The Interaction between Load Circuits and Decision of Frequency for Efficient Wireless Power Transfer

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Article Info

ABSTRACT

This paper derives an expression of efficiency of wireless power transfer on a situation that there are two devices towards one AC power supply. The interaction between a power supply and load is paid attention on a conventional wireless power transfer system, in contrast, the interaction between loads must be taken account of on the situation too. This is attributed to a possibility that a load disturbs the energy transmitted from a power supply to another load. Moreover each load needs different frequency of power supply for the ideal transfer since they have different natural frequencies on many situations. This paper models a circumstance that there are a power supply and two loads with a state space equation, and proposes how to decide a frequency of power supply to realize efficient transfer for each load.

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1. INTRODUCTION

Wireless Power Transfer (WPT) system is focused on the areas of energy, information, communication, and control, etc. In WPT, electric power is transferred on electromagnetic field by coils based on some electromagnetic theory. The basic theory of transfer electromagnetically was established by Nikola Tesla[1], and WPT is researched actively from the study of WPT on strongly coupled magnetic resonance[2].

The various methods are attempted according to some papers to improve efficiency and power, and to reduce energy loss of WPT. [3] focuses on circuit topology, and it mentions that the combination of series or parallel of circuit affects the intensity of power. A phase difference between the two transmission coils is controlled to increase efficiency[4]. The several coils are utilized for transmission of power to enlarge power, efficiency, and the distance between a power supply and loads[5][6].

Moreover WPT is applied for the various devices, places, and situations. In the medical area, a capsule endoscopy is charged from the outside of human body[7], and an effect to human body is investigated because the electromagnetic wave is generated on transmission wirelessly[8]. In the area of semiconductor, an application of WPT for the semiconductors is examined on low power and high frequency[9]. In terms of the long distance transmission, charge from a power supply on the ground to a balloon at high altitude is reviewed with electric propulsion[10].

As a merit of WPT, charging several devices simultaneously by closing them near a power supply is listed since WPT system does not need any connection of wire. However, when the device A is charged by a power supply, the other device B which is put near A may disturb exchanging energy between the power supply and A. Therefore the interaction between devices must be investigated in addition to the transmission of energy from power supply to loads[11].

In this paper, the exchange of energy between a power supply and two loads is analyzed with a mathematical model based on modern control theory. Especially the coupling coefficients between a transmitting coil and device coil, and between device coils are focused for finding transportation efficiency. Finally, the effect of

a load circuit which is not aimed to transfer energy is revealed, and the optimal angular frequency to maximize efficiency is obtained.

2. DESIGN OF WPT CIRCUIT AND DERIVATION OF MATHEMATICAL MODEL

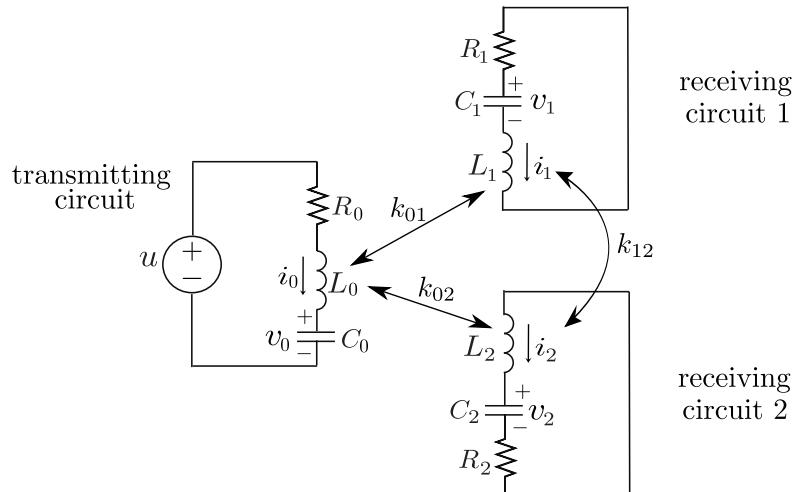At first, a circuit treated in this paper is shown as below.

