

## Nonlinear System Identification of Laboratory Heat Exchanger Using Artificial Neural Network Model

Nader Jamali Soufi Amlashi<sup>1</sup>, Amin Shahsavari<sup>2</sup>, Alireza Vahidifar<sup>3</sup>, Mehrzad Nasirian<sup>4</sup>

<sup>1</sup>Department of Computer and Electrical Engineering, University of Tabriz, Tabriz, Iran

<sup>2,3,4</sup>Malek-Ashtar University of Technology, Tehran, Iran

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### Article Info

#### Article history:

Received Dec 13, 2012

Revised Jan 14, 2013

Accepted Jan 24, 2013

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#### Keyword:

Heat exchanger

Nonlinear identification

Modeling

Neural networks

NARX

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### ABSTRACT

This paper addresses the nonlinear identification of liquid saturated steam heat exchanger (LSSHE) using artificial neural network model. Heat exchanger is a highly nonlinear and non-minimum phase process and often its working conditions are variable. Experimental data obtained from fluid outlet temperature measurement in laboratory environment is used as the output variable and the rate of change of fluid flow into the system as input too. The results of identification using neural network and conventional nonlinear models are compared together. The simulation results show that neural network model is more accurate and faster in comparison with conventional nonlinear models for a time series data because of the independence of the model assignment.

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### Corresponding Author:

Nader JamaliSoufiAmlashi,

Department of Computer and Electrical Engineering,

University of Tabriz,

Tabriz, Iran,

Email: Jamali.tabrizu@yahoo.com

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## 1. INTRODUCTION

Heat exchangers are used widely in most industries as an important part of the thermal energy recovery networks. Among the important applications of these converters can be referred to their role in natural gas processes, exothermic reactions, refrigeration, power plants, etc. Different types of heat exchangers can be mentioned shell heat exchanger, double-tube heat exchangers, shell heat exchanger and plate heat exchanger. Double-tube heat exchangers that suitable for high pressure applications are the most common types of heat exchangers used in oil refineries, chemical processing steam generators and condensers [1]. Another advantage of these converters is that for specific heat transfer occupies less area than the shell exchanger type. This heat exchanger consists of a large cylindrical tank (shell) in high pressure and some tubes inside it. Fluid moving inside the tube lead to the flowing of hot steam on the tubes and inside the shell. Due to the large number of tubes and their high exposure surface, steam heat is transferred to the liquid in the tubes and brings liquid to boil. On the other hand, this processes a highly nonlinear process due to a number of associated phenomena with the flow and heat transfer. Some of these are complicated heat and fluid flow, geometries, turbulence in the flow, non-uniform local heat transfer rates and fluid temperatures as well as temperature dependency on fluid properties, etc. On the other hand, heat exchanger is frequently operated under varying operating conditions in common practice. As a result, adequate nonlinear models are indispensable in the characterization of such processes for the appropriate use and efficient control of heat exchangers under varying conditions and significant influence of such nonlinear dynamics. Because of some changes in water flow, heat exchangers have delays that caused a time dependent non-minimum phase identification problems to control nonlinear systems. Due to Nonlinear identification and control of such systems, some different approaches are presented. Two methods for detecting of heat exchanger in tube and shell types presented in [3] that the first approaches modeling the real physical parameters of the converter,

using of Newton's laws of physics and the second approaches non-parametric system identification technique based on the data. Identification of a heat exchanger with three inputs and one output with linear least squares (LLS) algorithm to build a tree including functions of authentication for parameters estimation of linear models is considered [4]. Model parameters estimation of heat exchanger has been studied using least squares method and state space filters [5]. Extracting of NARX polynomial models using simulation error minimization, the effect of sampling time on the choice of structure for polynomial and comparison results with other detection methods such as prediction error minimization (PEM) has been presented [6]. System identification of heat exchanger based on hammerstein-wiener models with software simulations and compared with experimental data obtained in the laboratory environment has been performed [7]. In recent years, use of artificial neural networks and fuzzy logic developing because the relationship between system inputs and outputs requires no complicated mathematical formulas. The main problem in identifying with fuzzy approach is increasing the number of required laws knowledge of the qualitative behavior of the system, computation time for inference and conclusions with increasing of model complexity. This paper is organized as follows. Mathematical model of heat exchanger using heat transfer equations is presented in Section II. In the third part, the nonlinear system identification methods such as nonlinear ARX models and artificial neural network (ANN) models with theories and theoretical arguments are given. In the fourth part, sum of squares error (SSE) between each observed indicators as compared with the average of observations in a data set trial and mean square error (MSE) has been paid. Finally obtained results based on identification of the neural network approach with conventional techniques have been compared together.

