

# Robust Multi-Objective Control of Power System Stabilizer Using Mixed $H_2/H_\infty$ and $\mu$ Analysis

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## ABSTRACT

In order to study the dynamic stability of the system, having a precise dynamic model including the energy generation units such as generators, excitation system and turbine is necessary. The aim of this paper is to design a power stabilizer for Mashhad power plant and assess its effects on the electromechanical fluctuations. Due to lack of certainty in the system, designing an optimized robust controller is crucial. In this paper, the establishment of balance between the nominal and robust performance is done by the weight functions. In the frequencies where the uncertainty is high, in order to achieve to the robust performance of the controller,  $\mu$  analysis is more profound, otherwise, in order to achieve to nominal performance, robust stability, noise reduction and decrease of controlling signal, the impact of the controller  $H_2/H_\infty$  is more profound. The results of the simulation studies represent the advantages and effectiveness of the suggested method.

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## 1. INTRODUCTION

The operation of the power system is usually limited to the boundaries of the dynamic stability, which is far from the limits of thermal stability. Nowadays, to improve the attenuation of the oscillations with low frequencies, in most cases, power system stabilizers (PSS) are widely used in power systems. Designing a proper stabilizer could decrease the limitations due to dynamic issues and help the system to get closer to its nominal capacity. These stabilizers are usually designed according to the single machine, infinite bus of the system in a definite working point. Therefore, it is possible that the stability of the system to be threaten by changes in the parameters or equilibrium points of the system. In this paper, the system stability, in the other words the stability of power system's ability against the change of parameters are checked. As a result, three main controlling goals will be obtained: strengthening the closed loop system, lower cost designing strategies and improving the transient response.  $H_2$  control is used in the transient stage for presenting a fast dynamic response to minimize the response energy of the impulse. While  $H_\infty$  control is used in the stable stage for reduction in the disturbance and protecting against tracking; which in turn would guarantee the robust stability. In order to achieve to optimal performance, taking into account the effect of uncertainty during system design period is required. But on the other hand it can lead to severe restrictions on the controller that sometimes makes it an infeasible problem. So far, various studies have been conducted on the stability and controlling of power systems.

The first formulation of the  $H_\infty$  control problem was performed in 1981 by Zames. To date, large numbers of researches have been performed for study of the robust control, the  $H_2$  control and  $H_\infty$  control. Doyle has analyzed the state space by using  $H_\infty$  and  $H_2$  standard form and it's solving. The conditions of

solving problem and its solution using Hamiltonian matrix introduction are the highlights of this paper [1]. Also Doyle as well as a tutorial overview in the fractional linear transformations (LFTs) and the value of the unique structure,  $\mu$ , and linear matrix inequalities (LMIs) in the solution of LFT problems has offered [2].  $H_2/H_\infty$ , were combined by Rotea in this way, two important approaches were suggested, 1) optimal control limit of  $H_2$  and  $H_\infty$  (actually constrained optimization), and 2) at the same time optimal control of  $H_2/H_\infty$  [3]. Lanzon in his PHD thesis chooses the weight functions in  $\mu$  and  $H_\infty$  design [4]. Many of the power stabilizers proposed for systems of the single machine are not able to resolve the interaction problems; while some of the multi variable stabilizers are also lacking suitable robust stability. Studies on the stability are mostly conducted on two transient and steady states. At operation condition, a power system is in its permanent state [5]. When performance is in the permanent state, if a sudden change happens, the system will go toward the disturbance.

Investigation of the classic stability [5], the optimization method with the help of pareto multi-objective [6], the method of adaptive control [7], the nonlinear controller [8], using the parameters estimation [9], robust controller  $H_2/H_\infty$  [10], the pole placement and application of the linear matrix inequality [11], fuzzy and Neural network control [12], and Evolutionary algorithm [13] are among the works which had been done. The problem of closed-loop identification of the Heffron-Phillips model parameters is of practical importance since the data used for identification can be gathered when the machine is normally connected to the power system [14]. In this paper, at first the power system was modeled. Then, the problem was introduced. In this paper, at first,  $H_2/H_\infty$  controller was investigated with a new insight along with the new controller,  $\mu$ ; and then these two different controllers were combined via the use of the weight matrices. Solving this problem would be possible by application of the linear matrix inequality. The results indicate that, the goals of  $H_2/H_\infty/\mu$  combination, including elimination of the perturbation effect, reducing the controlling signal and accounting for the uncertainty during the system's functionality investigation, were properly realized.

