

1        **Optimisation of orthopaedic implant design**  
2        **using statistical shape space analysis based on**  
3        **level sets**

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10        **Abstract**

11        Statistical shape analysis techniques have shown to be efficient tools to build pop-  
12        ulation specific models of anatomical variability. Their use is commonplace as prior  
13        models for segmentation, in which case the instance from the shape model that  
14        best fits the image data is sought. In certain cases, however, it is not just the most  
15        likely instance that must be searched, but rather the whole set of shape instances  
16        that meet certain criterion. In this paper we develop a method for the assessment  
17        of specific anatomical/morphological criteria across the shape variability found in a  
18        population. The method is based on a level set segmentation approach, and used on  
19        the parametric space of the statistical shape model of the target population, solved  
20        via a multi-level narrow-band approach for computational efficiency. Based on this  
21        technique, we develop a framework for evidence-based orthopaedic implant design.  
22        To date, implants are commonly designed and validated by evaluating implant bone  
23        fitting on a limited set of cadaver bones, which not necessarily span the whole vari-  
24        ability in the population. Based on our framework, we can virtually fit a proposed  
25        implant design to samples drawn from the statistical model, and assess which range  
26        of the population is suitable for the implant. The method highlights which patterns  
27        of bone variability are more important for implant fitting, allowing and easing im-  
28        plant design improvements, as to fit a maximum of the target population. Results  
29        are presented for the optimisation of implant design of proximal human tibia, used  
30        for internal fracture fixation.

31        *Key words:* Statistical shape models, image registration, principal component  
32        analysis, level sets, orthopaedics, implant design

## 33 1 Introduction

34 Statistical shape analysis techniques have shown to be efficient tools to build  
35 population specific models of anatomical variability. Their flagship, the Ac-  
36 tive Shape Model (ASM), proposed by Cootes et al. (1995) provides a method  
37 to study the variability encountered across a population in a compact repre-  
38 sentation based on a decomposition via principal components analysis (PCA)  
39 (Bishop, 1995). Statistical shape models representing the variation of shape  
40 and gray-level appearance, namely Active Appearance Models (AAM) (Cootes  
41 et al., 2004; Cootes and Taylor, 2004), have been extensively used in image  
42 segmentation to locate structures of interest and to solve many medical image  
43 interpretation problems. For instance, they have been used to locate vertebrae  
44 in DXA images of the spine (Cootes and Taylor, 2004; Roberts et al., 2006;  
45 Smyth et al., 1996), structures in MR images of the brain (van Ginneken et al.,  
46 2002; Hill et al., 1994), the femoral head in MR images (Cootes and Taylor,  
47 2004), the prostate in MR images (Haslam et al., 1994), and the outlines of  
48 ventricles of the heart in echocardiograms (Hill et al., 1994; Mitchell et al.,  
49 2000), amongst others. A comprehensive review of statistical shape models  
50 for 3D medical image segmentation is given by Heimann and Meinzer (2009).  
51 More recently, statistical shape models have been used for shape estimation  
52 in image-free computer assisted surgery (Rajamani et al., 2007).

53 In all these applications, the approach is to find the instance in the statistical  
54 shape model that best approximates the input data, subject to some regular-  
55 isation constraints (Davies et al., 2002; Rajamani et al., 2007). Optimisation  
56 in shape space of more complex criteria based on clinically meaningful shape  
57 measures related to anatomical locations has not been fully explored. Sierra  
58 et al. (2006) formulate a minimisation process based on Lagrange multipliers  
59 to incorporate such additional constraints, and then optimise this criterion  
60 based on a gradient descent algorithm starting from the mean of the shape  
61 distribution. This is used in their application to generate virtual anatomical  
62 models for surgery simulation, instantiated by specifying clinical parameters,  
63 such as fundus/cervix length/width, that depend non-linearly on the shape  
64 coefficients. However, it is not guaranteed that their optimisation algorithm  
65 will produce the instance of the shape space that best meets the constraints.  
66 Further, in common to other existing works, the aim is to find a single instance  
67 from the statistical shape model as the solution to their problem. In certain  
68 cases, it may be interesting to find *all* instances of the shape model that meet

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69 a certain criterion. That is, one may be interested in estimating which range  
70 of the population falls within a given anatomical criterion, thus establishing  
71 a partition of the shape space into “valid” and “invalid” shapes.

72 In this work our aim is to develop a framework to evaluate a given anatomi-  
73 cal/morphological criterion across the full PCA shape space, in order to find  
74 the group of shape instances that satisfy the criterion. The method is based on  
75 level sets on the parametric domain of the shape coefficients. Level set methods  
76 define a powerful optimisation framework that, in combination with statistical  
77 shape priors, has been used to recover objects of interest by the propagation  
78 of curves or surfaces (Bresson et al., 2006; Chen et al., 2002; Cremers, 2006;  
79 Leventon et al., 2000; Rousson et al., 2004). However, these previous works  
80 are of a very different nature to ours, as they deal with the extraction of  
81 structures of interest in medical images, employing level sets as their choice  
82 of shape representation. The shape prior is thus defined as a PCA of levels  
83 set representations, and the segmentation method finds the most likely shape.  
84 In our case, we do not employ level sets as a shape modelling tool, but as an  
85 optimisation framework to assess complex criteria in PCA space. The level  
86 set is therefore defined in the parametric shape coefficient space, not in image  
87 space. The high dimensionality of level sets allows for the segmentation of the  
88 space of any dimension, determined by the number of principal components  
89 retained. Moreover, the ability to represent complex topologies can be used to  
90 identify disconnected subsets of the shape space that meet the criterion.

91 The ultimate goal of an orthopaedic implant is to stabilise the fractured bone,  
92 to enable fast healing of the injured bone, and to return early mobility and full  
93 function of the injured extremity. These aspects are related to the shape of the  
94 implant, its material and the mechanical response it produces to decrease the  
95 stress at the fracture site. Although these three aspects should be considered  
96 when designing an orthopaedic implant, in this work we focus on the shape of  
97 the implant, and its ability to fit to the bone surface. Mechanical and material  
98 aspects are out of the scope of the presented study, although some comments  
99 about mechanical considerations are included in the conclusions section.

100 Current practice in orthopaedic research involves the evaluation of implants for  
101 fracture fixation by manual fitting and fixation procedures, applied on a small  
102 set of cadaver bones in a trial-and-error process to find the optimal implant  
103 shape and position (Goyal et al., 2007). More recently, a noninvasive semi-  
104 automatic method for quantifying implant fitting was developed (Schmutz  
105 et al., 2008). Although the authors discussed recommendations for optimising  
106 fitting, there are no real results on how these modifications would improve the  
107 fitting. Moreover, the method was tested on a small set of 21 CT data sets.  
108 Using limited amount of CT data or cadaver specimens does not necessarily  
109 describe the diversity in a population, such as age, gender or ethnic origin.  
110 This diversity can be studied using statistical shape analysis techniques. In this

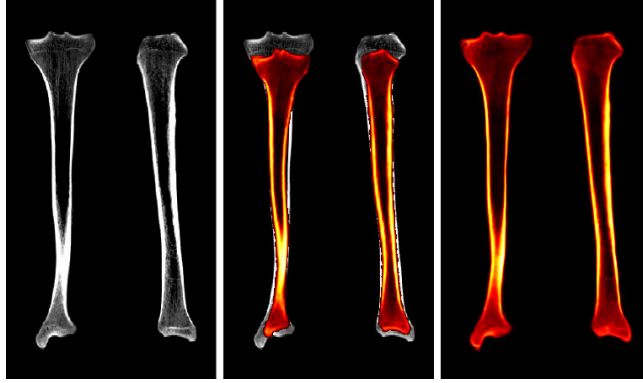


Fig. 1. Image registration. CT slice of the left human tibia, chosen as a reference bone; overlay of the rigid registration; overlay of the non-rigid registration matching the reference.

111 work, we show how our framework can be used as an evidence-based design  
 112 methodology, assessing implant fitting on samples drawn from a statistical  
 113 shape-and-intensity model by means of an automatic fitting procedure. We  
 114 thus evaluate which proportion of the whole population is correctly fit by  
 115 the proposed design. Then, by correlating segmented instances to the PCA  
 116 manifold we are able to propose modifications to implant shape design as to  
 117 fit a maximum of the target population.

118 Section 2 will briefly introduce the basic concepts behind statistical shape  
 119 models based on PCA. In section 3, the key idea will be presented, that is  
 120 the use of level set segmentation for PCA shape space optimisation. In section  
 121 4 we describe our framework for orthopaedic implant fitting assessment, and  
 122 show results on the optimisation of the design of human tibial plates. Finally,  
 123 discussion and conclusions are provided in section 5.

## 124 **2 Statistical shape model**

### 125 *2.1 Image registration*

126 The first step in generating a statistical model from a training set of images  
 127 or shapes is to establish correspondences across the samples in the training  
 128 set. Numerous approaches have been proposed in the literature, but since our  
 129 aim in this paper is to construct shape-and-intensity models we will focus on  
 130 non-rigid image registration techniques, and will illustrate the approach on  
 131 CT images of human tibiae.

132 First, an image from the training set is selected as the reference, using an  
 133 average box size as a reference, to which all other images will be registered.

134 In order to compensate for the different positioning during CT acquisition, we  
 135 spatially align the remaining images of the training data set with the selected  
 136 reference, via rigid registration. This allows to overcome the pose disparity  
 137 and to maintain the size variation of the tibia (Figure 1). The next step in  
 138 our model construction consists in warping the instances in the training set  
 139 to the reference image. To capture the entire anatomical variability, we ap-  
 140 ply an intensity-based non-rigid registration algorithm (Rueckert et al., 2001,  
 141 2003). This algorithm defines the deformation as a B-spline mapping, defined  
 142 by a uniformly-spaced grid of control points and the corresponding B-spline  
 143 coefficients.

144 For the registration of CT data sets in our particular application, we employ  
 145 sum of square distances (SSD) as the similarity metric, and gradient descent as  
 146 the optimisation function. Based on the deformation fields obtained from the  
 147 registration process, we build vectors of corresponding positions and image  
 148 intensities. The reference image can be described as in Generalized Image  
 149 Models (González et al., 2004):

$$v_R = (x_1, y_1, z_1, I_1, \dots, x_n, y_n, z_n, I_n), \quad (1)$$

150 where  $n$  is the number of voxels in the region of interest and  $I_i$  is the intensity  
 151 at voxel  $(x_i, y_i, z_i)$ . Similarly, each of the other images can be described as a  
 152 vector of the same length:

$$v_j = (x_1 + \Delta x_1^j, y_1 + \Delta y_1^j, z_1 + \Delta z_1^j, I_1^j, \dots, \\ x_n + \Delta x_n^j, y_n + \Delta y_n^j, z_n + \Delta z_n^j, I_n^j), \quad (2)$$

153 where  $(\Delta x_i^j, \Delta y_i^j, \Delta z_i^j)$  is the displacement vector at position  $(x_i, y_i, z_i)$ , and  
 154  $I_i^j$  is the intensity of the voxel  $(x_i + \Delta x_i^j, y_i + \Delta y_i^j, z_i + \Delta z_i^j)$  in image  $j$ .

## 155 2.2 Principal Component Analysis

156 The resulting image vectors described in Eq. (2) are high dimensional data,  
 157 because we consider every point coordinate in the region of interest. To reduce  
 158 the dimensionality of the data and obtain a compact parametric description,  
 159 we apply principal component analysis. PCA is a multivariate factor analysis  
 160 technique aiming at finding a low-dimensional manifold in the space of the  
 161 data, such that the distance between the data and its projection on the mani-  
 162 fold is small (Bishop, 1995). PCA is the best, in the mean-square error sense,  
 163 linear dimension reduction technique.

164 Given a set of training data  $\{\vec{t}_1, \vec{t}_2, \dots, \vec{t}_N\}$ , with  $\vec{t}_i = (\vec{x}_i, \vec{y}_i, \vec{z}_i)$  and  $N$  equal to  
 165 number of training instances, PCA finds a new orthonormal basis  $\{\vec{u}_1, \dots, \vec{u}_D\}$

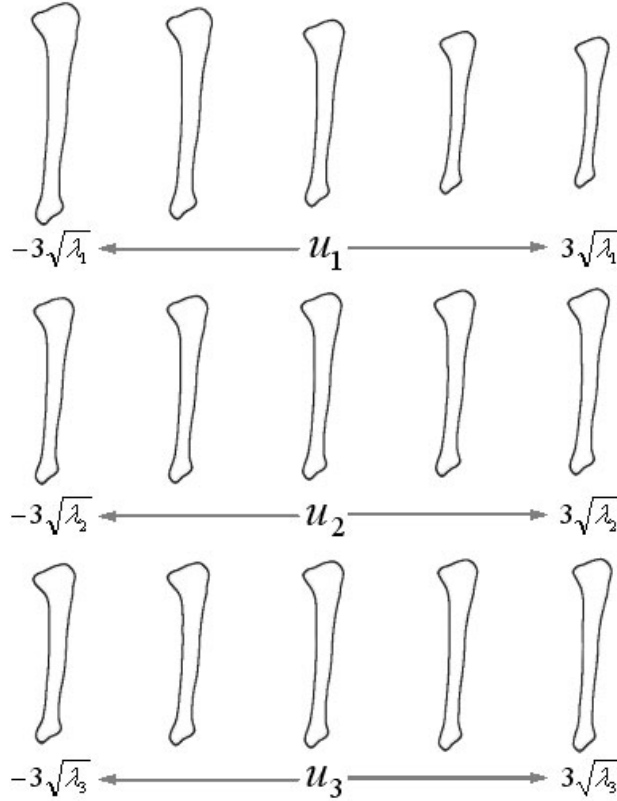


Fig. 2. The three first modes of variation for left human tibia are visualized individually. The first mode affects the change in the tibia length; the second mode influences the changes of the lateral condyle and a slight torsion of the lateral surface of the tibia; the third mode affects abduction of the medial condyle, changes of medial malleolus and in medial surface of the tibia.

166 with its axes ordered. This new basis is rotated such that the first axis is  
 167 oriented along the direction in which the data has its highest variance. The  
 168 second axis is oriented along the direction of maximal variance in the data,  
 169 orthogonal to the first axis. Similarly, subsequent axes are oriented so as to  
 170 account for as much as possible of the variance in the data, subject to the  
 171 constraint that they must be orthogonal to the preceding axes. Consequently,  
 172 these axes have associated decreasing “index”  $\lambda_d$ ,  $d = 1, \dots, D$ , corresponding  
 173 to the variance of the data set when projected on the axes. The principal  
 174 components are the set of new ordered basis vectors.

The principal components are found by computing the sample covariance matrix of the data set,  $\vec{S}$ , and then finding its eigenstructure

$$\vec{S}\vec{U} = \vec{U}\vec{\Lambda}.$$

175  $\vec{U}$  is a  $D \times D$  matrix which has the unit length eigenvectors  $\vec{u}_1, \dots, \vec{u}_D$  as  
 176 its columns, and  $\vec{\Lambda}$  is a diagonal matrix with the corresponding eigenvalues  
 177  $\lambda_1, \dots, \lambda_D$ . The eigenvectors are the principal components and the eigenvalues

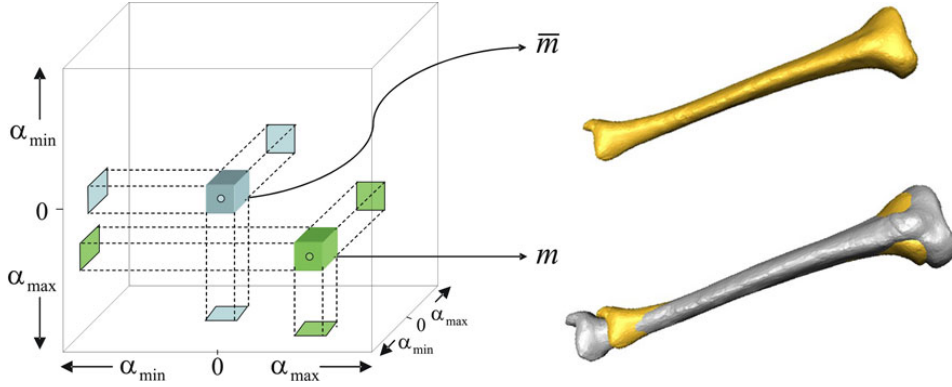


Fig. 3. Shape space defined by the three first principal components. The center element (labeled in the figure  $\bar{m}$ ) corresponds to the mean of the population. Each element in this shape space is formed by a linear combination of the principal components, in this case  $m = \bar{m} + \alpha_1\sqrt{\lambda_1}\vec{u}_1 + \alpha_2\sqrt{\lambda_2}\vec{u}_2 + \alpha_3\sqrt{\lambda_3}\vec{u}_3$ .

178 their corresponding projected variances (Figure 2).

### 179 3 Optimisation in PCA space using level sets

#### 180 3.1 PCA shape space mapping

181 Let us consider the shape space defined by the weighted linear combination of  
 182 the first  $L \leq D$  eigenvectors  $\vec{u}_1, \dots, \vec{u}_L$  of the PCA decomposition of a set of  
 183 training shapes in  $\mathcal{R}^D$ . Each element  $m \in \mathcal{R}^D$  in this shape space is defined  
 184 by a set of coefficients  $\alpha_1, \dots, \alpha_L$  (Figure 3):

$$m = \bar{m} + \sum_{i=1}^L \alpha_i \sqrt{\lambda_i} \vec{u}_i, \quad (3)$$

185 where  $\lambda_1, \dots, \lambda_L$  are the eigenvalues corresponding to each principal compo-  
 186 nent, and  $\bar{m}$  is the arithmetic mean of the training sets. Now let us consider  
 187 a scalar mapping  $\mathcal{M} : A = [\alpha_{min}, \alpha_{max}]^L \rightarrow \mathcal{R}$ . This mapping can represent a  
 188 clinically meaningful anatomical criterion derived from the shapes in the PCA  
 189 space (e.g. femoral inclination angle (Kozic et al., 2008)). We now would like  
 190 to find all instances in the shape space that meet a certain criterion dependent  
 191 on the scalar measure. This problem is approached as a segmentation in the  
 192 PCA shape space defined by the mapping  $\mathcal{M}$  defined above, and solved using  
 193 the level sets framework described in the following section.