## 2. MATHEMATICAL MODELING OF HEAT EXCHANGER TUBES

A simple heat exchanger is composed of straight tubes and fluid flow which is interconnected. Subscript  $i, j$  represents the pipes 1 or 2, Assuming the pipe length  $L$ , the heat capacity of the fluid  $C_i$  (energy per unit mass per unit temperature change) and the mass flow rate through the pipes fluids  $j_i$  (mass per unit time) the.

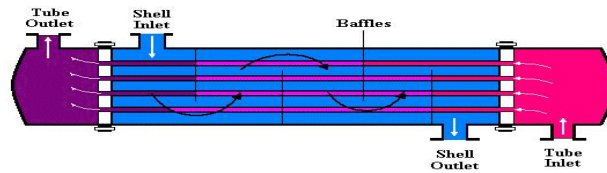


Figure 1. A view of a heat exchanger tubes.

Heat exchanger tube parameters for  $T_1(x)$  and  $T_2(x)$  are functions of  $x$  (distance along the tube). Using the concept of Newton's law, rate of change of energy in a small volume of fluid is proportional to the cooling temperature difference between the fluid in the pipe will be connected to the transducer [8]:

$$\frac{du_1}{dt} = \gamma(T_2 - T_1) \quad 1)$$

$$\frac{du_2}{dt} = \gamma(T_1 - T_2) \quad 2)$$

Where  $u_i(x)$  is heat energy per unit length and  $\gamma$  is a constant related to the heat between the two tubes per unit length. Since the time rate of temperature of the fluid is proportional to the mass variation thus Changes in fluid temperature can lead to changes in the internal energy of the fluid:

$$\frac{du_1}{dt} = J_1 \frac{dT_1}{dx} \quad 3)$$

$$\frac{du_2}{dt} = J_2 \frac{dT_1}{dx} \quad 4)$$

Where  $J_i = c_i j_i$  is flow rate of Thermal mass. Differential equations expressing the dynamic behavior of the heat exchanger can be written as follows:

$$J_1 \frac{\partial T_1}{\partial x} = \gamma(T_2 - T_1) \quad 5)$$

$$J_2 \frac{\partial T_2}{\partial x} = \gamma(T_1 - T_2) \quad (6)$$

Because the equation is written in the steady state conditions thus there is no derivative of temperature respect to time and any heat transfer at the end of the tube. So it not been seen any second derivative of  $x$  in the equation of heat. By solving these two coupled first order differential equation, we have:

$$T_1 = A - \frac{Bk_1}{k} e^{-kx} \quad (7)$$

$$T_2 = A + \frac{Bk_2}{k} e^{-kx} \quad (8)$$

Sok<sub>1</sub> =  $\frac{\gamma}{j_1}$ , k<sub>2</sub> =  $\frac{\gamma}{j_2}$ , k<sub>1</sub> + k<sub>2</sub> = 1 and A, B are constants of integration that not been determined yet. Assuming T<sub>10</sub>, T<sub>20</sub> and T<sub>1L</sub>, T<sub>2L</sub> are the temperatures of two tubes at  $x = 0$  and  $x = L$  respectively, the average temperatures in each pipe is calculated as follows:

$$\bar{T}_1 = \frac{1}{L} \int_0^L T_1(x) dx \quad (9)$$

$$\bar{T}_2 = \frac{1}{L} \int_0^L T_2(x) dx \quad (10)$$

By solving the above integrals, the temperatures will be obtained as follows:

$$T_{10} = A - \frac{Bk_1}{k} \quad T_{20} = A + \frac{Bk_2}{k} \quad (11)$$

$$T_{1L} = A - \frac{Bk_1}{k} e^{-kL} \quad T_{2L} = A + \frac{Bk_2}{k} e^{-kL} \quad (12)$$

$$\bar{T}_1 = A - \frac{Bk_1}{k^2L} (1 - e^{-kL}) \quad \bar{T}_2 = A + \frac{Bk_2}{k^2L} (1 - e^{-kL}) \quad (13)$$

