

Recent progress on supersymmetric effects in rare K decays

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The dominant MSSM effects in the rare K decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K_L \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \pi^0 \mu^+ \mu^-$, are discussed both within and without the minimal flavor violation hypothesis, at moderate and large $\tan\beta$. In each case, the sensitivities to MSSM soft-breaking terms are compared, laying emphasis on possible correlations among observables. In most scenarios, rare K decays offer unique windows into the $\Delta S = 1$ sector of the soft-breaking terms. Therefore, together with B-physics and collider observables, these modes will be essential for reconstructing the still elusive SUSY-breaking mechanism.

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1. Introduction

The FCNC-induced decays, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K_L \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \pi^0 \mu^+ \mu^-$, are very suppressed in the Standard Model (SM), where they can be predicted very accurately[1]. Therefore, these modes are ideal for probing possible New Physics effects[2]. In the present talk, the signatures of supersymmetry (SUSY), in its simplest realization as the MSSM, are reviewed. As is well-known, SUSY unifies matter (fermions) and interactions (bosons), and has a number of desirable features, e.g. it provides a dark matter candidate, helps unify the gauge couplings at high-energy and stabilizes the electroweak scale[3]. Even though the minimal supersymmetrization of the SM requires one super-partner for each SM particle (and two Higgs doublets), it is very constrained and involves only a few free parameters. The problem, however, is that SUSY must be broken, and the precise mechanism still eludes us. Therefore, in practice, an effective description is adopted, introducing all possible explicit soft-breaking terms allowed by the gauge symmetries. In the squark sector, there are LL and RR mass-terms and trilinear couplings giving rise to LR mass-terms after the Higgses acquire their VEV's, $\langle H_{u,d}^0 \rangle = v_{u,d}$:

$$\mathcal{L}_{soft}^{LL,RR} = -\tilde{Q}^\dagger \mathbf{m}_Q^2 \tilde{Q} - \tilde{U} \mathbf{m}_U^2 \tilde{U}^\dagger - \tilde{D} \mathbf{m}_D^2 \tilde{D}^\dagger, \quad \mathcal{L}_{soft}^{LR} = -\tilde{U} \mathbf{A}^U \tilde{Q} H_u + \tilde{D} \mathbf{A}^D \tilde{Q} H_d,$$

with $\tilde{Q} = (\tilde{u}_L, \tilde{d}_L)^T$, $\tilde{U} = \tilde{u}_R^\dagger$, $\tilde{D} = \tilde{d}_R^\dagger$. Obviously, $\mathbf{m}_{Q,U,D}^2$ and $\mathbf{A}^{U,D}$, which are 3×3 matrices in flavor-space, generate a very rich flavor-breaking sector as squark mass eigenstates can differ substantially from their gauge eigenstates.

What to expect from SUSY in rare K decays: In the SM, the Z -penguin is the dominant contribution, and is tuned by $\lambda_t = V_{ts}^* V_{td}$ (Fig.1a). The four MSSM corrections depicted in Figs.1b – e (together with box diagrams), represent the dominant corrections, and are thus the only MSSM effects for which rare K decays can be sensitive probes. Let us briefly describe each of them. First, there is the charged Higgs contribution to the Z -penguin (Fig.1b), which is, at moderate $\tan \beta = v_u/v_d$, aligned with the SM one ($\sim \lambda_t$). Then, there is the supersymmetrized version of Figs.1a – b, with charginos – up-squarks in place of W^\pm/H^\pm – up-quarks in the loop (Fig.1c), and which is sensitive to the mixings among the six up-squarks (Z^U), a priori not aligned with the CKM mixings. Another purely supersymmetric contribution, relevant only for charged lepton modes, is the gluino electromagnetic penguin (Fig.1d), sensitive to down-squark mixings (Z^D). The last class of effects consists of neutral Higgs FCNC (Fig.1e), and arises at large $\tan \beta \approx m_t/m_b \approx 50$. Indeed, the 2HDM-II structure of the Higgs couplings to quarks, required by SUSY, is not preserved beyond leading order due to \mathcal{L}_{soft} , and the “wrong Higgs”, H_u , gets coupled to down-type quarks, $\mathcal{L}_{eff} \supset \tilde{d}_R^\dagger Y_d^{ik} (H_d^0 + \epsilon Y_u^\dagger Y_u H_u^{0\dagger})^{kj} d_L^j$. Clearly, once the Higgses acquire their VEV's, there is a mismatch between quark mass eigenstates and Higgs couplings; both are no longer diagonalized simultaneously and Higgs FCNC are generated[4].