195 Segmentation techniques based on active contours, or deformable models, have  
 196 been widely used in image processing for different medical applications (Kass  
 197 et al., 1987; McInerney and Terzopoulos, 1996). The idea behind active con-  
 198 tours is to extract the boundaries of homogeneous regions within the image,  
 199 while keeping the model smooth during deformation. In such models, the ini-  
 200 tial contour, specified by the user, is evolved to the boundaries of the object  
 201 by balancing two energy forces. The first force, computed from image data,  
 202 represents external energy that attracts the curve toward image features, while  
 203 the second force, defined within the curve, represents the internal energy and  
 204 affects the smoothness of the curve. A particular instantiation of this paradigm  
 205 is that of active contours based on level sets (Chan and Vese, 2001; Chen and  
 206 Guan, 2004; Mumford and Shah, 1989; Tsai et al., 2001).

207 Let us consider a parameterized closed surface  $C(s) : S = [0, 1]^{L-1} \rightarrow \mathcal{R}^L$   
 208 defined in a bounded region  $\Omega \in \mathcal{R}^L$ . In order to segment the observed image  
 209  $\mu : \Omega \rightarrow \mathcal{R}$  we propose to minimize the following energy functional:

$$E(C) = a \int_{\omega} (\mu - \epsilon) \partial\Omega + b \int_S |C'| ds, \quad (4)$$

210 where  $\omega \subset \Omega$  and  $C = \partial\omega$  is the closed surface. The first term represents the  
 211 boundary force that attracts the evolving surface toward a predefined segmen-  
 212 tation constraint  $\epsilon = const$ , while the second term regulates the smoothness  
 213 of the surface. Here,  $a$  and  $b$  are positive scalar weights.

214 The energy functional proposed in Eq. (4) is not easy to solve because of  
 215 the unknown set of complex surfaces  $C$  and unidentified image topologies.  
 216 The segmentation algorithm developed in this work is based on the implicit  
 217 representation of deformable models implemented within the framework of  
 218 level sets. This implicit representation for evolving curves, introduced by Os-  
 219 her and Sethian (1988), allows automatic change of topologies without re-  
 220 parametrization. Using the level set formulation, the boundary surface  $C = \partial\omega$   
 221 can be modeled as a zero level set of a Lipschitz function  $\phi$ , defined on the  
 222 entire image domain  $\Omega$  as (Figure 4):

$$\begin{aligned} C = \partial\omega &= \{x \in \Omega : \phi(x) = 0\}, \\ inside(C) = \omega &= \{x \in \Omega : \phi(x) > 0\}, \\ outside(C) = \Omega \setminus \omega &= \{x \in \Omega : \phi(x) < 0\}. \end{aligned}$$

223 Having the Heaviside function  $H(\phi)$  defined on the whole image domain as  
 224  $\int_{\omega} \partial\Omega = \int_{\Omega} H(\phi) dx$ , for  $\omega \subset \Omega$ , and its corresponding Dirac function  $\delta(\phi) =$   
 225  $\frac{d}{d\phi} H(\phi)$ , we can replace the unknown variable  $C$  by the level set function  $\phi(x)$



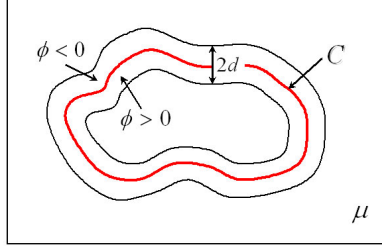


Fig. 4. Narrow band level set approach allows us to compute mapping values only for the points in a narrow band around the zero level set (red line).

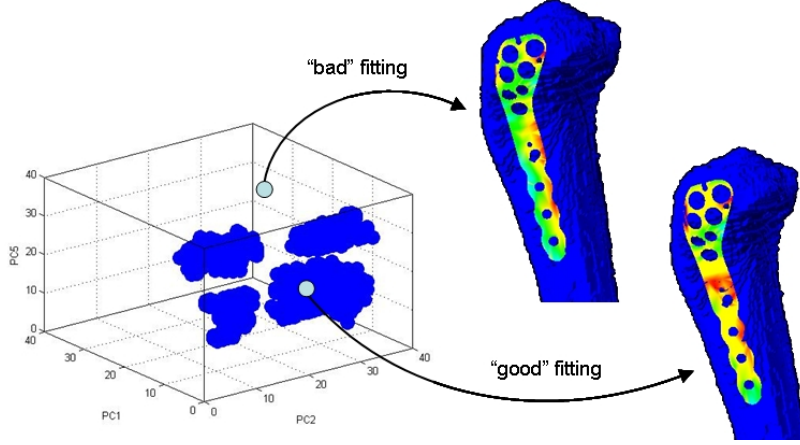


Fig. 5. Level set segmentation of the PCA shape space. Clinical criteria is chosen to be the implant fitting distance to the proximal human tibia. Segmented regions (in blue) satisfy the given segmentation criterion for the 'good' fitting (average fitting distance less than 1mm). Results of the fitting for the instances from 'good' and 'bad' fitting area in the PCA shape space are visualised.

226 as:

$$E(\phi) = a \int_{\Omega} (\mu - \epsilon) H(\phi) dx + b \int_{\Omega} \delta(\phi) |\nabla(\phi)| dx, \quad (5)$$

227 where the surface value  $|C(\phi = 0)| = \int_{\Omega} \delta(\phi) |\nabla(\phi)| dx$  is estimated directly  
 228 from the level set function (Evans and Gariepy, 1992). By minimizing the  
 229 energy functional with respect to  $\phi$  we get a model associated Euler-Lagrange  
 230 equation for boundary flow:

$$\frac{\partial \phi}{\partial t} = a (\mu - \epsilon) \delta(\phi) + b \operatorname{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) \delta(\phi), \quad (6)$$

231 where  $t$  is an artificial time  $t \geq 0$  for boundary flow and  $\int_{\Omega} |\nabla(\phi)| dx =$   
 232  $\operatorname{div} \frac{\nabla(\phi)}{|\nabla(\phi)|}$ .

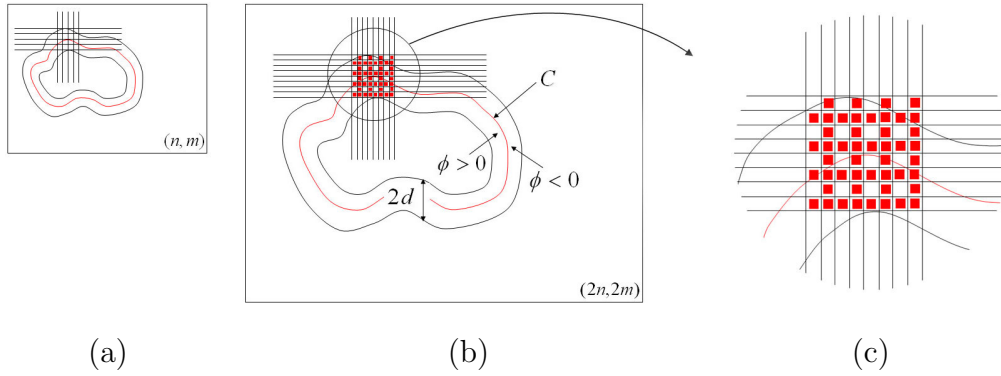


Fig. 6. Hierarchical approach to narrow band zero-level set evolution. (a) Initial low resolution 2D image space map with a stable zero level set in red colour and a narrow band around it. (b) Higher-resolution map with the augmented narrow band and zero level set, adopted from the low-resolution map. (c) The values of white pixels in the grid map come from the low resolution map, while the values of the red pixels that come from the augmented map still need to be calculated.

### 233 3.3 Level set optimisation in PCA shape space

234 In the framework of our application to the evaluation of anatomical criteria  
 235 in PCA shape space, shape space will be the  $L$ -dimensional “image”  $\mu$  to be  
 236 segmented, defined in the domain of shape coefficients  $\Omega = A$ . Thus, level  
 237 sets are used to find the region in the shape space defined by the weights  
 238 applied to the principal components, in which the criterion is met (Figure 5).  
 239 The flexibility of level sets allows to identify disconnected regions of the shape  
 240 space. Further, the generality of the method allows to define any criterion,  
 241 including complex functions that depend non-linearly on the shapes defined  
 242 by the principal components.

243 It must be noted that in this work level sets are not used as a shape repre-  
 244 sentation method, as is the case in all previous works that combine level sets  
 245 with statistical shape models (employed as prior in the segmentation process).  
 246 Rather, we do the analysis in the statistical shape space directly, not in im-  
 247 age space, and we deal with the identification of a population, rather than a  
 248 particular image.

### 249 3.4 Hierarchical approach to zero level set evolution

250 In order to decrease the computational complexity of the standard level set  
 251 method we extend a narrow band level set approach, which uses only the points  
 252 close to the evolving front at every time step (Adalsteinsson and Sethian, 1995)  
 253 to hierarchical narrow band level set (HNBLS) approach. First we initialize  
 254 our level set function using automatic seed initialisation on a low resolution

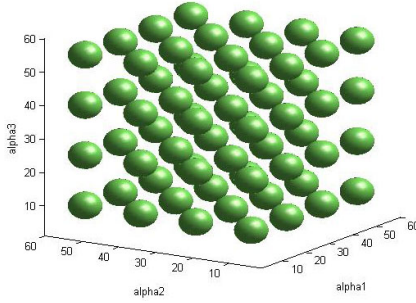


Fig. 7. Seed initialisation of the level set function.

255 image map. The seed initialisation consists of partitioning the data image  $u_0$   
 256 into  $N$  windows  $W_n$ ,  $n = 1..N$  (Figure 7). Windows are of predefined size  
 257 and do not overlap. The size is selected empirically to be  $dim(\mu)/15$  in order  
 258 to detect all the small “irregularities” in the image space and to decrease  
 259 computational time. Level set function is computed only in these seed points.  
 260 Then, minimisation of the energy functional (Eq. (5)) is performed to evolve  
 261 the surface towards the segmented region.

262 We define a thin band around the zero-level set, that contains the neighboring  
 263 points with distance to the zero-level less than  $d_{max}$  and we update the level  
 264 set only on these points (Eq.(6)), instead of re-calculating it for each grid  
 265 point (Figure 6a). As the zero-level set corresponding to the front evolves,  
 266 we must ensure that it stays within the band. We re-initialise the band after  
 267 10 iterations, when the front is close to the edge of the domain, using the  
 268 current zero-level set as the initial surface. Once the stable boundaries of the  
 269 low resolution map are reached we increase the resolution of the image space  
 270 and continue zero-level set surface evolution in the augmented low-resolution  
 271 narrow band (Figure 6b).

272 The hierarchical narrow band level set algorithm is as follows (Figure 8):

273 **Step 1.** Initialise the zero level set function  $\phi^0$ , as a corresponding circular  
 274 signed distance on each window  $W_n$ . Construct a thin band around zero-level  
 275 set  $\beta^0 = N(\phi^0)$ .

276 **Step 2.** Update  $\phi^{k+1}$  for all pixels on  $\beta^k$  (Eq.(6)). If  $k(mod10) = 0$  then go  
 277 to Step 4, else if  $k$  is equal to a maximum number of iterations, then stop.

278 **Step 3.** Update narrow band  $\beta^k$  and assign values to new pixels on narrow  
 279 band. Outside the domain the value is defined as:  $\phi^{k+1} = +d_{max}$  if the point  
 280 is inside of the curve and  $\phi^{k+1} = -d_{max}$  if the point is outside of the curve.

281 Go back to Step 2.

282 **Step 4.** Increase the resolution of the image space and compute the values of  
 283 the missing pixels in the augmented low-resolution narrow band. Go back to  
 284 Step 2 (Figure 6).

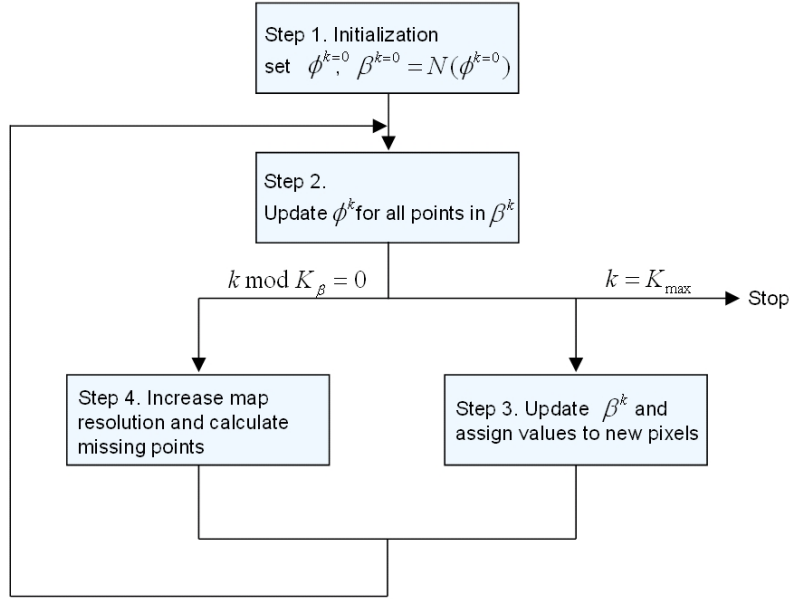


Fig. 8. Hierarchical level set segmentation algorithm.

## 285 4 Optimisation of orthopaedic implant designs

### 286 4.1 Clinical context

287 Since the late 1950s, open reduction and internal fracture fixation has been  
 288 used to restore bone anatomy and enable early mobilization. Internal fixa-  
 289 tion alters the biology of fracture healing and reduces strain at the fracture  
 290 site. Plate contouring is an important step in osteosynthesis. Plates are pre-  
 291 contouring before or during a surgery to match either patient-specific or an av-  
 292 erage bone anatomy. The safety and ease of this procedure depends on certain  
 293 material properties of the plate, such as the yield point and fatigue endurance  
 294 (Frankel and Burstein, 1970). In addition to this, contouring is affected by the  
 295 complexity of the bone shape to which the plate has to fit.

296 Nowadays, with an annual incidence of over a half million fractures of the tibia  
 297 and fibula in the US (Russell and Levine, 1996), manufactures are moving from  
 298 costly patient-specific implant design to the average implant shape that can  
 299 fit to a given population.

300 Currently in orthopaedic research, the evaluation of implants for fracture fix-  
 301 ation is done by manual fitting and fixation procedures, applied on a small set  
 302 of cadaver bones in a trial-and-error process to determine the optimal implant  
 303 shape and position (Figure 9). Goyal et al. (2007) investigated the accuracy of  
 304 the periarticular tibial plate fit using 101 cadaver specimens of human tibia, on  
 305 whom the implants were manually fixed by visually finding the best implant



Fig. 9. Internal fixation of the proximal tibia implant.

306 position. More recently, a noninvasive semi-automatic method for quantifying  
 307 implant fitting was developed (Schmutz et al., 2008). In this study the surface  
 308 of the plate was fitted to 21 computer tomography (CT) based 3D models of  
 309 human tibia. Although the recommendations for implant modifications were  
 310 discussed, there are no conclusive results on how these modifications would  
 311 improve fitting.

#### 312 4.2 Automatic implant fitting algorithm

313 A modified Iterative Closest Point (ICP) technique (Besl and McKay, 1992),  
 314 developed in our group (Reyes et al., 2008), was used for the specific task of  
 315 bone implant fitting. The method initialises the position of the implant close  
 316 to the bone surface and optimises its position as to fit the bone as closely as  
 317 possible, subject to specified positioning constraints. Based on this result, it  
 318 computes the distance map from each point in the implant to the closest point  
 319 in the bone surface.

320 In this work, the method is refined by a modified collision constraint to ensure  
 321 that no points in the implant mesh model fall inside the bone model. Colli-  
 322 sion detection is performed by tracking the change of direction between the  
 323 vector pointing from the inspected point to its closest point to the mesh and  
 324 its normal vector. In addition, fitting guidelines provided by the implant man-  
 325 ufacturer were included as fitting constraints, this in order to find plausible  
 326 implant fittings. These further specific constraints favor fittings of the implant  
 327 that are collinear with the bone main axis, and do not take place above the  
 328 bone plateau (Figure 10).

329 The constrained ICP algorithm is based on the optimization of the following  
 330 functional:

$$\operatorname{argmin}_i \sum W_i \|e_i\|, \quad (7)$$

331 where  $W_i$  and  $e_i$  are the corresponding weight and distance error for point  
 332  $i$  in the implant mesh model, respectively. The weights  $W_i$  are computed as

333 a linear combination of constraint-specific weights for collision  $W_i^C$ , implant-  
 334 bone collinearity  $W_i^{\parallel}$ , and tibia plateau  $W_i^p$ :

$$W_i = W_i^C + W_i^{\parallel} + W_i^p. \quad (8)$$

335 The collision weight  $W_i^C$  is computed as follows:

$$W_i^C = \begin{cases} 1 & p_i \notin V_{in} \\ k_i^c \|e_i\| & p_i \in V_{in} \end{cases}, \quad (9)$$

336 where  $V_{in}$  is the 3D space inside the bone model. To detect if a point  $p_i$  is  
 337 inside or outside the bone model, the sign of the dot product between the  
 338 normal vector on the bone surface closest to  $p_i$  and the vector formed by  $p_i$   
 339 and its closest point on the bone surface is computed.

340 In order to avoid biases due to the number of points inside and outside the  
 341 volume, the variable  $k_i^c$  in Eq. (9) is proposed by the following inequality:

$$k_i^c \geq (N_{tot} - N_{in}) / \sum_{i \in V_{in}} \|e_i\|, \quad (10)$$

342 with  $N_{tot}$  the number of points of the implant mesh, and  $N_{in}$  the number of  
 343 points falling inside the bone model. We have found that adjusting the weight  
 344  $k_i^C$  we avoid biases due to the number of points inside and outside as the  
 345 iterations proceed.

346 Similarly as for the collision constraint, weights  $W_i^{\parallel}$ , and  $W_i^p$  are computed as  
 347 follows:

$$W_i^{\parallel} = \begin{cases} 1 & \alpha \leq \alpha_{th} \\ k^{\parallel} \|\alpha_{th} - \alpha\| & \alpha > \alpha_{th} \end{cases}, \quad (11)$$

348

$$W_i^p = \begin{cases} 1 & p_i \in \Gamma \\ k_i^p \|p_i - \Upsilon\| & p_i \notin \Gamma \end{cases}, \quad (12)$$

349 where  $\alpha$  is the angle between the implant main axis and the bone main axis,  
 350  $\alpha_{th}$  is a threshold angle chosen by the user,  $k^{\parallel}$  is a scalar chosen empirically  
 351 and used to weigh the global effect of the parallelism constraint,  $\Upsilon$  is the z-  
 352 coordinate of the plateau region interface, and  $\Gamma$  is the 3D space above the  
 353 bone plateau (Figure 10).

