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Simulation Models

**INFORMATION ONLY**

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# Technical Notes



## MICRO-ANALYTICAL MODELS – VIRTUAL POPULATION

### ANALYSIS, THOMPSON & BELL

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The most methods currently used in temperate fisheries stock assessments rely on catch-at-age data and among them virtual population analysis (VPA) or cohort analysis is the most common method that calculates stock size based on catches with no underlying statistical assumptions. Virtual population analysis (VPA) calculates past stock abundances based on past catches. Once stock sizes are calculated, fishing size-selectivity as well as changes in vulnerability over time can be determined. The stock size estimates, which include recruitment estimates for each year, can be used for stock and recruitment analysis. VPA, also known as cohort analysis, is one of the most powerful techniques available for the analysis of fisheries data and forms the heart of many current assessment methods where catch-at-age data are available. Virtual population analysis or VPA is basically an analysis of the catches of commercial fisheries, obtained through fishery statistics, combined with detailed information on the contribution of each cohort to the catch, which is usually obtained through sampling programmes and age readings. The word “virtual”, introduced by Fry is based on the analogy with the “virtual image”, known from physics. A “virtual population” is not the real population, but it is the only one that is seen. The idea behind the method is to analyse that what can be seen, the catch, in order to calculate the population that must have been in the water to produce this catch.

VPA therefore looks at a population in an historic perspective. The advantage of doing a VPA is that once the history is known it becomes easier to predict the future catches, which is usually one of the most important tasks of fishery scientists.

Virtual population analysis calculates the number of fish alive in each cohort for each past year. It is also called cohort analysis because each cohort is analysed separately. VPA relies on a very simple relationship for each cohort. If we knew the initial cohort size, and the natural mortality rate, we could use the equation to calculate the number alive each year. Unfortunately, we rarely if ever know the initial number alive; this is in fact one of the things we want cohort analysis to tell us.

The basic equation for VPA is

$$\begin{array}{ccccccc} \text{Number alive} & & \text{Number alive} & & \text{Catch} & & \text{Natural} \\ \text{At Beginning} & = & \text{at beginning} & + & \text{this} & + & \text{mortality} \\ \text{Of this year} & & \text{of next year} & & \text{year} & & \text{this year} \end{array}$$

If we are willing to assume that at some age there are none alive (or that we know the number alive at some terminal age) and that we know the natural mortality rate, we can use above equation to iteratively calculate the number alive each year, starting from the oldest ages and moving backward to the youngest.

### **Age-based cohort analysis (Pope's cohort analysis)**

As derived from the catch equation, the VPA implied the solution by some numerical techniques (some trial and error method). This is a minor technical problem when one has access to a computer. However, the problem can be circumvented in an easy way, so that VPA can also be carried out on a pocket calculator. The version of VPA suitable for pocket calculators is the "cohort analysis" developed by Pope (1972).

Cohort analysis is conceptually identical to VPA, but the calculation technique is simpler. It is based on an approximation, illustrated which shows the number of survivors of a cohort during one year. The catch is taken continuously during the year, but in cohort analysis the assumption is made that all fish are caught on one single day..

Consequently in the first half year the fish suffer only natural mortality so the number of survivors becomes:

$$N(y, t + 0.5) = N(y, t) * \exp(-M/2)$$

Then, instantaneously, the catch is taken and the number of survivors becomes:

$$N(y, t) * \exp(-M/2) - C(y, t, t + 1)$$

This number of survivors then suffers further only natural mortality in the second half year and finally the number of survivors at the end of the year is:

$$N(y+1, t+1) = (N(y, t) * \exp(-M/2) - C(y, t, t+1)) * \exp(-M/2)$$

For convenience of calculation this equation is rearranged as:

$$N(y, t) = (N(y + 1, t + 1) * \exp(M/2) + C(y, t, t+1)) * \exp(M/2)$$

Note that the F that caused computational problems in the VPA equation does not occur here.

### **Jones' Length-based cohort analysis**

Keeping in view the difficulty in determination of ages for certain resources and also the fact that it is rather difficult to obtain age-frequency data for most of the tropical fish, cohort analysis described above is modified to make use of the length frequency data (length composition data for the total fishery are available for one year or the average length composition for a sequence of years). According to Sparre et.al ( ) the name "length-based cohort analysis" is somewhat misleading, as we are not dealing with real cohorts in the present analysis. The real cohort is replaced by a "pseudo-cohort" which is based on the assumption of a constant parameter system. Thus, it is assumed that the picture presented by all length (or age) classes caught during one year reflects that of a single cohort during its entire life span.

To convert the cohort analysis equation into a length-based version, only the term  $\exp(-Z \cdot t/2)$  has to be changed.