Figure 1. A WPT circuit which has two loads

In Figure 1, the left side is a transmitting circuit, and the circuits on the right side are receiving circuits, also two receiving circuits are distinguished as receiving circuit 1 and receiving circuit 2 each other. These three circuits are connected electromagnetically with three coils L_0, L_1, and L_2. R_0 is a parasitic resistance in the transmitting circuit, and R_1 and R_2 are the load resistances on each receiving circuit, and C_0, C_1, and C_2 are parasitic capacitances on each circuit. k_{01}, k_{02}, and k_{12} are the coupling coefficients between each coil.

In this section, the optimal angular frequency of power supply which maximizes efficiency is derived. Then a mathematical model is found to derive the expressions of power and efficiency in the following [12].

$$\dot{x} = Ax + Bu, \; x = \begin{bmatrix} v_0 & v_1 & v_2 & i_0 & i_1 & i_2 \end{bmatrix}^{\mathrm{T}} \tag{1}$$

$$A = \frac{1}{\Delta} \begin{bmatrix} 0 & 0 & 0 & \frac{\Delta}{C_0} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\Delta}{C_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\Delta}{C_2} \\ \Delta_{12} & \delta_2 & \delta_1 & \Delta_{12}R_0 & \delta_2 R_1 & \delta_1 R_2 \\ \delta_2 & \Delta_{02} & \delta_0 & \delta_2 R_0 & \Delta_{02}R_1 & \delta_0 R_2 \\ \delta_1 & \delta_0 & \Delta_{01} & \delta_1 R_0 & \delta_0 R_1 & \Delta_{01}R_2 \end{bmatrix}, \; B = \frac{1}{\Delta} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\Delta_{12} \\ -\delta_2 \\ -\delta_1 \end{bmatrix}$$

$$\Delta = L_0 L_1 L_2 (k_{01}^2 + k_{02}^2 + k_{12}^2 - 2k_{01}k_{02}k_{12} - 1)$$

$$\Delta_{01} = L_0 L_1 (1 - k_{01}^2)$$

$$\Delta_{02} = L_0 L_2 (1 - k_{02}^2)$$

$$\Delta_{12} = L_1 L_2 (1 - k_{12}^2)$$

$$\delta_0 = L_0 \sqrt{L_1 L_2}(k_{01}k_{02} - k_{12})$$

$$\delta_1 = L_1 \sqrt{L_2 L_0}(k_{12}k_{01} - k_{02})$$

$$\delta_2 = L_2 \sqrt{L_0 L_1}(k_{02}k_{12} - k_{01}).$$

This model is a state space equation to obtain the state solutions, and it has the state variables $v_0, v_1, v_2, i_0, i_1, i_2$, which are voltages and currents at each capacitor and coil. The above equation is solved with regard to these variables to find the state solutions, powers, and efficiencies.

3. FINDING OF EFFICIENCY FOR OBTAING THE OPTIMAL ANGULAR FREQUENCY

3.1. Derivation of the expressions of efficiencies

By the mathematical model, a mathematical expression of efficiency is obtained. Efficiency is defined as the ratio of the load power and the input power, and two efficiencies must be distinguished because there are two loads in Figure 1. In this paper, the two efficiencies η_1 and η_2 are written as below.

$$\eta_1 = \frac{P_1}{P_0}$$
$$\eta_2 = \frac{P_2}{P_0}. \tag{2}$$

Where P_0 is the power of power supply, and P_1 and P_2 are the load powers of R_1 and R_2 each other. From the equation (1), η_1 and η_2 are found theoretically, however it is difficult to describe all of the expression of η_1 and η_2, and therefore the second order approximation as regards k_{01}, k_{02}, and k_{12} is applied because these values are less than 1. The expressions of η_1 and η_2 with second order approximation are derived as follows.

$$\eta_1 = \frac{P_1}{P_0} = \frac{k_{01}^2 R_1 L_0 L_1 C_1^2 \omega^4}{R_0 L_1^2 C_1^2 \omega^4 + R_0 (R_1^2 C_1 - 2L_1) C_1 \omega^2 + R_0}$$
$$\eta_2 = \frac{P_2}{P_0} = \frac{k_{02}^2 R_2 L_0 L_2 C_2^2 \omega^4}{R_0 L_2^2 C_2^2 \omega^4 + R_0 (R_2^2 C_2 - 2L_2) C_2 \omega^2 + R_0}. \tag{3}$$

k_{12} is not contained in the expressions (3), hence it is obvious that its effect to efficiency is little compared to k_{01} and k_{02} in terms of efficiency. Moreover the optimal angular frequencies ω_{opt1} and ω_{opt2} which maximize η_1 or η_2 each other are obtained from expressions (3) as below.

