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Modal Coupling Coefficients and Frequency/Bias Planes for Gyromagnetic Boundary Value Problems

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ABSTRACT

In this paper, electromagnetic and uniform precession magnetostatic mode interaction theory is reformulated to include comprehensive electromagnetic modal impact in the determination of modal coupling calculations. For this purpose orthogonal electromagnetic and normal magnetostatic modes character is solved with coupled field Maxwell's equations and vectorized magnetization expression to model the interactions between electromagnetic modes and magnetostatic uniform precession mode. Calculations for modal coupling factors are presented here for the first time and frequency/ bias planes are constructed using the developed modal interaction formulation with an ameliorated accuracy. The proposed formulation is validated and tested against closed form frequency/ bias solutions concerning these gyromagnetic boundary value problems.

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1. INTRODUCTION

A meticulous conception regarding modal spectrum, and its frequency/magnetic bias dependency, is mandatory for efficient gyromagnetic device characterization at Watts and Kilowatts. A preliminary mode spectrum theory was developed by Auld [1] and Weiner [2] specializes Auld [1] theory to model interaction between electromagnetic modes to a single magnetostatic mode. The modal interactions between magnetostatic waves either in non-exchange and exchange limit with electromagnetic waves have been reported in literature for both linear and non-linear response [3-5]. In microwave ferrite devices, it is of interest to establish that how the magnetostatic uniform precession mode interacts with the electromagnetic modes at higher power thresholds. In this paper the coupling solving a uniform precession magnetostatic field expression and electromagnetic field equation in cylindrical coordinates with the magnetized vector expression develops expressions. By substituting the related field components of each boundary value problem in cylindrical coordinates to the filling factor expression and by imposing condition of orthogonality provides the required filling factor expressions for planar gyromagnetic disk, and axially magnetized circular gyromagnetic cavities. The proposed formulation is flexible and can be used to evaluate coupling factor expressions for any gyromagnetic boundary value problem having properly defined demagnetization factors, field expressions and orthogonality conditions.

2. MODAL COUPLING FORMULATION

The coupling factor B_{nv} specifies the coupling between v^{th} magnetostatic mode to the n^{th} electromagnetic mode and is given by [1];

$$B_{nv} = \mu_o \int_{ferrite} \overline{h_n^*} \cdot (\nabla\phi + \frac{\overline{m_v}}{\mu_o}) dv \quad (1)$$

where $\nabla\phi$ is the magnetostatic irrotational part, h_n normal electromagnetic magnetic field component, and $\frac{\overline{m_v}}{\mu_o}$ represents the normal magnetostatic part in the coupled field expressions. Similarly the factor, $K_{nv,}$ specifies the coupling between the nth electromagnetic mode to the vth magnetostatic mode and can be expressed as;

$$K_{vn} = -\frac{\omega_m}{\mu_o} \int_{ferrite} \overline{h_n} \cdot \overline{m_v^*} dv \quad (2)$$

2.1 Coupling Expressions for Electromagnetic/ Uniform Precession Mode Interactions

If the coupling factor is small electromagnetic field pattern is relatively uniform inside the sample and only the uniform mode among all the magnetostatic modes is coupled to the electromagnetic mode [1, 2]. The magnetostatic field for the uniform mode is given by;

$$\nabla\phi_{up} = [(\frac{\omega_o + \omega_{up}}{\omega_m})]\overline{m}_{up} / \mu_o \quad (3)$$

The resonant frequency for the uniform precession can be expressed as;

$$\omega_{up} = -\omega_o + 1/2\omega_m(3N_z - 1) \quad (4)$$

whereN_z is the demagnetization factor in the z-direction.

$$\overline{m}_{up} / \mu_o = (\overline{a}_x - j\overline{a}_y) / \sqrt{2V} e^{(j\omega_{up}t)} \quad (5)$$