Choosing any pair of above temperatures, the constants of integration will be disappearing and it helps to find unknown constants for other four temperatures also. The total energy transfer is calculated from sum of time rate of internal energy changes per unit length as follows:

$$\frac{dU_1}{dt} = \int_0^L \frac{du_1}{dt} dx = J_1(T_{1L} - T_{10}) = \gamma L(\bar{T}_2 - \bar{T}_1) \quad (14)$$

$$\frac{dU_2}{dt} = \int_0^L \frac{du_2}{dt} dx = J_2(T_{2L} - T_{20}) = \gamma L(\bar{T}_1 - \bar{T}_2) \quad (15)$$

According to the law of conservation of energy, the total energy of a system must be zero due to transmission and energy absorption.  $\bar{T}_2 - \bar{T}_1$  is known as average temperature difference and indicate the effectiveness of the heat exchanger in heat energy transfers process. To identify the heat exchanger considering to laboratory prototype, the flow rate of liquid mass as input parameters  $J_1$ , the temperature of outlet fluid (T<sub>1</sub>) as the output of the process has been considered. Experimental result has been adapted from database for the identification of systems Daisy.

### 3. IDENTIFICATION USING NONLINIER METHODS

Since the heat exchanger for internal heat and fluid interactions is a highly nonlinear and non-minimum phase system therefore nonlinear methods should be used to identify it. In this paper, two approaches for nonlinear identification of converter are presented.

#### 3-1 Nonlinear ARX Method

Nonlinear ARX (NARX) Model that was presented for the first time [9] proposed a nonlinear differential equations form as the follows:

$$y(t)=f(y(t-1),\dots, y(t-n_y),u(t-1),\dots,u(t-n_u)) \quad (16)$$

Where  $f$  represents the nonlinear mapping,  $u(t)$  and  $y(t)$  the sample input and output and  $n_u$  and  $n_y$  are the maximum time shift for input and output respectively. One of most common representation for NARX model in the above equation is polynomial form which function  $f$  is as a polynomial of degree 1 and presents the following form equation[10].

$$y(t)=\theta_0 + \sum_{i_1=1}^n f_{i_1}(x_{i_1}(t)) + \sum_{i_1=1}^n \sum_{i_2=1}^n f_{i_1 i_2}(x_{i_1}(t), x_{i_2}(t)) + \dots + \sum_{i_1=1}^n \dots \sum_{i_m=1}^n f_{i_1 i_2 \dots i_m}(x_{i_1}(t), x_{i_2}(t), \dots, x_{i_m}(t)) \quad (17)$$

$$f_{i_1 i_2 \dots i_m}(x_{i_1}(t), x_{i_2}(t), \dots, x_{i_m}(t)) = \theta_{i_1 i_2 \dots i_m} \prod_{k=1}^m x_{i_k}(t), \quad (18)$$

$$1 \leq m \leq l$$

Where  $\theta_{i_1 i_2 \dots i_m}$  are unknown parameters and  $n = n_u + n_y$  and we have:

$$x_k(t) = \begin{cases} y(t-k) & 1 \leq k \leq n_y \\ u(t-(k-n_y)) & n_y + 1 \leq k \leq n_y + n_u \end{cases} \quad (19)$$

Degree of a multivariate polynomial was determined considering to the biggest order among all terms. Since the NARX representation based on the parameters is linear, therefor linear regression can be used to estimate the parameters in selection of the determined structure. Identification of a NARX was performed in following stages:

- Parameter Estimation
- Select the structure that is divided into two parts:
  - 1 - Select the model order
  - 2 - Select which of the parameters are attended in model identification.

Selection of model order is considered as a part of the chosen structure. Determination of model order limits the selection existing terms in polynomial. For NARX models, order of system can be defined as follows:

$$O \equiv [n_u n_y l] \quad (20)$$

Where  $n_u$  and  $n_y$  are the maximum number of terms in NARX model with the dynamic terms and  $l$  is nonlinear order. As a result, the number of candidate terms for complex models will be numerous and it is difficult to select the optimal structure for the model. Choosing of the maximum number of terms pas a number of candidate terms for initial identification, we have:

$$P = \sum_{i=1}^l P_i \quad (21)$$

$$P_i = \frac{P_{i-1}(n_y + n_u + i - 1)}{i}, \quad P_0 = 1$$

It makes easy to determine necessary Candidate values for estimation of optimal and safe of this parameter. For most non-linear systems NARX representation requires to few limited components only. However, increasing the order of system caused to increasing number of candidate terms for identification according to equation (21) simultaneously.

