogga**, [e] **and Daniel R. Phillips**[f]

[a] *Helmholtz-Institut für Strahlen- und Kernphysik (Theorie), Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany*

[b] *Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics, Universität Bern, CH-3012 Bern, Switzerland*

[c] *Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany*

[d] *ITEP, 117218, B. Cheremushkinskaya 25, Moscow, Russia*

[e] *Forschungszentrum Jülich, Institut für Kernphysik, Jülich Center for Hadron Physics and Institute for Advanced Simulation, D-52425 Jülich, Germany*

[f] *Institute of Nuclear and Particle Physics and Department of Physics and Astronomy, Ohio University, Athens, OH 45701, USA*

E-mail: hoferichter@itp.unibe.ch

In recent years, high-accuracy data for pionic hydrogen and deuterium have become the primary source of information on the pion–nucleon scattering lengths. Matching the experimental precision requires, in particular, the study of isospin-breaking corrections both in pion–nucleon and pion–deuteron scattering. We review the mechanisms that lead to the cancellation of potentially enhanced virtual-photon corrections in the pion–deuteron system, and discuss the subtleties regarding the definition of the pion–nucleon scattering lengths in the presence of electromagnetic interactions by comparing to nucleon–nucleon scattering. Based on the $\pi^{\pm}p$ channels we find for the virtual-photon-subtracted scattering lengths in the isospin basis $a_{\not\gamma}^{1/2} = (170.5 \pm 2.0) \cdot 10^{-3} M_{\pi}^{-1}$ and $a_{\not\gamma}^{3/2} = (-86.5 \pm 1.8) \cdot 10^{-3} M_{\pi}^{-1}$.

The 7th International Workshop on Chiral Dynamics,
August 6 -10, 2012
Jefferson Lab, Newport News, Virginia, USA

[*]Speaker.

PoS(CD12)093

1. Introduction

Leading-order chiral perturbation theory (ChPT) predicts the low-energy theorem [1]

$$a^- = \frac{M_\pi}{8\pi(1+M_\pi/m_\mathrm{p})F_\pi^2} \approx 80\cdot 10^{-3} M_\pi^{-1} \tag{1.1}$$

for the isovector pion–nucleon (πN) scattering length a^-. While this prediction for a^- is very stable against higher-order corrections, which only enter at $\mathscr{O}(M_\pi^3)$ [2], the chiral expansion for its isoscalar counterpart a^+ vanishes at leading order and involves large cancellations amongst the subleading terms. Moreover, given precise input for a^-, the Goldberger–Miyazawa–Oehme (GMO) sum rule [3] relates the πN coupling constant to an integral over πN cross sections. Therefore, new, independent information on both the isovector and isoscalar scattering lengths becomes particularly interesting.

Such an approach is offered by hadronic atoms, more precisely high-precision measurements of the spectra of pionic hydrogen (πH) and deuterium (πD) [4, 5]. These systems, composed of a π^- and a proton/deuteron, are bound by electromagnetism, but strong interactions induce distortions of the pure QED spectrum. Accordingly, the level shift of the ground state is related to elastic $\pi^- p$ and $\pi^- d$ scattering, e.g. for the level shift ε_{1s} in πH (with reduced mass μ_H)

$$\varepsilon_{1s} = -2\alpha^3\mu_H^2 a_{\pi^- p}\big(1+2\alpha(1-\log\alpha)\mu_H a_{\pi^- p}+\cdots\big), \tag{1.2}$$

while the width of the πH ground state gives access to the charge-exchange reaction $\pi^- p \to \pi^0 n$ [6]. This leads to the following system for $a^\pm$

$$\begin{aligned} a_{\pi^- p} &= \tilde{a}^+ + a^- + \Delta\tilde{a}_{\pi^- p}, \qquad a_{\pi^- p\to\pi^0 n} = -\sqrt{2}a^- + \Delta a_{\pi^- p\to\pi^0 n},\\ \mathrm{Re}\, a_{\pi^- d} &= 2\frac{1+M_\pi/m_\mathrm{p}}{1+M_\pi/m_\mathrm{d}}(\tilde{a}^+ + \Delta\tilde{a}^+) + a^{(3)}_{\pi^- d}, \end{aligned} \tag{1.3}$$

