

A Novel Nonlinear Control of Boost Converter using CCM Phase Plane

Eko Setiawan¹ and Ichijo Hodaka²

¹Interdisciplinary Graduate School of Agriculture and Engineering, University of Miyazaki, Japan

²Department of Environmental Robotics, University of Miyazaki, Japan

Article Info

Article history:

Received Jan 20, 2018

Revised Jul 18, 2018

Accepted Aug 7, 2018

Keyword:

Nonlinear control

Boost converter

Continuous Conduction Mode

Phase plane

ABSTRACT

Boost converter is one of fundamental DC-DC converters and used to deliver electric power with boosted voltage in many electrical systems. Several control strategies have been applied to control a boost converter delivering a constant output voltage. Generally, boost converter works in two modes; one is called a Continuous Conduction Mode (CCM). Many researches use CCM model in the controller design, but they never ensure that the controller always works in CCM. This paper proposes novel nonlinear controller of boost converter designed using the modification of flow in phase plane. The proposed controller guarantees that the boost converter works only in CCM region. The simulation result confirms that our proposed controller brings the state variables from any initial point to a desired operating point successfully.

Copyright © 2018 Institute of Advanced Engineering and Science.

All rights reserved.

Corresponding Author:

Ichijo Hodaka

Department of Environmental of Robotics, University of Miyazaki, Japan

1-1, Gakuen Kibanadai-nishi, Miyazaki, 889-2192, Japan

Phone: +81 985 587 352

Email: [hijhodaka\[at\]cc.miyazaki-u.ac.jp](mailto:hijhodaka[at]cc.miyazaki-u.ac.jp)

1. INTRODUCTION

Over the last few decades, DC-DC converters have been the subject of great interest due to its extensive increment of utilization in different applications. Right now, they are popularized in standard and redone items that power an extensive variety of applications, for example, photovoltaic (PV) power systems [1, 2], wind turbines (WT) [3, 4], brushless DC (BLDC) motor [5, 6] etc. Among these converters, the boost converter is a fundamental controller which is used in many systems due to its simplicity.

In order to achieve the operating point, boost converter usually works with the controller techniques. There are two controllers for the DC-DC converter as pulse-width modulation (PWM) and phase-shift modulation (PSM). The PWM has been widely utilized to control of DC-DC converter in several applications. In the case of less number of components usage and high-reliability demand, the PWM control shows the better performance than PSM [7]. Proportional-integral-derivative (PID) and sliding-mode control (SMC) are used widely in a DC-DC converter. The PID control offers the good stability system, but it only operates on limited operating point. The SMC provides larger operating point than PID. The SMC works well at most operating point. However, the fundamental barrier for SMC execution is a marvel called 'chattering'.

Based on conduction mode, the DC-DC converter is analyzed in two modes as continuous-conduction mode (CCM) and discontinuous-conduction mode (DCM). The CCM is the most often used in DC-DC converter analysis to design a controller. Although it is often chosen, none of the previous literature examines that their proposed controller works only on CCM region. Since the design is based on CCM, the controller and system should work only in that region or the analysis and design may mislead the controller. Moreover, the controller of boost converter should be able to handle any operating points with correct design.

The rapid development of the very large scale integrated circuit technology brings digitally controlled DC-DC converter as hot topic [8]. Digital processors also have the advantage of being less susceptible to aging and environmental or parameter variations. In addition, the processor can monitor the system, perform self-

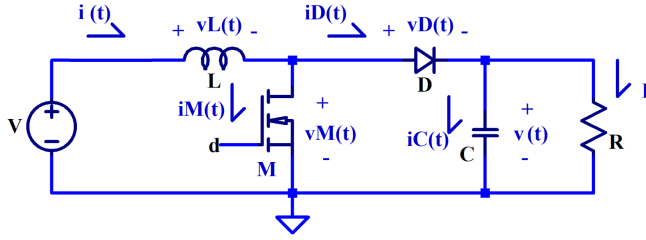


Figure 1. Boost converter

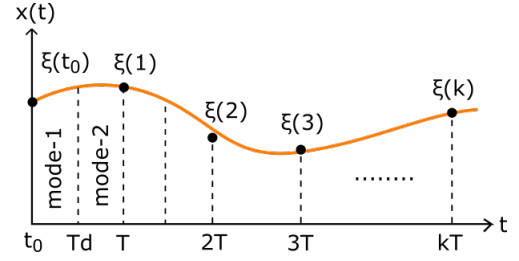


Figure 2. Discrete-time of state

diagnostics and tests, and communicate status to a display or a host computer [9]. Implementation of digital controller requires system analysis in discrete-time domain.

