

## High Resolution Method using Patch Circular Array

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### ABSTRACT

Smart antennas have recently received increasing for improving the performance of wireless radio systems. In this research article, we have used a patch antenna using uniform circular arrays (UCA) with central element for direction of arrival (DOA). A central element was added to arrays in order to increase steering capability of the proposed array. This geometry is used to determine the elevation and azimuth based on two famous algorithms of high resolution method: Matrix Pencil method (MP) and Multiple Signal Classification (MUSIC). The comparison results demonstrate clearly that the matrix pencil is more accurate and stable to estimation of direction of arrival compared to the MUSIC algorithm.

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## 1. INTRODUCTION

In the last few decades, smart antenna have received increasing interest for improving the performance of wireless radio systems[1]. Their application has been suggested for mobile communications systems, to resolve the problem of limited channel bandwidth and satisfying a growing demand for a large number of mobiles on communications channels. The smart antenna systems can be divided into two categories. These are: switched beam system, and adaptive arrays [2], [3].

The direction of arrival algorithms are classified as non subspace and subspace types. The Bartlett and Capon (Minimum Variance Distortion less Response) are non subspace algorithms. This methods are highly dependent on the physical size of the array aperture, which results in poor resolution and accuracy. The subspace based DOA estimation method is based on the eigen decomposition. The subspace based DOA estimation algorithms MUSIC and ESPRIT provide high resolution; they are more accurate, and not limited to the physical size of the array aperture [4-6].

There exist many algorithms to estimate the 2D-DoA of signals received by the antenna arrays such as Matrix Pencil(MP). Sarkar and Hua utilized the MP to get the DOA of the signals in a coherent multipath environment [7], [8]. The matrix pencil method, based on the spatial samples of the data, the analysis is done on a snapshot-by-snapshot basis, and hence is computationally quite efficient. A snapshot is defined as the voltages measured at the feed points of all the antenna elements in the array at a particular instance of time. Non-stationary in the data then has little effect for this method, as no assumption is made about the statistics of the environment.

Unlike the conventional covariance matrix techniques, the MP method can find DOA easily in the presence of multipath coherent signal without performing additional processing of spatial smoothing. This method is based on the assumption that each antenna element is isotropic point radiator and is spaced uniformly along a line for 1D-DOA and on plane for 2D-DOA.

The rest of the article is organized as follows. First, Design of uniform circular patch antenna arrays is presented in Section 2, followed by the description of MUSIC and Matrix Pencil algorithms for the estimation of azimuth and elevation angles direction investigated in Section 3. the results of the comparison of this algorithms are analysed and discussed based on the results of uniform circular arrays analyzed in Section 4. Finally, Section 5 concludes the article.

## 2. SMART ANTENNA DESIGN

A smart antenna is an antenna array system aided by some smart algorithm designed to adapt to different signal environments. Smart antenna is an antenna system that can modify its beam pattern or other parameters, by means of internal feedback control while the antenna system is operating. The basic idea behind smart antennas is that multiple antennas processed simultaneously allow static or dynamical spatial processing with fixed antenna topology. The pattern of the antenna in its totality is now depending partly on its geometry but even more on the processing of the signals of the antennas individually. Several algorithms have been developed based on different criteria to compute the complex weights [9],[10].

### 2.1. Design of Rectangular Patch Antenna

The dielectric material selected for design is FR4 having a dielectric constant of 4.4. A substrate having a high dielectric constant should be selected because if the dielectric constant is small, the dimensions of the antenna are small [10],[11].

In many applications it is necessary to design antennas with very directive characteristics to meet the demands of long distance communication. This can be achieved by forming an assembly of radiating elements in electrical and geometrical configuration, which is referred to as an array. For the microstrip patch antenna that is used in wireless communication, it is essential that the antenna is not cumbersome. Hence the height of the dielectric substrate must be small; the effect of the height is discussed in [11].

The *FR4 epoxy* substrate used in this work has a standard height of 1.6 mm. The essential parameters for the design are:

- Resonance frequency  $f_r = 6.4$  GHz.
- Dielectric constant of the substrate  $\epsilon_r = 4.4$ .
- Height of the dielectric substrate  $h = 1.6$  mm.

The following steps are followed to design our proposed design the rectangular antenna and the layout was presented in Figure 1.

