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Fractional-order sliding mode controller for the two-link robot arm

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ABSTRACT

This study presents a control system of the two-link robot arm based on the sliding mode controller with the fractional-order. Firstly, the equations of the two-link robot arm are analyzed, then the author proposes the controller for each joint based on these equations. The controller is a sliding mode controller with its order is not an integer value. The task of the control system is controlling the torques acted on the joints so that the response angle of each link equal to the desired angle. The effectiveness of the proposed control system is demonstrated through Matlab-Simulink software. The robot model and controller are built for investigating the efficiency of the system. The result shows that the system quality is very good: there is not the chattering phenomenon of torques, the response angle of two links always follow the desired angle with the short transaction time and the static error of zero.

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1. INTRODUCTION

Robots have been applied for industrial production in the 1950s with the role of replacement of human labor, improving the product quality and the efficiency of production. There are common missions, which the robot address very well, such as manufacturing, material handling, and painting, etc. Recently, robots are more and more playing an important role in life and industrial production. Therefore, it is very important to develop the performance of the robot application. There is some research on robot control [1-3], the controller that is most used for the robot is the proportional–integral–derivative (PID) one. However, because of the nonlinear characteristic of the robot, so the PID controller is not suitable, the effectiveness is low.

There are some nonlinear methods for robot control, for example the linearization technique [4], the back-stepping control method [5], etc. However, these methods have the drawback, i.e., the control signal has the chattering. A number of approaches have been contributed to robot control, which includes adaptive control [6], optimal control [7], robust control [8], and intelligent control [9, 10]. Whereas, these mentioned methods endure some shortcomings that is the complexity of the computation. In order to minimize this limitation, fuzzy control, which is considered an efficient and simple solution, was utilized based on the experiences of a human [11, 12] but its sustainability is quite low. With the intention of enhancing sustainability, the fuzzy control associated with the PID controller was presented [13, 14], however, the deficiency of this proposal is that the output response is not quick.

The study [15] introduced a sliding mode controller to solve the above constraints. In recent years, the sliding mode controller, which is powerful in controlling both linear and nonlinear objects, has been examined by various researchers. Emelyanov and his co-workers first introduced this method in the early 1950s. This controller offers some advantages involving the quick output response, the robustness and stability, the simple control algorithm, and the good transient performances. However, the limitation of the sliding mode controller is the high frequency chattering of the control signals, which leads to undesirable loads on control actuators [16, 17]. In order to diminish the above cons, the author presents the control system for the two-link robot arm using the fractional-order sliding mode control.

The fractional-order sliding mode controller is a sliding mode controller where the order of sliding surface is a fractional value. The fractional-order sliding mode controller has many advantages compared to the traditional sliding mode controller. It can even control the objects with uncertain dynamical model [18]. Hence, recently, the fractional-order sliding mode controller has been investigated and applied into many system controls such as: single-link flexible manipulator [19], antilock braking systems [20, 21], speed control system for permanent magnet synchronous motor [22]. As a result, the systems with the fractional-order sliding mode controller are of superior quality to traditional controllers. Because of the above advantages, in this paper, based on the dynamic model of the two-link robot arm, the author will build a suitable fractional-order sliding mode controller and prove the stability of the whole system through the theory of Lyapunov. The achieved findings will be transparently displayed through Matlab Simulink. The results will indicate that there is not a high-frequency chattering in the control signals as well as a static error, the quality of the control system is adequate, and the response angle of two links quickly approaches the desired angle.

The remains of the paper are as follows: Section 2 presents the calculus of derivatives and integrals of fractional order, the kinematic equation of two-link robot arm, and the fractional-order sliding mode controller. Section 3 presents the results and analysis. Finally, The conclusions are presented in section 4.