This paper proposes nonlinear controller which be able to handle any initial point and keep the system works in CCM. This paper focuses on one type DC-DC converter which is boost converter but the idea has possibility to be applied on other converter. The proposed controller design is based on flow modification of phase plane. The analysis is conducted in discrete time-domain which is required by a digital controller due to the popularity of digital controller. Section 2 explains analysis of boost converter in discrete time-domain. Digital implementation of controller requires indirectly the discrete time-domain analysis. Section 3 shows the control system specification in CCM of boost converter. Section 4 proposes the nonlinear control which its design based on phase plane examination of any initial condition. Section 6 simulates the system on several initial points. Finally, Section 7 tells the important point of this paper.

2. DYNAMIC MODEL OF BOOST CONVERTER

The boost converter consist of inductor L , capacitor C , MOSFET M and diode D as shown in Figure 1. The circuit equation of boost converter is derived as follows.

$$\begin{cases} V = L\dot{i}(t) + v_M(t), & v_M(t) = v_D(t) + v(t) \\ i(t) = i_M(t) + i_D(t), & i_D(t) = C\dot{v}(t) + v(t)/R \end{cases} \quad (1)$$

The diode voltage and MOSFET voltage are denoted as v_D and v_M respectively. A resistive load R is connected to the boost converter. A voltage source V supplies a constant voltage for boost converter. The converter is controlled by duty-ratio d of PWM.

In CCM, there are two modes which work alternately. In the first mode or mode-1, the MOSFET M is conducted and the diode D is disconnected. We assume that the MOSFET voltage is constant V_M during mode-1 and there is no current on diode ($i_D(t) = 0$). The second mode or mode-2 occurs when the MOSFET is off and diode is conducted. We assume that the diode voltage is constant V_D and there is no current on MOSFET ($i_M(t) = 0$) during mode-2. The equation of boost converter becomes as follows.

$$\text{mode-1} \begin{cases} L\dot{i}(t) = V - V_M \\ C\dot{v}(t) = -v(t)/R \end{cases} \quad (2)$$

$$\text{mode-2} \begin{cases} L\dot{i}(t) = V - V_D - v(t) \\ C\dot{v}(t) = i(t) - v(t)/R. \end{cases} \quad (3)$$

In this section, we introduce a non-dimensional variable x_1 and x_2 which is described as follows.

$$\begin{cases} x_1 = \frac{v(t) - V + V_D}{V} \\ x_2 = \frac{i(t)}{V} \sqrt{\frac{L}{C}} \end{cases} \quad (4)$$

The derivative of non-dimensional state on each mode can be calculated by substituting $\dot{i}(t)$ and $\dot{v}(t)$ of equation (2) and (3) as follows.

$$\text{mode-1} \begin{cases} \dot{x}_1 = \frac{\dot{v}(t)}{V} = \frac{1}{V} \times \frac{-(Vx_1 + V - V_D)}{RC} = -\frac{1}{RC}x_1 - \frac{1}{RC} \left(1 - \frac{V_D}{V}\right) \\ \dot{x}_2 = \frac{\dot{i}(t)}{V} \sqrt{\frac{L}{C}} = \frac{1}{V} \sqrt{\frac{L}{C}} \times \left(\frac{V - V_M}{L}\right) = \frac{1}{\sqrt{LC}} \left(1 - \frac{V_M}{V}\right) \end{cases} \quad (5)$$

$$\text{mode-2} \begin{cases} \dot{x}_1 = \frac{\dot{v}(t)}{V} = \frac{1}{V} \times \left(\frac{i(t)}{C} - \frac{v(t)}{RC}\right) = \frac{1}{\sqrt{LC}}x_2 - \frac{1}{RC}x_1 - \frac{1}{RC} \left(1 - \frac{V_D}{V}\right) \\ \dot{x}_2 = \frac{\dot{i}(t)}{V} \sqrt{\frac{L}{C}} = \frac{1}{V} \sqrt{\frac{L}{C}} \times \frac{V - V_D - v(t)}{L} = \frac{-1}{\sqrt{LC}}x_1 \end{cases} \quad (6)$$