Substituting Eq. (4) & Eq. (5) in Eq. (3), the magnetostatic field for the uniform mode becomes;

$$\nabla\phi_{up} = 1/2(3N_z + 1)\frac{(\overline{a}_x - \overline{a}_y)}{\sqrt{2V}} e^{(j\omega_{up}t)} \quad (6)$$

The electromagnetic field, $h_{n,}$ can be expressed in Cartesian coordinates as;

$$h_n = (h_{nx}\overline{a}_x + h_{ny}\overline{a}_y + h_{nz}\overline{a}_z) \quad (7)$$

Eqs. (8-9) are obtained by expressing a linear combination of rotational and irrotationalmagnetostatic fields. Adding Eq. (5) to Eq. (6), and by making use of the electromagnetic field expression in Eq. (7), and finally substituting these results in Eqs. (1-2) yieldsEqs. (8-9).Eq. (1) & Eq. (2) describes the coupling between electromagnetic and magnetostatic modes in the modal spectrum plane. Similarly the modal interaction formulation between an electromagnetic mode to uniform precession modes is given by;

$$B_{no} = -\mu_o\sqrt{1/8V}(3N_z + 1) \int_{ferrite} (h_{nx} + jh_{ny}) dv \quad (8)$$

$$K_{on} = -\sqrt{1/2V}\omega_m \int_{ferrite} (h_{nx} + jh_{ny}) dv \quad (9)$$

The Filling factor (F), which is simply the multicative product of the coefficients determined in Eq. (8) and Eq. (9), ($F = B_{no}K_{on}/\omega_m$) can be obtained by using Eqs. (8-9) and is given by;

$$F = \mu_o / 4V(3N_z + 1)\left| \int_{ferrite} (h_{nx} + jh_{ny}) dv \right|^2 \quad (10)$$

3. RESULTS & ANALYSIS

By employing standard coordinate transformation from Cartesian to cylindrical coordinates gives the magnetization of the uniform mode as;

$$\overline{m}_{up} / \mu_o = \frac{(\overline{a}_r - j\overline{a}_\theta)}{\sqrt{2V}} e^{j\theta} e^{(j\omega_{up}t)} \quad (11)$$

The electromagnetic field expression in Eq. (7) can be expressed in cylindrical coordinates as;

$$h_n = (h_{nr}(r,z)\overline{a}_r + h_{n\theta}(r,z)\overline{a}_\theta + h_{nz}(r,z)\overline{a}_z)e^{j\theta} \tag{12}$$

Similarly the filling factor Expression in Eq. (10) takes the form;

$$F = \mu_o / 4V(3N_z+1)\left|\int_{ferrite}(h_{nr}+jh_{n\theta})\exp(j\theta)rdrd\theta dz\right|^2 \tag{13}$$

Filling factors acutely influence the modal spectrum and are presented here for the axially magnetized planar and circular gyromagnetic cavities.

3.1 Axially magnetized Circular Disk

A schematic illustration of axially magnetized ferrite filled cavity of circular cross-section is presented in Fig.1.

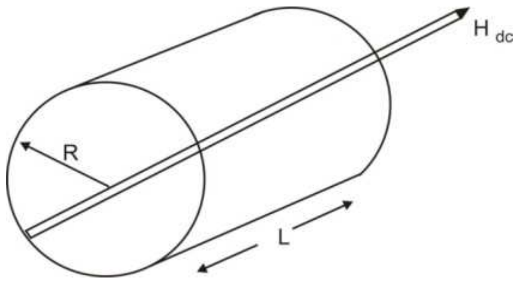

Figure 1. A schematic illustration of an axially magnetized circular cavity.