It is convenient to use a symbol instead of this complicated term, therefore we introduce the symbols:

$$\begin{aligned}
 N(L1) = N(t(L1)) &= \text{the number of fish that attain length } L1 \\
 &= \text{the number of fish that attain age } t(L1) \\
 &\quad (\text{also called the number of survivors}) \\
 N(L2) = N(t(L1 + \Delta t)) &= \text{the number of fish that attain length } L2 \\
 &= \text{the number of fish that attain age } t(L2) \\
 &\quad (= t(L1 + \Delta t)) \\
 C(L1, L2) = C(t, t + \Delta t) &= \text{the number of fish caught of lengths between} \\
 &\quad L1 \text{ and } L2 \\
 &= \text{the number of fish caught of ages between } t \\
 &\quad (L1) \text{ and } t(L2)
 \end{aligned}$$

Now equation can be rewritten using these length-based symbols, as:

$$N(L1) = (N(L2) * H(L1, L2) + C(L1, L2)) * H(L1, L2)$$

The calculation procedure of equation is similar to that of the age-based cohort analysis. We start with the last group and use the length-based form of the catch equation

$$C(L1, L2) = N(L1) * C/Z * (1 - \exp(-Z * \Delta t))$$

### Thompson and Bell model

The first predictive model developed much earlier than the Beverton and Holt model was by Thompson and Bell (1934). The Thompson and Bell model is the exact opposite of the VPA and cohort analysis. It is used to predict the effects of changes in the fishing effort on future yields, while VPA and cohort analysis are used to determine the numbers of fish that must have been present in the sea, to account for a known sustained catch, and the fishing effort that must have been expended on each age or length group to obtain the numbers caught. Therefore, VPA and cohort analysis are called historic or retrospective models, while the Thompson and Bell model is predictive.

The Thompson and Bell model is a very important tool for the fishery scientist to demonstrate the effect that certain management measures, such as changes in the minimum mesh size, decreases or increases of fishing effort, or closed seasons will have on the yield, the biomass and the value of the catch. Since a large number of calculations is required, it is essential to use computers.

An important aspect of the Thompson and Bell model is that it allows for the incorporation of the value of the catch. Therefore, the model has become the basis for the development of so-called bio-economic models, which are extremely useful for the provision of predictions needed for management decisions.

## Age based model

The Thompson and Bell method consists of two main stages: 1) Provision of essential and optional inputs and 2) the calculation of outputs in the form of predictions of future yields, biomass levels and even the value of the future yields.

- 1) Provision of inputs: The main input is a so-called “reference F-at-age-array”, an array of F-values per age group. In principle any F-array could be used as input, but, of course, not just any F-array will produce results which are related to the real situation of a fishery. Therefore, it is customary to use an F-array that has been obtained from an analysis of historical data, in other words from a VPA or a cohort analysis

Another important input parameter is the number of recruits, which may also be obtained from VPA or cohort analysis. This input is needed to obtain predictions of yields etc. in absolute quantities. However, if this input is not available the Thompson and Bell model can still be used to provide relative figures as output, for example, in the form of units “per 1000 recruits”.

The model further requires a “weight-at-age-array”, the weights of individual fish per age group. For economic analyses the model also requires inputs of value, usually in the form of the price per kg by age group. (For the length-based Thompson and Bell model the same type of input is required per length group).

- 2) Outputs: The output of the model is in the form of predictions of the catch in numbers, the total number of deaths, the yield, the mean biomass and the value, all per age group, related to values of F for each age group. New values of F can be obtained by multiplying the reference F-array as a whole by a certain factor, usually called X, or by applying such factors only to a part of the reference F-array. The latter is applied, for example, in the case of a change in the minimum mesh size, or to separate the effect of fleets with different characteristics (e.g. artisanal and industrial) on a particular stock. By carrying out a whole series of calculations with different values for X (F-factors), graphs can be drawn that illustrate clearly the effects of changes in F on the yield, the average biomass and the value of the catch.

The “length-based Thompson and Bell model” takes its inputs from a length-based cohort analysis. The inputs consist of the fishing mortalities by length group, the F-at-length-array, the number of fish in the smallest length group, the growth parameter K and the natural mortality factor H by length group, which must be the same as the ones used in the cohort analysis. Additional inputs are the parameters of a length-weight relationship (or the average weight of a single fish or shrimp by length group) and the average price per kg by length group.

The outputs are the same as for the age-based model, viz., for each length group the number at the lower limit of the length group,  $N(L1)$ , the catch in numbers, the yield in weight, the biomass multiplied by  $\Delta t$ , i.e. the time required to grow from the lower limit to the upper limit of the length group and the value. Finally, the totals of the catch, yield, mean biomass  $\times \Delta t$  and value are obtained. The calculations are repeated for a range of X

values (F-arrays) and the final results (totals) are plotted in graphs. The principle is the same as that described above for the age-based models, only the formulas are slightly different. They can be derived from those used for Jones' length-based cohort analysis.

Since the length-based Thompson and Bell analysis is derived from Jones' length-based cohort analysis which in turn is based on Pope's age-based cohort analysis, the length-based Thompson and Bell method has the same limitations as Pope's age-based cohort analysis. The approximation to VPA in the predictive mode is valid for values of  $F \cdot t$  up to 1.2 and of  $M \cdot t$  up to 0.3 (Pope, 1972, as quoted by Sparre and Venema, 1992). If the F's are high, nonsensical results will come out of the analysis, such as negative stock numbers. If that is the case, smaller length groups and hence, smaller  $t$  values, are required.

**For further reading:**

Gulland, J.A 1983. Fish Stock Assessment: A manual of basic methods: Chichester, U.K.

Wiley Interscience, FAO/Wiley Series of Food and Agriculture Vol. 1, 223 pp.

Hillborn, R and Walters C.J. 1992. Quantitative Fisheries Stock Assessment. Choice, Dynamics & Uncertainty. Kluwer Academic Publishers, Boston/ Dordrecht/ London, 570 pp.

Sparre, P and S.C. Venema 1992. Introduction to Tropical Fish Stock Assessment. Part I. Manual. FAO Fisheries Technical paper, No. 306.1. Rev1. Rome. FAO. 376 pp.