where

$$\tilde{a}^+ = a^+ + \frac{1}{4\pi(1+M_\pi/m_\mathrm{p})}\left\{\frac{4(M_\pi^2-M_{\pi^0}^2)}{F_\pi^2}c_1 - 2e^2 f_1\right\} \tag{1.4}$$

includes isospin violation at leading order in ChPT [7], $\Delta\tilde{a}_{\pi^- p}$, $\Delta a_{\pi^- p\to\pi^0 n}$, and $\Delta\tilde{a}^+$ denote further isospin-breaking corrections [8, 9], and $a^{(3)}_{\pi^- d}$ incorporates three-body contributions in πD. The theoretical tool to evaluate these corrections is chiral effective field theory [10 – 17]. From the uncertainty estimate for the isospin-conserving three-body contributions of $1\cdot 10^{-3} M_\pi^{-1}$ [16], compared to $\mathrm{Re}\, a_{\pi^- d} \sim -25\cdot 10^{-3} M_\pi^{-1}$, it follows that only effects significantly below that threshold may be ignored. In particular, at the level of accuracy required for the interpretation of the hadronic-atom data it becomes mandatory to investigate the role of isospin violation in $a^{(3)}_{\pi^- d}$ [16].

2. Isospin breaking in threshold $\pi^- d$ scattering

Isospin breaking is generated both by the difference between the light quark masses and virtual-photon effects, see Fig. 1 for representatives of each class. Mass-difference insertions are

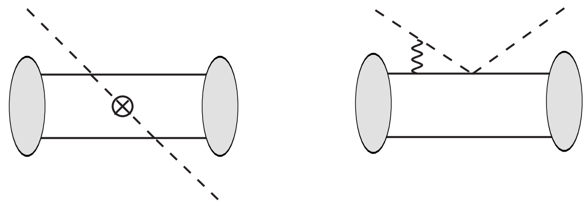

Figure 1: Mass-difference and virtual-photon effects in $\pi^- d$ scattering. Solid, dashed, and wiggly lines denote nucleons, pions, and photons, while the cross and the blobs refer to a mass-difference insertion and the deuteron wave function, respectively.

numerically relevant only for the leading, double-scattering diagram (first diagram in Fig. 1), to which they contribute in the combination

$$\rho = 2M_\pi \Delta_\mathrm{N} - \Delta_\pi, \qquad \Delta_\mathrm{N} = m_\mathrm{n} - m_\mathrm{p}, \qquad \Delta_\pi = M_\pi^2 - M_{\pi^0}^2. \tag{2.1}$$

Since the leading-order pion mass difference is caused solely by electromagnetic effects, this implies that the quark mass difference only enters at subleading orders, and indeed Δ_π is responsible for the bulk of the total 2% correction. In addition, isospin violation at the vertices, which may again be related to isospin breaking in the πN scattering lengths, generates another 1% effect.

The calculation of virtual-photon contributions to $a^{(3)}_{\pi^- d}$ proves particularly challenging due to the presence of various momentum scales: $p \sim \alpha M_\pi$ ("hadronic-atom regime"), $p \sim M_\pi$ ("chiral regime"), $p \sim \sqrt{m_\mathrm{p}\epsilon}$ (deuteron wave function), and $p \sim \sqrt{M_\pi \epsilon}$ (three-body dynamics), with the deuteron binding energy ϵ. While the hadronic-atom regime is already included in the calculation that leads to (1.2), the remaining scales might lead to an enhancement $\sim \sqrt{M_\pi/\epsilon}$ of virtual-photon effects. For isovector πN interactions the pertinent integral takes the form