The state space equation of non-dimensional variable can be written as follows.

$$\dot{x} = \underbrace{\begin{bmatrix} \frac{-1}{RC} & 0 \\ 0 & 0 \end{bmatrix}}_{A_1} x + \underbrace{\begin{bmatrix} \frac{-1}{RC} \left(1 - \frac{V_D}{V}\right) \\ \frac{1}{\sqrt{LC}} \left(1 - \frac{V_M}{V}\right) \end{bmatrix}}_{b_1} \quad (7)$$

$$\dot{x} = \underbrace{\begin{bmatrix} \frac{-1}{RC} & \frac{1}{\sqrt{LC}} \\ \frac{-1}{\sqrt{LC}} & 0 \end{bmatrix}}_{A_2} x + \underbrace{\begin{bmatrix} \frac{-1}{RC} \left(1 - \frac{V_D}{V}\right) \\ 0 \end{bmatrix}}_{b_2} \quad (8)$$

As shown in Figure 2, mode-1 works from the beginning t_0 until $t = t_0 + Td$. Based on general solution of state space equation, the last state of mode-1 can be written as follows.

$$\begin{aligned} x(t_0 + Td) &= e^{A_1(t_0 + Td - t_0)} x(t_0) + \int_{t_0}^{t_0 + Td} e^{A_1(t_0 + Td - p)} b_1 dp \quad \text{where } \begin{cases} q = \frac{p - t_0}{T} \\ p = qT + t_0 \\ \frac{dp}{dq} = T \end{cases} \\ &= e^{A_1 Td} x(t_0) + \int_0^d e^{A_1 T(d - q)} b_1 T dq \end{aligned} \quad (9)$$

In CCM, the boost converter has two state-space equations (7) and (8). The last state of mode-1 is equal to the initial state of mode-2. On the next mode, the last state of mode-2 will be as the initial state of the next period mode-1. These phenomena occurs repeatedly. Based on these facts, the solution of boost converter per period (T) can be obtained by substituting the last state of mode-1 ($x(t_0 + Td)$) into the general solution of mode-2 as follows.

$$\begin{aligned} x(t_0 + T) &= e^{A_2 T(1-d)} x(t_0 + Td) + \int_d^1 e^{A_2(t_0 + T - (qT + t_0))} b_2 T dq \\ &= e^{A_2 T(1-d)} e^{A_1 Td} x(t_0) + e^{A_2 T(1-d)} \int_0^d e^{A_1 T(d - q)} b_1 T dq + \int_d^1 e^{A_2 T(1 - q)} b_2 T dq \end{aligned} \quad (10)$$

Let us assume that the sensor measures every end of switching period (T). This measurement is shown as dot-point in Figure 2. We will introduce new variable ξ to distinguish with the continuous-time variable x . Then, the discrete representation of solution (10) is defined as follows:

$$\xi(k + 1) = x(t_0 + (k + 1)T) = e^{\tilde{A}_2(1-d)} e^{\tilde{A}_1 d} \underbrace{x(t_0 + kT)}_{\xi(k)} + e^{\tilde{A}_2(1-d)} \int_0^d e^{\tilde{A}_1(d - q)} \tilde{b}_1 dq + \int_d^1 e^{\tilde{A}_2(1 - q)} \tilde{b}_2 dq \quad (11)$$

where

$$\begin{aligned} \tilde{A}_1 &= \begin{bmatrix} -\varepsilon_1 & 0 \\ 0 & 0 \end{bmatrix}, \tilde{A}_2 = \begin{bmatrix} -\varepsilon_1 & \varepsilon_2 \\ -\varepsilon_2 & 0 \end{bmatrix}, \tilde{b}_1 = \begin{bmatrix} -\varepsilon_1 \beta \\ \varepsilon_2 \alpha \end{bmatrix}, \tilde{b}_2 = \begin{bmatrix} -\varepsilon_1 \alpha \\ 0 \end{bmatrix}, \\ \varepsilon_1 &= \frac{T}{RC}, \varepsilon_2 = \frac{T}{\sqrt{LC}}, \alpha = \left(1 - \frac{V_M}{V}\right), \text{ and } \beta = \left(1 - \frac{V_D}{V}\right) \end{aligned} \quad (12)$$

