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APPLICATION OF VON BERTALANFFY'S GROWTH MODEL TO *SETIPINNA PHASA* (HAMILTON) WHEN GROWTH IS ALLOMETRIC

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ABSTRACT

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A shape factor was introduced in the growth equation for *S. phasa* (Ham.) based on Von Bertalanffy's growth model and allometric growth. The estimates thus obtained fit the observed values better than those obtained with the assumption that growth is isometric. A new method of evaluating shape factor is described.

INTRODUCTION

Growth equations in ichthyological studies are often derived from Von Bertalanffy's growth model, since the parameters occurring therein are considered to have biological significance. This growth model has the form

$$\frac{dW}{dt} = HS - DW \qquad (1)$$

where W is the weight, S the effective physiological surface; H and D are constants associated with the anabolic and catabolic rates of the animal respectively. The functional relationship of l, the length of the animal, with S and W can be expressed in the form

$$S = C_1\ l^m \text{ and } W = C_2\ l^n \qquad (2)$$

where C_1, C_2, m and n are constants. Many workers (Beverton and Holt, 1957) have used (1) assuming isometric growth, taking $m = 2$ and $n = 3$, and obtained

$$W_t = W_\infty\ (1 - \exp\{-K(t-t_o)\}\ )^3 \qquad (3)$$

where W_∞ is the weight of fish as $t \to \infty$; t_o is the time when it has zero weight, and K is the catabolic coefficient. The catabolic coefficient K has much physiological significance since it is also taken as an index of mortality (Gulland, 1969). Hence, a precise estimate of K and thus a better approximation to the growth pattern in fish, is required for biological studies of fish populations.

In fish, the exponent n occurring in the weight—length relationship has been found to vary between 2 and 4 (Ricker, 1958). Thus, assuming isometric growth and giving the value 3 for n may be incorrect. Taking this fact into consideration, the yield equation has been suitably modified by Jones (1957) and Paulik and Gales (1964). Using the modification of Paulik and Gales, Krishnan Kutty (1968, a and b) states that the general form of (3) for any value of n is

$$W_t = W_\infty \left(1 - \exp\{-K(t-t_o)\}\right)^n \quad (4)$$

It has been shown by Taylor (1962) that (4) is not the general form and when allometric growth is assumed the general form is

$$W_t = W_\infty \left(1 - \exp\{-Kd(t-t_o)\}\right)^{n/d} \quad (5)$$

where $d = n - m$. Thus the form (4) is a particular case of (5) when $d = 1$. Assuming that d lies in $0 \leqslant d \leqslant 1$, Taylor (1962) also showed that in the absence of data on m, d can be estimated from data on length and age by a trial and error method. However, Bhattacharya (1964) and Southward and Chapman (1965) have dealt with the problem of estimation of all parameters using a least squares technique. Paulik and Gales (1964) have defined "Representative cross sectional area" by considering the width and depth of fish. They assumed that this representative cross sectional area is proportional to S in (1) and $S = C\,W/l$ where C is a constant. In this case $d = 1$ and the growth equation reduces to (4). However, the examples worked out by Alagaraja (1973) and the example considered in the present paper do not fit in with this relationship.

Alagaraja (1973) has suggested a method of estimating m by taking the outer shape of fish into consideration with the assumption that this area is directly proportional to S in (1). He suggests that the fish under study may be placed over a transparent sheet and its outer surface traced. While doing this, dorsal and ventral fins should be bent towards the fish and the tracing stopped where the caudal fin starts both above and below. Then these ends are connected by a straight line. This traced-out figure is placed over graph paper, or any squared paper, and the number of squares lying inside the figure are taken to be proportional to S. However, no example was worked out by Alagaraja (1973) using data on length, weight, age and shape since data on shape were not then available.

MATERIAL AND METHODS

In the present paper data on length, weight, age and shape available for *Setipinna phasa* (Ham.) have been collected for the purpose of evaluating the parameters and comparing them with those obtained by Jhingran (1971) from the same data applied to (3). Jhingran estimated growth parameters but did not consider the shape factor. The data on length, weight and age are taken from Jhingran (1971). For the shape factor, 54 specimens of *S. phasa* were collected and the shapes recorded directly on graph paper after wiping and drying the specimens thoroughly on blotting paper.

Two methods have been used to calculate the area of each traced-out figure. The first one has been explained in the introduction. The squares falling within the figure are counted. While counting, care is taken to see that a square having half or more of its area inside the figure is taken as one, whereas a square having less than half of its area inside the figure is omitted. Thus, this method is an approximate one for determining the area inside the traced-out figure.

The second method seems to be more precise than the first one. The traced-out figures are cut to their shapes and each cut piece is weighed in a sensitive balance and the weight of each piece noted. The weight of a sheet of paper having known area — the same paper as that on which specimens are traced — is also noted. With the help of this, the corresponding area of each piece can be calculated from its weight as follows: Let the known area be A with weight W. Let the weight of a traced-out figure be w. Then its area is $A \times w/W$. In this case care should be taken to see that the sheets used for this purpose are of the same quality in thickness, weight etc., so that the simple proportional relationship given above is in no case vitiated.