The filling factor expression in Eq. (10) can be re-expressed in simplified cylindrical co-ordinates form as;

$$F = \frac{\mu_o}{4V}(3N_z+1)\left|I\right|^2 \tag{14}$$

where I^2 needs to be determined. The magnetic field expressions in cylindrical coordinates can be given as;

$$h_r = -\frac{j\beta}{k_c^2}\frac{\partial h_z}{\partial r} + \frac{j\omega\varepsilon_o\varepsilon_r}{k_c^2}\frac{\partial e_z}{r\partial\theta} \tag{15}$$

$$h_\theta = -\frac{j\beta}{k_c^2}\frac{\partial h_z}{r\partial\theta} - \frac{j\omega\varepsilon_o\varepsilon_r}{k_c^2}\frac{\partial e_z}{\partial r} \tag{16}$$

$$h_z = AJ_n k_c r e^{-jn\theta}e^{-j\beta z} \tag{17}$$

The dominant mode for this problem is TE_{111} and factor *I* in Eq. (14) is given by;

$$I = \int_0^L\int_0^{2\pi}\int_0^R (h_{nr}+jh_{n\theta})e^{j\theta}rdrd\theta dz \tag{18}$$

where *I* can be determined as;

$$I = \frac{4\pi AR^3}{(1.84)^3}[J_1(1.84)1.84] \tag{19}$$

The orthogonality condition to be imposed is given by Eq. (20);

$$\iiint \varepsilon_o\varepsilon_r[e_r e_r^* + e_\theta e_\theta^*] + \mu_o[h_r h_r^* + h_\theta h_\theta^* + h_z h_z^*]rdrd\theta dz = 1 \tag{20}$$

By using Eq. (20) the factor *I* can be expressed as;

$$I = I_E + I_H \quad (21)$$

where

$$I_E = \int_z\int_\theta\int_r \varepsilon_o\varepsilon_r[e_r e_r^* + e_\theta e_\theta^*]rdrd\theta dz \tag{22}$$

The required fied expressions to determine I_E follows;

$$e_r = \frac{A\omega\mu_o}{k_c}\frac{J_1(k_c r)}{(k_c r)}e^{j\theta}e^{-j\frac{\pi}{L}z} \tag{23}$$

$$e_\theta = \frac{A\omega\mu_o}{k_c}\frac{J_1^{'}(k_c r)}{(k_c r)}e^{j\theta}e^{-j\frac{\pi}{L}z} \tag{24}$$

And $I_H$become;

$$I_H = \int_z\int_\theta\int_r \mu_o[h_r h_r^* + h_\theta^*]rdrd\theta dz \tag{25}$$

Substituting the field components in Eq. (23-25) yields;

$$I_H = \frac{1.19}{k_c^{\,2}}\mu_o A^2 J_1^{\,2}(1.84)[1+(\frac{\pi}{k_c L})^2]2\pi L \tag{26}$$

Using Bessel integral expressions and evaluated Eq. 29 yields factor Aas;

$$A^2 = \frac{(1.84/R)^2}{1.19\mu_o J_1(1.84)[1+(\frac{\pi R^2}{1.84L})2\pi L]} \tag{27}$$

Finally the filling factor expression in Eq. (14) comes out to be;

$$F_{11} = \frac{0.248(3N_z+1)}{[(\frac{L}{R})^2+2.915]} \tag{28}$$

The dispersion expression governing the interaction between an electromagnetic and magnetostatic mode for different values of coupling factor (F_{nv}) becomes;

$$\omega = (1/2)[\omega_n+\omega_v+\omega_m F]\pm\sqrt{1/4(\omega_n-\omega_v)^2+4(\omega_n+\omega_v)\omega_m F+(1/4)\omega_m^{\,2}F^2} \tag{29}$$

$$\frac{\partial^2 h_z}{\partial r^2}+\frac{1}{r}\frac{\partial h_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 h_z}{\partial\theta^2}+k_c^{\,2}h_z = 0 \tag{30}$$