354 For the computation of  $\alpha$  the main axis of the implant model and the bone  
 355 are required. This is performed through a Oriented-Bounding-Box (OBB) de-  
 356 composition of both shapes (Figure 11b). Furthermore, for the implant model,  
 357 only the lower region is used in order to improve the alignment between the  
 358 bone shaft and the implant.

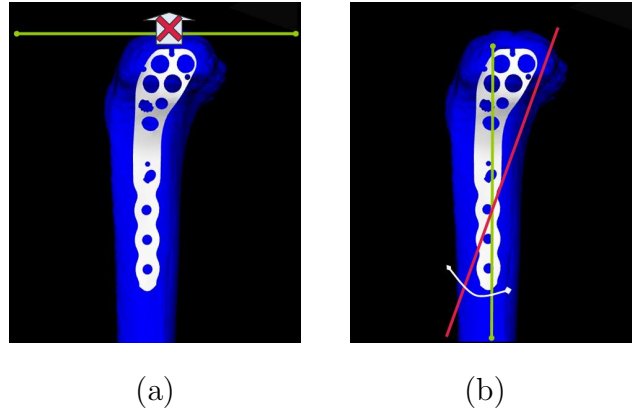


Fig. 10. The constraints proposed by the implant manufacturer: (a) plateau constraint and (b) parallelism constraint.

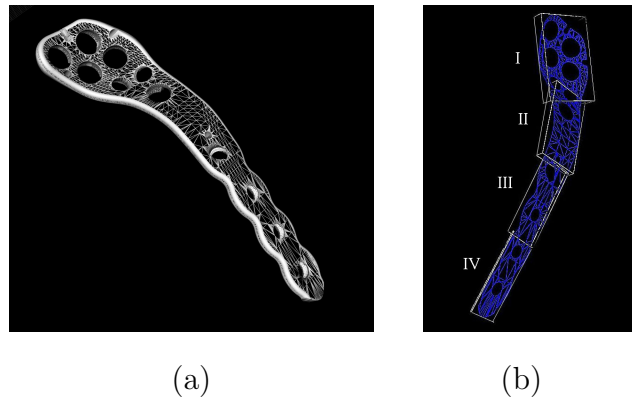


Fig. 11. (a) The original implant model and the extracted inner surface. (b) The Oriented-Bounding-Boxes of the implant.

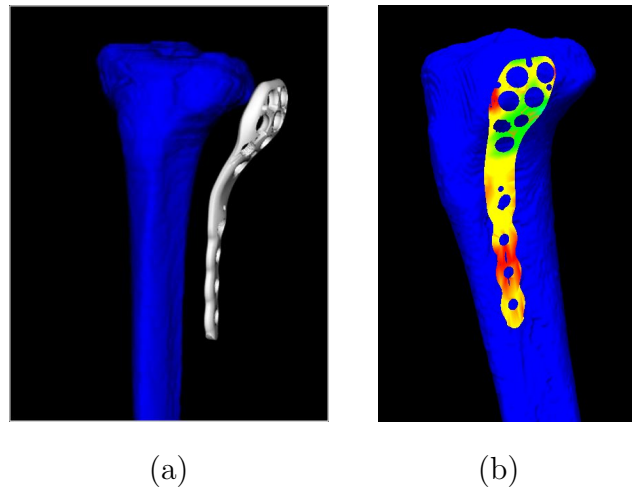


Fig. 12. (a) The initialisation of the implant fitting. (b) The final result of the implant fitting shows the distance map of fitting error, where the red colour represents the perfect fitting of the implant to the bone and the green colour represents the distance of 3mm to the bone surface.

359 Figure 12 shows the initialisation step of the automatized implant fitting pro-  
360 cedure and the final result of the fitting where the colour map of the implant  
361 represents the distance map of the fitting error.

### 362 4.3 Population-based evaluation of implant fitting

363 We apply our method to evaluate the performance of orthopaedic implants,  
364 used for internal fracture fixation of the proximal tibia, within the context  
365 of the PCA space level set evaluation framework described in Section 3, to  
366 optimise the implant shape as to fit a majority of the target population. Figure  
367 13 illustrates the complete procedure.

368 We present results obtained from a training set of tibia surface models ex-  
369 tracted from CT data. The training set consists of 92 left human tibiae from  
370 which Asian, Caucasian, male and female are equally present. Statistical shape  
371 modeling was then performed, as explained in Section 2. We retain the first  
372 five principal components, which account for 92% of shape variability in the  
373 population. Using more than five modes to explain the statistical model would  
374 give us more subtle changes which, however, do not bring modifications in the  
375 area of implant placement (Figure 14). We define the mapping transformation  
376  $\mathcal{M}$  as the mean error distance from 844 points sampled on the implant surface  
377 to their corresponding best fitted points on the bone surface. The PCA shape  
378 space is then built by sampling the space of shape coefficients, generating the  
379 corresponding shape, and then computing the mapping  $\mathcal{M}$  to obtain the mea-  
380 sure of interest. We use the range  $-3 \leq \alpha_i \leq 3$  for every shape coefficient.  
381 This accounts for 99.7% of the shape variability encompassed in each principal  
382 component.

383 We start with a low resolution sampling of the PCA space, namely a sampling  
384 step of  $\Delta\alpha_i = 0.1$  for each principal component (which would result in a map  
385 of 60x60 instances if two principal components were retained). We initialise  
386 the zero level set by applying seed initialisation on the PCA shape space, and  
387 then proceed with the hierarchical narrow-band zero level set evolution, as  
388 explained in Section 3.4. We do not need to explicitly generate all instances  
389 and compute mean error fitting for every point in the shape space, but only  
390 in the narrow band around the evolving zero level set. We continue with a  
391 hierarchical narrow band approach by reducing successively the sampling step  
392 to  $\Delta\alpha_i = 0.05$  and  $\Delta\alpha_i = 0.025$ , respectively. As sampling resolution in-  
393 creases, the narrow band level set approach becomes mandatory to decrease  
394 high computation times and to reduce the search space of shape parameters.

395 For the given implant fitting problem, which includes space optimisation, in-  
396 stance creation and fitting without manual initialisation, we need less than



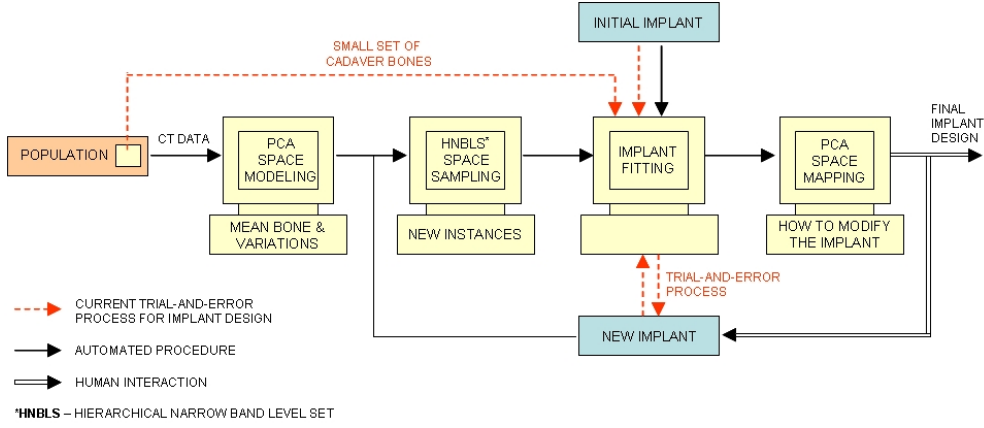


Fig. 13. The complete procedure of the implant design process. First a statistical model of a given population is computed and new instances are created (PCA shape space modeling and HNBL'S 'sampling'). The implant is automatically fitted to these virtual bones and a mean error of the fitting is computed (PCA shape space 'mapping'). Using hierarchical narrow band level set segmentation 'good' fitted bones are selected. From observing the selected instances and the fitting results on them we could propose modifications to the implant shape. Finally, we repeat the process of automatized fitting for new implant to verify its performance.

397 1 minute per bone (Dual CPU @2.2 GHz, RAM 2GB). In combination with  
 398 hierarchical narrow band and a given segmentation criterion the fitting pro-  
 399 cess was performed on 1'504, 1'168 and 4'904 instances, respectively for the 3  
 400 resolution levels mentioned above. This results in reducing the computation to  
 401 only 13.5% of the whole shape space (i.e. 57'600 instances), which drastically  
 402 reduces the computation time.

403 The segmented areas in Figure 15a represent the range of parametric values  
 404 that generate tibia shapes satisfying the segmentation criterion that was pro-  
 405 vided as a requirement from the implant designer, i.e. mean fitting error of  
 406 less than 1mm. The 2D shape space map is built using 2 principal compo-  
 407 nents,  $u_1$  and  $u_2$ , in order to illustrate the strong effect of the first principal  
 408 component for the implant shape design. Figure 15b shows an example of a  
 409 construction of a 3D PCA shape space (i.e. using 3 principal components to  
 410 generate the shape instances) and the result of the level set optimisation for  
 411 the fitting error less then 1mm. It can be visualised that the first and fifth  
 412 PCs have higher influence on the implant shape design, whereas the second  
 413 PC does not interfere much as it covers the whole space  $-3\sqrt{\lambda_2}, +3\sqrt{\lambda_2}$ . We  
 414 decided to exclude principal components  $u_3$  and  $u_4$  since their variations do  
 415 not affect the bone in the area of the implant placing (Figure 14).

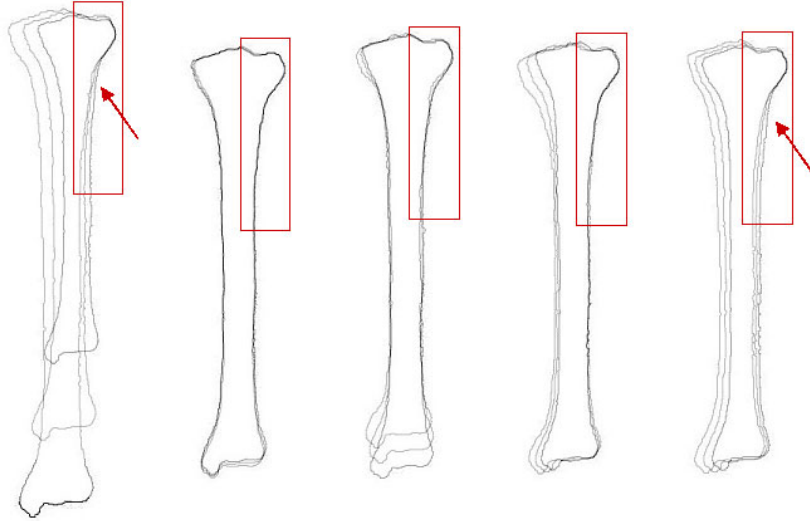


Fig. 14. The first five modes of variation for the left human tibia (anterior view) are visualised. For each principal component, we show  $\bar{m} - 3\sqrt{\lambda_i u_i}$ ,  $\bar{m}$ , and  $\bar{m} + 3\sqrt{\lambda_i u_i}$ , where  $\bar{m}$  represents the mean bone. The arrows point to the area of implant placement, which is most affected by the first and fifth principal component. The first five modes account for 71, 11, 6, 3 and 1% of shape variability in the population, respectively.

#### 4.4 Implant design modifications and analysis

Implant design modifications followed analysis of the segmented spectrum of shapes. It can be seen in Figure 15a that the result of the fitting depends mostly on the first principal component, as the segmented area falls in the negative values of  $u_1$ . Since the negative values of the first principal component favor 'good' fitting, this leads to the conclusion that the implant works better for longer bones. Our aim is to optimise the fitting as to cover the whole shape space, i.e. the majority of the population. Having the measure of variation between positive and negative values for the first principal component (Figure 14), it can be concluded that changes of the length of the tibia affect the result of the fitting, since these changes affect as well changes of the oblique line of the tibia and a slight torsion of the lateral surface of the tibia. In other words, the analysis allows us to conclude that the angles and curvatures in the first and second OBB of the implant geometry (Figure 11b) as well as the curvature of the third and fourth OBB of the implant are responsible for the fitting.

In agreement with the previous conclusions we proceed with the optimisation of implant shape design by applying the following modifications. First, we decrease the angles and flatten the curvatures in the first and second bounding box of the implant (Figure 16a), to follow the oblique line of the mean tibia

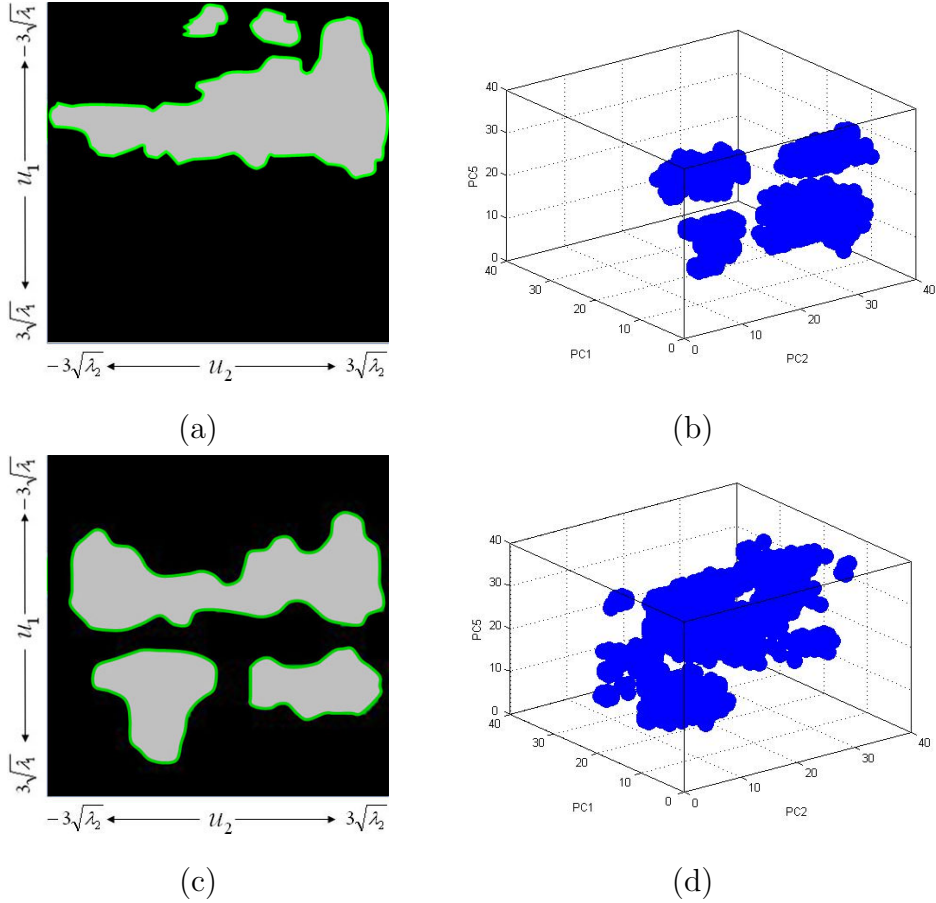


Fig. 15. (a) Automatic hierarchical 2D level set segmentation gives the spectrum of shapes that have fitting error less than 1mm for the implant given by the manufacturer. (b) 3D level set segmentation gives the spectrum of shapes that have the fitting error less than 1mm for the implant given by the manufacturer. (c) Spectrum of shapes that have fitting error less than 1mm for the modified implant design. (d) 3D level set segmentation for the modified implant design.

436 bone (Figure 14). In addition, we follow the distance dimensions between  
 437 bone head and implant from the implant fitting distance map (Figure 12b).  
 438 We apply further modifications to the implant shape by increasing the torsion  
 439 of the distal part of the plate. We rotate the third and fourth bounding box  
 440 along the center of the plate to bring the left anterior edge of the implant  
 441 closer to the lateral surface of tibia (Figure 16b).

442 To evaluate the new design we perform a re-fitting in the PCA shape space us-  
 443 ing the modified implant shape. The results of the segmented space are shown  
 444 in Figures 15c and 15d. It can be seen that the modified implant expands the  
 445 space of segmented bones by covering different shape variability and therefore  
 446 fits better to the majority of the population. With the new implant design we  
 447 found that there is an increase of 40% on the number of instances that satisfy  
 448 the given fitting criterion.

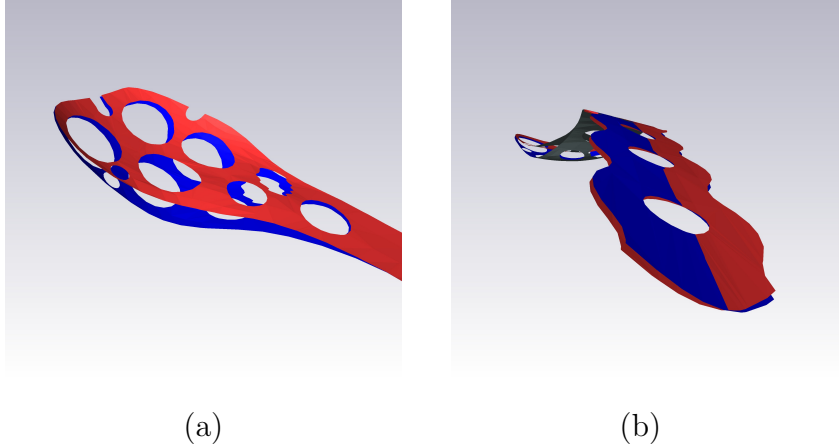


Fig. 16. New implant design (in red). (a) Curvatures in the first and second bounding box of implant are flattened. (b) Implant surface in the third and fourth Oriented-Bounding-Box is twisted inside.