## 2. POWER SYSTEM MODELLING

The stabilizers of the power systems are designed with the aim of improving the attenuation of the low frequency oscillations of the system, based on the single machine, infinite bus model. The power system stabilizer is a traditional and economic controller whose aim is to increase the dynamic stability of the power system. By creating the damping electrical torque, the stabilizer of the power system will improve the deviations of the rotors rotations. The mentioned equipment also optimizes and tunes the exciting voltage, by creating the suitable voltage. The power plant of Mashhad city is located at the eastern part of the city at the beginning of Sarakhs Boulevard. This is the oldest power plant of Khorasan province and has 8 electricity generating units, 4 of them are working with steam and the other 4 ones are gaseous. The steam units consist of two ELIN and two SKODA units, and the gaseous units include two BBC units and two ALSTOM units. This power plant was established in 1964 and started its work in 1968. The exciting system of ALSTOM gaseous units of the power plant of Mashhad are classified as the static type. Feeding of such exciting system is done via power voltage transformer and three current transformers [15] with the capability of being saturated. The controller part of the stimulation system of ALSTOM gaseous units includes 3 main control modules. By elimination of the three controlling modules, in order to attenuate the oscillations, the power system stabilizers could be applied. In studying the dynamic stability of the power networks, and also in the cases where the changes and disturbances of the network are mainly partial and slow, the linear generator model could be employed. In order to consider a synchronous generator, we use 3 rd order synchronous generator model called Heffron-Phillips model [11]. This model contains 3 state variables:  $\Delta\omega_r$ ,  $\Delta\delta$ ,  $\Delta E_q$ .

Considering the exciter model will lead to the introduction of the fourth state variable  $\Delta E_b$ . In this model, governing differential equations are linear around operating point. Figure 1 shows block-diagram of linear mode of Heffron-Phillips model along with exciter and AVR Regarding the generator parameters Heffron-Phillips coefficients could be obtained by (1) [16]:

$$\begin{aligned}
 K_1 &= \frac{E_b E_q \cos\delta}{X_q + X_e} + \frac{X_q - X'_d}{X_e + X'_d} E_b \sin\delta, K_2 = \frac{X_q + X_e}{X_e + X'_d} i_q, K_4 = \frac{X_d - X'_d}{X_e + X'_d} E_b \sin\delta \\
 K_3 &= \frac{X'_d + X_e}{X_e + X_d}, K_5 = -\frac{E_b X_q \cos\delta}{(X_q + X_e)V_t} - \frac{X'_d}{(X_e + X'_d)V_t} E_b \sin\delta, K_6 = \frac{X_e E_q}{(X_e + X'_d)V_t}
 \end{aligned} \tag{1}$$

State space of equation of Figure 1 shows in (2).

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}, x = \begin{bmatrix} \Delta\omega_r \\ \Delta\delta \\ \Delta E_q \\ \Delta E_b \end{bmatrix}, u = \begin{bmatrix} V_{ref} \\ \Delta T_m \end{bmatrix}, A = \begin{bmatrix} -\frac{K_D}{2H} & -\frac{K_1}{2H} & -\frac{K_2}{2H} & 0 \\ \omega_b & 0 & 0 & 0 \\ 0 & -\frac{K_4}{T'_{do}} & \frac{1}{T'_{do}K_3} & \frac{1}{T'_{do}} \\ 0 & -\frac{K_A K_5}{T_A} & -\frac{K_A K_6}{T_A} & -\frac{1}{T_A} \end{bmatrix}, B = \begin{bmatrix} 0 & \frac{1}{2H} \\ 0 & 0 \\ 0 & 0 \\ \frac{K_A}{T_A} & 0 \end{bmatrix} \quad (2)$$

