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# A SIMPLE MODEL TO TEST EQUAL CATCHABILITY OF MARKED AND UNMARKED ANIMALS IN CAPTURE-RECAPTURE STUDIES

In capture-recapture studies it is generally assumed that both tagged and untagged ones have equal catchability. No test however is available to verify this assumption. Ricker[1] has enumerated the effects of tagging and noted that these effects will in general be hard to detect. Darroch[2] has pointed out the absence of such a test and stated "we hope to fill this gap at a later date". But it appears that this has not been done so far. Seber[3] has also remarked the absence of such tests in general in concluding chapter of his remarkable book. Here we shall indicate a model, to detect the differential catchability.

When the population is 'closed' a model similar to that considered by Darroch[4] and Seber[3] is shown to give the required test. Let us consider 'n' coins, the outcome of each is independent of the rest. Let $S_1$ and $F_1$ denote the events of appearance of head and tail respectively in each coin in the first experiment. Similarly $S_2$ and $F_2$ denote that of head and tail respectively in each coin for the same 'n' coins in the second experiment.

Let $P(S_1)=p$ ; $P(F_1)=q$ ; $P(S_2/S_1)=p^1$ ; $P(S_2/F_1)=p$ ; $P(F_2/S_1)=q^1$ and $P(F_2/F_1)=q$ for each coin. Then $P(S_1F_2)=pq^1$ ; $P(S_1S_2)=pp^1$ ; $P(F_1F_2)=q^2$ and $P(F_1S_2)=qp$ for each coin where $O\leqslant p, p^1\leqslant 1$ Φ and $p+q=p^1+q^1=1$. The out come of tossing each of 'n' coins twice will result in one of the four mutually exclusive and exhaustive groups *viz.*—$S_1F_2$ ; $S_1S_2$ ; $F_1F_2$ and $F_1S_2$. Let the number of coins falling in $S_1F_2$ be denoted by $n(S_1F_2)=n_{21}$. Similarly $n(S_1S_2)=n_{22}$ ; $n(F_1F_2)=n_{23}$ and $n(F_1S_2)=n_{24}$. Then the probability of getting such $n_{2i}$ $(i=1, 2, 3, 4)$ is

$$\frac{n!}{\prod_{i=1}^{4} n_{2i}!} \prod_{i=1}^{4} P_{2i}^{n_{2i}} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (1)$$

Where $P_{21}=pq^1$ ; $P_{22}=pp^1$ ; $P_{23}=q^2$ and $P_{24}=qp$.

Now this model can be extended to a population of 'n' animals where the number of success in the first experiment is the number of animals tagged and $p^1$ and $p$ are the probabilities of catching a tagged and untagged one respectively in the second experiment. The process of catching remains the same in both the experiments so that difference in probability of an animal being caught lies only in whether that animal is tagged or not and not in the process.

Let T be the total number tagged, in other words the catch in the first experiment. Hence $T=n_{21}+n_{22}$. Let C be the catch in the second experiment consisting t tagged animals and u untagged ones. Then $C=n_{22}+n_{24}$ ; $t=n_{22}$ and $u=n_{24}$.

Considering (1) the moments as well as mximum likelihood gives $\hat{p}^1=t/T$ and $\hat{p}=T/n$ or $\frac{u}{n-T}$ or $\frac{T+u}{2n-T}$.

It can be seen whether

$\hat{p}=T/n$ or $\frac{u}{n-T}$ or $\frac{T+u}{2n-T}$.

$\hat{n}=T^2/(T-u)$ provided $T>u$

$=\hat{n}_1$ ( say ) ... ... ... ... ... ... (2)

This follows from E (T) E (U) E (u)=n where U=n−T. When $P=P^1$ the Peterson estimate is obtained and $\hat{n}=T.C/t=\hat{n}_2$ ( say ). Now $\hat{n}_2 \geqslant \hat{n}_1$ when $T \geqslant C$ and $\hat{n}_1 > \hat{n}_2$ when $T < C < T+t$.

**Large sample test :** To test equal catchability amounts to testing $p=p^1$, A simple test that immediately comes to our mind is to test $P_{21}=P_{24}$. The statistic to be used for the test is $(n_{21}-n_{24})$ whose variance V (say) is $\Phi\ n(p_{21}\ q_{21}+p_{24}\ q_{24}+2P_{21}\ P_{24})$. Hence under null pypothosis $t=\frac{(n_{21}-n_{24})}{\sqrt{\hat{V}}}$ is N (0.1)

for large n where $\hat{V}=\hat{n}_2(\hat{P}_{21}\ \hat{q}_{21}+\hat{P}_{24}\ \hat{q}_{24}+2\hat{p}_{21}\ \hat{p}_{24})$.

Let us take up the data, from Table 4.5 pp 144, considered by Seber (*op. cit.*) which satisfies the required conditions for the above model in that the population is 'closed' and the process of catching remains the same in both the experiments. In his notation $n_1=109$, $n_2=133$ and $m_2=15$. In our notation $n_{21}=94$, $n_{22}=15$, $n_{24}=118$. Hence $\hat{n}_2=920$ ; $\hat{P}_{21}=94/920$ ; $\hat{P}_{24}=118/920$ ; $\hat{V}=211.37$ and $t=1.65$. Here equal catchability is not ruled out at 5% level of significance.

Let $Y=n_{21}+n_{24}$, $x=n_{21}$ and $p=P_{21}/(P_{21}+p_{24})$. Thus from Seber[3] p.16, $P\ (x/y)=\binom{Y}{X} p^X(1-p)^{Y-X}$. Testing $P_{21}=P_{24}$ implies testing $P=\frac{1}{2}$. From the above example $y=212$, $x=94$ and $p=\frac{94}{212}$. Referring to Binomical tables the observed value of p is not significantly different from $\frac{1}{2}$ at 5% significance level.

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## References


1. Ricker, W. E. (1958). *Bull. Fish. Res. Bd. Canada*, No. 119.
2. Darroch, J. N. (1959). *Biometrika*, **46** : 336-51.
3. Seber, G. A. F. (1973). The estimation of animal abundance and related parameters. Griffin, London.
4. Darroch, J. N. (1958). *Biometrika*, **45** : 343-59.


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