### 3-2 Using Artificial Neural Network

In this paper, an offline system identification based on single-input single-output (SISO) neural network model is considered for a laboratory heat exchanger. This model is based on autoregressive with the

independent inputs(ARX). Neural network is trained using Back Propagation algorithm. A general black box model for linear dynamical system is as follows:

$$A(q)y(t) = \frac{B(q)}{F(q)}u(t) + \frac{C(q)}{D(q)}e(t) \quad (21)$$

Where

$$\begin{aligned} A(q) &= 1 + a_1q^{-1} + a_2q^{-2} + \dots + a_{na}q^{-na} \\ B(q) &= b_1q^{-1} + b_2q^{-2} + \dots + a_{nb}q^{-nb} \\ C(q) &= 1 + c_1q^{-1} + c_2q^{-2} + \dots + a_{nc}q^{-nc} \\ D(q) &= 1 + d_1q^{-1} + d_2q^{-2} + \dots + a_{nd}q^{-nd} \\ F(q) &= 1 + f_1q^{-1} + f_2q^{-2} + \dots + a_{nf}q^{-nf} \end{aligned} \quad (22)$$

$u(t)$ System input,  $y(t)$  corresponding output of the system input,  $e(t)$  white noise with variance  $\lambda$  and  $na$ ,  $nb$ ,  $nc$ ,  $nd$  and  $nf$  are the related polynomials order. Auto regressive Function model (AR), autoregressive moving average model (ARMA) and the state space with invariant forms (SSIF) model are special cases for presented model in equation (1). An autoregressive model with independent inputs (ARX) is presented as follows:

$$y(t) + a_1y(t-1) + \dots + a_{na}y(t-n_a) = b_1u(t-n_k) + b_2u(t-n_k-1) + \dots + b_{nb}u(t-n_k-n_b+1) \quad (23)$$

$n_k$  is a delay from input to output. Calculated output of a nonlinear model can be obtained also as follows:

$$\hat{y}(t|\theta) = g[y(t-1), \dots, y(t-n_a), u(t-n_k), \dots, u(t-n_k-n_b+1)] \quad (24)$$

Where  $\theta$  is coefficient matrix for  $u$  and  $y$  terms in equation 23 and  $g$  is a nonlinear function. Equation 23 can be wrote a function of vector of coefficients  $\theta$  and a recursive function  $\varphi$ :

$$\begin{aligned} \theta &= (a_1, \dots, a_{na}, b_1, \dots, b_{nb}) \\ \varphi(t) &= (y(t-1), \dots, y(t-n_a), u(t-n_k), \dots, u(t-n_k-n_b+1)) \end{aligned} \quad (25)$$

$$g(t, \theta, \varphi(t)) = -a_1y(t-1), \dots, -a_{na}y(t-n_a) + b_1u(t-n_k) + \dots + b_{nb}u(t-n_k-n_b+1)$$

In the above equation, regression function  $\varphi(t)$ is depends on previous and present inputs and outputs. The calculated output value is as a function of  $\theta$  and  $\varphi(t)$ that is written as follows:

$$\hat{y}(t) = \hat{y}(t|\hat{\theta}) = g(\hat{\theta}, \varphi(t)) \quad (26)$$

In the above equation  $\theta$  represents the best possible value of weight coefficients. Selectivity of coefficients matrix  $\hat{\theta}$ for a nonlinear system with similar dynamics to a heat exchanger is very frustrating and can lead to decrease predicted model performance. This process will be more complicated for a MIMO system. There fore, recurrent feed forward (RFF) network can be used for modeling of heat exchanger including a hidden layer and an output layer. In this case, the output of the network is determined by the following function:

$$\hat{y}_i(t) = F_i\left(\sum_{j=0}^{l_1} W_{1ij}G_i\left(\sum_{k=1}^{l_2} W_{2jk}x_k(t) + W_{2j0}\right) + W_{1i0}\right) \quad (27)$$