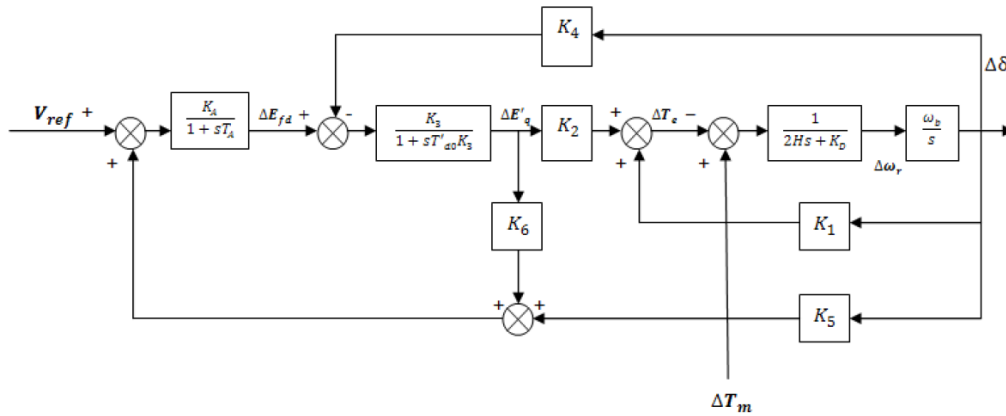


Figure 1. Heffron-Phillips model

3. PROBLEM STATEMENT

3.1. H<sub>2</sub>/H<sub>∞</sub> Controller

Existence of uncertainty created due to an uncertain and erratic input (noise and disturbance) and Un-modeled dynamic cannot be described completely and precisely as a true system by a mathematical modeling. On the other hand, a true system should contain the following important objects: robust stability, robust and nominal performance, settling time, maximum over shoot and etc which try to gain these objectives of the controlling problem [4]. The type of uncertainty is another important factor in the system analysis. Consider additive uncertainty shown in Figure 3.

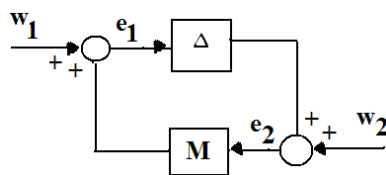


Figure 2. M – Δ Model

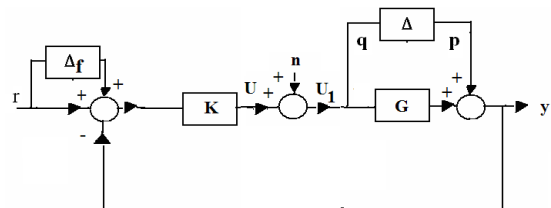


Figure 3. Additive uncertainty

**Objective 1:** if  $\Delta = 0$  then  $\|FS\|_{\infty} < 1$  (nominal performance).  $S = (I + GK)^{-1}$  (S is sensitivity function).

**Objective 2:** if  $\Delta \neq 0$  then system is robust stability.  $M = (I + KG)^{-1} K$ ,

$$\text{if } \bar{\sigma}(\Delta(j\omega)) \leq \gamma(j\omega) \Rightarrow \|\gamma(S)M\|_{\infty} < 1 \quad (3)$$

**Objective 3:** n is white noise with one PSD (power spectral density). H<sub>2</sub> norm, caused due to decrease in the controlling signal.  $\|RT_{nU_1}\|_{H_2} < 1$  (To minimize U<sub>1</sub> variance with noise input). F(s), R(s) and  $\gamma(s)$  are weighting function) from Parseval equation and objective 3. Then we have three tasks for controller design

$$(\|FS\|_\infty < 1, \|\gamma(S)M\|_\infty < 1, \|T_{nU_1}\|_\infty < 1), \text{ such that, } \left\| \begin{bmatrix} FS(K,G) \\ \gamma M(K,G) \\ RT_{nU_1}(K,G) \end{bmatrix} \right\|_\infty < 1 \tag{4}$$

Problem (4) shown in Figure 4. Rotea and Doyle offer two other methods for solve this problem. [1]-[3]. A large class of system with uncertainty can be treated as LFT (Linear fractional Transformation). LFT model is shown in Figure 3. W: the disturbance signals to the system which won't be a function of states of system, Z: the variable that will be controlled, P: the nominal open loop system, Y: the system measurable output. To transform the changed diagram of Figure 4 to the LFT model, we will write the problem in the standard form, and then solve it by using of Riccati equation [17]. The (4) LFT model is practicable in Figure 5 and can be used to design a controller. State space of Figure 5 is written in (5).