## 449 5 Conclusions

450 In this paper we have presented a methodology for the evaluation of a func-  
 451 tional criterion (that could represent an anatomical/physiological measure)  
 452 across a target population. Our framework is based on building a statistical  
 453 model via PCA and finding the region of the parametric space defined by the  
 454 principal component weights that matches the criterion. The mechanism to  
 455 search for this partition is based on a level set evolution in parametric space,  
 456 optimised via a multi-level narrow-band approach for computational efficiency.  
 457 To our knowledge, this is the first work that tackles the issue of finding a par-  
 458 tition of PCA space based on a criterion, and the first time that level sets are  
 459 used within this context. Existing previous works combining PCA and level  
 460 sets used the later as a shape representation, and evolve the level set in image  
 461 space. This is fundamentally different to our work.

462 Current evaluation and optimisation of orthopaedic implants is done by man-  
 463 ual fitting and fixation procedures, applied on a small set of cadaver bones  
 464 in a trial-and-error process. The method that we propose allows to virtually  
 465 test the implants on a representative set of bones generated by sampling the  
 466 statistical model. Using level sets a spectrum of shapes is segmented in the  
 467 PCA shape space, based on a given fitting criterion. By correlating the prin-  
 468 cipal components of the selected instances to the given implant geometry the  
 469 modifications to the implant design/geometry can be assessed directly from  
 470 the segmented map. The proposed method highlights which patterns of bone  
 471 variability are more important for implant fitting, allowing and easing im-  
 472 plant design improvements, as to fit a maximum of the target population. A  
 473 hierarchical narrow band approach is used to avoid exhaustive search of the  
 474 instances in the high resolution space, and to search for the instances only in

475 the neighborhood of the zero level set and not in the whole shape space.

476 To our knowledge this is the first research into the problem of estimating  
477 how a given implant fits to the wide population and how the morphological  
478 features in implant design can be improved. The practical use of the proposed  
479 concept are of great importance for the implant manufacturer, due to the huge  
480 potential benefits in terms of patient satisfaction and financial gains in this  
481 high-volume market. Further validation of the method is ongoing work.

482 Future work will include automatic correlation of the principal components  
483 to the given implant geometry, so that the modifications to the implant de-  
484 sign/geometry could be assessed directly from the segmented map and au-  
485 tomatically proposed. A parametric model for the implant design could be  
486 established, including design parameters such as diameters, lengths, positions  
487 of the holes, etc. Such parameters could be automatically optimised by max-  
488 imising the fitted volume in the PCA space.

489 Furthermore, we intend to include the application of the proposed method to  
490 bone implant fitting assessment taking into account shape and biomechanical  
491 properties. A combined shape and intensity statistical bone model will be  
492 built, and the intensity values, which are linked to bone density, will be used  
493 to do a finite element analysis of the performance of the implant (Belenquer  
494 et al., 2006), which will be used as the criterion to be evaluated in the level  
495 set evolution.

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Figure1

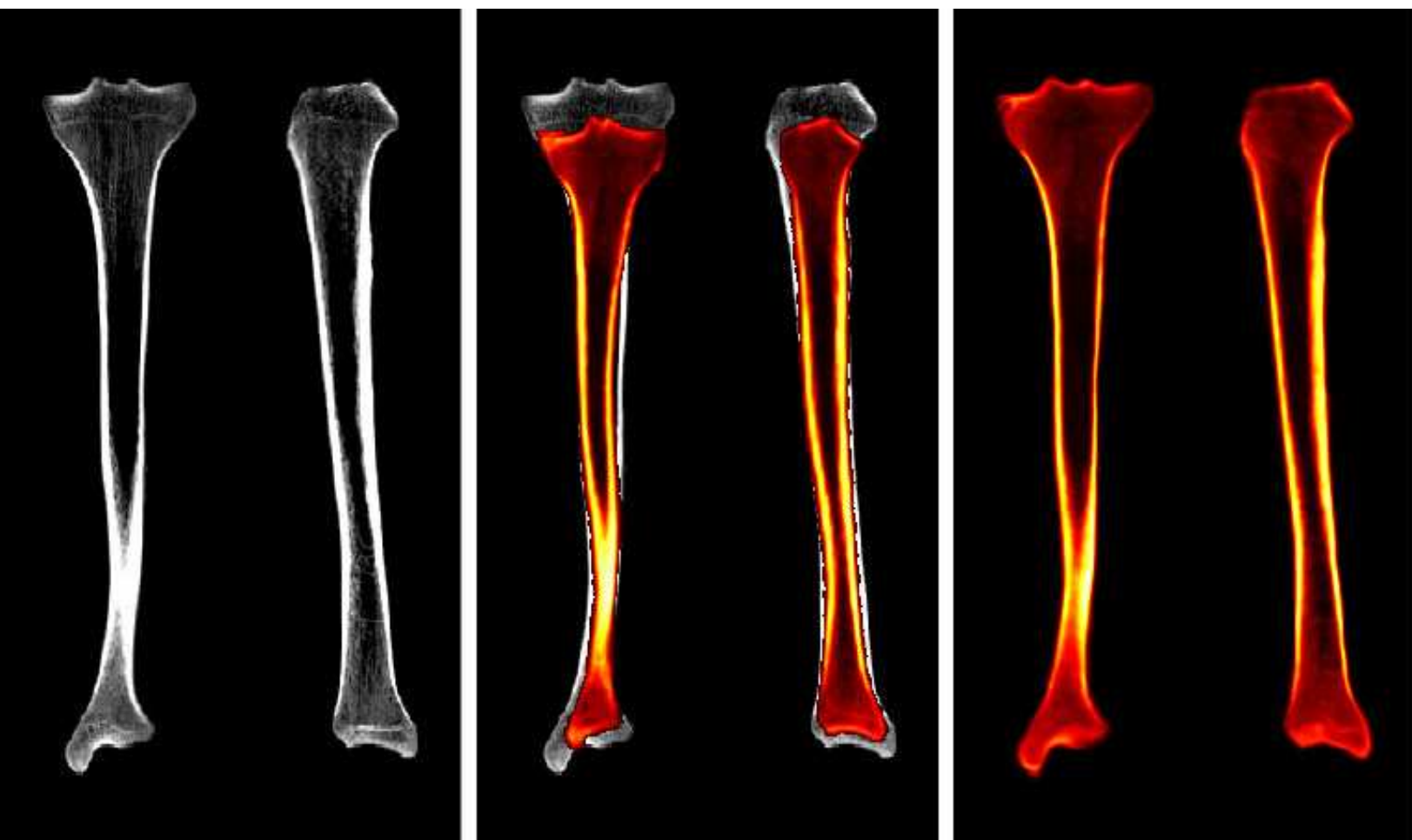


Figure2

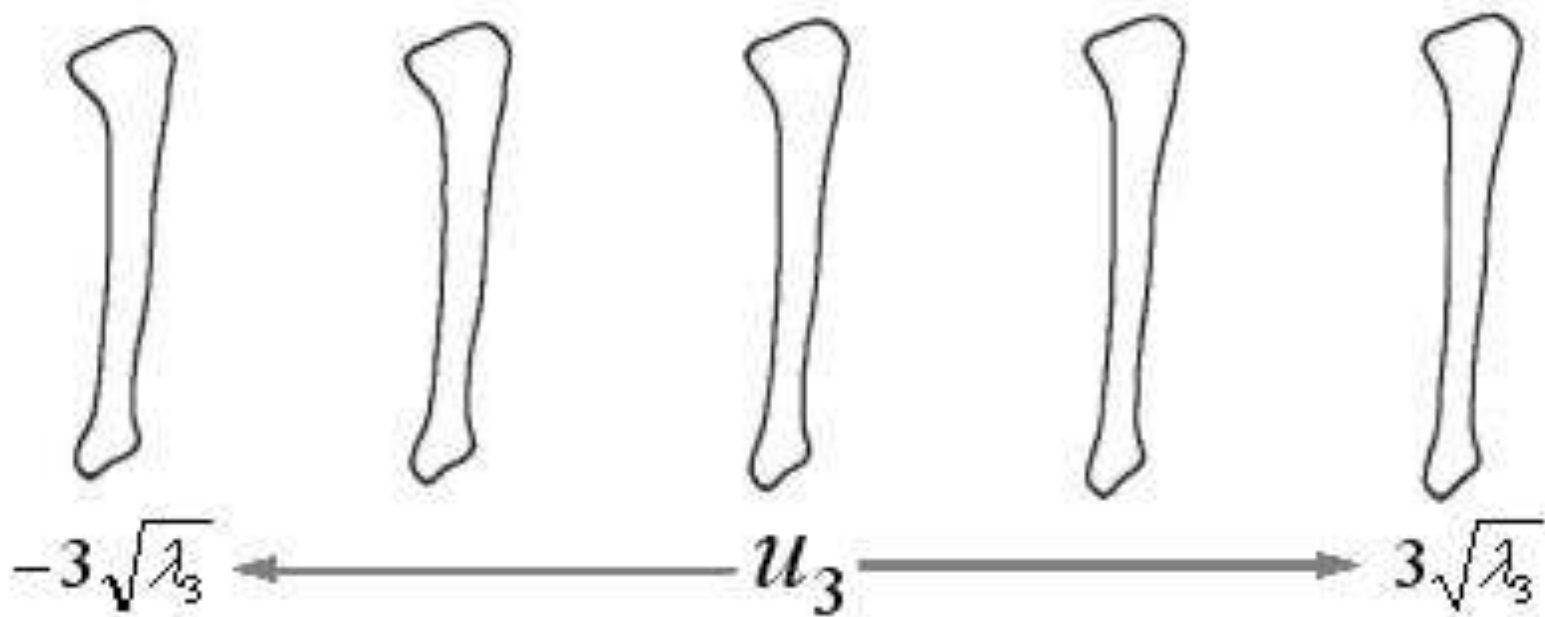
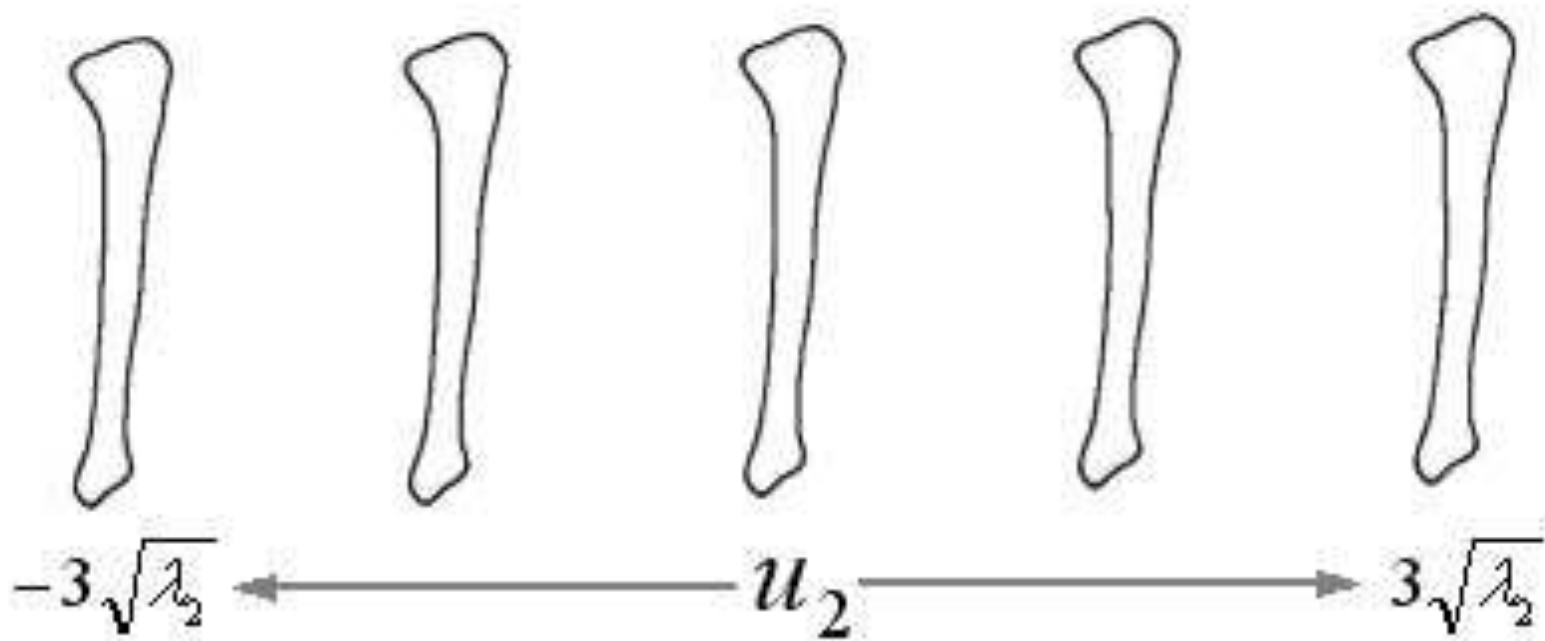
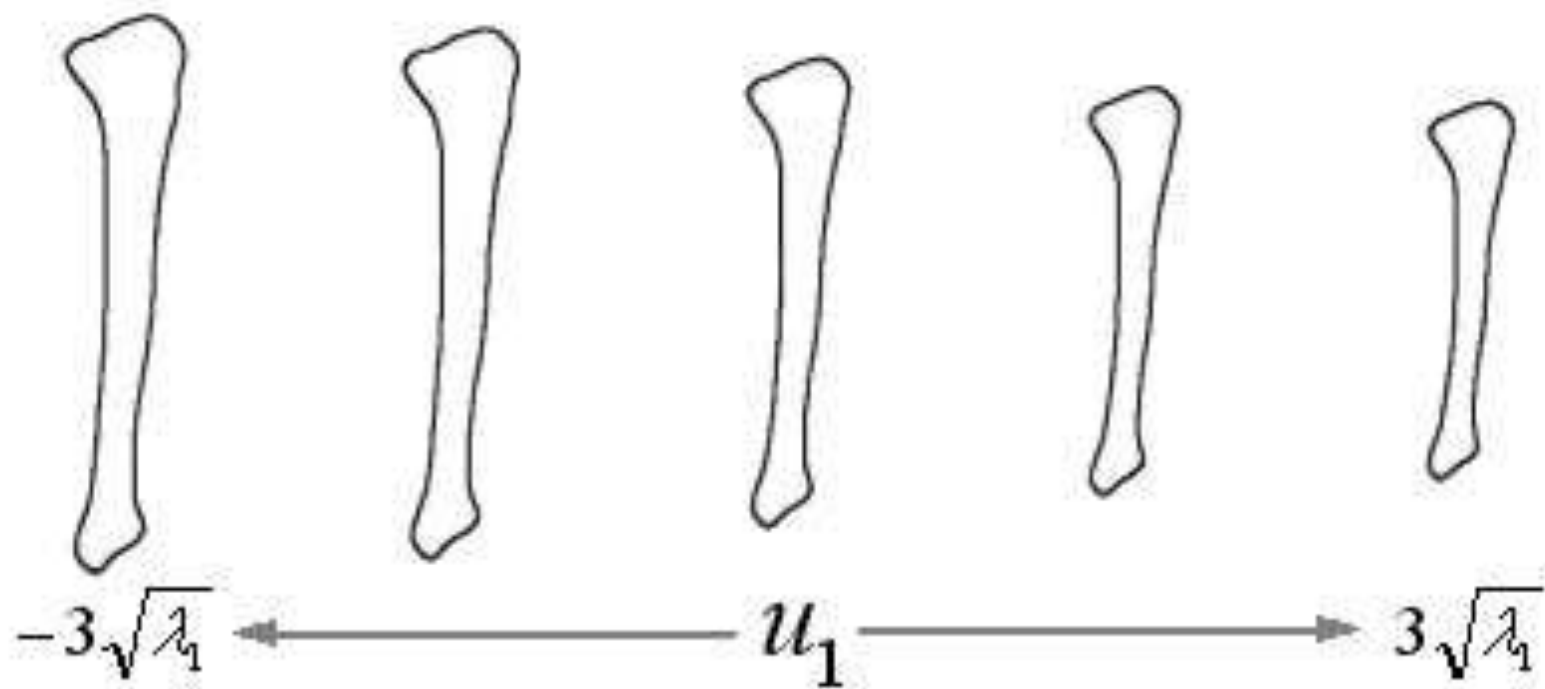