W1 and W2 are weighting coefficients for two layers. x network input and F and G are the activation functions of neurons in the output and hidden layer respectively. The first step in the identification algorithm is choice of model that is very important. Various model compounds can be considered with constraints of error minimization. The second step is to build a regression matrix that its structure depends on chosen AR model. The third step is Network training. It is possible with different training algorithms. It can be shown that back propagation algorithm uses less time training comparing with other training methods and has more resistant towards identified noisy. Therefore, the regression function and its corresponding output data set are used for feed forward network training using back Propagation method. This method operates based on the minimization of the defined cost function by error between the actual output of system and the computed output by the network. The cost function to be minimized is calculated as follows:

$$\text{Cost function} \equiv E_p = \frac{1}{2} \text{Trace} (D \cdot D^T) \tag{28}$$

As D is defined as follows:

$$D = \text{Target} - Y_3$$

Where Target the actual output of system and y3 is the calculated output by neural network. Due to implement a Back Propagation training algorithm in a multi-layer network a Flowchart is proposed as shown in Figure 2.

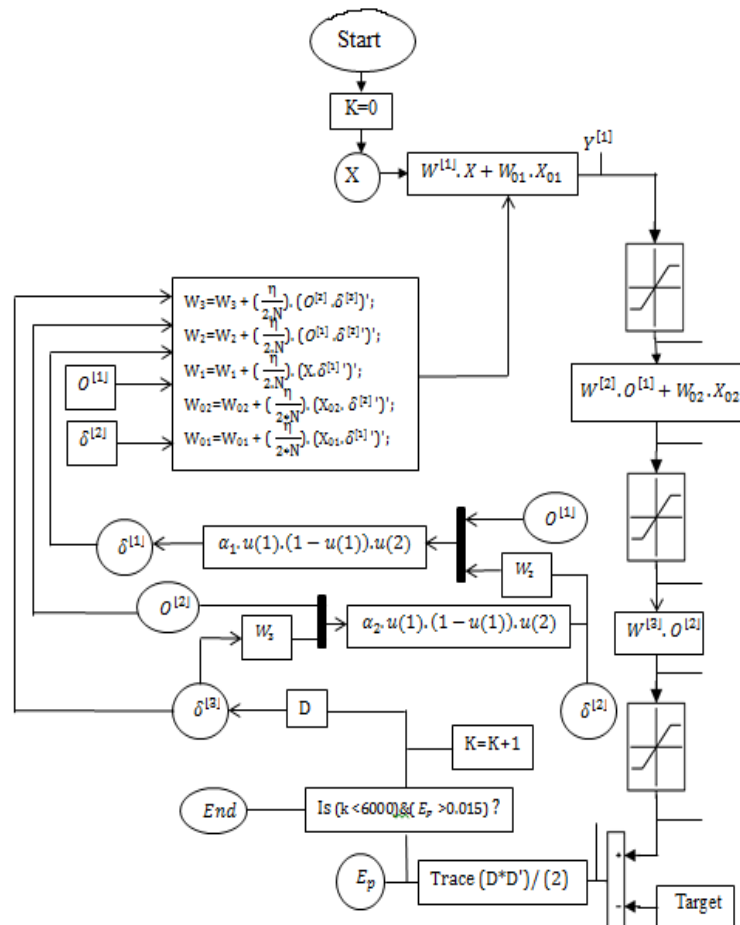


Figure 2. Provided Flowchart to implement back propagation training algorithm

#### 4. PERFORMANCE INDEXES

##### 4.1 SSE

Sum of Squared Error (SSE) is defined as difference between each observation with the mean of the observations in a data set. The criterion is used to measure the variance value in a set that is defined as follows:

$$SSE = \sum_{i=1}^n (x_i - \bar{x})^2 \quad (29)$$

Where  $n$  is number of observations,  $x_i$  the value of each observation and  $\bar{x}$  is the mean of all of observations in data set.

#### 4.1 MSE

In this paper, Back Propagation training algorithm has been used in network training. During the training process, the weights and bias values for network are tuned to minimize the mean square error between experimental data and predicted values. Mean square error criterion is defined as follows [11]:

$$MSE = \frac{1}{N} \sum_{k=1}^N D(k)^2 = \frac{1}{N} \sum_{k=1}^N (\text{target}(k) - \hat{Y}(k))^2 \quad (30)$$

Where  $N$  the number of collected data,  $D(k)$  neural network error,  $\text{Target}(k)$  experimental values and  $\hat{Y}(k)$  predicted values by the network. In this study, considered options for completion of the training process  $EP < 0.015$  and the number of iteration are set equal to 6000. In addition, MSE is second moment of error and thus is including both the variance of estimator and its bias according to the following equation:

$$MSE(\hat{Y}) = \text{Var}(\hat{Y}) + (\text{Bias}(\hat{Y}), \text{Target})^2 \quad (31)$$

MSE is equal to zero that means the estimator anticipate a desired output with high and ideal resolution that this is not the case in practice.