Assuming the period T is small then the element of \tilde{A}_1 , \tilde{A}_2 , \tilde{b}_1 , and \tilde{b}_2 become small too. The exponential part in (11) can be calculated using the definition of exponential as follows.

$$\begin{aligned} e^M &= \mathbf{I} + M + \frac{1}{2!} M^2 + \dots \\ e^{\tilde{A}_1 d} &= \mathbf{I} + \tilde{A}_1 d + \underbrace{\frac{\tilde{A}_1^2 d^2}{2} + \dots}_{\text{neglected}} \simeq \mathbf{I} + \tilde{A}_1 d \\ e^{\tilde{A}_2(1-d)} e^{\tilde{A}_1 d} &\simeq \mathbf{I} + \tilde{A}_1 d + \tilde{A}_2(1 - d) \\ e^{\tilde{A}_2(1-d)} \int_0^d e^{\tilde{A}_1(d - q)} \tilde{b}_1 dq &\simeq (\mathbf{I} + \tilde{A}_2(1 - d)) \int_0^d (\mathbf{I} + \tilde{A}_1(d - q)) \tilde{b}_1 dq = \tilde{b}_1 d \\ \int_d^1 e^{\tilde{A}_2(1 - q)} \tilde{b}_2 dq &\simeq \int_d^1 (\mathbf{I} + \tilde{A}_2(1 - q)) \tilde{b}_2 dq = \tilde{b}_2(1 - d) \end{aligned} \quad (13)$$

Thus, the discrete-time solution of state (11) is simplified as follows

$$\xi(k+1) = \underbrace{(I + \tilde{A}_1 d + \tilde{A}_2(1-d))}_A \xi(k) + \underbrace{\tilde{b}_1 d + \tilde{b}_2(1-d)}_b \quad (14)$$

The equation (14) is used to simulate the boost converter on the next sections.

3. CONTROL PROBLEM

A boost converter in continuous-conduction mode (CCM) has the limitation such as:

1. the output voltage $v(t)$ must be greater than the input voltage V subtracted by the diode voltage V_D . This is the principle of boost. In the non-dimensional state, ξ_1 never be negative,
2. the inductor current $i(t)$ must be greater than zero as the definition of CCM. It means that the state ξ_2 must not negative, and
3. the two previous conditions are satisfied for any initial states.

Breaking the limitation means the model is not proper anymore in the controller design. Keeping the wrong model may mislead the analysis of controller design. The three limitations will be used as control specification in this paper.

Table 1. The parameter of Boost Converter

Parameter	Symbol	Value
Input voltage	V	10 V
Inductor	L	300 μ H
Capacitor	C	100 μ F
MOSFET voltage	V_M	162 mV
Diode voltage	V_D	0.5 V
Load	R	10 Ω
Switching period	T	20 μ s
Reference voltage	V_{ref}	16 V

Let us simulate the behavior of boost converter using parameters of boost from [12] as shown in Table 1. The parameters are also used in the next sections. We examines the behavior of boost converter without the controller called as open-loop response. The boost converter is given a constant equilibrium duty ratio notated as D_{eq} . The mathematics software (e.g. Wolfram Mathematica) finds the value of equilibrium duty-ratio (D_{eq}) by solving the duty-ratio when equation (14) is equal to $[\xi_1, \xi_2]^T$ as follows.

$$D_{eq} = \frac{\xi_{ref}}{\alpha + \xi_{ref}} \quad (15)$$

where $\xi_{ref} = (V_{ref} - V + V_D)/V$. In order to examine the behavior of boost for any initial condition, let us utilize phase plane. The phase plane of open-loop response is shown in Figure 3. The dashed line in Figure 3 shows the set of equilibrium condition. Let us focus on the phase trajectory of two initial conditions which are notated as A and B. It shows that the system can achieve equilibrium point well for both initial conditions. For the initial condition B, the system enters the negative region of ξ_1 and ξ_2 which breaks the limitation of CCM boost converter. In order to avoid this situation, a proper controller is required.