Bottom-up approach and Minimal Flavor Violation: There are obviously too many parameters in \mathcal{L}_{soft} to have any hope to fix them all from rare K decays. At the same time, however, observed FCNC transitions and CP-violation seem to indicate that new physics induces only small departures with respect to the SM. Therefore, one starts from a lowest-order basis in which the flavor-breakings due to $\mathbf{m}_{Q,U,D}^2$ and $\mathbf{A}^{U,D}$ are minimal. This can take the form of $mSUGRA$, alignment of squarks with quarks or the Minimal Flavor Violation hypothesis (MFV). In a second stage,

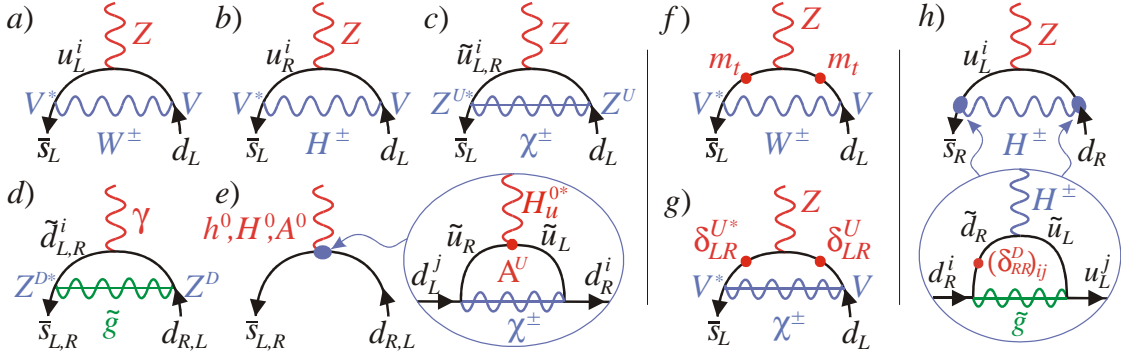


Figure 1: *a – e)* Dominant MSSM contributions to rare K decays. *f – g)* Dominant sources of $SU(2)_L$ -breaking in the Z -penguin. *h)* Schematic representation of the H^\pm contribution to the Z -penguin at large $\tan\beta$.

one probes the possible signatures of departures from this minimal setting. The goal being, ultimately, to constrain SUSY-breaking models, which imply specific soft-breaking structures. At that stage, information from rare K decays, colliders and B -physics must of course be combined.

Here we adopt MFV as the lowest order basis, i.e. we impose that the SM Yukawas $\mathbf{Y}_{u,d}$ are the only sources of flavor-breaking[5]. In practice, this means that \mathcal{L}_{soft} terms can be expanded as $(a_i, b_i \sim O(1), A_0$ and m_0 setting the overall mass-scale as in $mSUGRA$)

$$\mathbf{m}_Q^2 = m_0^2(a_1 \mathbf{1} + b_1 \mathbf{Y}_u^\dagger \mathbf{Y}_u + b_2 \mathbf{Y}_d^\dagger \mathbf{Y}_d + b_3 (\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d)), \mathbf{m}_U^2 = m_0^2(a_2 \mathbf{1} + b_4 \mathbf{Y}_u \mathbf{Y}_u^\dagger), \\ \mathbf{m}_D^2 = m_0^2(a_3 \mathbf{1} + b_5 \mathbf{Y}_d \mathbf{Y}_d^\dagger), \mathbf{A}^U = A_0 \mathbf{Y}_u (a_4 \mathbf{1} + b_6 \mathbf{Y}_d^\dagger \mathbf{Y}_d), \mathbf{A}^D = A_0 \mathbf{Y}_d (a_5 \mathbf{1} + b_7 \mathbf{Y}_u^\dagger \mathbf{Y}_u),$$