## 5. SIMULATION RESULTS

### 5-1 Nonlinear Identification Result

The data used in this simulation have been achieved from a liquid saturated steam heat exchanger which water using high pressure saturated steam inside of heated copper tubes. In this experiment, the temperatures of the vapor and input channels liquid are kept constant in nominal values. Some considerations performed in two methods for identification, such as sampling time equal to 1 second and the number of collected samples equal to 4000 and choice of liquid flow rate ( $J$ ) as heat exchanger input and the temperature of the extracted liquid from channels tube as the system output ( $T$ ).

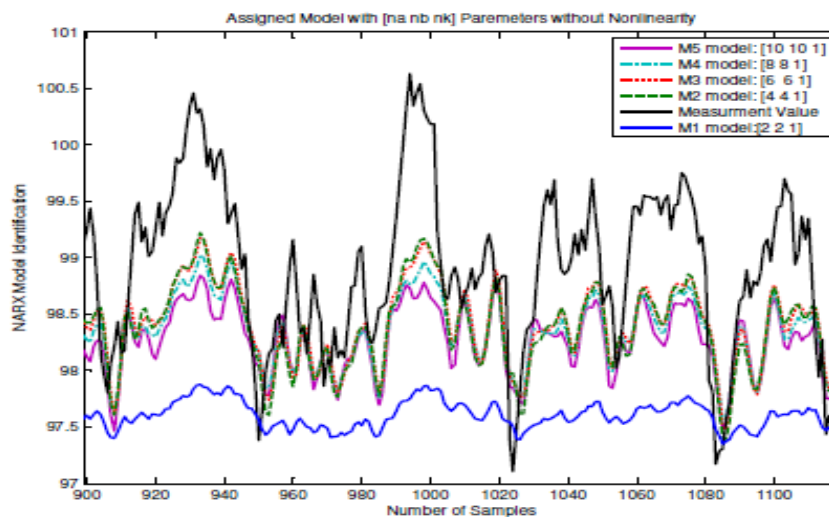


Figure 3. Nonlinear identification of heat exchanger without selecting of nonlinearity method

The nonlinear identification results with assignment a model to system are presented without nonlinearity in figure 3, wavelet nonlinearity in figure 4 and sigmoid nonlinearity in figure 5 respectively. Because of the dependence of conventional nonlinear methods to model assignment for prediction, the obtained Best Fit is 63.95% for models with  $n_a = 8$ ,  $n_b = 8$ ,  $n_k = 1$ .

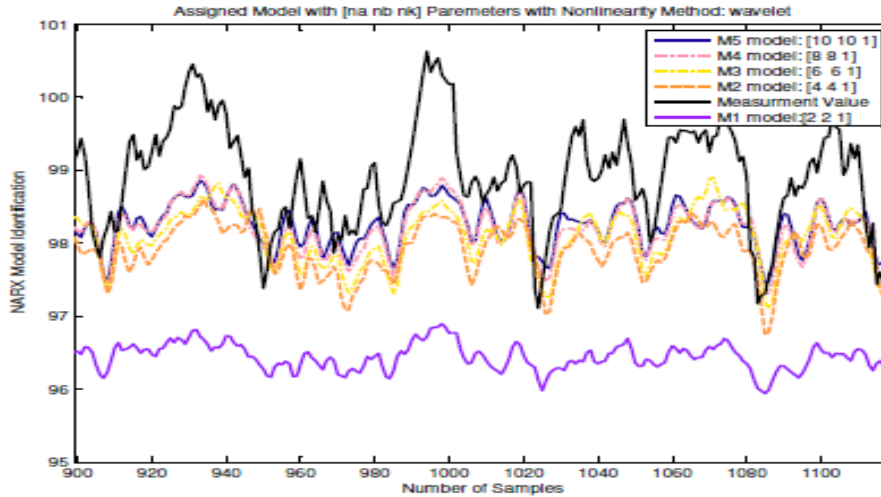


Figure 4. Nonlinear identification of heat exchanger using "wavenet" method.