4. PID CONTROLLER

Among the several controllers of boost converter, PID controller is the mature controller which well-explained in several papers such as [7, 8, 11, 13, 14]. This section discusses the implementation of PID controller and its characteristic on boost converter. The PID controller in discrete-time domain is expressed as follows [11].

$$u(k) = K_P \left[e(k) + \frac{T}{T_I} \sum_{j=0}^k e(j) + \frac{T_D}{T} \{e(k) - e(k-1)\} \right] \quad (16)$$

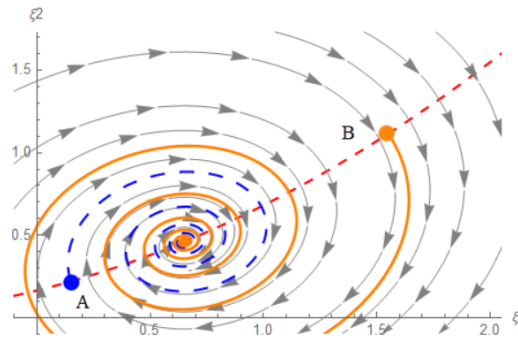


Figure 3. Phase plane of open-loop response

The recursive expression of PID control in discrete-time is formed by difference between simultaneous input ($\Delta u(k) = u(k) - u(k-1)$). The previous PID control can be expressed as follows [11, 12]:

$$\begin{aligned} u(k) &= u(k-1) + \Delta u(k) \\ &= d(k-1) + (K_P + K_I + K_D)e(k) - (K_P + 2K_D)e(k-1) + K_De(k-2) \end{aligned} \quad (17)$$

where $K_I = \frac{K_P T}{T_I}$, $K_D = \frac{K_P T_D}{T}$, $e(k) = V_{ref} - v(k)$, $e(k-1) = V_{ref} - v(k-1)$, and $e(k-2) = V_{ref} - v(k-2)$.

PID controller or called compensator in some references is usually designed by small-signal model of DC-DC converter [10]. The small-signal model is derived by adding small perturbation on inductor current $i(t)$, capacitor voltage $v(t)$ and duty-ratio d . Since the PID controller is designed by linearization around the equilibrium point, implementation of PID controller needs equilibrium duty-ratio D_{eq} as described in following equation [13].

$$d(k) = D_{eq} + u(k) \quad (18)$$

Let us examine the behavior of PID controller in the phase plane. The parameter of PID needs to be tuned before used. Based on [14], the Ziegler-Nichols (ZN) has the best performance comparing with the others. The simulation shows that the boost system achieves ultimate gain (K_U) and ultimate period (T_U) on 0.06 and 1,8 ms respectively. According to the ZN table on [14], the P-gain (K_P), D-gain (K_D), and I-gain (K_I) are 0.036, 8×10^{-4} and 0.405 respectively. Let us draw the phase trajectory of PID controller on several initial condition $v(0)$ s and $i(0)$ s. The phase trajectory of PID controller is observed by applying (14), (17), (18) and (15) on the several initial points. Figure 4 shows the phase trajectory of PID controller for several initial conditions. Based on Figure 4, the flow of state tends to go to negative area of ξ_2 at first. Most of tested initial state enters the negative area of ξ_1 and ξ_2 which breaks the limitation of Boost converter in CCM. This fact shows that the PID does not guarantee the boost converter works always in CCM. It means that the controller design using CCM is not suitable with the implementation.

5. PROPOSED CONTROLLER

This paper proposes the nonlinear feedback control which is designed based on manipulation of flow in phase plane. The flow is consisted from two vectors which are $(\xi_1(k) - \xi_1(k-1))$ and $(\xi_2(k) - \xi_2(k-1))$ as shown in Figure 5. The flow is forced to has a specific direction. In that condition, the following equality works.

$$(\xi_1(k) - \xi_1(k-1)) \cos \theta + (\xi_2(k) - \xi_2(k-1)) \sin \theta = 0 \quad (19)$$

The proposed controller is designed by solving the duty ratio d in the equation (19). Then, a proportional controller is added to push the controller to the reference $\xi_{ref} = (v_{ref} - V + V_D)/V$. Finally, the overall proposed controller is described as follows.

$$d_{proposed}(k) = k(\xi_{ref} - \xi_1(k)) + \frac{\varepsilon_2 \xi_1(k) \sin \theta - (\varepsilon_1 \beta + \varepsilon_1 \xi_1(k) - \varepsilon_2 \xi_2(k)) \cos \theta}{\varepsilon_2 [\xi_2(k) \cos \theta + (\alpha + \xi_1(k)) \sin \theta]} \quad (20)$$