$$a_{T=1} \propto a^- \int d^3p\, d^3q \frac{\big(\Psi^\dagger(\mathbf{p}-\mathbf{q}) - \Psi^\dagger(\mathbf{p})\big)\Psi(\mathbf{p})}{\mathbf{q}^2\big(\mathbf{q}^2 + 2M_\pi(\epsilon + \mathbf{p}^2/m_\mathrm{p})\big)}, \tag{2.2}$$

with the deuteron wave function $\Psi(\mathbf{p})$. Indeed, the individual terms corresponding to $\Psi^\dagger(\mathbf{p}-\mathbf{q})$ and $\Psi^\dagger(\mathbf{p})$ scale with $\sqrt{M_\pi/\epsilon}$, but such contributions cancel in their difference [16]. The occurrence of this cancellation can be traced back directly to the Pauli principle, which forces the intermediate NN pair to be in a P-wave and thus leads to the relative sign in (2.2).

In the isoscalar case intermediate-state S-wave NN interactions are now permitted, in particular the deuteron pole that is already included in (1.2) needs to be separated. Expressed in terms of overlap integrals between the deuteron and continuum wave functions $\Psi(\mathbf{q})$ and $\Psi^s_\mathbf{p}(\mathbf{q})$, the contribution to the $\pi^- d$ scattering length then becomes

$$a_{T=0} \propto a^+ \int \frac{d^3k}{(2\pi)^3} \frac{1}{\mathbf{k}^2} \left\{ \frac{|F(\mathbf{k})|^2 - 1}{\mathbf{k}^2/2M_\pi - i\eta} + \frac{1}{2} \int \frac{d^3p}{(2\pi)^6} \frac{G^s_\mathbf{p}(\mathbf{k})\big(G^s_\mathbf{p}(\mathbf{k}) + G^s_\mathbf{p}(-\mathbf{k})\big)}{\epsilon + \mathbf{p}^2/m_\mathrm{p} + \mathbf{k}^2/2M_\pi - i\eta} \right\},$$
$$F(\mathbf{k}) = \int d^3q\, \Psi^\dagger(\mathbf{q})\Psi(\mathbf{q}-\mathbf{k}/2), \qquad G^s_\mathbf{p}(\mathbf{k}) = \int d^3q\, \Psi^\dagger(\mathbf{q})\Psi^s_\mathbf{p}(\mathbf{q}-\mathbf{k}/2), \tag{2.3}$$

which, by virtue of the normalization of $\Psi(\mathbf{q})$ and the orthogonality of bound-state and continuum wave functions for vanishing momentum transfer

$$|F(\mathbf{k})|^2 - 1 = \mathscr{O}(\mathbf{k}^2), \qquad G^s_\mathbf{p}(\mathbf{k}) = \mathscr{O}(\mathbf{k}), \tag{2.4}$$

isospin limit	channel	scattering length	channel	scattering length
$a^+ + a^-$	$\pi^- p \to \pi^- p$	86.1 ± 1.8	$\pi^+ n \to \pi^+ n$	85.2 ± 1.8
$a^+ - a^-$	$\pi^+ p \to \pi^+ p$	-88.1 ± 1.8	$\pi^- n \to \pi^- n$	-89.0 ± 1.8
$-\sqrt{2}\,a^-$	$\pi^- p \to \pi^0 n$	-121.4 ± 1.6	$\pi^+ n \to \pi^0 p$	-119.5 ± 1.6
a^+	$\pi^0 p \to \pi^0 p$	2.1 ± 3.1	$\pi^0 n \to \pi^0 n$	5.5 ± 3.1

Table 1: πN scattering lengths for the physical channels in units of $10^{-3} M_\pi^{-1}$, Table taken from [16].