2. DESIGNING THE CONTROLLER

2.1. The calculus of derivatives and integrals of fractional order

Leibniz and L'Hopital propose fractional calculus on the basis of the integer-order calculus in 1695 [23]. The first-order derivative is determined by the (1):

$$f'(t) = \lim_{h\to 0}\frac{f(t)-f(t-h)}{h} \tag{1}$$

Therefore, the second-order and n-order derivative are determined as follows:

$$f''(t) = \lim_{h\to 0}\frac{f'(t)-f'(t-h)}{h} = \lim_{h\to 0}\frac{f(t)-2f(t-h)+f(t+2h)}{h^2} \tag{2}$$

$$f^{(3)}(t) = \lim_{h\to 0}\frac{f''(t)-f''(t-h)}{h} = \lim_{h\to 0}\frac{f(t)-3f(t-h)+3f(t-2h)-f(t-3h)}{h^3} \tag{3}$$

$$f^{(n)}(t) = \lim_{h\to 0}\frac{\sum_{i=0}^{n}(-1)^i\binom{n}{i}f(t-i.h)}{h^n} \tag{4}$$

where:

$$\binom{n}{i} = \frac{n!}{i!(n-i)!} \tag{5}$$

In (5), n is a natural number. If $n = z$ is not a natural number, (5) is rewritten as follows:

$$\binom{z}{i} = \frac{\Gamma(z+1)}{\Gamma(i+1)\Gamma(z-i+1)} \tag{6}$$

where $\Gamma(.)$is the gamma function.

For generalizing, Riemann-Liouville and Caputo have defined the fractional order for the calculus of derivatives and integrals as follows [24]:

$$f^{(z)} = \frac{1}{\Gamma(n-z)}\int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{z+n-1}}d\tau \tag{7}$$

where: $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt, n-1 < z < n$ (n is an integer value). If $z < 0$then the calculus (7) is integration, else if $z > 0$ then the calculus (7) is differentiation. Especially, if $z = 1$ the calculus (7) is first order derivative function.

2.2. The model of the two-link robot arm

Figure 1 illustrates the two-link robot arm model, in which m_1 and l_1 represent Link1's mass and length; m_2, l_2 stand for Link2's mass and length; T1, T2 represent the torque of Link1 and Link2; θ_1 and θ_2 represent the Link1 angle and Link2 angle.

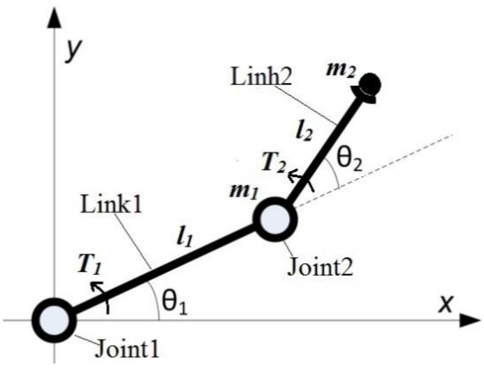


Figure1. The model of two-link robot arm

The dynamic equation of the two-link robot arm is as the following [25, 26]:

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \tag{8}$$

where:

$$D_{11} = (m_1 + m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 \cos(\theta_2)$$
$$D_{12} = D_{21} = m_2 l_2^2 + m_2 l_1 l_2 \cos(\theta_2)$$
$$D_{22} = m_2 l_2^2$$
$$G_1 = (m_1 + m_2) g l_1 \cos(\theta_1) + m_2 g l_2 \cos(\theta_1 + \theta_2)$$
$$G_2 = m_2 g l_2 \cos(\theta_1 + \theta_2)$$
$$C_1 = -m_2 l_1 l_2 \sin(\theta_2)\dot{\theta}_2 - m_2 l_1 l_2 \sin(\theta_2)(\dot{\theta}_1 + \dot{\theta}_2)$$
$$C_2 = m_2 l_1 l_2 \sin(\theta_2)\dot{\theta}_1$$

Rewriting (8), we get:

$$\tau = D.\ddot{\theta} + C.\dot{\theta} + G \tag{9}$$

where:

$$\tau = \begin{bmatrix} T_2 \\ T_2 \end{bmatrix};\ D = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix};\ \ddot{\theta} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix};\ C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix};\ \dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix};\ G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}$$

2.3. The control system of two-link robot arm based on the fractional-order sliding mode controller

Setting the fractional-order sliding surface as follows:

$$s = \lambda_1.e + \lambda_2.e^{(-\alpha)} + \lambda_3.e^{\beta} + \dot{e} \tag{10}$$

where e is the error:

$$e = \theta_{set} - \theta$$

$\theta_{set} = \begin{bmatrix} \theta_{1\_set} \\ \theta_{2\_set} \end{bmatrix}$ is the vector of desired angle values;

$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$ is the vector of response angle values;

α, β are fractional oders, those values are positive real values.