The proposed controller consists of two parts. The first part is proportional term and the second part is modification of flow. The parameter k and θ needs to be tuned, which represent speed of achievement reference

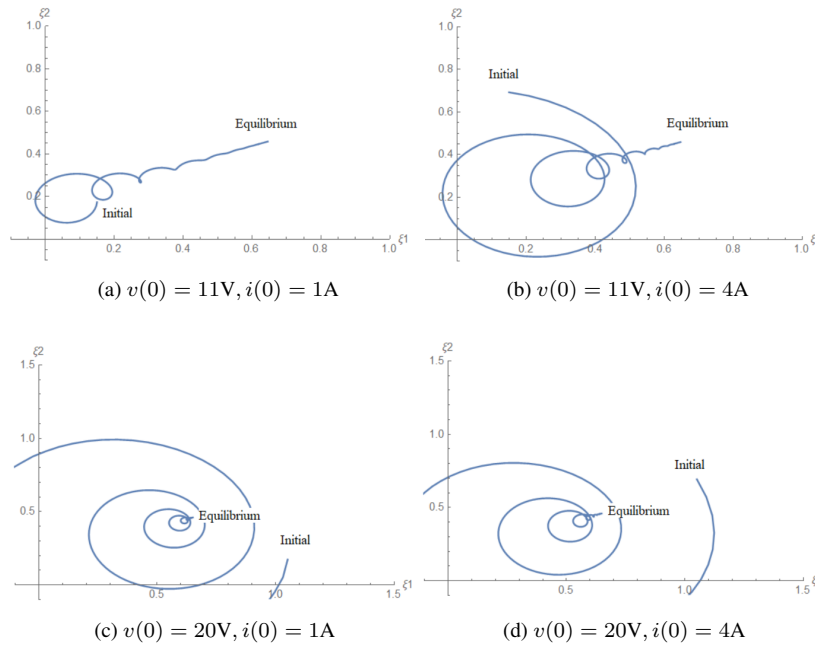


Figure 4. Phase trajectory of PID controller

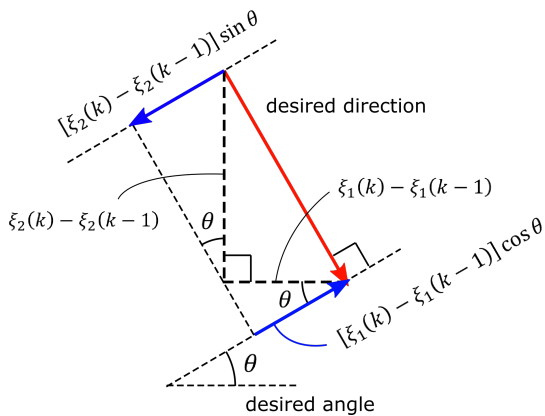


Figure 5. Modification of flow

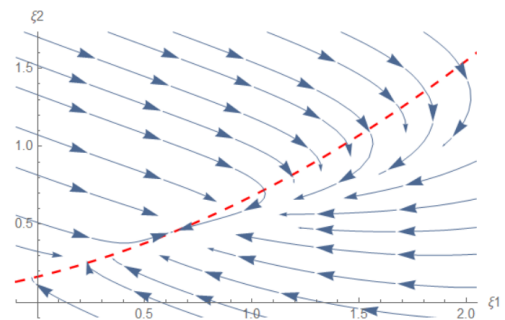


Figure 6. Phase plane of proposed controller ($V_{ref} = 16V$)

and flow direction respectively. The best value of k and θ are 0.06 and (-0.35π) respectively. The phase plane of proposed controller is shown in Figure 6. The phase plane shows that the controller can handle any initial point on CCM boost converter. Any initial points are pushed into the equilibrium line (dashed line in Figure 3) without passing the negative area of ξ_1 and ξ_2 .

6. SIMULATION

This section simulates the proposed controller response on time domain. The proposed controller (20) is compared with PID controller (18) which described in Section 4. The simulation is conducted in several initial points. The comparison between proposed and PID controller are shown in Figure 7. The PID controller response is shown in dashed-line while the proposed controller is expressed in solid blue line. The proposed controller show smother response than PID controller. In the PID controller, the output voltage goes to less than input voltage 10 V. This condition is not proper for boost converter characteristic which must greater than the input voltage. Moreover, the phase plane of proposed controller on various reference point is shown in Figure 8. Figure 8 gives clear information that the proposed controller has capability to handle any references point without entering the negative area of states.