such that all FCNC's and CP-violation are still essentially tuned by the CKM matrix. For example, the dominant contributions to the Z -penguin are those breaking the $SU(2)_L$ gauge-symmetry[6, 7]. In the SM, this breaking is achieved through a double top-quark mass insertion (Fig.1f). Similarly, in the MSSM, it is the double $\tilde{t}_L - \tilde{t}_R$ mixing via the \mathbf{A}^U trilinear terms which plays the dominant role (Fig.1g in the sCKM basis)[8]. Within MFV, this gives a factor $m_t^2 \lambda_t |a_4 - \cot\beta \mu^*|^2$ [9], still enhanced by m_t^2 and tuned by λ_t .

2. Supersymmetric effects in $K \rightarrow \pi \nu \bar{\nu}$

SUSY effects in the SM-like operators. $(\bar{s}d)_{V\pm A} (\bar{\nu}\nu)_{V-A}$, cannot be distinguished since only $(\bar{s}d)_V (\bar{\nu}\nu)_{V-A}$ contributes to the $K \rightarrow \pi \nu \bar{\nu}$ matrix-element. All MSSM effects are thus encoded into a single complex number, $X^\nu \equiv y_L^\nu + y_R^\nu$ [7]:

$$\mathcal{H}_{eff} = y_L^\nu (\bar{s}d)_{V-A} (\bar{\nu}\nu)_{V-A} + y_R^\nu (\bar{s}d)_{V+A} (\bar{\nu}\nu)_{V-A} \rightarrow (y_L^\nu + y_R^\nu) (\bar{s}d)_V (\bar{\nu}\nu)_{V-A}.$$

At moderate $\tan\beta$, the dominant MSSM contribution comes from chargino penguins because of their quadratic sensitivity to up-squark mass-insertions (Figs.1c,1g). Within MFV, this means, given the m_t enhancement present in the δ_{LR}^U sector, that $K \rightarrow \pi \nu \bar{\nu}$ are particularly sensitive. Still, a significant enhancement would require a very light stop and chargino[9], mostly because of the constraint from $\Delta\rho$ [10]. Any enhancement $\gtrsim 5\%$ would thus falsify MFV if sparticles are found

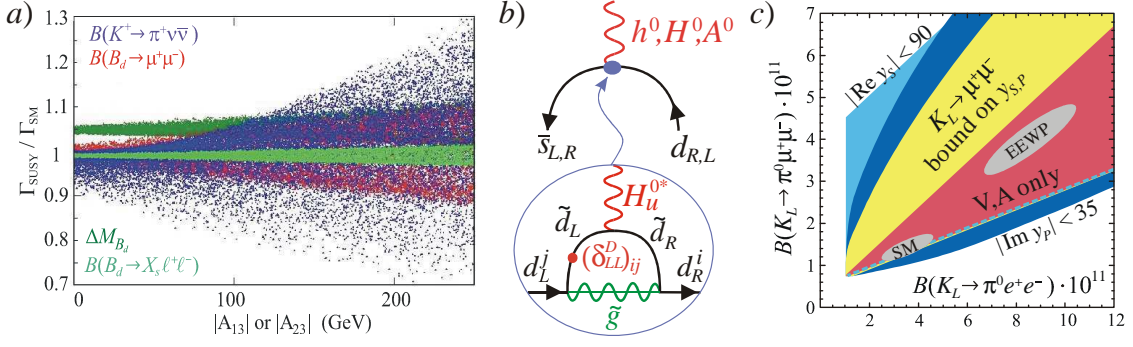


Figure 2: a) Sensitivity of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ to \mathbf{A}^U terms, compared to B -physics observables. b) Schematic representation of the neutral Higgs FCNC beyond MFV, at large $\tan \beta$. c) Impacts of dim-6 FCNC operators in the $\mathcal{B}(K_L \rightarrow \pi^0 \mu^+ \mu^-)$ vs. $\mathcal{B}(K_L \rightarrow \pi^0 e^+ e^-)$ plane.