Figure3

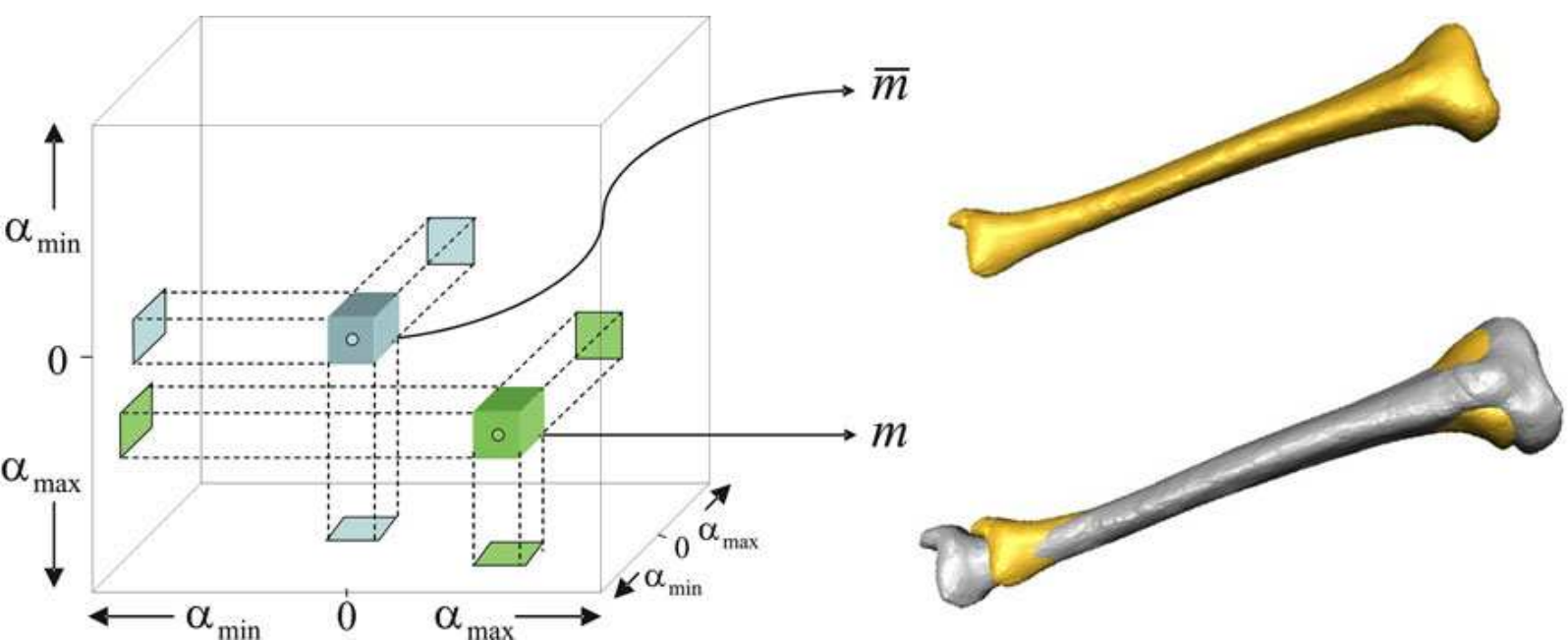


Figure4

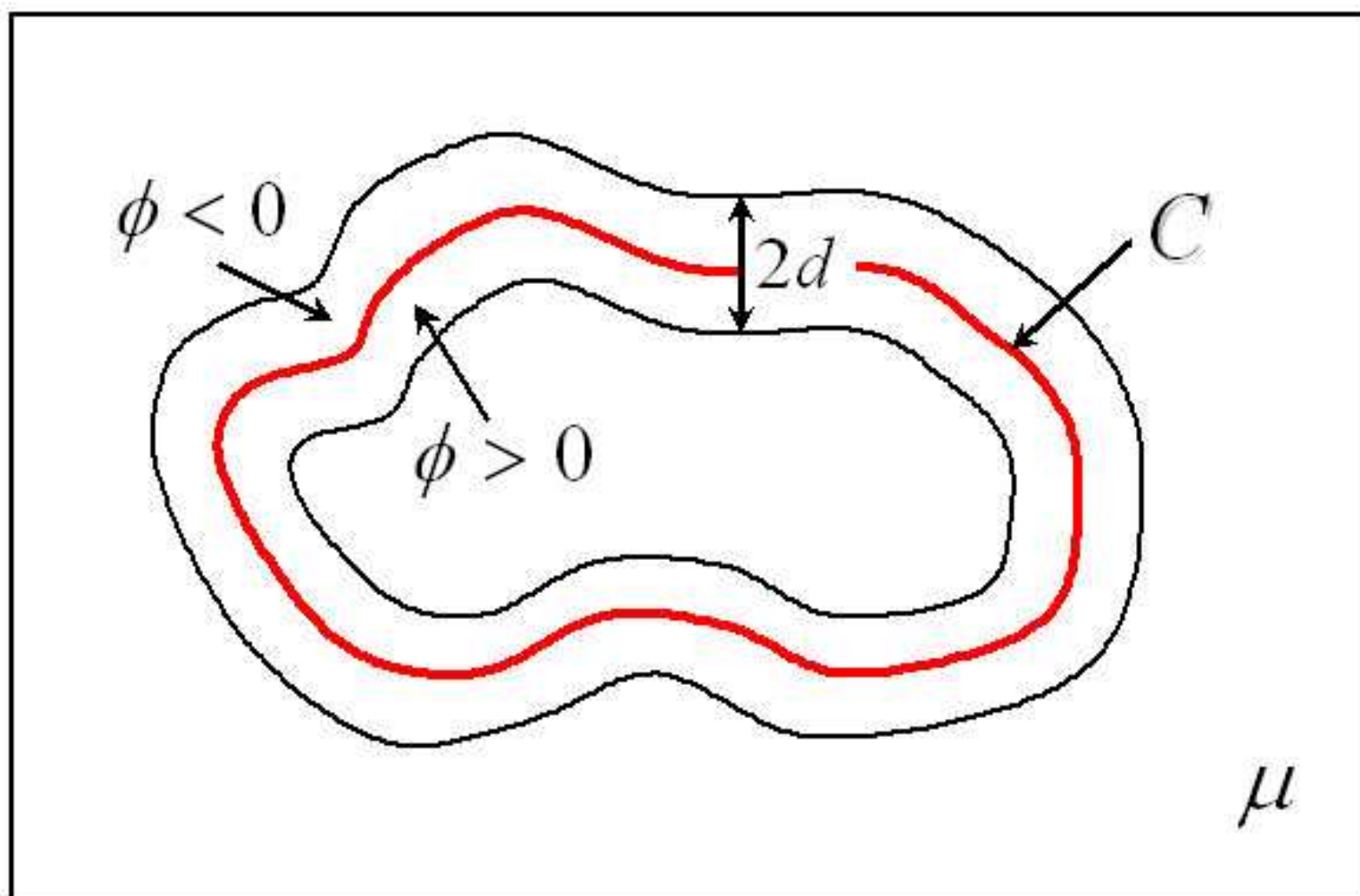


Figure5

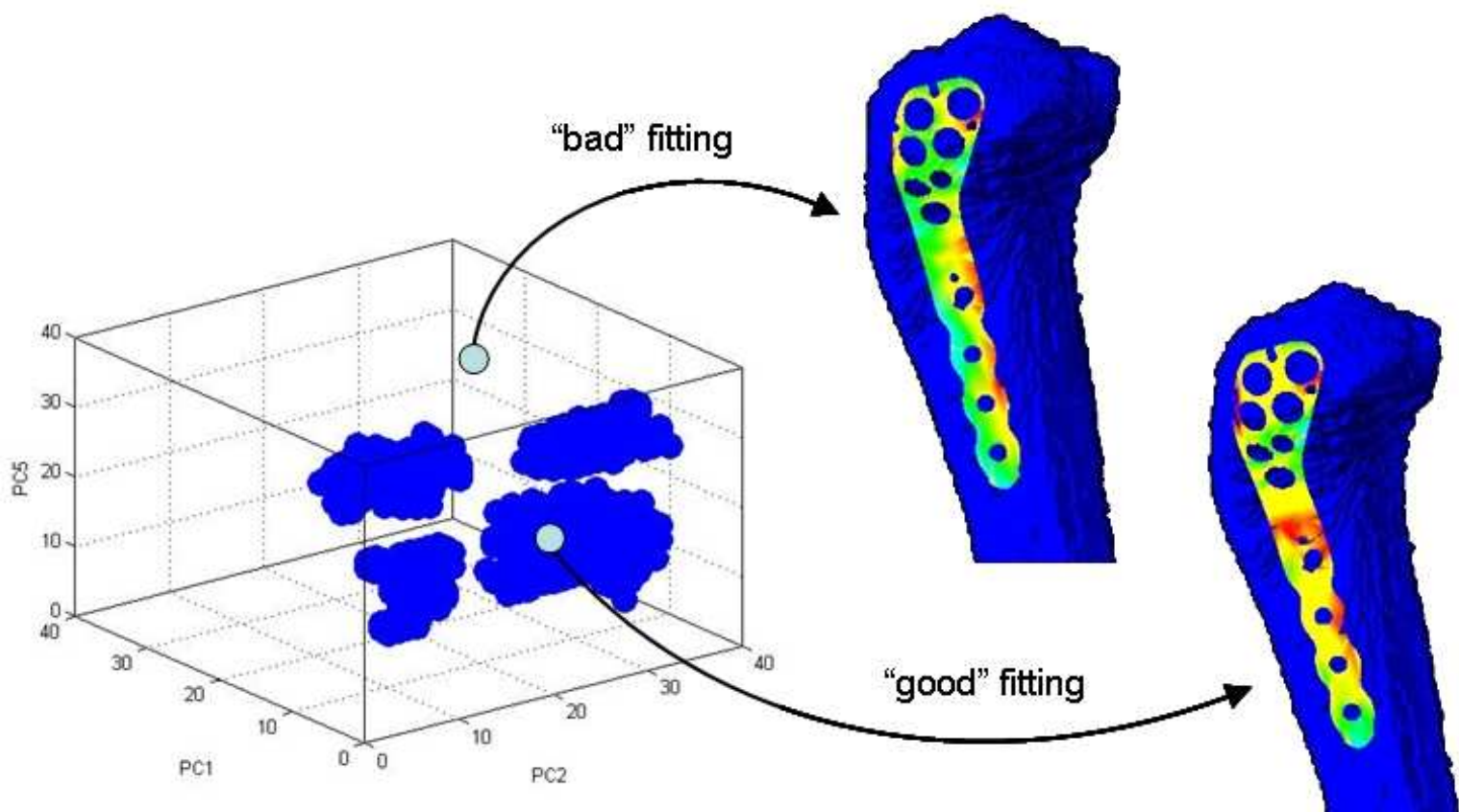
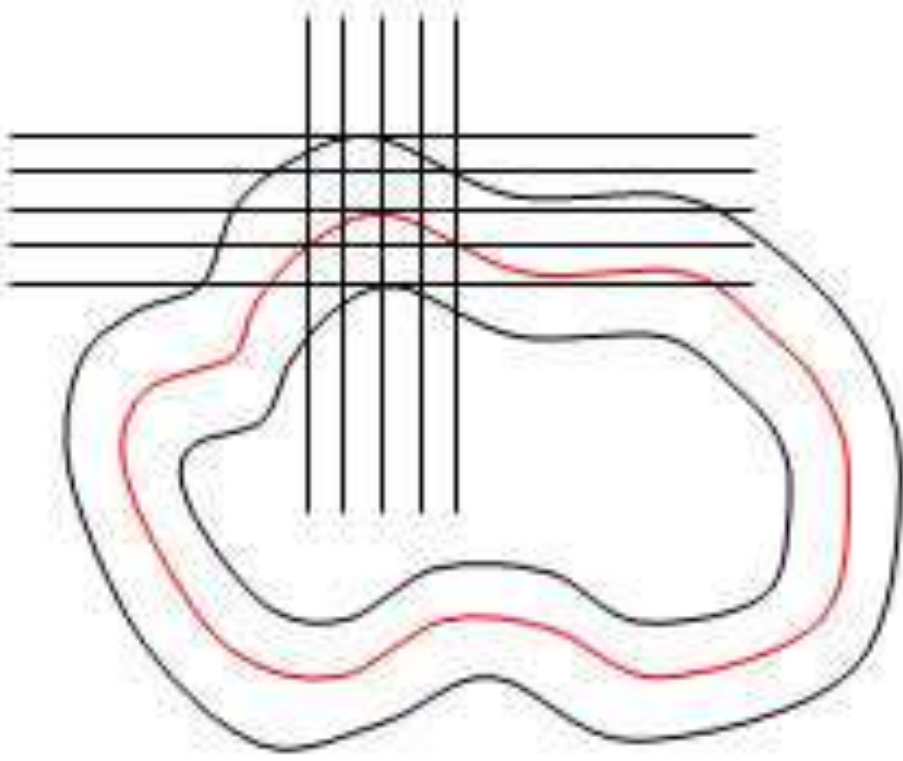