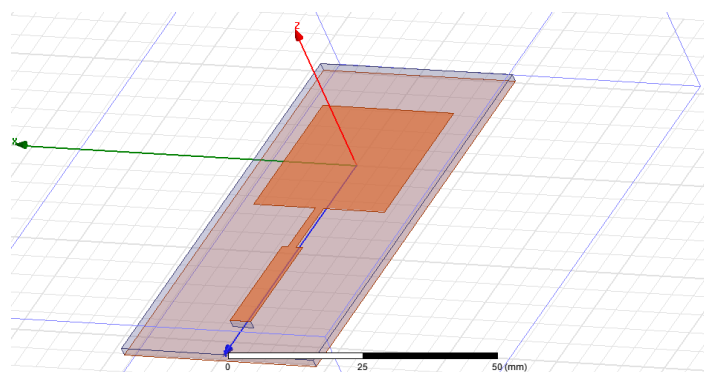


Figure 1. Rectangular patch antenna designed by microstrip line feed

The width  $W$  of the patch antenna is calculated as follows:

$$w = \frac{c}{2f_r \sqrt{\epsilon_r + 1}} = 17.73 \text{ mm} \quad (1)$$

With  $C$  = free space velocity of light.

The effective dielectric constant is:

$$\epsilon_{r\text{eff}} = \frac{\epsilon r + 1}{2} + \frac{\epsilon r - 1}{2} \left[ 1 + \frac{12h}{w} \right]^{-\frac{1}{2}} = 5.92 \quad (2)$$

The computation of the effective length is given by the following equation:

$$L_{\text{eff}} = \frac{c}{2fr\sqrt{\epsilon_{r\text{eff}}}} = 11.61 \text{ mm} \quad (3)$$

The extension of the length and the length patch:

$$\Delta L = 0.412 * h \frac{(\epsilon_{\text{eff}} + 0.3) \left( \frac{W}{L} + 0.264 \right)}{(\epsilon_{\text{eff}} - 0.258) \left( \frac{W}{L} + 0.8 \right)} = 0.47 \text{ mm} \quad (4)$$

$$L = L_{\text{eff}} - \Delta L = 11.14 \text{ mm} \quad (5)$$

The length and width of the ground plane:

$$Lg = 6 * h + L = 20.74 \text{ mm} \quad (6)$$

$$wg = 6 * h + w = 21.68 \text{ mm} \quad (7)$$

## 2.2. Proposed Uniform Circular Array Structure

The planar arrangements can be sub-divided into three other categories; circular, rectangular, and square. Among these three categories, the circular arrays do not have edge elements. Without edge constraints, the beam pattern of a circular array can be electronically rotated. Besides, the circular arrays also have the capability to compensate the effect of mutual coupling by breaking down the array excitation into a series of symmetrical spatial components [11], [12]. Figure 2 presents the geometry of circular array antenna.

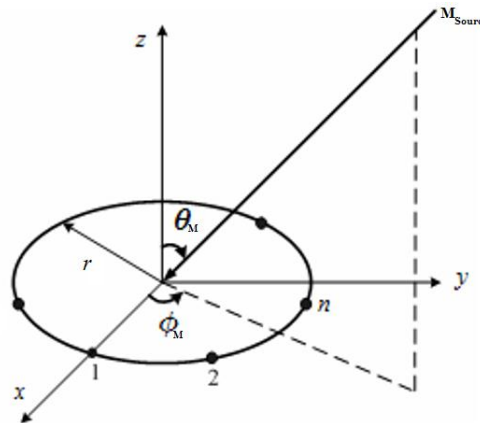


Figure 2. N elements arranged in Uniform Circular Array (UCA) with central element

## 3. DIRECTION OF ARRIVAL ALGORITHM USING UCA

### 3.1. Music Algorithm

MULTiple Signal Classification (*MUSIC*) is one of the high-resolution subspace DOA algorithms that based on eigen decomposition of the autocorrelation matrix of the received signal [13], [14]. Figure 2 present a uniform circular array with incident plane waves from various directions. The data model is given by:

$$X(t) = AS(t) + N(t) \quad (8)$$

The signal received by the array is defined in (8) , where  $A=[a(\theta_1, \varphi_1), \dots, a(\theta_2, \varphi_2)]$  is a  $(N \times M)$  matrix of the  $M$  steering vectors and  $S=[S_1(t), \dots, S_M(t)]^T$  is a signal source vector of order  $(M \times N)$ .  $[ ]^T$  denote *Transpose* of a matrix and  $N(t)$  is  $N(t) = [n_1(t), n_2(t), \dots, n_N(t)]^T$  is the  $t^{\text{th}}$  snapshot of either zero mean stationary complex additive white gaussian noise (AWGN). The correlation matrix of received vector can be written as:

$$\begin{aligned} R_x &= E [XX^H] \\ &= E [ASS^H + NN^H] \\ &= AVA^H + \sigma^2 I \\ &= R_s + \sigma^2 I \end{aligned} \tag{9}$$