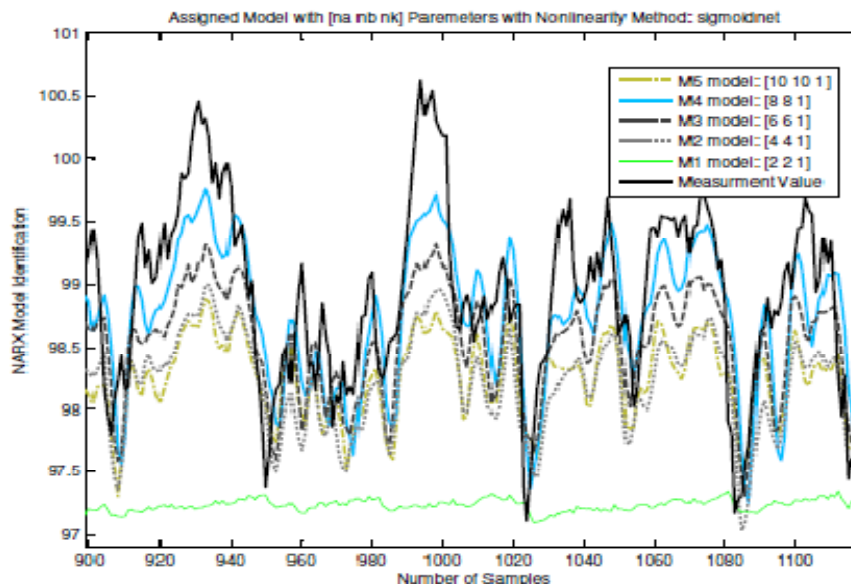


Figure 5. Nonlinear identification of heat exchanger using "Sigmoid" method.

The obtained results of nonlinear identification for assigned different models and also for identifying allocated for different nonlinearity methods are presented in Table 1.

### 5-2 Identification Results Using Artificial Neural Network

The obtained results for the prediction of the data derived from laboratory exchanger with 6000 iteration and 5 neurons in the first hidden layer and 12 neurons in the second hidden layer using the Back propagation training algorithm and activation function sigmoid is presented in Figure 6. The best prediction and identification obtained for learning rate  $\eta$  equal to 0.4, 0.6 and 0.8. However, the prediction accuracy is much lower for  $\eta=0.2$  value. Since the primary goal of an optimal estimation is minimization of the cost function defined based on the estimation error for the considered problem thus. The obtained results for the



error value for  $\eta$  equal to 0.2, 0.4, 0.6 and 0.8 are shown in Figure 7. As shown, the error value for the learning rate of 0.4, 0.6 and 0.8 is very low meanwhile for learning rate of 0.2 is very high.

Results of neural identification for different learning rates based on performance indexes like cost function, the mean of error square and the sum of error squared resulting in estimation are given in Table 2.

Table 1. Results of nonlinear identification using NARX methods

l nk]	Mode [nanb	Nonlinear ARX with Delay=1 s Using		PE	loss	est Fitting	B
		Different Methods Meth od					
1]	[2 2	-----	wave	.1832	(	(	2
		let	sigm	.1813	(	(	2
		oident	-----	.1828	(	(	2
1]	[4 4	-----	wave	.1738	(	(	5
		let	sigm	.1693	(	(	5
		oident	-----	.1718	(	(	5
1]	[6 6	-----	wave	.1719	(	(	5
		let	sigm	.1712	(	(	5
		oident	-----	.1677	(	(	5
1]	[8 8	-----	wave	.1708	(	(	5
		let	sigm	.1727	(	(	5
		oident	-----	.1574	(	(	6
10 1]	[10	-----	wave	.1692	(	(	5
		let	sigm	.1708	(	(	5
		oident	-----	.1687	(	(	5