$\lambda_1, \lambda_2, \lambda_3$are vectors of positive gain parameters.

The differential of the (10) is as:

$$\dot{s} = \lambda_1.\dot{e} + \lambda_2.e^{(1-\alpha)} + \lambda_3.e^{(1+\beta)} + \ddot{e} \quad (11)$$

Setting the control signal $u = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$so that$\dot{s} = -K.sign(s)(K > 0)$. The torque force is:

$$\tau = D.(\lambda_1.\dot{e} + \lambda_2.e^{(1-\alpha)} + \lambda_3.e^{(1+\beta)} + K.sign(s)) + C.\dot{\theta} + G \quad (12)$$

Choosing Lyapunov function $V = \frac{1}{2}s^2$.

If the control signal is set up according to the equation (12), $\dot{s} = -K.sign(s)$. We have:

$$\dot{V} = s.\dot{s} = -K.|s| < 0\forall s$$

Thus, the system will attain the asymptotical stability at the equilibrium point $s = 0$, which indicates that e and all derivative of e will equal zero.

3. THE RESULTS AND ANALYSIS

According to the dynamic (8) and the model introduced in Figure 1, the model of the two-link robot arm is constructed on Matlab-Simulink and presented in Figure 2. Parameters of robot are set up as the followings: $l_1 = 1$ (m); $l_2 = 0.6$ (m); $m_1 = 0.7$ (kg); $m_2 = 0.4$ (kg). The primary angles of two links:

$$\underline{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -\pi/6 \\ -\pi/12 \end{bmatrix} (rad)$$

Figure 3 presents the diagram of the controller which is constructed following the (12). We set parameters of the controller as below:

$$\lambda_1 = \begin{bmatrix} 4 \\ 4 \end{bmatrix}; \lambda_2 = \begin{bmatrix} 6 \\ 6 \end{bmatrix}; \lambda_3 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}; \underline{K} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}; \alpha = 0.4; \beta = 0.6$$

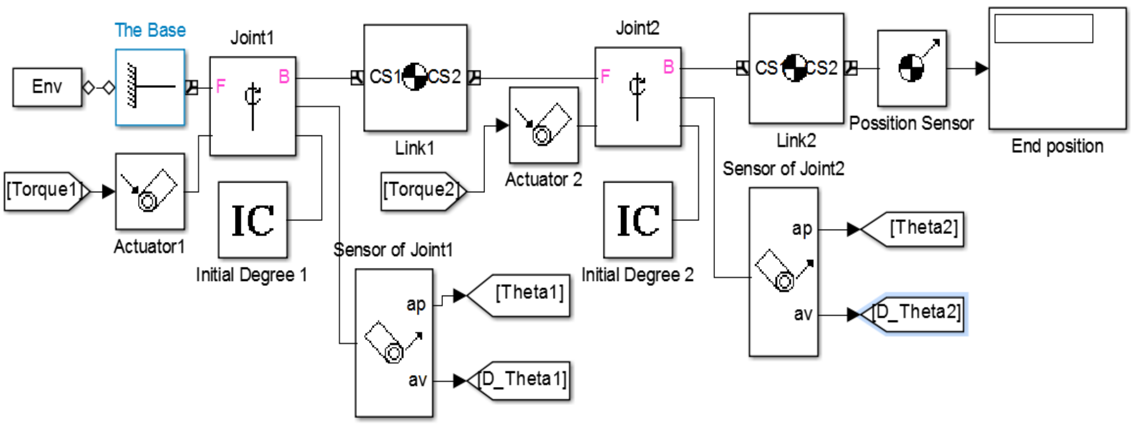


Figure 2. The two-link robot arm model

Running this system, the achieved results are presented in Figures 4-7. The phase-status trajectories of two joints is displayed in Figure 4. The results of simulation indicate that all status trajectories approach the origin *O(0,0)* without oscillations. Thus, the response angles (θ) reach the expected angles(θ_{set}).