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{x}_f \\ \dot{x}_\gamma \\ \dot{x}_R \end{bmatrix} &= \underbrace{\begin{bmatrix} A & 0 & 0 & 0 \\ -B_f C & A_f & 0 & 0 \\ 0 & 0 & A_\gamma & 0 \\ 0 & 0 & 0 & A_R \end{bmatrix}}_{A_{CL}} \begin{bmatrix} x \\ x_f \\ x_\gamma \\ x_R \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & B & B \\ B_f & -B_f D & -B_f D \\ 0 & 0 & B_\gamma \\ 0 & B_R & B_R \end{bmatrix}}_{\begin{bmatrix} B_1 & B_2 \end{bmatrix}} \begin{bmatrix} r \\ n \\ u \end{bmatrix} \\ \dot{x} &= Ax + B_1 W + B_2 u \\ z &= C_1 x + D_{11} W + D_{12} u \\ y &= C_2 x + D_{21} W + D_{22} u \\ Z &= \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} FS \\ \gamma M \\ RT_{nU_1} \end{bmatrix} \\ \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ y \end{bmatrix} &= \underbrace{\begin{bmatrix} -D_f C & C_f & 0 & 0 \\ 0 & 0 & C_\gamma & 0 \\ 0 & 0 & 0 & C_R \\ -C & 0 & 0 & 0 \end{bmatrix}}_{\begin{bmatrix} C_1 \\ C_2 \end{bmatrix}} \begin{bmatrix} x \\ x_f \\ x_\gamma \\ x_R \end{bmatrix} + \underbrace{\begin{bmatrix} D_f & -D_f D & -D_f D \\ 0 & 0 & D_\gamma \\ 0 & D_R & D_R \\ 1 & -D & -D \end{bmatrix}}_{D_{CL}} \begin{bmatrix} r \\ n \\ u \end{bmatrix} \end{aligned} \tag{5}$$

Determining three weight functions, specified in Figure.4, contain special importance. Using robust optimal state feedback method for (4) equations.

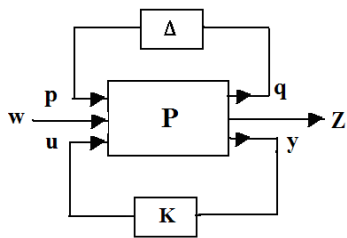


Figure 4. LFT Model

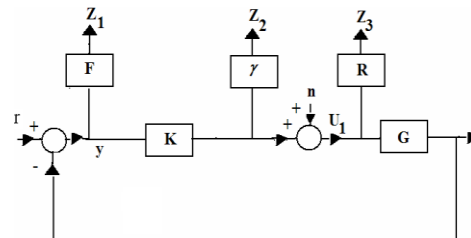


Figure 5. Graphical model of problem (4)

### 3.2. μ Controller

Here we try to assess robust performance of this closed-loop system by using μ-analysis. Robust performance condition is equivalent to the following structured singular value μ test [2].

$$\|T_{wz}(M, \Delta)\|_\infty < \gamma^{-1} \quad \forall \|\Delta\|_\infty < \gamma \Leftrightarrow \mu_{\Delta P}(M) < \gamma \quad \forall W \tag{6}$$

The complex structured singular value  $\mu_{\Delta(M)}$  is defined as  $\mu_{\Delta(M)} = \frac{1}{\min \{ \bar{\sigma}(\Delta) | \det(I - M\Delta) = 0 \}}$  Lower and

Upper bond of μ can be shown to be  $P(UM) \leq \mu_{\Delta}(M) < \min \bar{\sigma}(DMD^{-1})$ .