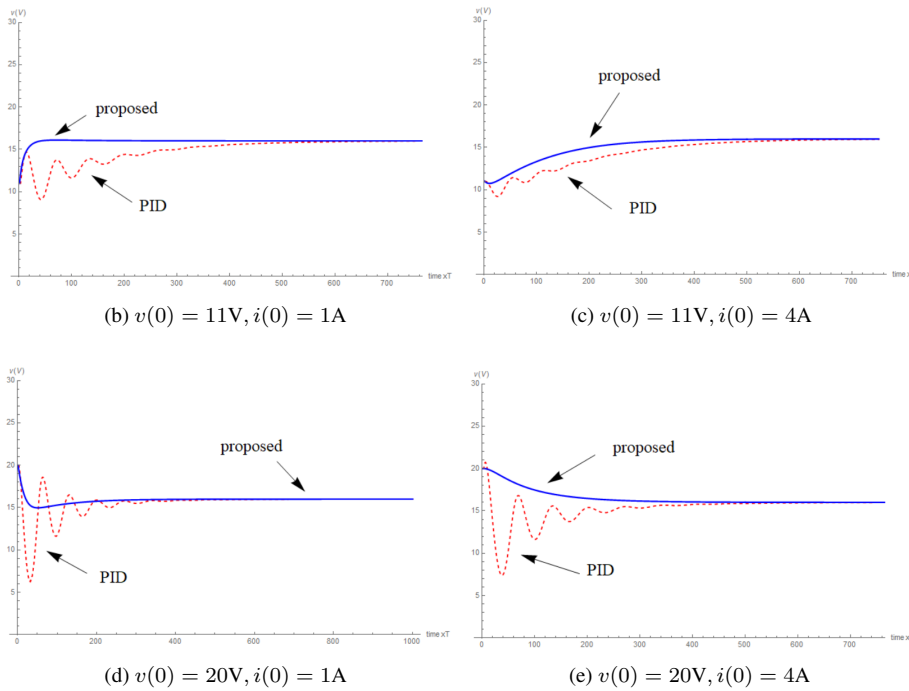


Figure 7. Comparison between proposed and PID on several initial points

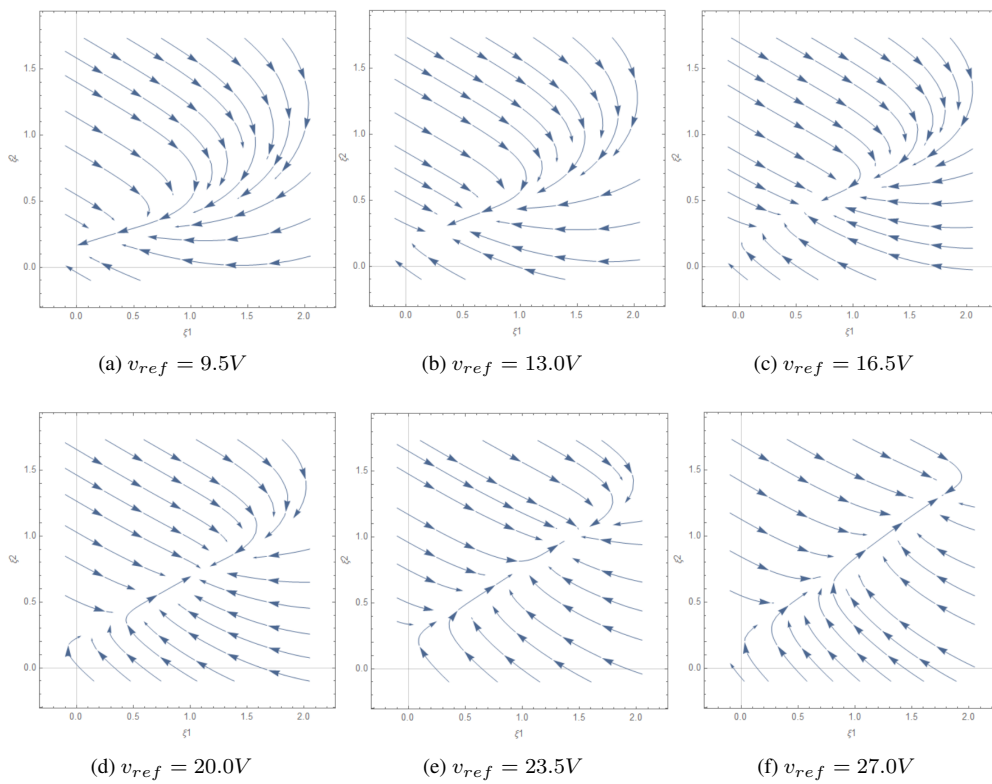


Figure 8. Proposed phase plane of the various reference point

7. CONCLUSION

This paper has pointed out that the PID control does not guarantee that boost converter always works within CCM. This paper has formulated a discrete approximation of CCM boost converter, and proposed a

novel nonlinear controller that guarantees that boost converter always works properly within CCM for any initial condition. Moreover, the proposed controller has advantage more than PID that it can handles any reference point without tuning the parameters again.

REFERENCES

- [1] S.D. Stallon, K.V. Kumar, S.S. Kumar, "High Efficient Module of Boost Converter in PV Module," *International Journal of Electrical and Computer Engineering*, vol. 2, pp. 758-781, 2012.
- [2] S. Bhat, H.N. Nagaraja, "Effect of Parasitic Elements on the Performance of Buck-Boost Converter for PV Systems," *International Journal of Electrical and Computer Engineering*, vol. 4, pp. 831-836, 2014.
- [3] A. Boulahia, K. Nabti, H. Benalla, "Direct Power Control for AC/DC/AC Converters in Doubly Fed Induction Generators Based Wind Turbine," *International Journal of Electrical and Computer Engineering*, vol. 2, pp. 425-432, 2012.
- [4] T.Z. Khaing, L.Z. Kyin, "Control Analysis of Stand-Alone Wind Power Supply System with Three Phase PWM Voltage Source Inverter and Boost Converter," *International Journal of Electrical and Computer Engineering*, vol. 5, pp. 798-809, 2015.
- [5] V. Ramesh, Y.K. Latha, "An Interleaved Boost Converter Based PFC Control Strategy for BLDC motor," *International Journal of Electrical and Computer Engineering*, vol. 5, pp. 957-966, 2015.