proves the cancellation of the leading infrared enhanced contributions also for isoscalar πN interactions. Explicit calculation shows that the infrared enhancement of momenta $\sim \sqrt{m_\mathrm{p}\epsilon}$ is too weak to become numerically relevant, so that in the end the only non-negligible correction due to virtual photons is generated by residual isovector terms dominated by momenta $\sim M_\pi$ [16]

$$a^{\mathrm{EM}} = (0.95 \pm 0.01) \cdot 10^{-3} M_\pi^{-1}. \tag{2.5}$$

These findings vindicate a posteriori the application of a chiral power counting and imply that the main impact of isospin violation for the extraction of the πN scattering lengths is due to next-to-leading order isospin-breaking corrections, in particular the large shift $\Delta\tilde{a}^+ = (-3.3 \pm 0.3) \cdot 10^{-3} M_\pi^{-1}$, in the πN amplitude [18].

The final results for the combined analysis of πH and πD for the πN scattering lengths are [16]

$$a^+ = (7.6 \pm 3.1) \cdot 10^{-3} M_\pi^{-1}, \qquad \tilde{a}^+ = (1.9 \pm 0.8) \cdot 10^{-3} M_\pi^{-1}, \qquad a^- = (86.1 \pm 0.9) \cdot 10^{-3} M_\pi^{-1}, \tag{2.6}$$

which, in combination with the isospin-breaking corrections from [8], lead to the results for the physical channels given in Table 1.

3. Modified effective range expansion and subtraction of virtual-photon effects

To illustrate the issues regarding the definition of a scattering length for charged particles in the presence of electromagnetic interactions we first consider the example of proton–proton scattering. First, the pure Coulomb phase shift σ^C is removed from the total phase shift, so that the remainder δ^C_{pp}, related to the strong amplitude $T_{pp}(k)$ by

$$k\big(\cot\delta^C_{pp} - i\big) = -\frac{4\pi}{m}\frac{e^{2i\sigma^C}}{T_{pp}(k)}, \qquad k = |\mathbf{k}|, \tag{3.1}$$

obeys the modified effective range expansion [19]

$$k\Big[C_\eta^2\big(\cot\delta^C_{pp} - i\big) + 2\eta H(\eta)\Big] = -\frac{1}{a^C_{pp}} + \frac{1}{2} r_0 k^2 + \mathcal{O}(k^4), \tag{3.2}$$

$$C_\eta^2 = \frac{2\pi\eta}{e^{2\pi\eta}-1}, \qquad \eta = \frac{\alpha m}{2k}, \qquad H(\eta) = \psi(i\eta) + \frac{1}{2i\eta} - \log(i\eta), \qquad \psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}.$$

The removal of the residual Coulomb interactions to define a purely strong scattering length a_{pp} is a scale-dependent procedure [20, 21]

$$\frac{1}{a_{pp}} = \frac{1}{a^C_{pp}} + \alpha m\bigg[\log\frac{\mu\sqrt{\pi}}{\alpha m} + 1 - \frac{3}{2}\gamma_E\bigg], \tag{3.3}$$

since the Coulomb-nuclear interference depends on the short-distance part of the nuclear force. Stated differently, for a consistent subtraction of virtual photons the electromagnetic coupling should be switched off also in the running of operators, which requires the knowledge of the underlying theory [22, 23]. For pp scattering such residual Coulomb effects induce a huge difference between a_{pp} and a^C_{pp} [24, 25]

$$a^C_{pp} = (-7.8063 \pm 0.0026)\,\text{fm}, \qquad a_{pp} = (-17.3 \pm 0.4)\,\text{fm}. \tag{3.4}$$