#### 3.2.1. D-K iteration

Unfortunately, it is not known how to obtain a controller's achieving path directly to the structured singular value test. But we can obtain the lower and upper bounds of μ. This method taken here is the so-

called *D-K* iteration procedure. The *D-K* iteration involves a sequence of minimizations over either *K* or *D* while holding the other fixed, until a satisfactory controller is constructed. First, for  $D = I$  fixed, the controller *K* is synthesized using the well-known state-space  $H_\infty$  optimization method. LFT form of Figure 3 is written in equations (7) [17], [18].

$$\dot{x} = Ax + \begin{bmatrix} 0 & 0 & B \end{bmatrix} \begin{bmatrix} P \\ W \\ U \end{bmatrix}, \quad \begin{bmatrix} q \\ z \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ -C \\ -C \end{bmatrix} x + \begin{bmatrix} 0 & 0 & I \\ -I & I & -D \\ -I & I & -D \end{bmatrix} \begin{bmatrix} P \\ W \\ U \end{bmatrix} \tag{7}$$

**3.3. New approach:  $H_2/H_\infty, \mu$  combination**

Now, we tend to synthesize two controllers according to Figure 6. As mentioned before, the availability of robust performance causes extreme limitation on the controller, which sometimes prevents it from reaching a possible condition. Also, availability of nominal performance means considering operation without uncertainty, and it is usual that the essence of uncertainty has decisive effect on the operation. So, we tend to balance between robust and nominal performance.  $W_1$  and  $W_2$  are weight functions. Having this data, we can determine which frequencies have more uncertainty effect, with regard to the controller effect of  $\mu$ . Of course, it is of importance to mention that robust performance contains nominal performance, so, controller coefficient of  $\mu$  should be smaller than  $H_2/H_\infty$  controller coefficient.

**Problem 1:** Determine  $W_1$  and  $W_2$ , in a way that an additive uncertainty system contains robust stability.

$$M = (W_1 K_1 G + W_2 K_2 G + I)^{-1} (W_1 K_1 + W_2 K_2) \tag{8}$$

$$\|M\|_\infty < 1$$

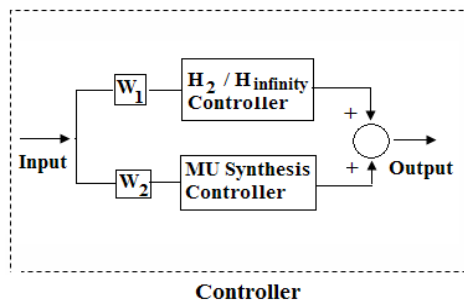


Figure 6. Controller  $H_2/H_\infty/\mu$

**3.3.1. Robust optimal state feedback with  $H_2/H_\infty, \mu$  combination**

We now attempt to follow the analysis of the conditioning of the pole placement problem. Researchers shown a number of robust performance indices have been considered in optimization approaches for control system design [18]. In robust control using  $H_\infty$  optimization, the objectives are expressed in terms of the  $H_\infty$ -norm of transfer functions. One of the objectives is the following:

$$\min_K \sup \bar{\sigma} \left\{ \begin{bmatrix} S \\ KS \end{bmatrix} \right\}, \quad \text{where } S = [I - (j\omega I - A)^{-1} BK]^{-1}. \text{ In this paper we assume that the state of the generalized}$$

plant *G* is available for feedback. To be more precise let a state-space description of *P* (figure 3) is given by (LFT Model):

$$\begin{aligned} \dot{x} &= AX + BU + B_W W \\ Y &= X \\ U &= KX \\ \dot{x} &= (A + BW_1 K_1 + BW_2 K_2)X + B_W W \end{aligned} \tag{9}$$

The signal *W* denotes disturbance. The signals *U* and *Y* denote the control input and the measured output, respectively. Next to gaining  $K_1$  by  $H_2/H_\infty$  and  $K_2$  by  $\mu$  analysis, we tend to determine weight functions, using linear matrix inequality.