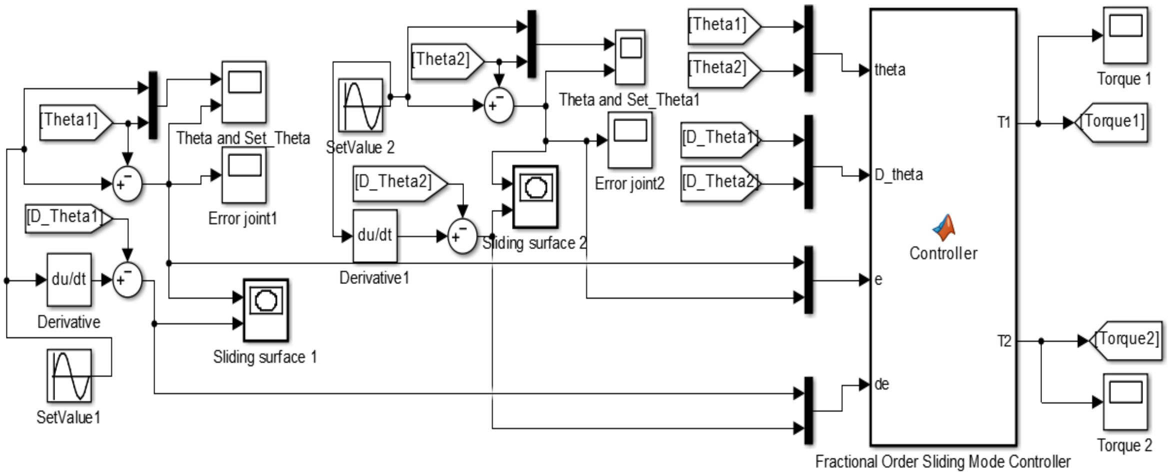
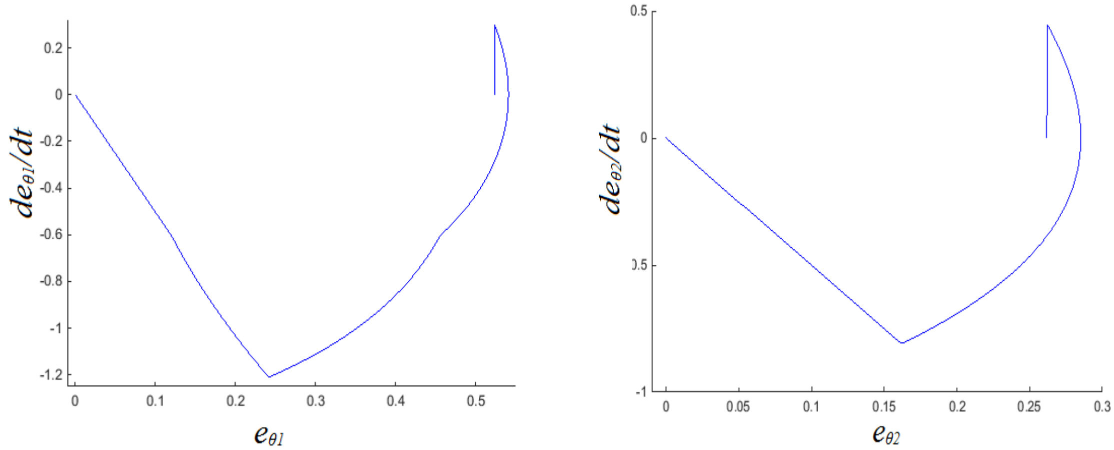


Figure 3. The model of controller



(a) (b)