$$\omega_{\text{optn}} = \sqrt{\frac{2}{(2L_\text{n} - R_\text{n}^2 C_\text{n}) C_\text{n}}}. \tag{4}$$
$$n = 1 \text{ or } 2$$

The desirable angular frequency for efficient WPT has been revealed in this section. Furthermore a numerical calculation is shown by the expression (4) in the following section.

3.2. Numerical calculation of efficiencies

In the former section, we find the optimal frequency by the mathematical model (1). In this section, the variation of efficiency with respect to angular frequency is investigated based on the expression (4). From the equation (3), a numerical calculation of η_1 and η_2 with practical values of circuit elements (Table 1) is shown in Figure. 2.

Table 1. values of elements

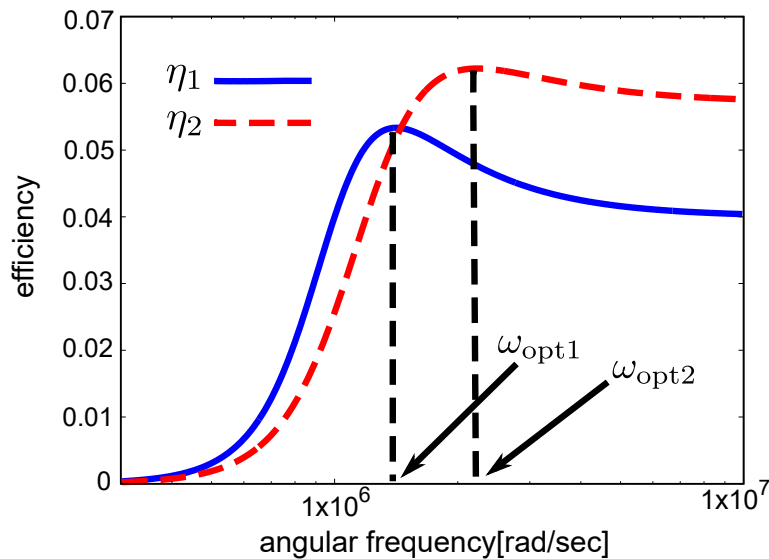|			
R_0	1Ω	C_0	1nF
R_1	10Ω	C_1	100nF
R_2	100Ω	C_2	10nF
L_0	1μH	k_{01}	0.2
L_1	10μH	k_{02}	0.2
L_2	70μH	k_{12}	0.2

Figure 2. A numerical calculation of efficiency

By the expression (4), we find that $\omega_{\mathrm{opt1}} = 1.41 \times 10^6 [\mathrm{rad/sec}]$, $\omega_{\mathrm{opt2}} = 2.24 \times 10^6 [\mathrm{rad/sec}]$, and η_1 and η_2 are maximized with these angular frequencies each other. In many cases, the load circuits have its unique optimal frequency which maximizes efficiency, and therefore it is rare to maximize efficiencies from a power supply to the several loads simultaneously as shown in Figure 2. And it is seen that η_1 and η_2 converge to 0 at low frequency, and constant value at high frequency. The correctness is proved by taking the limit of the expression (3) with regard to ω.

4. CONCLUSION

In this paper, we supposed the situation that there are two loads towards one power supply, and found the optimal angular frequency for efficient WPT. From the expression (4), it is obvious that the optimal angular frequency is not affected by the elements on transmitting circuit. Thus the elements in receiving circuits are significant when efficient WPT is desired, and the ideal frequency is different with each receiving circuit. Moreover the interaction between the coils on receiving circuits is little compared with the interaction between the coils on transmitting and receiving circuit. Hence the coupling between a transmitting coil and each receiving coil is more significant than the coupling between receiving coils.

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