**Lemma1:** (bounded-real lemma) given a constant  $\gamma > 0$ , for system,  $M(s) = (A, B, C)$  the following two statements are equivalent, 1) this system is stable  $\|M(s)\|_{\infty} < \gamma$ , 2) there exists a symmetric positive definite matrix  $Q$ , such that: [19]

$$\begin{bmatrix} A^T Q + QA & QB_p & C_q^T \\ B_p^T Q & -\gamma^{-1}I & D_q^T \\ C_q & D_q & -\gamma^{-1}I \end{bmatrix} < 0 \quad (10)$$

$$Q > 0$$

**Lemma2:** Consider the feedback system of Figure.3, where  $G$  is given by (9). Then, a given controller  $K$  is admissible and close loop system is robust stability and desired performance if and only if there exists  $W_1$  and  $W_2$  solving the following LMI problem:

$$\begin{bmatrix} A\Omega + BY_1 + BY_2 + Y_1^T B + Y_2^T B + \Omega A^T & B_W & \Omega C^T \\ & B_W & -\beta I & D^T \\ & C\Omega & D & -\beta I \end{bmatrix} < 0$$

$$\Omega > 0, \quad Y_1 > 0, \quad Y_2 > 0$$

Where,  $W_1 = Y_1 \Omega^{-1} K_1^{-1}$ ,  $W_2 = Y_2 \Omega^{-1} K_2^{-1}$ .  $K_1$  and  $K_2$  Design with equations 5 and 7 and controller achieves  $K = W_1 K_1 + W_2 K_2$ .

Lemma 2, it helps to solve of problem 1. Mashayekhifard et al. presented Robust multi-objective static output feedback with  $H_2/H_{\infty}$ ,  $\mu$  combination [20].

#### 4. METHODOLOGY

a. To design the  $H_2/H_{\infty}$  for the process with uncertainty. (It helps to select the weighting function properly).

For  $H_2/H_{\infty}$  design can use Rotea and Doyle method. ([3], [8]) or use  $\left\| \begin{bmatrix} FS(K, G) \\ \gamma M(K, G) \\ RT(K, G) \end{bmatrix} \right\|_{\infty} < 1$  and obtained  $K_1$ .

For  $F, \gamma$  and  $R$  we use inverse sensitivity function. Or use Automatic Weight Selection Algorithm [4], [21].

b. To design the  $\mu$  controller for the process with uncertainty (if the process is unstable, at first must be stabilize). D-K iteration method can be used to improve the performance of the controller design for the system. Peak value of the  $\mu$  (D-K iteration) bound should be less than one, and obtained  $K_2$ .

c. Order reduction method can be used to reduce the order of the  $K_1, K_2$ .

d.  $W_1, W_2$  are given with LMI (12) then the robust stability of the system has to be established.

e. H infinity norm of  $w_2$  must be smaller than  $w_1$ .

f.  $K = W_1 K_1 + W_2 K_2$ .

This controller ( $K$ ) has robust stability and desired performance.

#### 5. RESULTS OF SIMULATION

First  $H_2/H_{\infty}$  controller and then  $\mu$  is designed. After that the order of  $I+GK$  was reduced by the help of the residual method. Regarding the practical considerations and by application of the inverse of the sensitivity functions, the weight functions were selected with the form of  $R = \frac{s+2}{s+200} I$ ,  $F = \frac{4(0.1s+1)}{0.5s+4} I$ ,

$\gamma = \frac{s+100}{s+10} I$ .  $K_1$  and  $K_2$  are determined according to the equations 5 and 7, while  $w_1$  and  $w_2$  were defined

regarding equation 12. According to Figure 6 and section 3.A and 3.B,  $k$  was designed. The simulations were done by MATLAB software and toolboxes of LMI [22], Robust multiobjective control toolbox [23] and  $\mu$

[24] were employed. In the designing process, we used Heffron-Phillips model which is a reduced order model. In order to estimate our designs through simulations, we use complete model of power system containing synchronous generator, exciter system, governor, turbine, 3-phase transformer, transmission line, load and infinite bus. For comparison purposes, we compare the variations of before and after 3-phase fault occurring in the middle of transmission line. Three-phase fault occurs at 0.5 sec. and is gone within 0.55 sec. In addition, the comparison of the singular values for controlling signals related to three types of design is depicted in Figure 7. The results show that the largest amount of the control signal is related to  $\mu$  controller and the lowest amount was associated with  $H_2/H_\infty$ . Step response of the closed loop system for the three controllers shown in Figure 8.