Where  $\sigma^2$  is the variance of white gaussian noise vector (W),  $V$  is covariance matrix of signal vector (S) which is a full rank matrix of order  $(M \times M)$  given by

$$V = E [SS^H] = \begin{bmatrix} E[|S_1|^2] & \dots & \dots & 0 \\ 0 & E[|S_2|^2] & \dots & 0 \\ \vdots & \ddots & \dots & \vdots \\ 0 & 0 & \dots & E[|S_M|^2] \end{bmatrix} \tag{10}$$

Where the statistical expectation is denoted by  $E [ ]$ ,  $R_s$  is a signal covariance matrix of order  $(N \times N)$  with rank  $M$  given by

$$R_s = \begin{bmatrix} E[|S_1|^2] & \dots & \dots & 0 & \dots & 0 \\ 0 & E[|S_2|^2] & \dots & 0 & \dots & 0 \\ \vdots & \ddots & \dots & \vdots & \dots & 0 \\ 0 & 0 & \dots & E[|S_M|^2] & \dots & 0 \\ 0 & 0 & \dots & 0 & \dots & 0 \end{bmatrix} \tag{11}$$

$R_s$ , has  $(N-M)$  eigenvectors corresponding to zero eigen values. We know that steering vector  $a(\theta_1, \varphi_1)$  which is in the signal subspace is orthogonal to noise subspace let  $Q_n$  be such an eigenvector.

$$R_s Q_n = AVA^H Q_n = 0 \tag{12}$$

Since  $V$  is a positive definite matrix

$$a^H(\theta_i, \varphi_i) Q_n = 0 \tag{13}$$

This implies that signal steering vectors are orthogonal to eigenvector corresponding to noise subspace [15]. So the MUSIC algorithm searches through all angles and plots the spatial spectrum

$$P_{\text{MUSIC}(\theta, \varphi)} = \frac{1}{(a^H(\theta, \varphi) Q_n Q_n^H a(\theta, \varphi))} \tag{14}$$

### 3.2. Matrix Pencil Method

#### 3.2.1. Matrix Pencil for Uniform linear array

The Matrix Pencil formulations use the real matrices to estimate the *DOA* of multiple signals simultaneously impinging on the ULA [16],[17]. The vector  $x(n)$  is the set of voltages measured at the feed point of antenna element of the ULA. Therefore,  $x(t)$  can be modeled by a sum of complex exponentials. The observed voltage is given by

$$y(t) = x(t) + n(t) = \sum_{i=1}^N R_i e^{s_i t} + n(t) \tag{15}$$

- $y(t)$  = observed voltages at a specific instance  $t$
- $n(t)$  = noise associated with the observation
- $x(t)$  = actual noise free signal

Therefore, one can write the sampled signal as,

$$y(p) = \sum_{i=1}^N R_i z_i^p + n(p), \text{ for } p= 0, 1, \dots, N - 1 \tag{16}$$

Where,

$$Z_i = e^{j\frac{2\pi}{\lambda} d \sin(\theta)}, \text{ for } i = 1, 2, \dots, N \tag{17}$$

In this presentation, it has been assumed that the damping factor  $\alpha_0 = 0$ . The objective is to find the best estimation for  $\theta$ . Let us consider the matrix  $Y$  which is obtained directly from  $x(p)$ .  $Y$  is a Hankel matrix, and each column of  $Y$  is a windowed part of original data vector,  $\{x(0) \ x(1) \ x(2) \ \dots \ x(N-1)\}$ .

$$Y = \begin{bmatrix} x(0) & x(1) & \dots & x(L-1) \\ x(1) & x(2) & \dots & x(L) \\ \vdots & \vdots & \ddots & \vdots \\ x(N-L) & x(N-L+1) & \dots & x(N-1) \end{bmatrix} \tag{18}$$