above $\sim 200 \text{ GeV}$, and if $\tan \beta \gtrsim 5$ (to get rid of the H^\pm contribution). Turning on generic A^U terms, the largest deviations arise in $K \rightarrow \pi \nu \bar{\nu}$, see Fig.2a[9]. Further, the decoupling is slower than for observables sensitive to chargino boxes like ε_K . All in all, given that $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ has already been seen, how large the effect could be for $K_L \rightarrow \pi^0 \nu \bar{\nu}$? By an extensive, adaptive scanning over the MSSM parameter space, it has been shown[11] that it is possible to saturate the GN model-independent bound[12], which represents a factor ~ 30 enhancement of $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ over the SM.

At large $\tan \beta$, the chargino contributions decouple, both within and without MFV, while the Higgs FCNC obviously does not contribute (Fig.1e). However, higher order effects in the H^\pm contribution to the Z -penguin (Fig.1h), sensitive to δ_{RR}^D , can become sizeable beyond MFV[13]. Further, this contribution is slowly decoupling as M_H increases compared to tree-level neutral Higgs exchanges, as for example in $B_{s,d} \rightarrow \mu^+ \mu^-$.

SUSY effects in other dimension-six operators, $(\bar{s}d)(\bar{\nu}(\mathbf{1}, \gamma_5)\nu)$ and $(\bar{s}\sigma_{\mu\nu}d)(\bar{\nu}\sigma^{\mu\nu}(\mathbf{1}, \gamma_5)\nu)$, require active right-handed neutrinos and will not be discussed here[14]. Another possible class of operators, since the neutrino flavors are not detected, are $(\bar{s}\Gamma^A d)(\bar{\nu}^i \Gamma^B \nu^j)$ with $i \neq j$ and $\Gamma^{A,B}$ some Dirac structures. In the MSSM, such lepton flavor violating operators arise only from suppressed box diagrams, and cannot lead to significant effects[15]. However, they could be sizeable in the presence of R-parity violating terms[15, 16].

3. Supersymmetric effects in $K_L \rightarrow \pi^0 \ell^+ \ell^-$

Though the SM predictions for these modes are less accurate than for $K \rightarrow \pi \nu \bar{\nu}$, they are sensitive to more types of New Physics operators[17]. Indeed, the final-state leptons are now charged and massive. Therefore, besides electromagnetic effects, common to both the muon and electron modes, the relatively large muon mass opens the possibility to probe a whole class of helicity-suppressed effects.

SUSY effects in the QCD operators, i.e. in the chromomagnetic $\bar{s}\sigma_{\mu\nu}dG^{\mu\nu}$ or four-quark operators, have no direct impact on $K_L \rightarrow \pi^0 \ell^+ \ell^-$. Indeed, the two-photon CP-conserving contribution is fixed entirely in terms of the measured $K \rightarrow \pi\pi\pi, \pi\gamma\gamma$ modes[18], while the indirect

	$K \rightarrow \pi v \bar{v}$	$K_L \rightarrow \pi^0 \ell^+ \ell^-$
MFV, $\tan \beta \approx 2$	Best sensitivity, but max. enhancement $< 20\text{-}25\%$	Less sensitive, but precisely correlated with $K \rightarrow \pi v \bar{v}$
MFV, $\tan \beta \approx 50$	Negligible effects	
General, $\tan \beta \approx 2$	Best probes of δ_{LR}^U (quadratic dependence in δ_{LR}^U)	δ_{LR}^U : correlated with $K \rightarrow \pi v \bar{v}$ δ_{LR}^D : correlated with ϵ'/ϵ (but cleaner)
General, $\tan \beta \approx 50$	Good probes of δ_{RR}^D (slow decoupling as $M_H \rightarrow \infty$)	Good probes of $\delta_{RR,LL}^D$, corr. with $K_L \rightarrow \mu^+ \mu^-$ (but cleaner)

Table 1: Sensitivity of rare K decays to MSSM effects, within and without the MFV hypothesis, and with moderate and large $\tan \beta$. The dominant contributions can come from single, $(\delta_j^i)_{12}$, and/or double (e.g. $(\delta_j^i)_{32}^* (\delta_j^i)_{31}$) mass insertions, see text for the precise dependences.