Figure 8 indicates that the best function of the controller is for  $\mu$  while  $H_2/H_\infty$  shows the weakest performance. It could also be noted that since the system has multiple inputs and outputs, the sensitivity and weight functions have the matrix form. The results verify the success of combining the robust and nominal performance with each other. Reaching to the mentioned objectives with the minimum controlling signal is one of the advantages of  $H_2/H_\infty/\mu$  controller. Most of the robust controllers have high orders and controlling signals. But this new approach did well in this regard.  $H_2/H_\infty$  controller has the order of 7, and  $\mu$  controller's order in 10, due to use of order reduction method, the order of the  $H_2/H_\infty/\mu$  controller is 5. For further investigation of three controller, the form of the waves related to rotor angle and speed are shown according to 2% p.u increase in the input voltage of the system in Figure 9 and 10, respectively. The mentioned figures indicate for  $H_2/H_\infty/\mu$  controller the attenuation rate of 3 s and low oscillation. Variations of rotor speed before and after 3-phase fault shown in Figure 11.

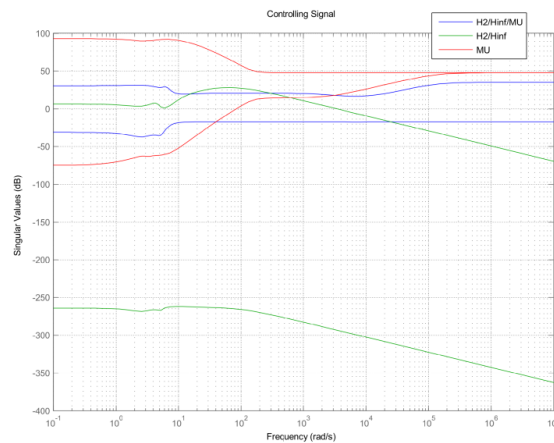


Figure 7. Singular value for controlling signal ( $H_2/H_\infty$ ,  $\mu$ ,  $H_2/H_\infty/\mu$ )

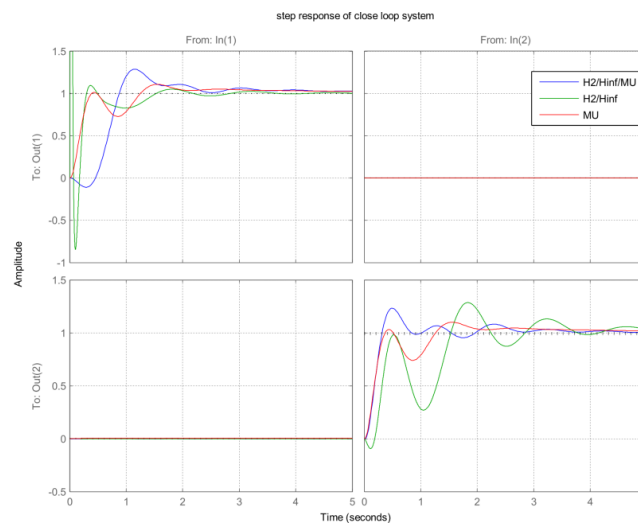


Figure 8. Step response of close loop system ( $H_2/H_\infty$ ,  $\mu$ ,  $H_2/H_\infty/\mu$ )

