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Winter School on

Towards Ecosystem Based Management of Marine Fisheries – Building Mass Balance Trophic and Simulation Models

INFORMATION ONLY

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Technical Notes

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MICRO-ANALYTICAL MODELS – RELATIVE YIELD PER RECRUIT

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Beverton and Holts yield per recruit model is a steady state model. A model that describes the state of the stock and yield when the fishing pattern has been the same over a long period so that all recruited fish alive are exposed to fishing is termed as a steady state model. The Beverton and Holts model makes the following assumptions.

1. Recruitment is constant
2. All fish belonging to a cohort are born on the same day
3. Recruitment and selection are knife-edged
4. Fishing and natural mortalities are constant through out the phase after recruitment
5. There is complete mixing within the stock
6. Growth in weight is isometric. That is $b ? 3$ in $W_t ? a L_t^b$

At the age at recruitment t_r of the cohort, numbers recruited is

$$R ? N_{t_r}$$

Number of survivors at age t_c, age at first capture, is

$$N_{t_c} ? R e^{? M (t_c ? t_r)}$$

because only natural mortality operates on the cohort between age t_r and t_c.
Number of survivors of the recruited cohort at age *t* is give by the equation

$$N_t ? N_{t_c} e^{? (M ? F)(t ? t_c)}$$
$$? R e^{? M (t_c ? t_r) ? (M ? F)(t ? t_c)}$$

The fraction of the recruits surviving up to age *t* is given by

$$\frac{N_t}{R} ? e^{? M (t_c ? t_r) ? (M ? F)(t ? t_c)}$$

The numbers caught between a very small interval $(t,\ t ? ?)$ is given by

$$C(t, t ? ?) ? ? F N_t$$

To obtain the yield we have to multiply this with the weight of the animal and hence we get the expression for yield from the cohort during this short period as

$$Y(t,t ? ?) ? ? \; F \; N_t \, W_t \text{ where } W_t \; ? \; a \, L_t^3$$

The relative yield is then obtained as

$$\frac{Y(t,t ? ?)}{R} ? ? \; F \frac{N_t}{R} W_t$$

which is the relative yield from the recruited cohort during the period $(t, \; t ? ?)$.
To obtain the total relative yield from the cohort during the entire life span of the cohort we have to add such quantities from age t_c onwards. The expression for the total relative yield then is

$$\frac{Y}{R} ? \overset{n}{\underset{i?1}{?}} \frac{Y(t ? (i ? 1)? , t ? i?)}{R}$$

where n is choosen sufficiently large to cover the entire life span of the cohort. The above sum finally reduce to the following form after a set of substitutions and simplifications.

$$\frac{Y}{R} ? \; F \; e^{?M(t_c ? t_r)} \, W_? \; \frac{?1}{?Z} ? \frac{3S}{Z ? K} ? \frac{3S^2}{Z ? 2K} ? \frac{S^3}{Z ? 3K} ? \text{ where}$$

$$S \; ? \; e^{?K(t_c ? t_0)}$$

$K, t_0, W_?$ are Von Bertalaffy growth parameters

t_c : age at first capture

t_r : age at recruitment

F : fishing mortality

M : natural mortality

$\frac{Y}{R}$: the yield per recruit in grams per recruit.

It is important to note that t_c and *F* are the two parameters over which the fishery managers have control. Fishing mortality *F* is proportional to effort and t_c is a function of gear selectivity which in turn is related to mesh size. Hence $\frac{Y}{R}$ can be considered as a function of *F* and t_c, and often $\frac{Y}{R}$ values are calculated for varying inputs of *F* and plotted for finding optimum value of *F*. This curve is known as yield per recruit curve and it often has a maximum that corresponds to the Maximum Sustainable Yield (MSY). This maximum changes as the value of t_c used is changed. By varying *F* and t_c simultaneously we can obtain a combination of *F* and t_c which gives the highest value for MSY. The above model is based on age of the cohorts. A similar version based on length of the cohort is given below.

When growth is isometric Beverton and Holt obtained the relation

$$\left(Y/R\right)' = E\, U^{M/K}\left[1 - \frac{3U}{1+m} + \frac{3U^2}{1+2m} - \frac{U^3}{1+3m}\right] \qquad (1)$$

where

$$m = \frac{1-E}{M/K} = \frac{K}{Z}$$

$$U = 1 - \frac{L_c}{L_\infty}$$

$E = \frac{F}{Z}$ is the exploitation rate (fraction of deaths due to fishing).

K, L_∞ : Von Bertalanffy growth parameters

F : Fishing mortality coefficient

M: Natural mortality coefficient

The relation between relative yield per recruit and $\left(Y/R\right)'$ is

$$\left(Y/R\right) = W_\infty\, e^{M(t_r - t_0)} \left(Y/R\right)' \qquad (2)$$

Using the above relation the Maximum Sustainable Yield can be obtained in the following steps.

- For the present level of exploitation (the estimated F which corresponds to the present level of exploitation, $E = \frac{F}{Z}$) calculate the quantity $\left(Y/R\right)'$ using equation (1).
- Calculate corresponding $\left(Y/R\right)$ using equation (2).
- This will be the yield per recruit for the present level of exploitation and since we know the present yield Y (catch) we can obtain the recruitment R by dividing yield by yield per recruit.
- Estimate $\left(Y/R\right)'$ for different levels of exploitation rates E and from the plot of $\left(Y/R\right)'$ against E or other wise find the maximum sustainable value of $\left(Y/R\right)'$, say MS $\left(Y/R\right)'$.
- Calculate MS $\left(Y/R\right)$ using equation (2) corresponding to the value of MS $\left(Y/R\right)'$.
- To get MSY multiply MS $\left(Y/R\right)$ by R.

Jones Modified Method of Yield Per Recruit:

The major flow in Beverton and Holts method is that it requires isometric growth. Jones (1957) suggested a general procedure for estimating yield per recruit as follows.

The instantaneous rate of yield when $W_t = aL_t^b$ is

$$\frac{dY_t}{dt} = F\ N_t\ a\ L_\infty^b\ [1 - e^{-K(t-t_0)^b}]$$

Jones obtained by transformation and integration that

$$\frac{Y}{R} = \frac{F}{K}\ a\ L_\infty^b\ e^{-M(t_c-t_r)}\ e^{(F+M)(t_c-t_0)} \int e^{-K(t_c-t_0)}\ x^{\frac{F+M}{K}-1}\ (1-x)^b\,dx$$

The integral part in the above equation is in the form of an incomplete beta function given by

$$\beta(z;p;q) = \int_0^z x^{p-1}\ (1-x)^{q-1}\ dx$$

where $p = \frac{F+M}{K} = \frac{Z}{K}$, $q = b+1$ and $z = e^{-K(t_c-t_0)}$. Values of beta function are tabulated and will be available in statistical tables.