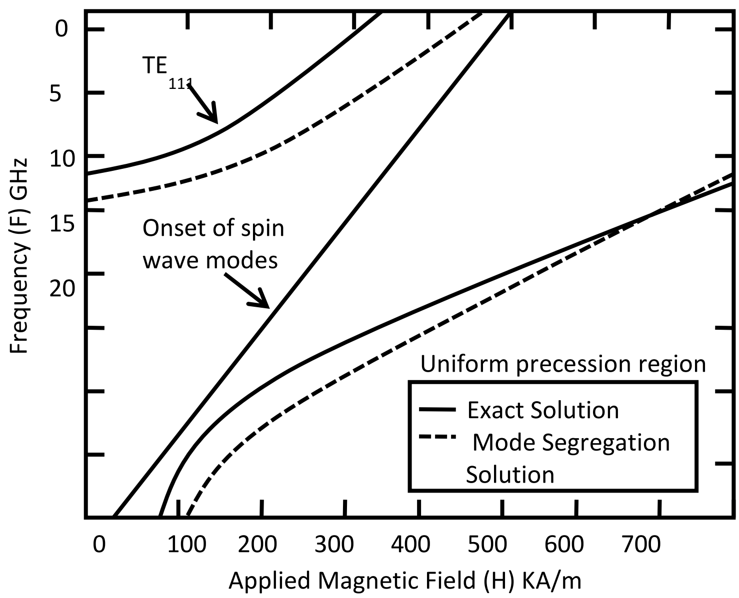

Figure 2. Comparison of developed mode interaction theory with the resonant frequencies of characteristic equation for a gyromagnetic filled circular cavity [Magnetization (M) =140 KA/m, Cavity Radius (R) = 3 mm, Cavity Length (L) = 4mm, ε_r= 15, Resonant Frequency (F_r) = 12.3 GHz, and the Demagnetization Factor (N_z)= 1.

$$k_c^{\ 2} = \omega^2 \varepsilon_o \mu_o \varepsilon_r - \frac{1}{\mu_o}(\frac{\pi}{L})^2 \quad (31)$$

Using Eq. 24 and $e_\theta=0$. At the outer radius of the cavity $r = R$, yields a value of k_c for the dominant mode i.e., $k_c = 1.84/ R$. Subsituting this value of k_c in Eq. (30) yields;

$$(\frac{\pi}{L})^2 = [\omega^2 \varepsilon_o \mu_o \varepsilon_r - (1.84 / R)^2][1 + \frac{\omega_m}{\omega_o \pm \omega}] \quad (32)$$

The related frequency/ bias plot is presented in Fig. 2 by making use of Eq. (29) and Eq. (32) and comparitive analysis included.

The spin wave frequency in Fig. 2 can be determined using Eq. 33 and is given by; [6]

$$\omega_{swm} = \sqrt{[(\omega_o - N_z \omega_m + \omega_{ex} a^2 k^2)(\omega_o - N_z \omega_m + \omega_{ex} a^2 k^2 + \omega_m \sin^2 \theta_k)]} \quad (33)$$

where lattice spacing (a) is 5x 10^{-8}cm, Exchange field (H_{ex}) = 397500 A/m, θ_k =0° and θ_k = 90° represents the bottom and top of spin wave manifold. The exchange frequency, $\omega_{ex} = \gamma H_{ex}$ where $\gamma = 2.21 \times 10^{-5}$ rad m/ A sec.By employing a similar procedure the filling factor expression for a planar gyromagnetic resonator presented in Fig. 3 becomes;