The standard ChPT convention for the πN scattering lengths [6]

$$e^{-2i\sigma^C} T_{\pi^- p} = \frac{\pi\alpha\mu_H a_{\pi^- p}}{k} - 2\alpha\mu_H \big(a_{\pi^- p}\big)^2 \log\frac{k}{\mu_H} + a_{\pi^- p} + \mathcal{O}(k,\alpha^2), \tag{3.5}$$

with the Coulomb pole $\propto 1/k$ and the term $\propto \log k/\mu_H$ first generated at one- and two-loop level, can be matched to the modified effective range expansion (3.2) by expanding first in α, then in k

$$e^{-2i\sigma^C} T_{\pi^- p} = \frac{\pi\alpha\mu_H a^C_{\pi^- p}}{k} - 2\alpha\mu_H \big(a^C_{\pi^- p}\big)^2 \left(\gamma_E + \log\frac{k}{\alpha\mu_H}\right) + a^C_{\pi^- p} + \mathcal{O}(k,\alpha^2), \tag{3.6}$$

and thus

$$a_{\pi^- p} = a^C_{\pi^- p} + 2\alpha\mu_H \big(a^C_{\pi^- p}\big)^2 (\log\alpha - \gamma_E) + \mathcal{O}(\alpha^2). \tag{3.7}$$

The correction term, involving the same $\log\alpha$ already present in (1.2), numerically evaluates to $-0.5\cdot 10^{-3} M_\pi^{-1}$, which is still appreciably smaller than the uncertainty in $a_{\pi^- p}$ itself (see Table 1).

The πN scattering lengths are of particular interest to help determine subtraction constants that appear in the GMO sum rule or, more generally, a dispersive analysis of πN scattering, see e.g. [16, 26]. In these applications, the derivation of the dispersion relations relies on the analyticity properties of the strong amplitude, so that the scattering lengths should be purified from any virtual-photons effects. Strictly speaking, the discussion of the pp case shows that this cannot be achieved completely model-independently unless the underlying theory is known. In ChPT these subtleties appear in the regularization of UV divergent virtual-photon diagrams, where the separation between mass-difference and virtual-photon contributions to the low-energy constants requires the choice of a scale. However, taking as an example the combination $a_{\pi^- p} - a_{\pi^+ p}$ needed for the GMO sum rule, we find for the virtual-photon corrections

$$a^\gamma_{\pi^- p} - a^\gamma_{\pi^+ p} = (2.1 \pm 1.8)\cdot 10^{-3} M_\pi^{-1}, \tag{3.8}$$

so that the scale dependence of the virtual-photon-subtracted scattering lengths

$$a^{\not\gamma}_{\pi^- p} - a^{\not\gamma}_{\pi^+ p} = (171.3 \pm 2.4)\cdot 10^{-3} M_\pi^{-1} \tag{3.9}$$

should be entirely negligible. These effects are so much smaller than in pp scattering since πN scattering is perturbative, whereas the fine tuning in the nucleon–nucleon potential enhances any residual virtual-photon contributions.

Using (3.9) as input for the GMO sum rule we find for the πN coupling constant $g_c^2/4\pi = 13.7 \pm 0.2$ [16]. Finally, we give the virtual-photon-subtracted scattering lengths in the isospin basis as derived from elastic $\pi^\pm p$ scattering

$$a^{1/2}_{\not\gamma} \doteq \frac{1}{2}\big(3a^{\not\gamma}_{\pi^- p} - a^{\not\gamma}_{\pi^+ p}\big) = (170.5 \pm 2.0)\cdot 10^{-3} M_\pi^{-1}, \qquad a^{3/2}_{\not\gamma} \doteq a^{\not\gamma}_{\pi^+ p} = (-86.5 \pm 1.8)\cdot 10^{-3} M_\pi^{-1}, \tag{3.10}$$

which are needed as input for a dispersive analysis of πN scattering [26].

References

[1] S. Weinberg, Phys. Rev. Lett. **17** (1966) 616.

[2] V. Bernard, N. Kaiser and U.-G. Meißner, Phys. Lett. B **309** (1993) 421 [hep-ph/9304275].

[3] M. L. Goldberger, H. Miyazawa and R. Oehme, Phys. Rev. **99** (1955) 986.

[4] D. Gotta *et al.*, Lect. Notes Phys. **745** (2008) 165.

[5] T. Strauch *et al.*, Eur. Phys. J. A **47** (2011) 88 [arXiv:1011.2415 [nucl-ex]].