Figure 6a



$(n, m)$

Figure6b

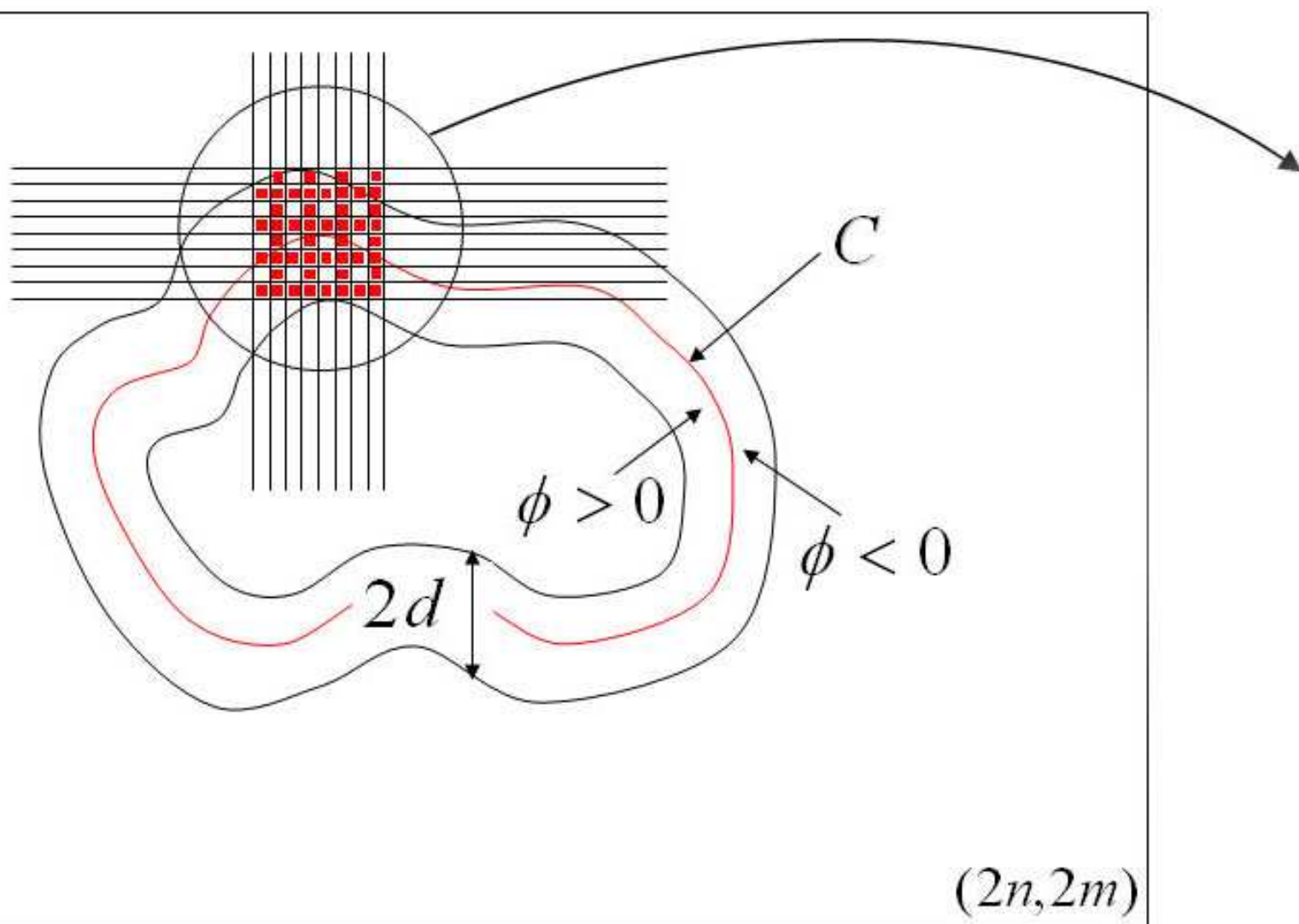


Figure6c

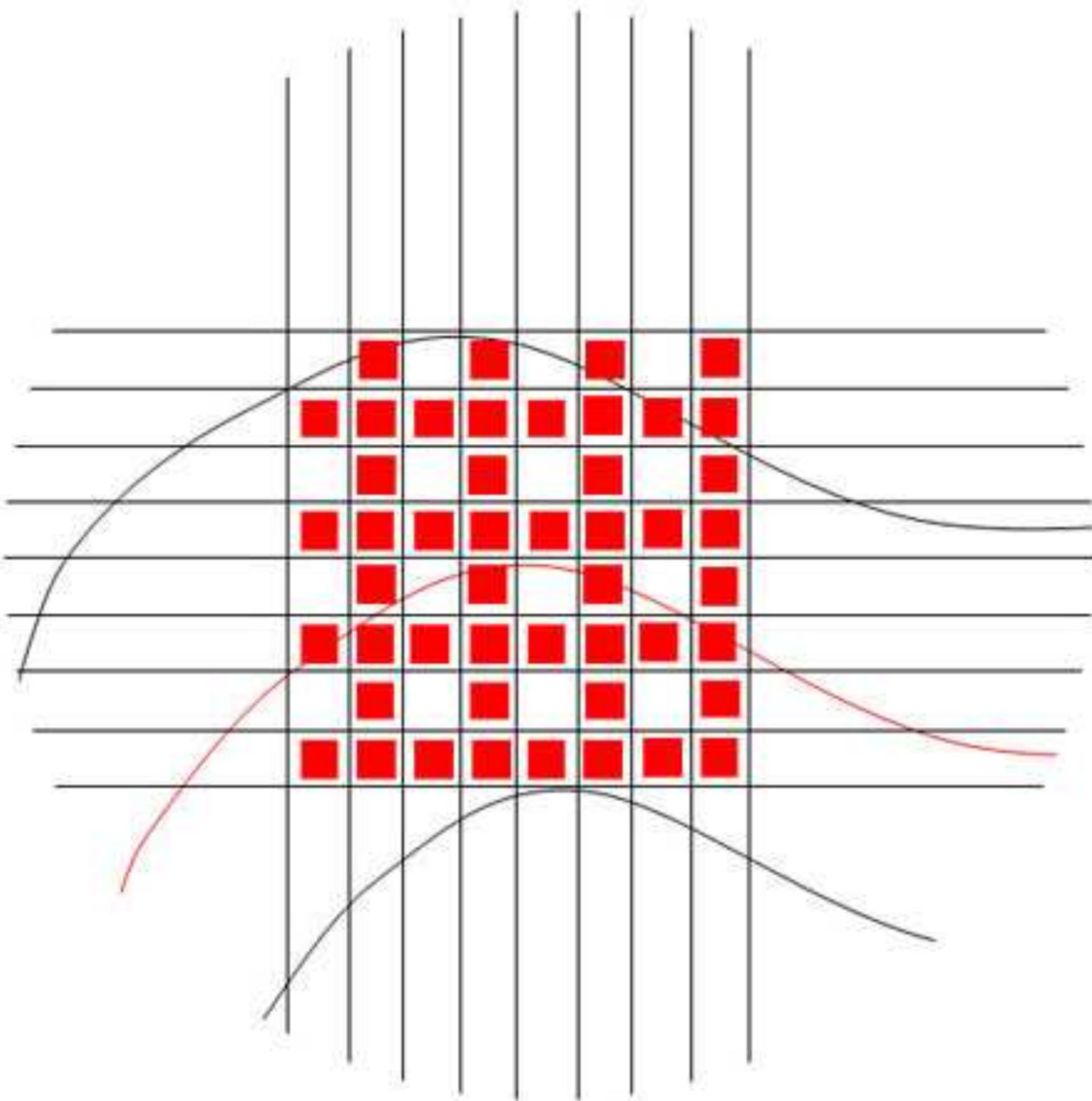




Figure7

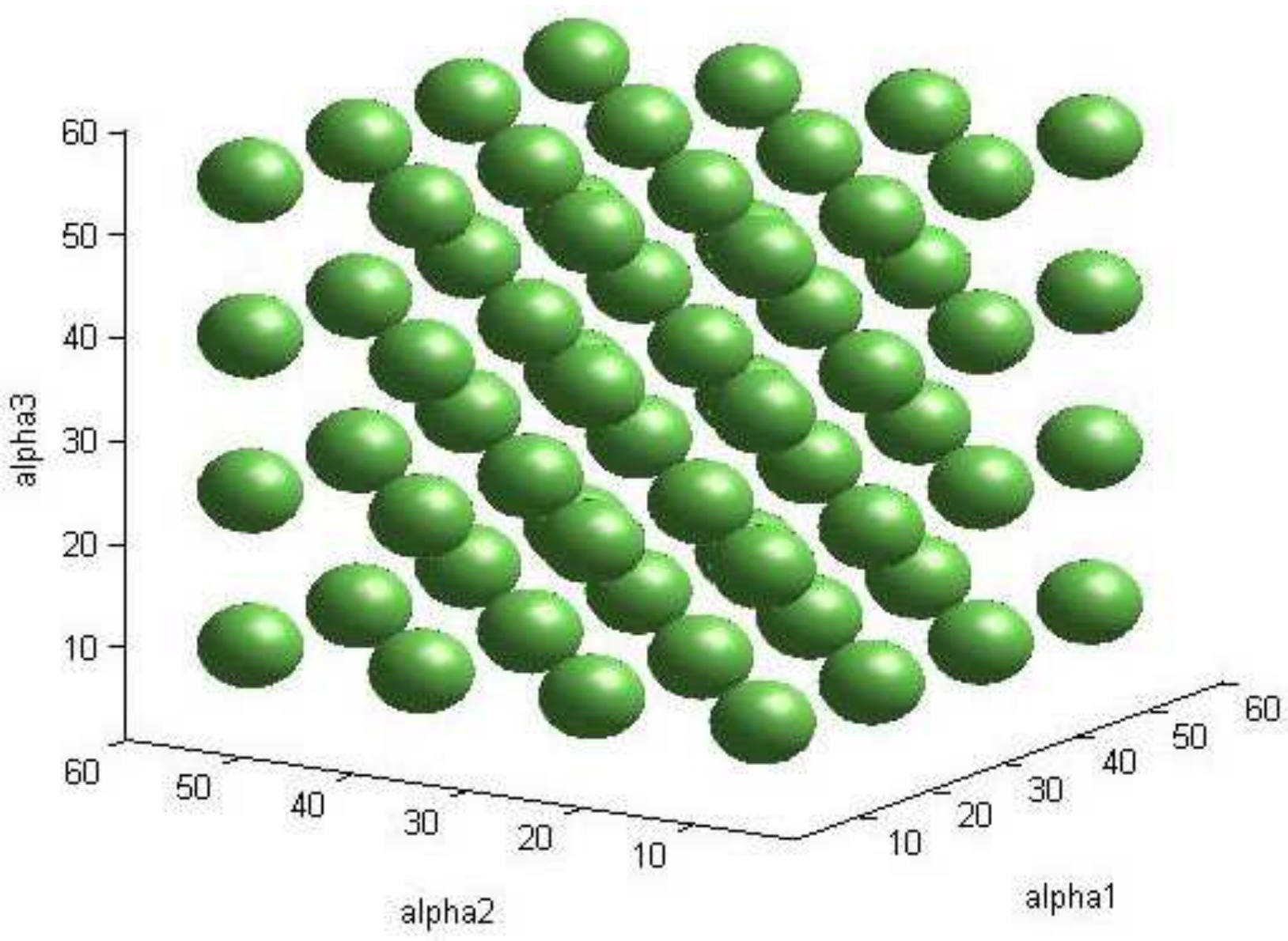


Figure8

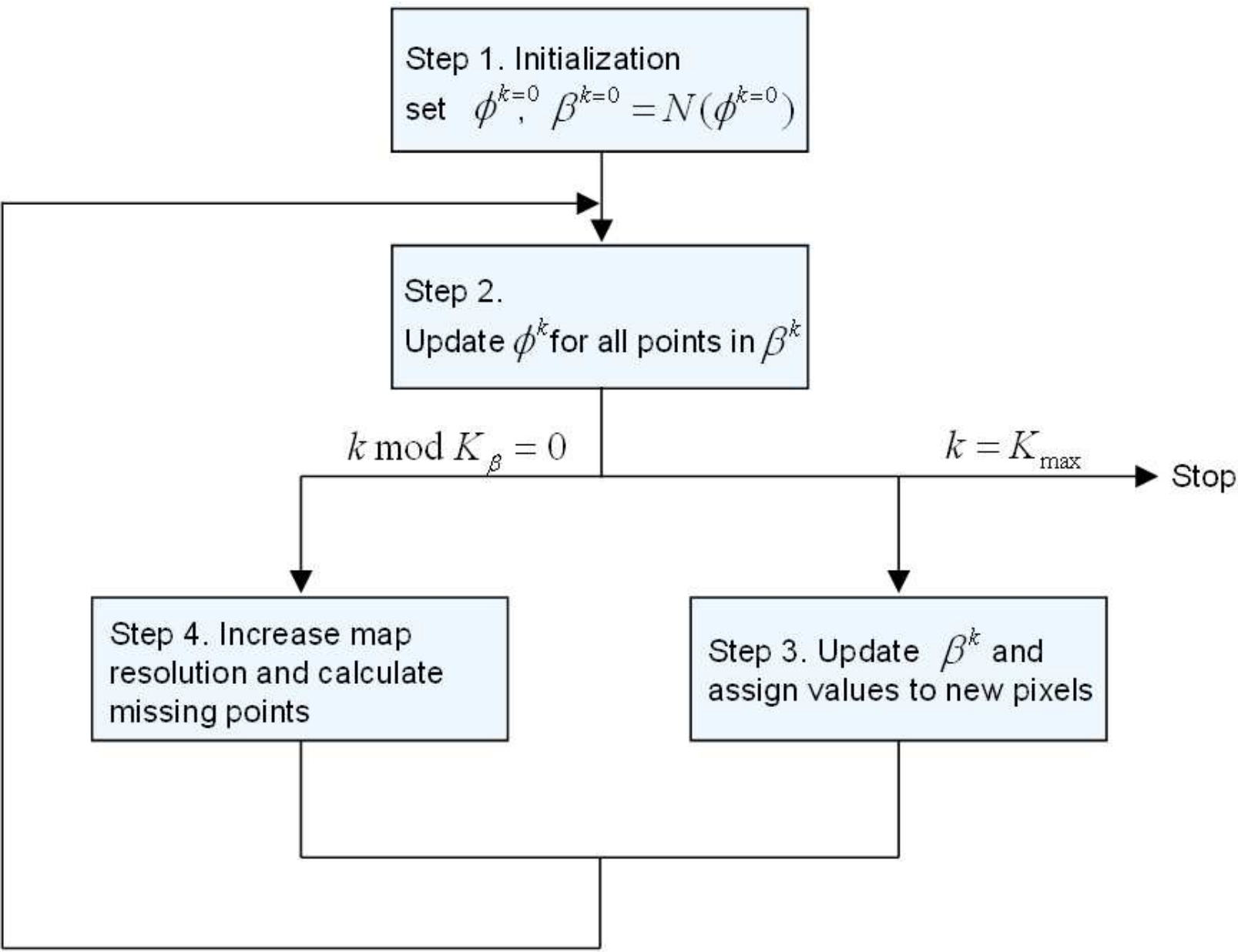


Figure9



Figure10a

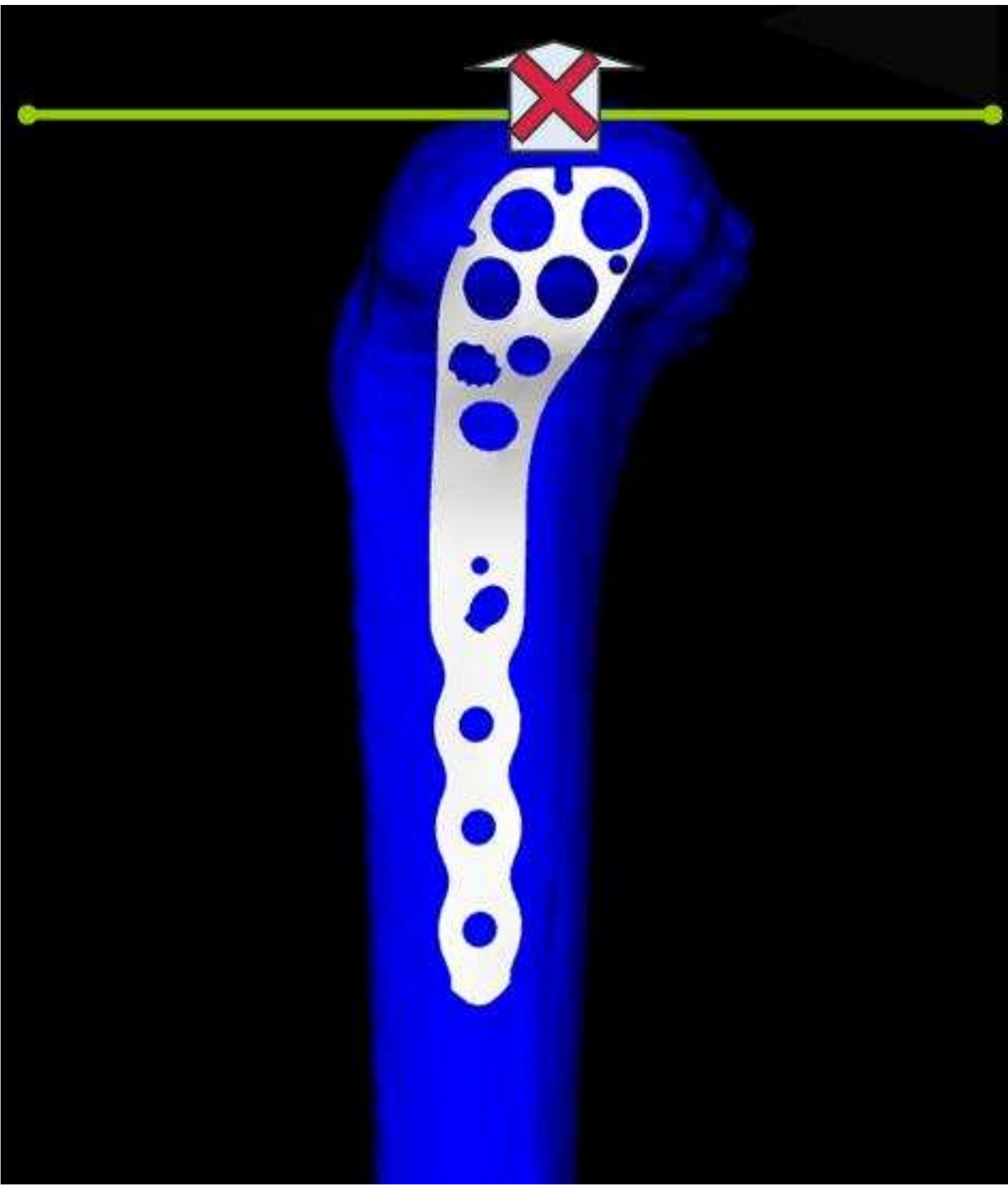


Figure10b

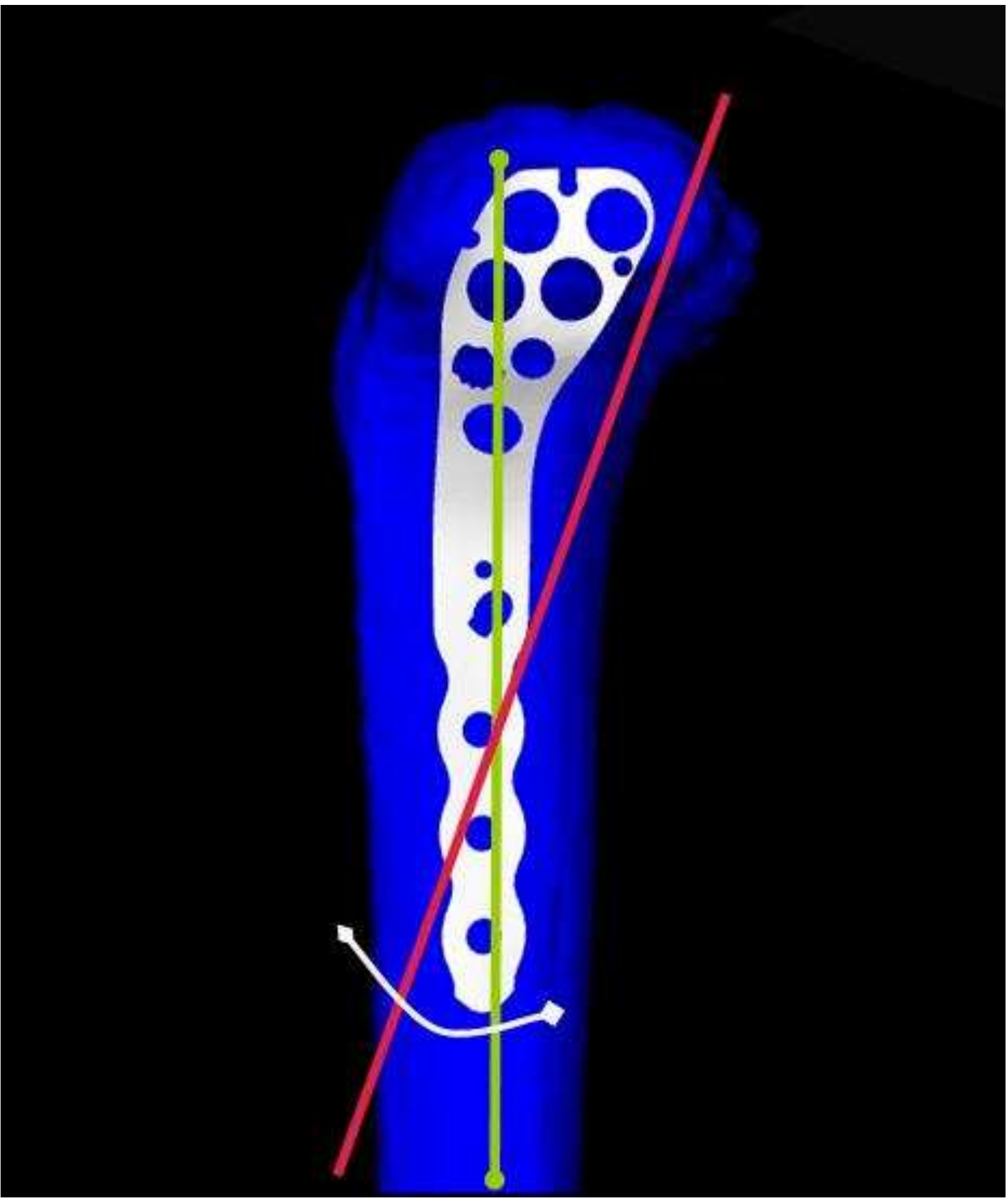
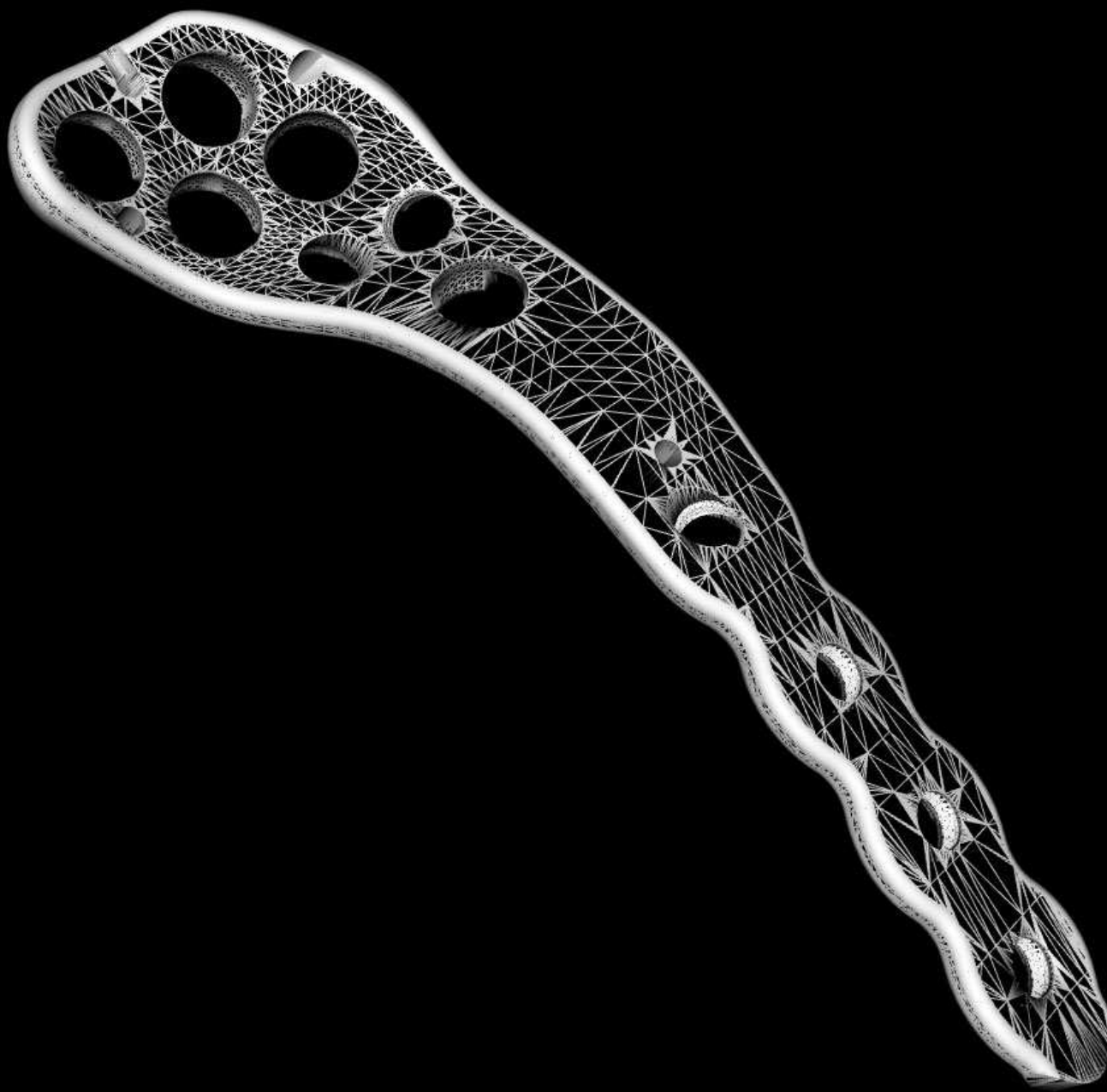
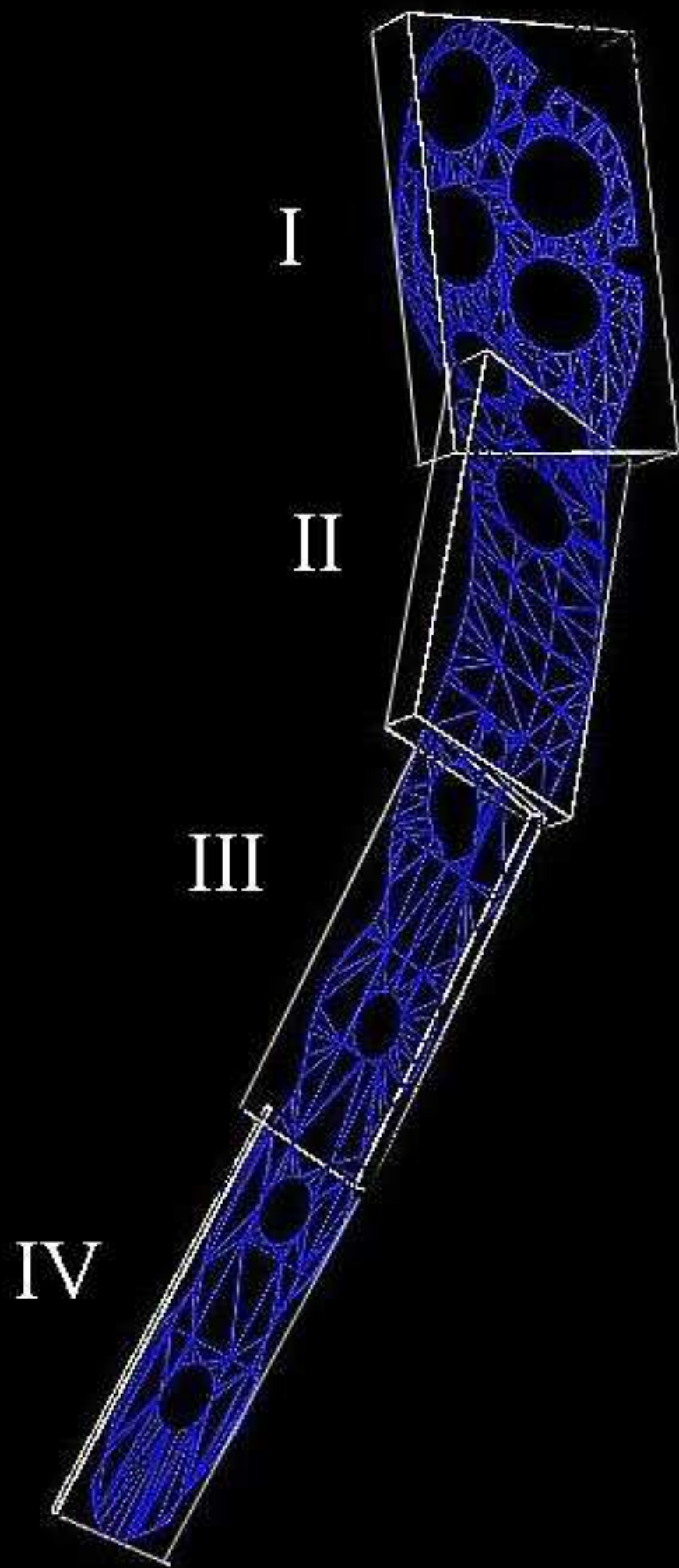