Figure 4. The phase-status trajectories, (a) the joint1, (b) the joint2

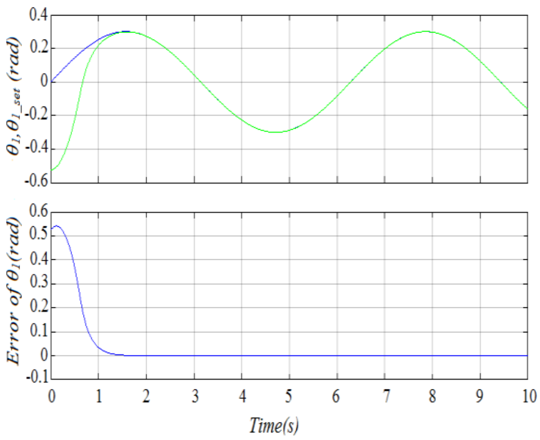


Figure 5. The response angle and control error of the joint 1

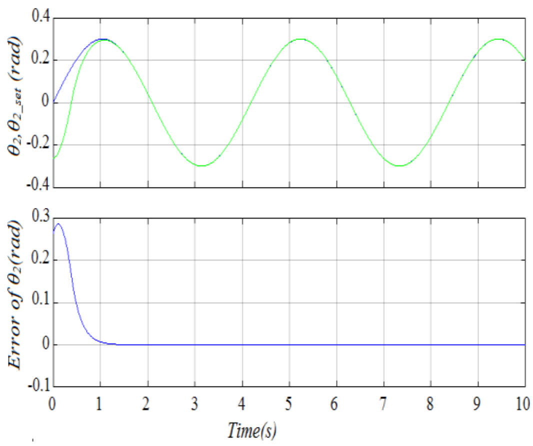
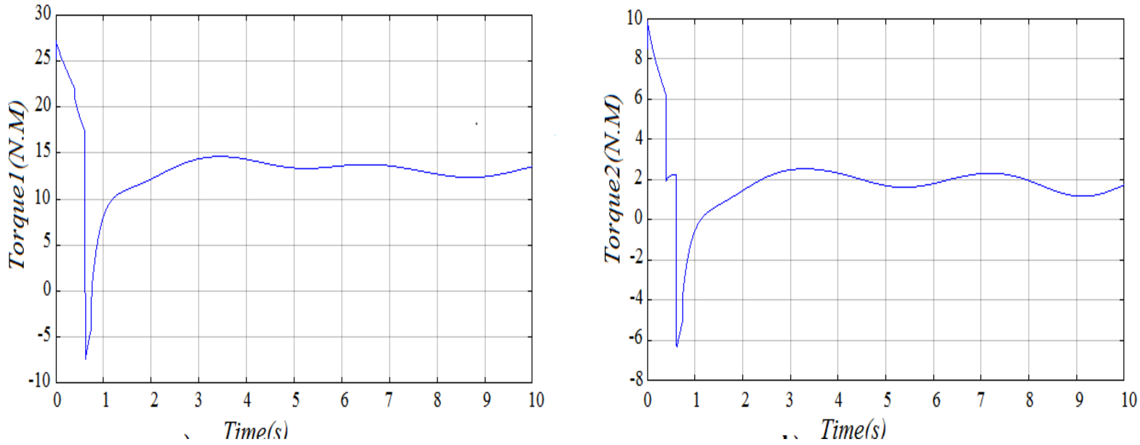


Figure 6. The response angle and control error of the joint 2

Figures 5 and 6 show the time characteristics of Link1 and Link2 respectively. Each characteristic includes the desired angleθ_{set}, the response angle(θ), and the error$(\theta_{set} - \theta)$. The outcomes of simulation demonstrate that: firstly, the response angles are not equal to the expected angles, then the response angles meet the expected values after a short time (approximately 1s), and finally, the response angles always adhere the desired values, the errors (e_1, e_2) are zero. Accordingly, it can be asserted that the control system has a high quality. The control torque of each joint is shown in Figure 7. The simulation results show that there is not the chattering phenomenon of torques, which is necessary for the system to assure the sustainability of actuators and devices.



(a) (b)

Figure 7. The control torque of two joints, (a) the joint1, (b) the joint2

4. CONCLUSION

In the present research, the author gained success in establishing the control system for the two-link robot arm utilizing the fractional-order sliding mode controller. The simulation outcomes displayed that the control system has remarkably high quality, the response angle of two links always reaches the desired angles in a short duration and the static error equals zero. Notably, the chattering phenomenon does not exist in the torques and therefore the lifespan of the controller and actuator is increased. Since the algorithms of the controller are analyzed specifically, the proposed controller is established quite easily in practice. The success of this proposed algorithm is the basis for the authors to conduct experiments in further studies.

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