Though the data pertaining to length, weight and age are taken from earlier collections and the data on shape factor from later collections, it is assumed that in the absence of any ecological changes during the interval the difference in time will not affect the analysis. The closeness of the estimated values to the observed ones supports this assumption.

It is strange that the results of Taylor (1962) escaped the notice of later workers. The results of Bhattacharya (1964) are not new from these of Taylor. Krishnan Kutty (1968 a and b) has also failed to take note of the results of Taylor. The results of Alagaraja (1973) also suffer to some extent from a similar oversight. Taylor (1962) has conjectured that the values of d are in range between 0 and 1, as stated earlier. However, Alagaraja has found $d > 1$ in the example considered by him. Taking note of this and other available data from Taylor, the range of d can safely be confined to $0 \leqslant d \leqslant 2$. This and other related problems of evaluatating d from weight and age data alone, will be considered subsequently and the work is in progress.

From (5) expressing weight in terms of length we have

$$l_t = l_\infty \, (1-\exp\{-\mathrm{K}d\,(t-t_0)\}\,)^{l/d} \tag{6}$$

from (6) we get

$$l_{t+1}^d = l_\infty^d \,(1-\exp\,(-\mathrm{K}d)\,) + l_t^d \exp\,(-\mathrm{K}d) \tag{7}$$

and

$$\log\,(1-\,(l_t/l_\infty)^d\,) = \mathrm{K}d\;t_0-\mathrm{K}dt \tag{8}$$

It may be noted that (6) represents the well-known time—length relationship when $d = 1$.

That is

$$l_t = l_\infty (1 - \exp\{-K(t - t_0)\}) \tag{9}$$

It is clear that (9) holds good irrespective of whether the growth is isometric or allometric if and only if $d = 1$. Now (7) is linear in l_t^d and l_{t+1}^d and (8) is linear in t and $\log(1 - (l_t/l_\infty)^d)$. Hence least square estimates of K, t_0 and l_∞ can be obtained from (7) and (8).

At this stage it may be noted that there are methods to solve (5). Abramson (1963) has given a computor programme. Southward and Chapman (1965) have also used a similar programme to evaluate the parameters. Bhattacharya (1964) has solved it using an iterative procedure of finding the least squares solution. This procedure requires at least five observations to estimate the parameters. When data are available for not more than five age groups, which is normal, the method given in this paper may give more precise estimates of parameters than the other methods. Moreover, available statistical techniques can be used to test d. The shape factor, as envisaged here, may also be profitably utilized for studying variations in population studies.

Jhingran (1971) has shown the length—time relationship of *S. phasa* to be

$$l_t = 446.7\,(l - \exp\{-0.1381\,(t + 0.3661)\}) \tag{10}$$

Having confirmed the reliability of age determination from examining fish scales, Jhingran noted lengths at different ages of *S. phasa* and obtained (10).

TABLE I

Comparison of estimates of parameters

	Estimates of			
	t_0	K	l_∞	d
Jhingran	−0.3661	0.1381	446.7	1.0000
Present method	−0.7996	0.2435	407.5	0.7541

TABLE II

Comparison of lengths at different ages. (Ages for the observed length were determined by examination of fish scales)

	Age (years)							
	1	2	3	4	5	6	7	8
Observed length (mm)	76.0	121.9	162.8	201.6	231.6	259.4	285.4	303.3
Estimated length (mm)								
Jhingran	71.2	126.5	166.5	201.1	231.5	258.2	281.6	302.1
Present method	75.8	121.6	163.7	200.4	232.6	260.3	283.8	303.8

The estimates of parameters available in (10) are compared here with those that are obtained using the shape factor and (6), (7) and (8). All these are given in Table I for comparison. The estimated lengths by the present method and those given by Jhingran (1971) are also compared and are presented in Table II.

CONCLUSIONS

Estimates of parameters correlate more closely with the observed values using the method suggested here than the method followed in other ichthyological studies. This method strengthens the assumption that the effective physiological surface of a fish is directly proportional to the surface area traced as explained in this paper, so far as the data on *S. phasa* are concerned. This conclusion supports the conjecture that the effective physiological surface need not have the form $S = C\, W/l$ that Paulik and Gales (1964) have defined. Of the two methods of evaluating the area of the traced-out figure, the values obtained by weighing are free from personal bias and are thus more precise than the method based on counting the number of squares falling within the figure. However, less equipment is required for this latter method.

Whether $d = 1$ or not can always be tested by usual techniques since

$$\log (W/S) = C + d \log l \tag{11}$$

and this is linear in $\log l$ and $\log (W/S)$. This test was done for *S. phasa* and it was found that d is significantly different from unity at the 1% level. Whenever d is found to be not significantly different from unity then the time—length relationship (9) can be used without reservation. Apart from this, precise estimates of K, the catabolic coefficient, can be obtained using this procedure. Present estimates of K are more effectively compared than the earlier estimates of K, because the earlier estimates of K based on (9) may be over-estimates or underestimates of true K, depending on whether d is less than or more than unity.

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