$(N - (L + 1) \times L)$

The parameter  $L$  is called the pencil parameter, it is chosen between  $N/3$  and  $N/2$  for efficient noise filtering [16], [17]. The variance of the estimated values of  $R_i$  and  $Z_i$  will be minimal if the values of  $L$  are chosen in this range [18]. The synthesis was to choose an  $L$  closest to  $N/2$  in order to eliminate the unavailable lobes in the simulation. From the matrix  $Y$ , we can define two sub-matrices :

$$Y_a = \begin{bmatrix} x(0) & x(1) & \dots & x(L-1) \\ x(1) & x(2) & \dots & x(L) \\ \vdots & \vdots & \ddots & \vdots \\ x(N-L-1) & x(N-L) & \dots & x(N-2) \end{bmatrix} \tag{19}$$

$(N - L) \times L$

$$Y_b = \begin{bmatrix} x(1) & x(1) & \dots & x(L-1) \\ x(2) & x(2) & \dots & x(L) \\ \vdots & \vdots & \ddots & \vdots \\ x(N-L) & x(N-L+1) & \dots & x(N-1) \end{bmatrix} \tag{20}$$

$(N - L) \times L$

We can also write

$$Y_a = Z_a R Z_b \tag{21}$$

$$Y_b = Z_a R_0 Z_0 Z_b \tag{22}$$

$$Z_a = \begin{bmatrix} 1 & 1 & \dots & 1 \\ Z_1 & Z_2 & \dots & Z_M \\ \vdots & \vdots & \ddots & \vdots \\ Z_1^{(N-L-1)} & Z_2^{(N-L-1)} & \dots & Z_M^{(N-L-1)} \end{bmatrix} \tag{23}$$

$(N - L) \times L$

$$Z_b = \begin{bmatrix} 1 & Z_1 & \dots & Z_1^{(L-1)} \\ 1 & Z_2 & \dots & Z_2^{(L-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & Z_M & \dots & Z_M^{(L-1)} \end{bmatrix} \tag{24}$$

$(M \times L)$

$$Z_0 = \text{diag} [Z_1, Z_2, \dots, Z_M] \tag{25}$$

$$R_0 = \text{diag} [R_1, R_2, \dots, R_M] \tag{26}$$

Considering the matrix pencil

$$Y_b - \lambda Y_a = Z_a R_0 [Z_0 - \lambda I] Z_b \tag{27}$$

Where  $I$  is the  $(M \times M)$  identity matrix, one can show that the rank of  $Yb - \lambda Ya$  will be  $M$ , provided that  $M \leq L \leq N - M$ . However, if  $\lambda = Zi, i=1, 2, \dots, M$  the  $i$ th row of  $[Z0 - \lambda I]$  is zero, then the rank of this matrix is  $(M \times 1)$ . Therefore, the parameters  $Zi$  can be found as the generalized eigenvalues of the matrix pair  $\{Ya + Yb - \lambda I\}$  where  $Ya+$  is the Moore-Penrose pseudo inverse of  $Ya$ , which is defined as :

$$Y_a^+ = \{Y_a^H Y_a\}^{-1} Y_a^H \tag{28}$$

The DOA is obtained from :

$$\theta_i = \sin^{-1} \left( \frac{\text{Im}(\log Z_i)}{\pi d} \right) \tag{29}$$

Where  $Zi$  is defined in (17)

**4. RESULTS AND DISCUSSION FOR THE DOA**

In this section, we use MUSIC and Matrix Pencil to determinate DOA of signals impinging on the proposed Uniform circular arrays (4+1) and (8+1) geometry [11]. The radius of circular array is chosen to get an inter-element distance of  $0.5\lambda$ . In order to demonstrate the numerical properties, a comparative study was made between the Matrix Pencil and MUSIC algorithm indicate at [21], [22] for the proposed structures.