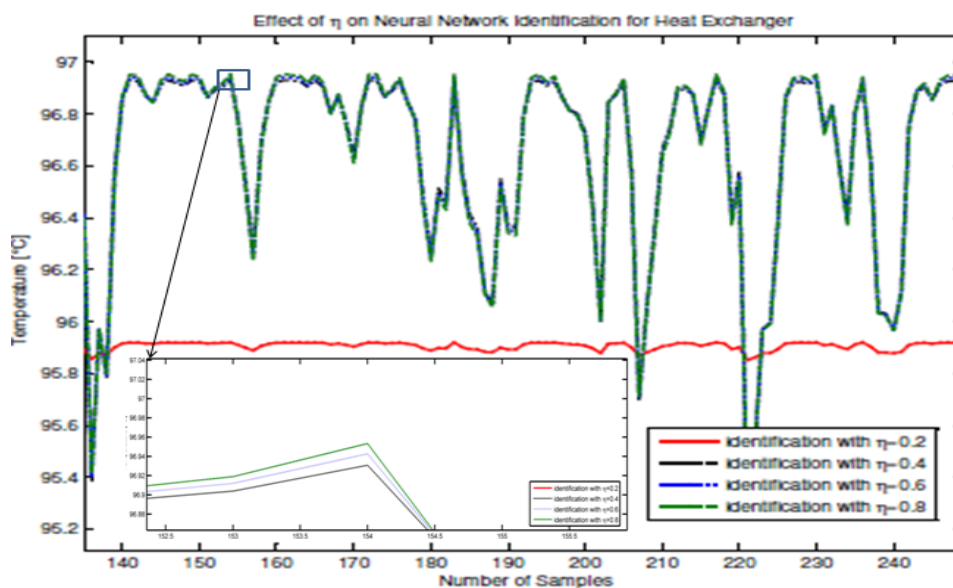


Figure 6. The impact parameter  $\eta$  on forecasting real output

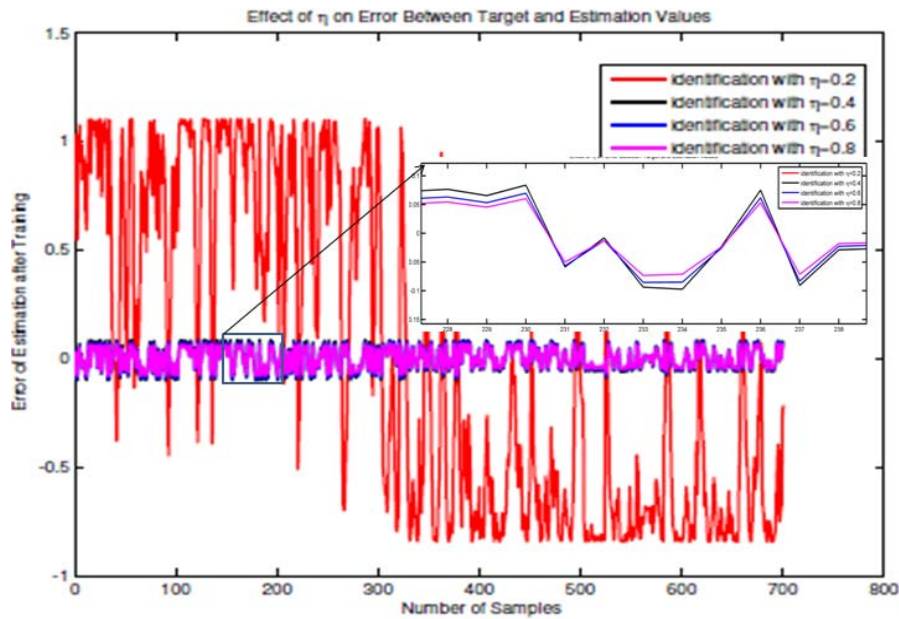


Figure 7. The effect of learning rate parameter ( $\eta$ ) of the predicted error

Table 2. Identification of heat exchanger using neural network

Learning factor	Nonlinear identification using ANN		
	Lost function	MSE	SSE
$\eta=0.2$	191.2503	0.0340	382.5006
$\eta=0.4$	1.1631	1.0310e-006	2.3262
$\eta=0.6$	0.9108	1.0992e-006	1.8216
$\eta=0.8$	0.6771	5.3209e-007	1.3541

## 6. CONCLUSION

In this paper, the artificial neural network model was used to predict the heat exchanger considering to temperature of the outlet water as output system and rate of changes in inlet water as input. In comparison with conventional nonlinear models like NARX, this method do not need to model assignment and with considering to performance like MSE and SSE and the cost function has a quick and accurate prediction. Consequently, the artificial neural networks model can be used as efficient tool in identifying heat exchanger for design objectives and design controller.

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