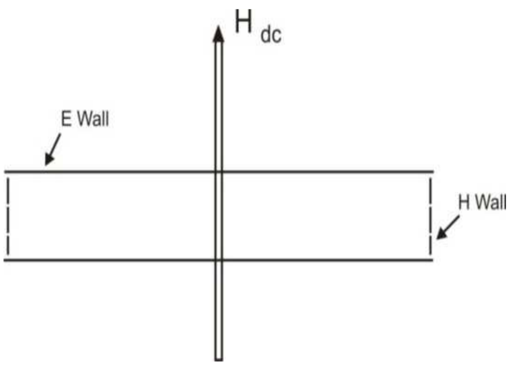

Figure 3. A schematic illustration of planar gyromagnetic disk

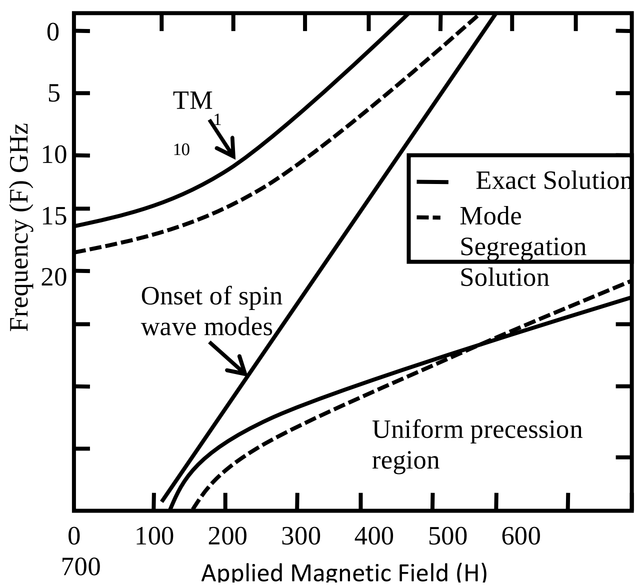

Figure 4. Comparison of developed mode segregation theory with the resonant frequencies of characteristic equation for a planar gyromagnetic disk [Magnetization (M) =140 KA/m, Disk Radius (R)= 2.52 mm, ε_r= 15.9, Resonant Frequency (F_r) = 9 GHz, and the Demagnetization Factor (N_z)= 1.

$$F = (3N_z + 1)\frac{\mu_o}{4V}[\frac{4\pi L^2 R^2}{4.77\mu_o \pi L}] \quad (34)$$

The expression in Eq. (34) can be further simplified to;

$$F_{11} = 0.209(3N_z + 1) \quad (35)$$

The frequency/ bias plot is developed and is illustrated in Fig. 4.

4. CONCLUSION

The proposed formulation is flexible and can be used to evaluate coupling factor expressions for any gyromagnetic boundary value problem having properly defined demagnetization factors, field expressions and orthogonality conditions. This technique has been employed very recently by the authors to develop a four-port X- band differential ferrite circulator [6] (1.2 MW peak power, and 2KW average power operational capability), to be used for air traffic surveillance radar system.

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J. Zafar was born on April, 1983, Lahore Pakistan. He has done his BSc in Electrical Engineering from the University of Engineering & Technology, Lahore with first division. He has been awarded prestigious overseas research fellowship in 2006 by Higher Education Commission, Pakistan to purse for a PhD degree. He was conferred upon a PhD in electrical & Electronic Engineering, by the University of Manchester, UK.
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Dr. Engr. Zafar is an active member of Pakistan Engineering Council and Pakistan Engineering Congress. He has been bestowed with the best paper research award in 2009, by the Higher Education Commission Pakistan. He has won twice the travel grants from the European Research Commission to attend professional training course in Ispra, Italy and Helsinki, Finland. Furthermore he has served as reviewer to various IEEE & IET Journals and Magazines.

H. Zafar was born in Lahore, Pakistan on 15th March, 1985. He has done his BSc in Electrical Engineering, from the University of Engineering & Technology, Lahore with flying colors. He has earned his collaborative "Masters of Science in Photonics" from St. Andrew's University, UK and Heriot Watt University, UK in 2010. He has done his "MSc in Photonics" from VUB University and Ghent University, Brussels in 2011.
Engr. Ir. Haroon Zafar is a member of International Society for Optics & Photonics (SPIE), Pakistan Engineering Council & Congress. In 2009, he has been awarded with a prestigious "Erasmus Mundus MSc in Phtonics Scholarship". Recently he was honored with a Hardimann Research Scholarship Award by the National University of Ireland, Galway. He has produced a range of high citation index publications and presented his work at different international platforms. Currently he is working on photonic and high power vacuum devices and correlation mapping methods for generating microcirculation morphology from optical coherence tomography (OCT) intensity images.