Figure11a









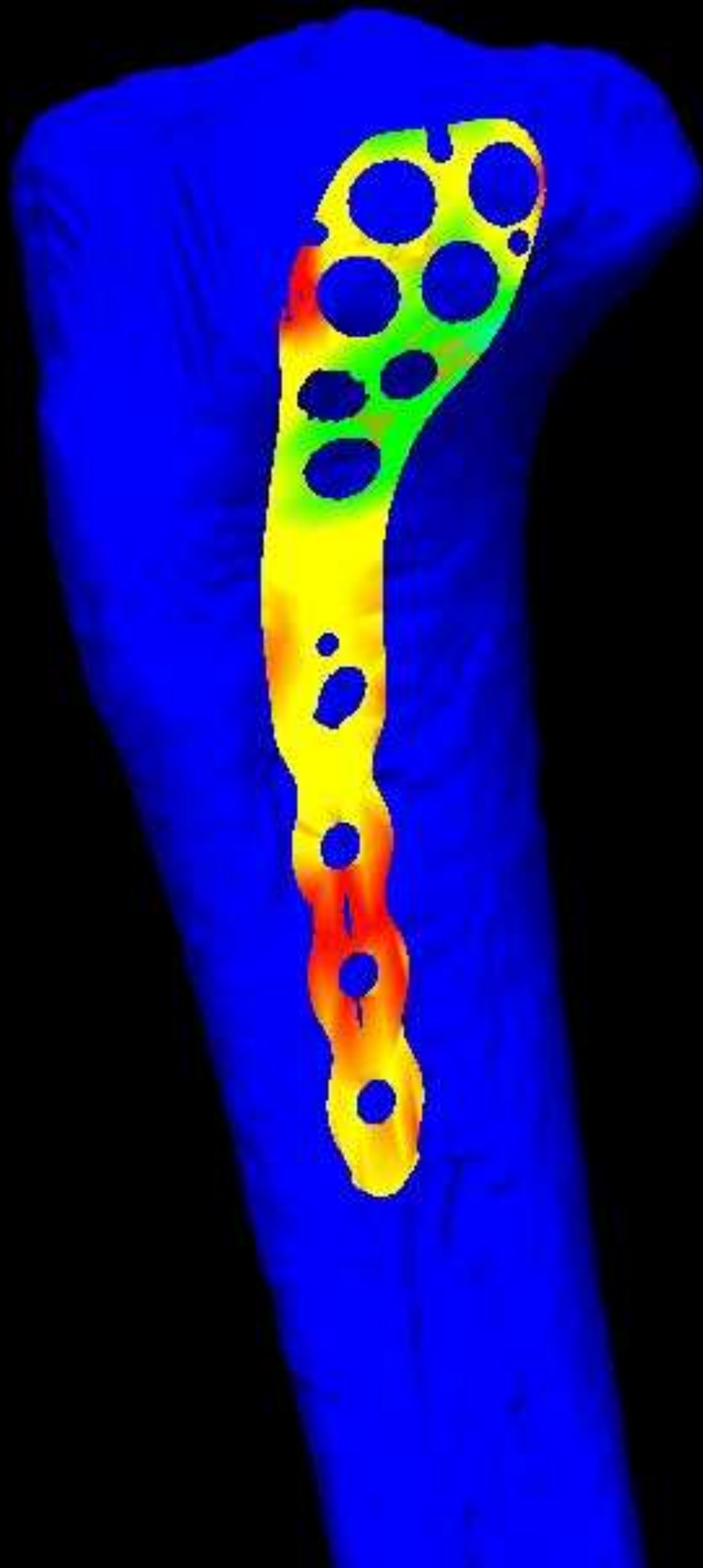


Figure13

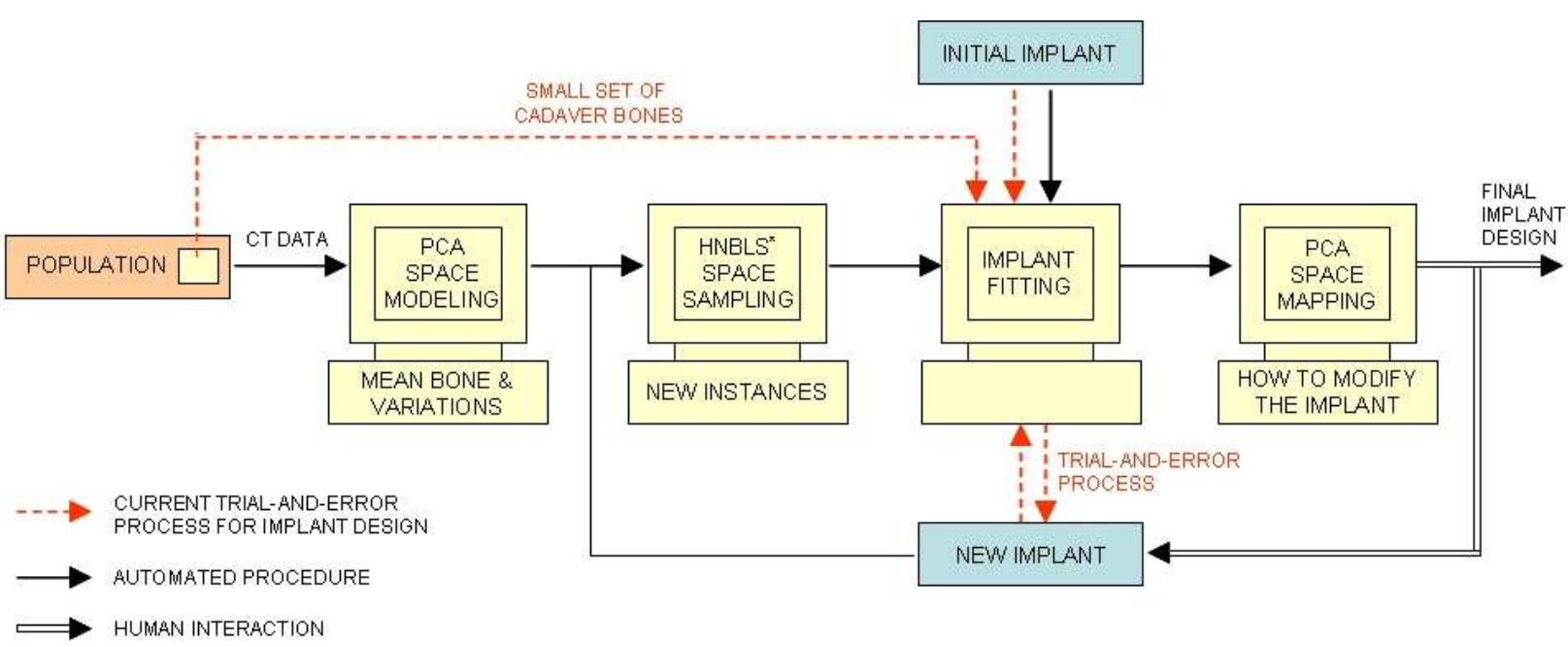


Figure14

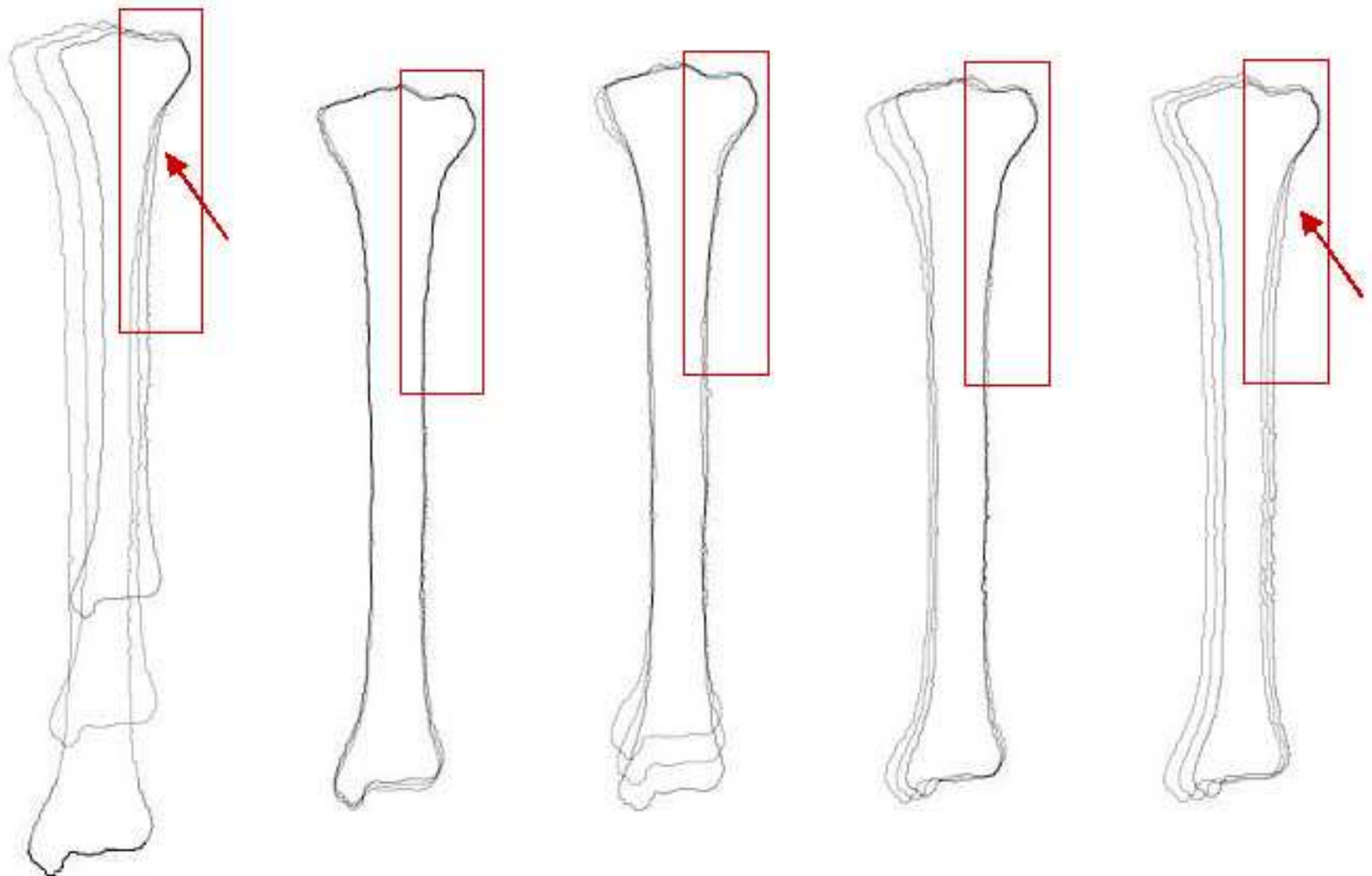


Figure15a

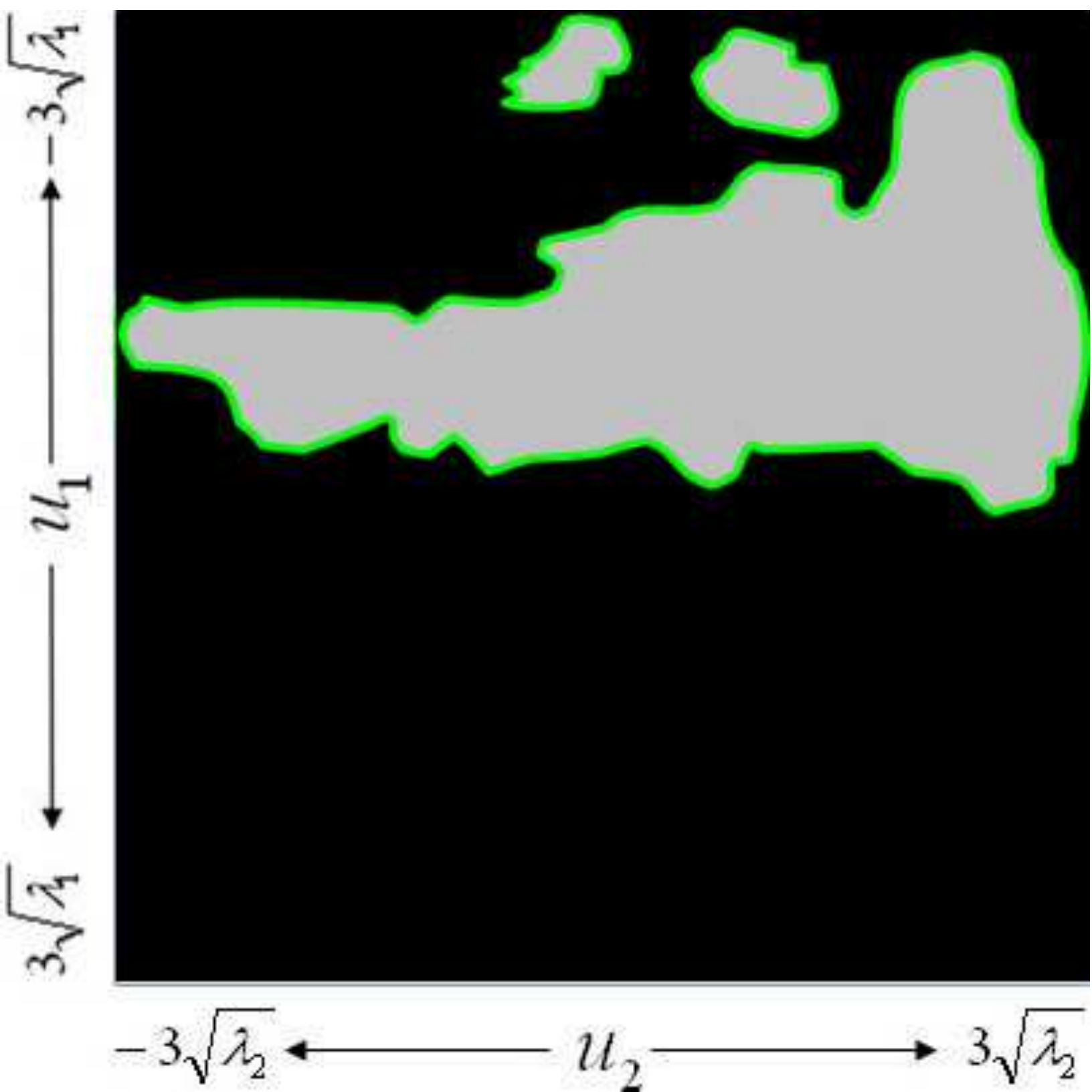


Figure15b

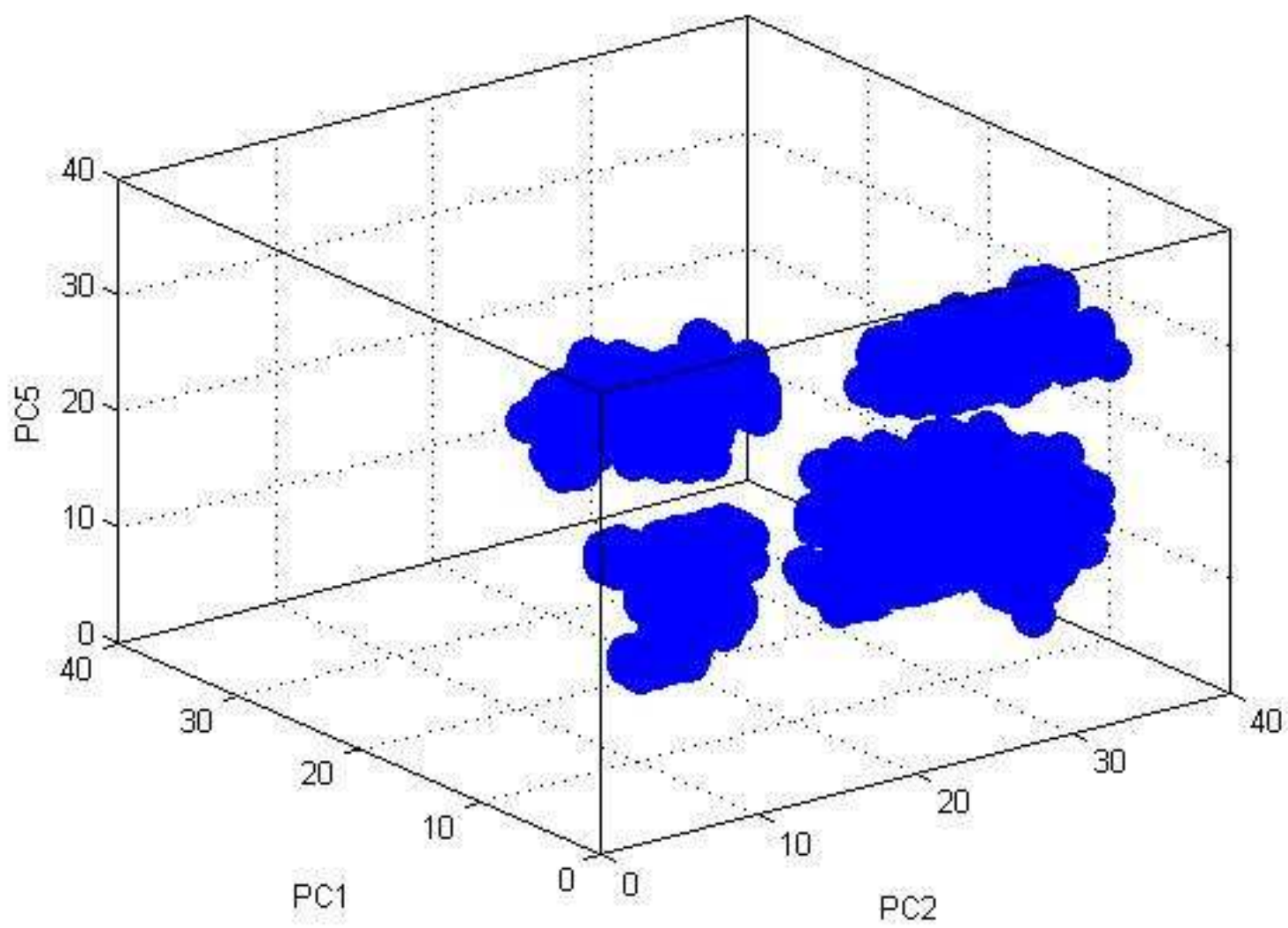