[6] V. E. Lyubovitskij and A. Rusetsky, Phys. Lett. B **494** (2000) 9 [hep-ph/0009206]; J. Gasser, V. E. Lyubovitskij and A. Rusetsky, Phys. Rept. **456** (2008) 167 [arXiv:0711.3522 [hep-ph]].

[7] U.-G. Meißner, U. Raha and A. Rusetsky, Eur. Phys. J. C **41** (2005) 213 [nucl-th/0501073].

[8] M. Hoferichter, B. Kubis and U.-G. Meißner, Phys. Lett. B **678** (2009) 65 [arXiv:0903.3890 [hep-ph]]; Nucl. Phys. A **833** (2010) 18 [arXiv:0909.4390 [hep-ph]].

[9] J. Gasser, M. A. Ivanov, E. Lipartia, M. Mojžiš and A. Rusetsky, Eur. Phys. J. C **26** (2002) 13 [hep-ph/0206068].

[10] S. Weinberg, Phys. Lett. B **295** (1992) 114 [hep-ph/9209257].

[11] S. R. Beane, V. Bernard, T. S. H. Lee and U.-G. Meißner, Phys. Rev. C **57** (1998) 424 [nucl-th/9708035].

[12] S. R. Beane, V. Bernard, E. Epelbaum, U.-G. Meißner and D. R. Phillips, Nucl. Phys. A **720** (2003) 399 [hep-ph/0206219].

[13] V. Baru, C. Hanhart, A. E. Kudryavtsev and U.-G. Meißner, Phys. Lett. B **589** (2004) 118 [nucl-th/0402027].

[14] V. Baru *et al.*, Phys. Lett. B **648** (2007) 46 [nucl-th/0608042]; Phys. Lett. B **659** (2008) 184 [arXiv:0706.4023 [nucl-th]].

[15] S. Liebig, V. Baru, F. Ballout, C. Hanhart and A. Nogga, Eur. Phys. J. A **47** (2011) 69 [arXiv:1003.3826 [nucl-th]].

[16] V. Baru, C. Hanhart, M. Hoferichter, B. Kubis, A. Nogga and D. R. Phillips, Phys. Lett. B **694** (2011) 473 [arXiv:1003.4444 [nucl-th]]; Nucl. Phys. A **872** (2011) 69 [arXiv:1107.5509 [nucl-th]].

[17] V. Baru, E. Epelbaum, C. Hanhart, M. Hoferichter, A. E. Kudryavtsev and D. R. Phillips, Eur. Phys. J. A **48** (2012) 69 [arXiv:1202.0208 [nucl-th]].

[18] M. Hoferichter, B. Kubis and U.-G. Meißner, PoS **CD09** (2009) 014 [arXiv:0910.0736 [hep-ph]].

[19] H. A. Bethe, Phys. Rev. **76** (1949) 38.

[20] J. D. Jackson and J. M. Blatt, Rev. Mod. Phys. **22** (1950) 77.

[21] X. Kong and F. Ravndal, Nucl. Phys. A **665** (2000) 137 [hep-ph/9903523].

[22] J. Gegelia, Eur. Phys. J. A **19** (2004) 355 [nucl-th/0310012].

[23] J. Gasser, A. Rusetsky and I. Scimemi, Eur. Phys. J. C **32** (2003) 97 [hep-ph/0305260].

[24] J. R. Bergervoet, P. C. van Campen, W. A. van der Sanden and J. J. de Swart, Phys. Rev. C **38** (1988) 15.

[25] G. A. Miller, B. M. K. Nefkens and I. Šlaus, Phys. Rept. **194** (1990) 1.

[26] C. Ditsche, M. Hoferichter, B. Kubis and U.-G. Meißner, JHEP **1206** (2012) 043 [arXiv:1203.4758 [hep-ph]]; JHEP **1206** (2012) 063 [arXiv:1204.6251 [hep-ph]].

PoS(CD12)093