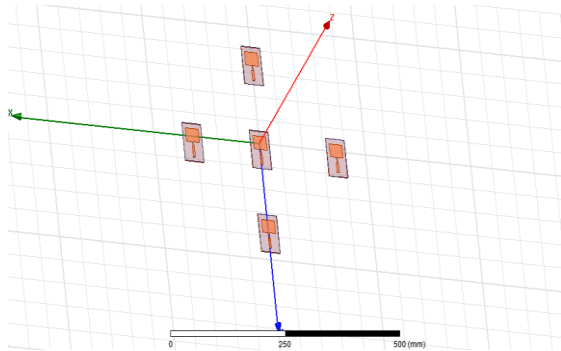


Figure 3. Proposed antenna arrays with (4+1) elements

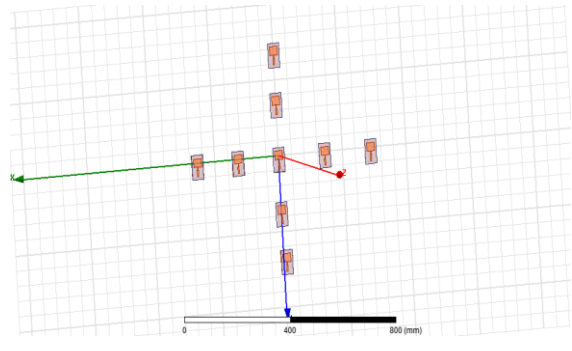


Figure 4. Proposed antenna arrays with (8+1) elements

The Figure 5 demonstrate the return loss simulated for proposed rectangular microstrip antenna and presents an antenna resonates at four frequencies: 4.88 GHz, 5.24 GHz, 6.98 and 7.26 GHz with return loss between -17.06 dB and -29.94 dB. we can say that proposed antenna exploit well the C-band even if antenna element change.

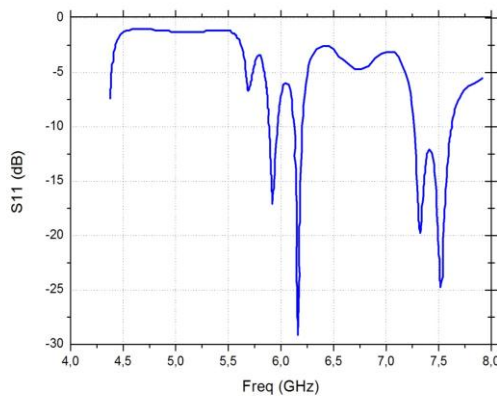


Figure 5. Return loss of (4+1) and (8+1) structure in C-band

In Figure 6 and Figure 7. The radiation pattern is traced in *E-plan* of proposed structure antenna. The main lobe is directive and there are no secondary lobes. The comparison between the two figures illustrate that the radiation pattern for the (8+1) structure is more directive and sharper compared to the lobe of (4+1) geometry, we conclude that if even if the element of antenna increase in the system the main lobe become more directive.

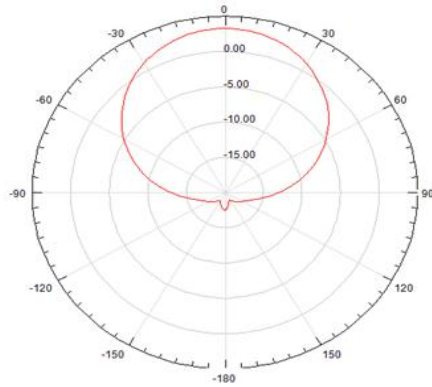


Figure 6. Radiation pattern of (4+1) element circular array

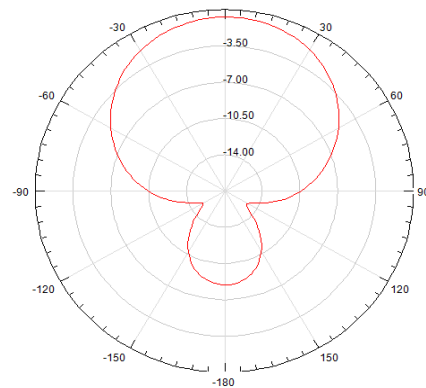


Figure 7. Radiation pattern of (8+1) element circular array

Figure 8 and 9 present that the direction of arrival of desired signal was accurately estimated by MUSIC method for the frequencies 5.24 GHz and 7.26 GHz with high amplitude for the angles  $\theta=[10^\circ, 80^\circ]$  and  $\varphi=[100^\circ, 160^\circ]$  using the UCA structure. This simulation demonstrates that the proposed structure is feasible for integration on smart antenna systems.