- [6] T. Raghu, S.C. Sekhar, J.S. Rao, "SEPIC Converter based-Drive for Unipolar BLDC Motor," *International Journal of Electrical and Computer Engineering*, vol. 2, pp. 159-165, 2012.
- [7] M.Z. Hossain, N.A. Rahim, J.a. Selvaraj, "Recent progress and development on power DC-DC converter topology, control, design and applications: A review," *Renewable and Sustainable Energy Reviews*, vol. 81, pp. 205-230, 2017.
- [8] Z. Shen, N. Yen, H. Min, "A Multimode Digitally Controlled Boost Converter with PID Autotuning and Constant Frequency/ Constant Off-time Hybrid PWM Control," *IEEE Transaction on Power Electronics*, vol. 26, pp. 2588-2598, 2011.
- [9] L.Guo, M.Aqil, D.S. Zinger, J. Wang, "Design of a digital control system for DC-DC converter to power electromagnets," *IEEE Industry Applications Society Annual Meeting*, pp. 1-5 2013.
- [10] R. W. Erickson, D. Maksimovic, "Fundamental of Power Electronics," Kluwer Academic Publishers, Netherland, 2001.
- [11] S. Chander, P. Agarwal, I. Gupta, "FPGA-based PID Controller for DC-DC Converter," *IEEE Joint International Conference on Power Electronics, Drives and Energy Systems & Power India*, 2010.
- [12] M.M. Peretz, S. Ben-Yaakov, "Time-Domain Design of Digital Compensators for PWM DC-DC Converters," *IEEE Transaction on Power Electronics*, vol. 27, pp. 284-293, 2012.
- [13] Y.I. Son, I.H. Kim, "Complementary PID Controller to Passivity-Based Nonlinear Control of Boost Converters With Inductor Resistance," *IEEE Transactions on Control Systems Technology*, vol. 20, pp. 826-834, 2012.
- [14] O. Ibrahim, N.Z. Yahaya, N. Saad, "Comparative studies of PID controller tuning methods on a DC-DC boost converter," *IEEE International Conference on Intelligent and Advanced Systems (ICIAS)*, August 2016.