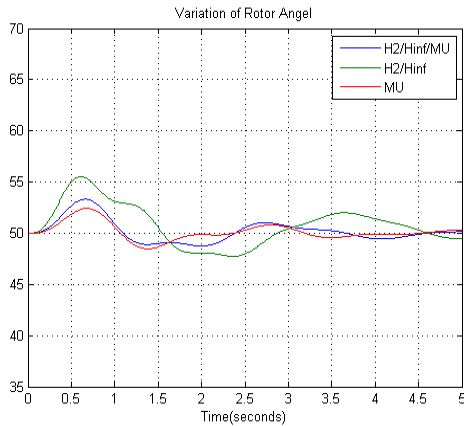


Figure 9. Rotor angle with 2% (p.u) change

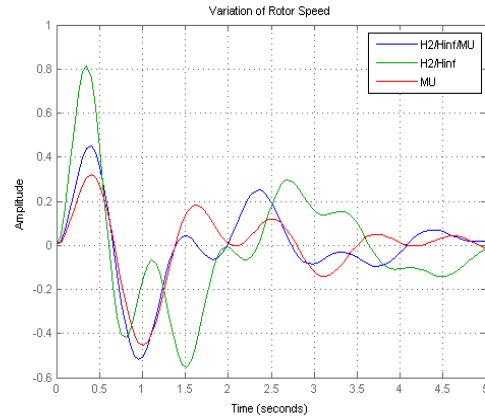


Figure 10. Rotor speed with 2% (p.u) change

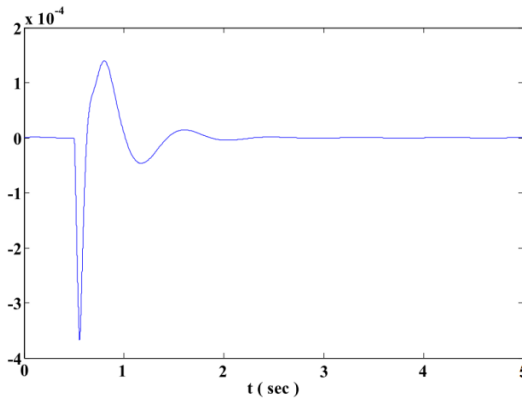


Figure 11. Variations of rotor speed before and after 3-phase fault

## 6. CONCLUSION

Providing the spare parts and resolving the errors in the excitation system are among the most important problems of the old power plants. For this reason, replacement of the control section of the excitation system seems necessary. For attenuating the oscillations by controlling the excitation process, the stabilizers of the power systems are used. The aim of this paper is to design a robust stabilizer of the power system for the power plant of Mashhad city. First the parameters of Hefron Philips model was derived and obtained, since there is no certain model of the system in hand, the robust performance is considered. By robust performance, it means by consideration of the uncertainty the errors of the system be minimized. In order to investigate the robust performance,  $\mu$  analysis was used. Generally, existence of the robust performance results in the severe limitations on the controller which is sometimes making it an unfeasible issue, and if it could be feasible the order of the controller would become higher and the resulted control signal would be increased which would lead to saturation of the actuator. In order to decrease the controlling signals, it is needed to use to controllers of  $\mu$  and  $H_2/H_\infty$  for the performance of robust and its stability. Designing the filters or in the other words weight functions have also crucial role in determination of the closed loop response. In this content, first, three weight functions were designed for  $H_2/H_\infty$  controller and then two weight functions by LMI were designed for balancing between  $H_2/H_\infty$  and  $\mu$ . Due to multi variable system of the weight functions, they were plotted in the form of matrix and the singular values. The results show that the closed loop was stabilized despite of the existence of uncertainty and has the desirable performance. Moreover, the response of the closed loop and controlling signal of the combined controller ( $H_2/H_\infty/\mu$ ), is between the two other controllers. The angle and speed of rotor verifies the effectiveness and advantages of the suggested method.



## 7. APPENDIX

### A. Nomenclature

$X_d$	direct axis reactance of synchronous machine (p.u)	$H$	inertia constant
$X'_d$	direct axis transient reactance of synchronous machine (p.u)	$E_b$	exciter Output Voltage
$X_q$	quadrature axis reactance of synchronous machine	$E_q$	voltage proportional to direct axis Flux linkages
$X'_q$	quadrature axis transient reactance of synchronous machine	$\delta(t)$	rotor angle
$X_e$	transmission line reactance	$\omega_r(t)$	speed of the rotor
$T'_{do}$	direct-axis transient open circuit time constant	$T_m$	mechanical/electrical torque
$K_1$ to $K_6$	Heffron-Phillips model coefficient	$I_q$	generator stator current
$K_A$	DC gain of the AVR	$V_t$	Terminal voltage of synchronous machine(p.u)
$T_A$	time constant of the AVR	$\Delta$	Denotes small perturbation in the variable from steady state value
$K_D$	PSS gain	$f_b$	Synchronous Generator

### B. Machine data

$X_d$	$X'_d$	$X_q$	$X'_q$	$X_e$	$T'_{do}$	$K_1$	$K_2$
2.013	0.3	1.76	0.65	0.68	0.53	0.55	1.2
$K_3$	$K_4$	$K_5$	$K_6$	$K_A$	$T_A$	$K_D$	$H$
0.66	0.7	0.095	0.815	50	0.5	7.1	3.5

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