Figure15c

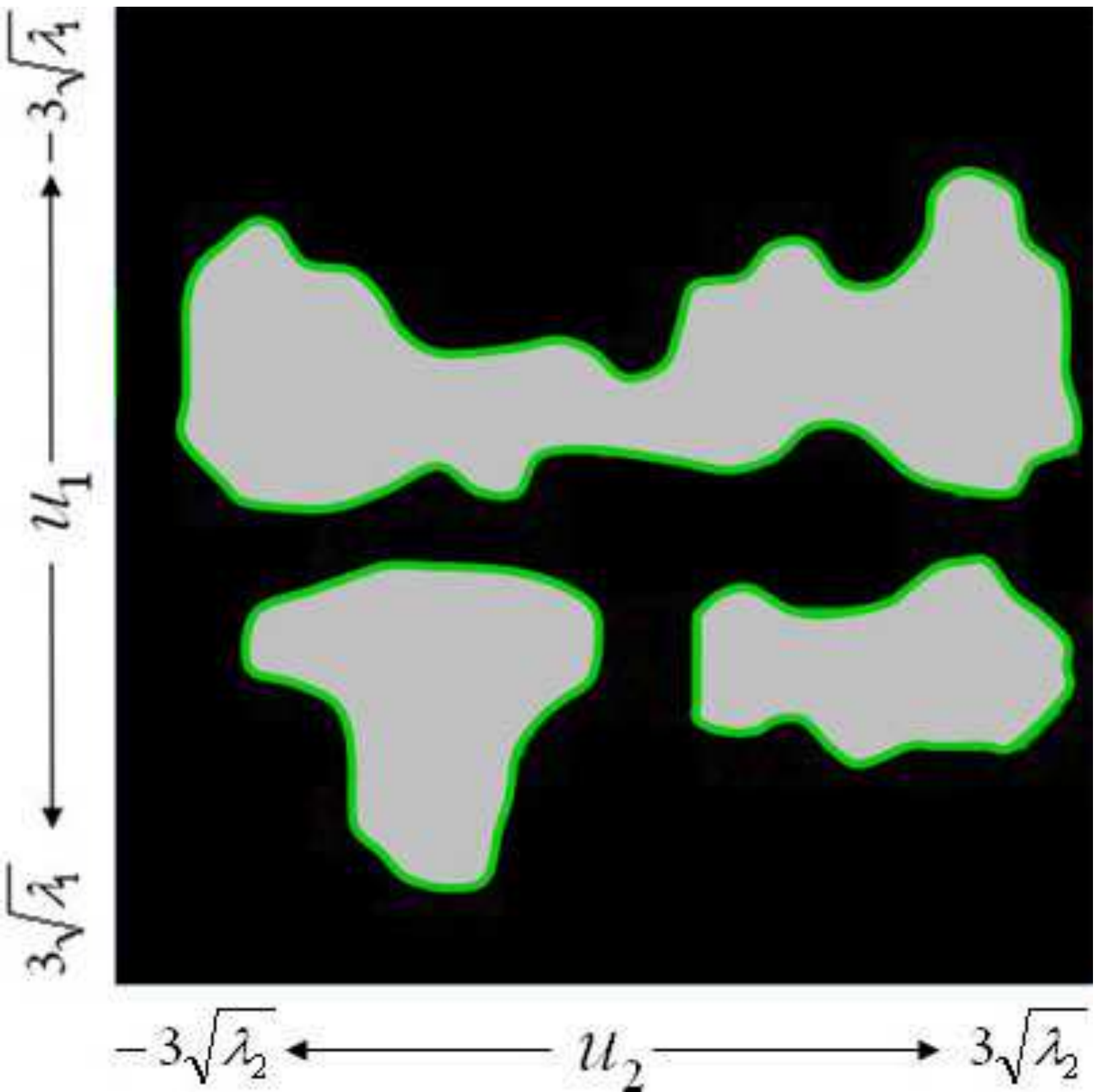


Figure15d

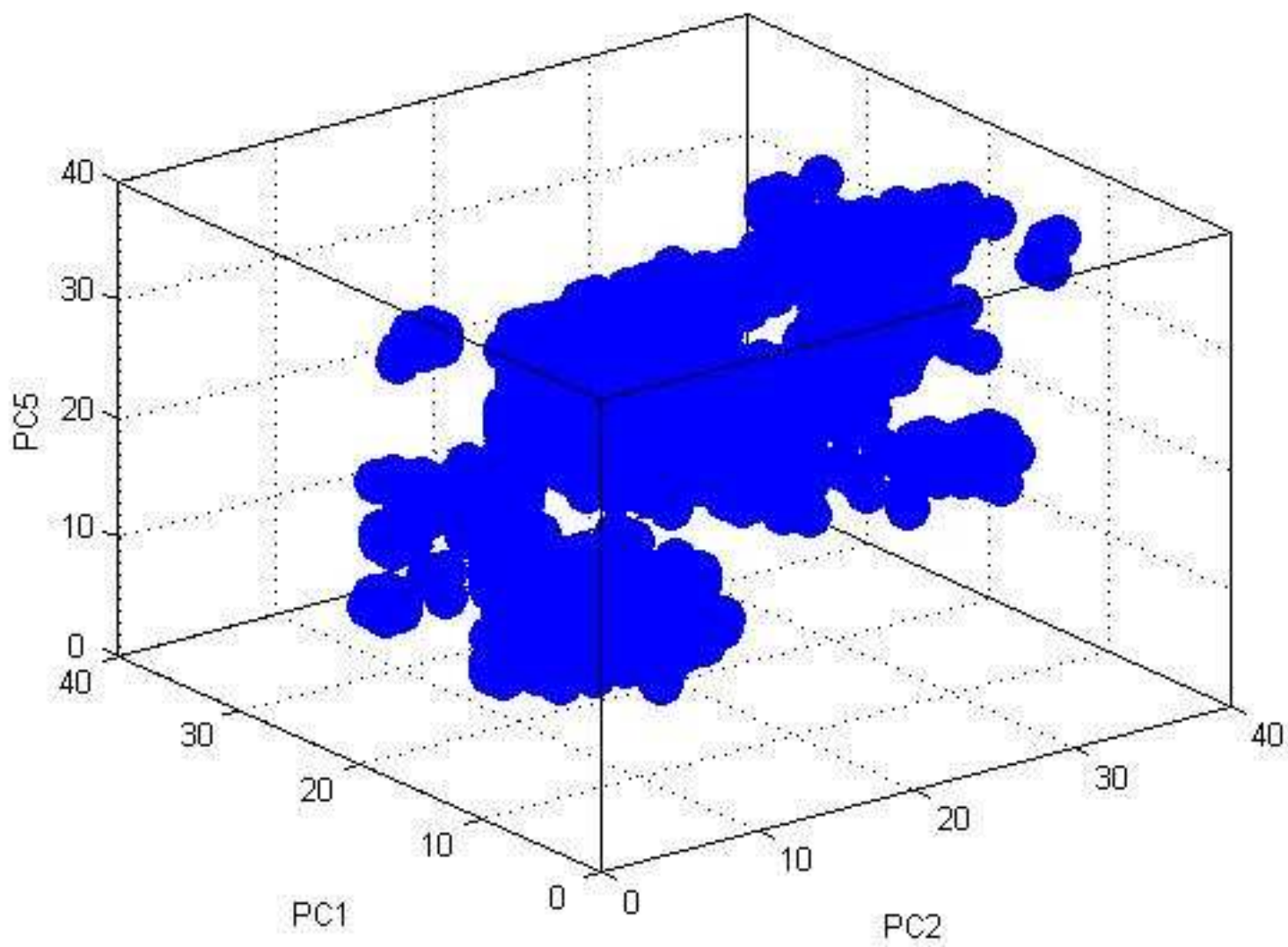


Figure16a

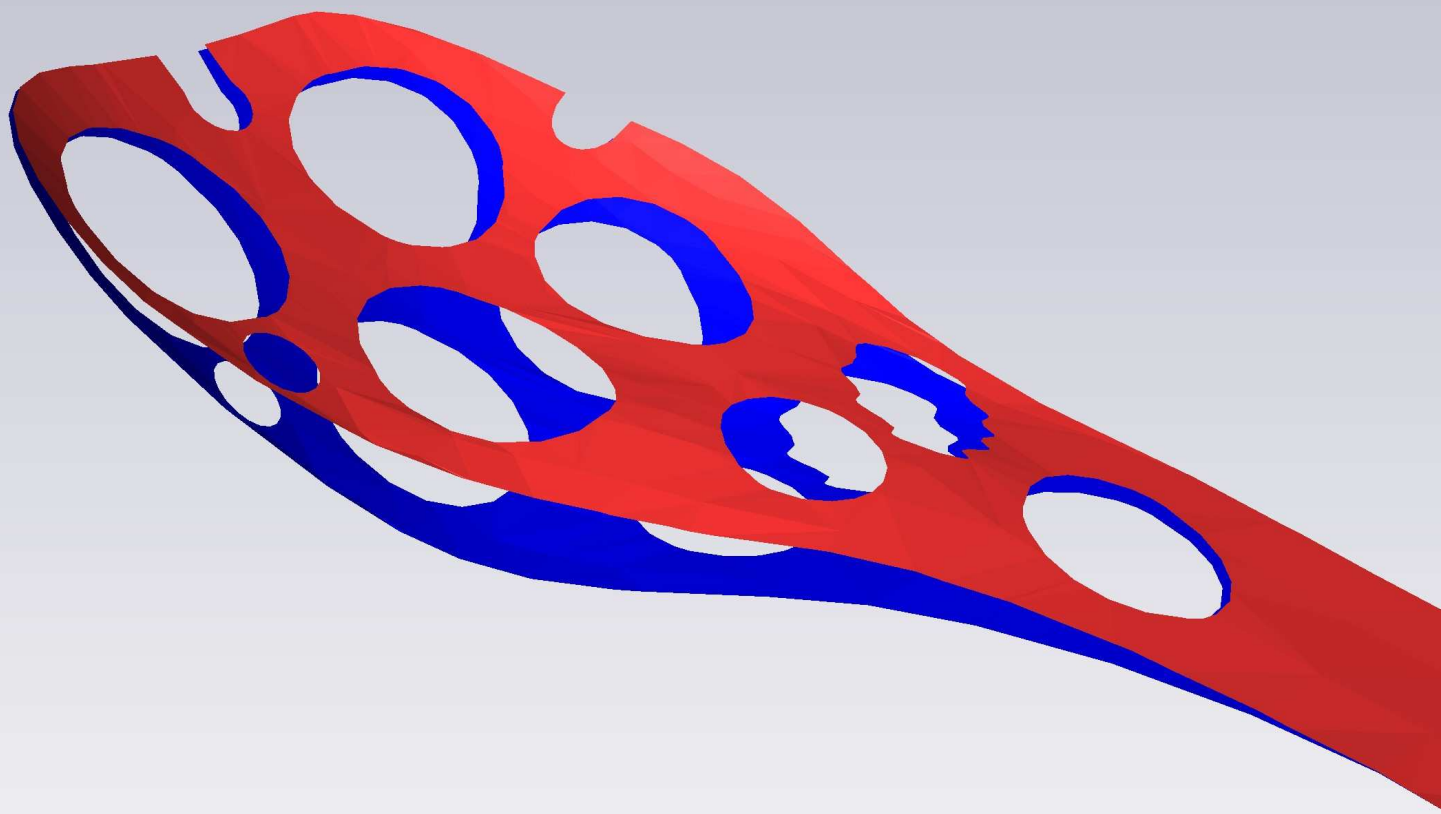
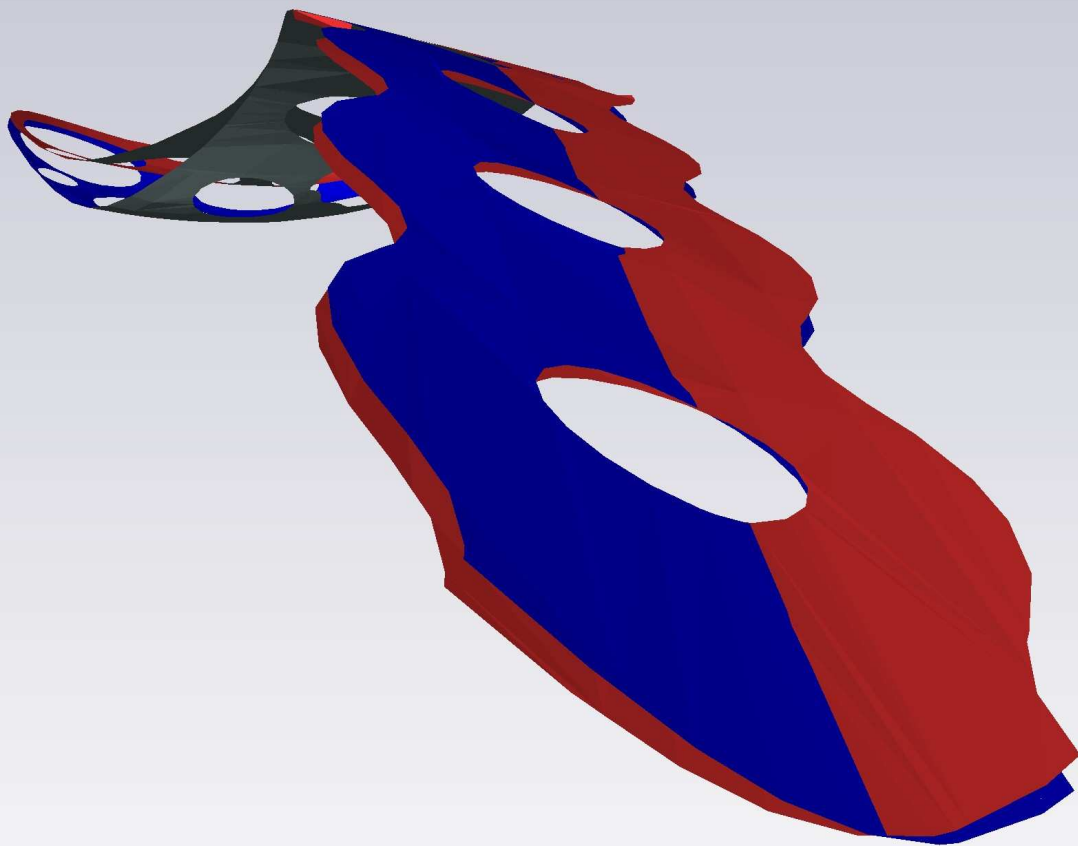




Figure16b



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