CP-violating contribution is fixed from the measured ϵ_K and $\mathcal{B}(K_S \rightarrow \pi^0 \ell^+ \ell^-)$ [19]. New physics can thus explicitly enter through short-distance semi-leptonic FCNC operators only.

SUSY effects in the SM operators, which are the vector and axial-vector operators

$$\mathcal{H}_{eff} = y_{7V} (\bar{s}d)_V (\bar{\ell}\ell)_V + y_{7A} (\bar{s}d)_V (\bar{\ell}\ell)_A ,$$

can in principle be disentangled thanks to the different sensitivities of the two modes to the axial-vector current (it also produces $\ell^+ \ell^-$ in a helicity-suppressed 0^{-+} state). Various MSSM contributions can enter in y_{7A} and y_{7V} . First, chargino contributions to the Z -penguin (Fig.1c) enter as $y_{7A}, y_{7V} \sim (\delta_{RL}^U)_{32}^* (\delta_{RL}^U)_{31}$, and are thus directly correlated to the corresponding contribution to $K \rightarrow \pi v \bar{v}$ discussed previously [9, 20]. Within MFV, the maximal effect for $K_L \rightarrow \pi^0 \ell^+ \ell^-$ is about one third of the one for $K_L \rightarrow \pi^0 v \bar{v}$, hence may be inaccessible due to theoretical uncertainties. Secondly, gluino contributions to the electromagnetic operator $\bar{s} \sigma_{\mu\nu} d F^{\mu\nu}$ (Fig.1d) can be absorbed into $y_{7V} \sim (\delta_{RL}^D)_{12}$. Even if directly correlated with ϵ'/ϵ , sizeable effects in $K_L \rightarrow \pi^0 \ell^+ \ell^-$ are still possible [21]. Finally, H^\pm contributions arise at large $\tan \beta$ (Fig.1h), with $y_{7A}, y_{7V} \sim (\delta_{RR}^D)_{12}$, and are directly correlated with those for $K \rightarrow \pi v \bar{v}$ [13].

SUSY effects in the (pseudo)-scalar operators, which can be helicity-suppressed (i.e., $y \sim m_\ell$) or not:

$$\mathcal{H}_{eff} = y_S (\bar{s}d) (\bar{\ell}\ell) + y_P (\bar{s}d) (\bar{\ell}\gamma_5 \ell) + y'_S (\bar{s}\gamma_5 d) (\bar{\ell}\ell) + y'_P (\bar{s}\gamma_5 d) (\bar{\ell}\gamma_5 \ell) .$$

The first (last) two operators contribute to $K_L \rightarrow \pi^0 \ell^+ \ell^-$ ($K_L \rightarrow \ell^+ \ell^-$). In the MSSM at large $\tan \beta$, they arise from Higgs FCNC [22], and are thus helicity-suppressed (Fig.2b). Sizeable effects for the muon mode are possible beyond MFV, where they are sensitive to $(\delta_{RR,LL}^D)_{12}$ and $(\delta_{RR}^D)_{23} (\delta_{LL}^D)_{31}$ mass-insertions. Also, even if this contribution is correlated to the one for $K_L \rightarrow \mu^+ \mu^-$, given the large theoretical uncertainties for this mode, a factor ~ 4 enhancement is still allowed (Fig.2c) [17]. On the other hand, helicity-allowed contributions to these operators do not arise in the MSSM, but could be generated from R-parity violating couplings. Still, a precise fine-tuning of these couplings would be needed to have an effect for $K_L \rightarrow \pi^0 \ell^+ \ell^-$ without overproducing $\mathcal{B}^{\text{exp}}(K_L \rightarrow e^+ e^-) = 9_{-4}^{+6} \cdot 10^{-12}$ [17].