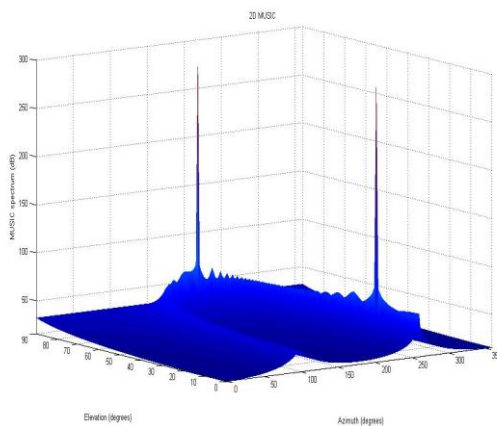


Figure 8. DOA estimations for resonance frequency 5.24 GHz in 3-D

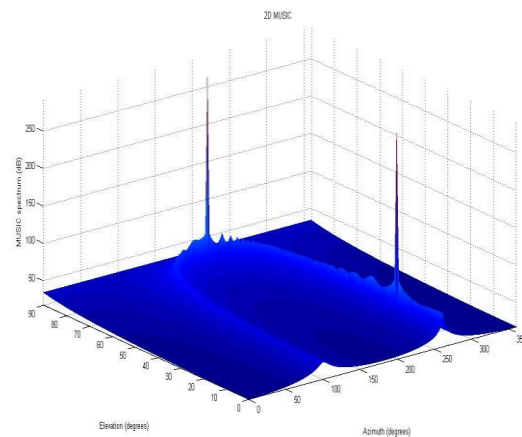


Figure 9. DOA estimations for resonance frequency 7.26 GHz in 3-D

In the following step a comparison between the Matrix Pencil method and Music method [21] under the same conditions shown in Table 1. It can be seen that the Matrix Pencil estimates three angles more accurately while MUSIC algorithm cannot detect angles when the Number of signals exceeds two signals. The proposed one gives a less error margin to estimate DOA.

Table 1. Pencil and MUSIC for different signals using UCA

Signal	$\theta_{in}$	$\theta_{out}$	$\Delta\theta_{out}$	$\varphi_{in}$	$\varphi_{out}$	$\Delta\varphi_{out}$				
1	MUSIC [21]	78 84	78.5 82.5	+0.0064	0	128.4 116.0 129.5 117	+0.0082	+0.0086		
	Matrix Pencil	78 84	78 84	0	0	128.4 116.0 128.1 116.2	-0.0023	+0.0017		
2	MUSIC [21]	77.0 85.8	76.0 86.5	-0.0065	0	128.2 120 129.5 121.5	77.0	85.8		
	Matrix Pencil	77.0 85.8	77.0 85.8	0	0	128.2 120 128.2 120.1	00	+0.0008		
3	MUSIC [21]	78.6 82.4	0 0	-1	-1	133.6 137.8	0	0	-1	-1
	Matrix Pencil	78.6 82.4	79.0 85.4	+0.005	+0.036	133.6 137.8	134	137	+0.0029	-0.0028

Table 2 illustrate the comparison between Matrix pencil and MUSIC [22] based on UCA for different values of SNR with two angles -38 and -57. We observe that when the SNR equal to -3 dB, the matrix pencil remains stable even if the conditions change. The comparison shows that Pencil based on UCA give good results for detecting angles with a maximum error 0.30 % and a minimum error 0.17 %.

Table 2. Pencil and MUSIC for different SNR using UCA

Signal	SNR(dB)	UCA angles(deg)	UCA error(deg)	Average of UCA(deg)	
1	Pencil	9	-37.900001 56.800002	0.099999 -0.100008	0.1000001
	MUSIC [22]	9	-37.899998 57.200001	0.100002 +0.200001	0.1500002
2	Pencil	3	-37.800000 57.300000	0.300000 +0.300000	0.300000
	MUSIC [22]	3	-37.799999 57.600002	0.200001 +0.600002	0.400002
3	Pencil	0	-37.700000 57.100002	0.400000 +0.100002	0.200000
	MUSIC [22]	0	-37.599998 58.000004	0.400002 +1.000004	0.700003
4	Pencil	-3	-37.600000 56.500000	0.600000 -0.500000	0.550000
	MUSIC [22]	-3	-37.399998 58.800003	0.600002 +1.800003	1.200003

## 5. CONCLUSION

This article presents the results of direction of arrival estimation using uniform circular antenna array for the C-band [4–8] GHz. A geometry modification was proposed in the placement of one of the antenna elements at the centre of the array. This element modifies the overall radiation pattern in such a way that the directivity is increased whilst the half-power beam width angle is reduced. For the detection of the DOA we recommend Matrix Pencil for different reasons: it is easier to arrange the circular array for wireless communication, the signals with the same frequency are independent for aerial reflectors around the antenna, the high SNR of airborne antennas can heighten the spectrum peak, increase probability of signal and the superiority of the pencil algorithm compared to the experimented ones. The matrix Pencil methods has a greater resolution and accuracy than MUSIC, for a finite number of snapshots, and number of elements of array decrease and the distance equal to  $0.5\lambda$ , the errors of angle of arrival decreases the Matrix Pencil detect clearly the angles. Therefore, the pencil method is highly efficient; with an error not exceeding 0.3%.

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