SUSY effects in the (pseudo-)tensor operators, $(\bar{s}\sigma_{\mu\nu}d)(\bar{\ell}\sigma^{\mu\nu}(\mathbf{1},\gamma_5)\ell)$, the last possible dimension-six semi-leptonic FCNC operators, are helicity-suppressed in the MSSM[23] and, being also phase-space suppressed, do not lead to any significant effect[17]. Further, they cannot arise from R -parity violating couplings.

4. Conclusion

The four rare K decay modes, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K_L \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \pi^0 \mu^+ \mu^-$, are the only theoretically clean windows into the $\Delta S = 1$ sector. If SUSY is discovered, the pattern of deviations they could exhibit with respect to the SM (see Table 1) will be essential to constrain the MSSM parameter-space, and hopefully unveil the nature of the SUSY-breaking mechanism.

References

- [1] U. Haisch, these proceedings.
- [2] For a review of new physics signals, see C. Tarantino, these proceedings, [*hep-ph*]0706.3436.
- [3] For an introduction, see for example S. P. Martin, *hep-ph/9709356*.
- [4] K. S. Babu, C. Kolda, *Phys. Rev. Lett.* **84** (2000) 228.
- [5] L. J. Hall, L. Randall, *Phys. Rev. Lett.* **65** (1990) 2939;
G. D'Ambrosio, G.F. Giudice, G. Isidori, A. Strumia, *Nucl. Phys.* **B645** (2002) 155.
- [6] Y. Nir, M.P. Worah, *Phys. Lett.* **B423** (1998) 319.
- [7] A.J. Buras, A. Romanino, L. Silvestrini, *Nucl. Phys.* **B520** (1998) 3.
- [8] G. Colangelo, G. Isidori, *JHEP* **09** (1998) 009.
- [9] G. Isidori *et al.*, *JHEP* **08** (2006) 064.
- [10] A. J. Buras *et al.*, *Nucl. Phys.* **B592** (2001) 55.
- [11] A. J. Buras, T. Ewerth, S. Jager, J. Rosiek, *Nucl. Phys.* **B714** (2005) 103.
- [12] Y. Grossman, Y. Nir, *Phys. Lett.* **B398** (1997) 163.
- [13] G. Isidori, P. Paradisi, *Phys. Rev.* **D73** (2006) 055017.
- [14] Y. Kiyo *et al.*, *Prog. Theor. Phys.* **101** (1999) 671; G. Perez, *JHEP* **9909** (1999) 019.
- [15] Y. Grossman, G. Isidori, H. Murayama, *Phys. Lett.* **B588** (2004) 74.
- [16] N. G. Deshpande, D. K. Ghosh, X. G. He, *Phys. Rev.* **D70** (2004) 093003;
A. Deandrea, J. Welzel, M. Oertel, *JHEP* **0410** (2004) 038.
- [17] F. Mescia, C. Smith, S. Trine, *JHEP* **08** (2006) 088.
- [18] G. Buchalla, G. D'Ambrosio, G. Isidori, *Nucl. Phys.* **B672** (2003) 387;
G. Isidori, C. Smith, R. Unterdorfer, *Eur. Phys. J.* **C36** (2004) 57.
- [19] G. D'Ambrosio, G. Ecker, G. Isidori, J. Portoles, *JHEP* **08** (1998) 004.
- [20] P. L. Cho, M. Misiak, D. Wyler, *Phys. Rev.* **D54** (1996) 3329.
- [21] A. J. Buras *et al.*, *Nucl. Phys.* **B566** (2000) 3.
- [22] G. Isidori, A. Retico, *JHEP* **0111** (2001) 001; *JHEP* **0209** (2002) 063.
- [23] C. Bobeth, A. J. Buras, F. Kruger, J. Urban, *Nucl. Phys.* **B630** (2002) 87.