

## Robust Multivariable Controller Design with the simultaneous $H_2/H_\infty/\mu$ for a Single Person Aircraft

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### ABSTRACT

In a physical system several targets are normally being considered in which each one of nominal and robust performance has their own strengths and weaknesses. In nominal performance case, system operation without uncertainty has decisive effect on the operation of system, whereas in robust performance one, operation with uncertainty will be considered. The purpose of this paper is a balance between nominal and robust performance of the state feedback. The new approach of present paper is the combination of two controllers of  $\mu$  and  $H_2/H_\infty$ . The controller for robust stability status, nominal performance, robust performance and noise rejection are designed simultaneously. The controller will be achieved by solving constraint optimization problem. The paper uses a simultaneous  $H_2/H_\infty/\mu$  robust multivariable controller design over an X-29 Single Person aircraft. This model has three inputs and three outputs, where the state space equations of the system correspond to an unstable one. Simulation results show the effectiveness and benefits of the method.

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### 1. INTRODUCTION

Unmodeled dynamics, non linearity of systems and the availability of disturbances usually cause that linear control systems theory never reaches the ideal solution. For this, several targets are attended in a control system. 1) Robust Stability: meaning that system will be stable with uncertainty. 2) Nominal performance: without considering uncertainty, the fault of system will be minimized. 3) Robust performance: with considering uncertainty, the fault of system will be minimized. To consider robust performance, we use  $\mu$  analysis. Usually, the availability of robust performance causes extreme limitations on the controller and prevents it from reaching its feasible condition, and in the case of achieved feasible condition, it increases the grade of the controller, and the resulted controlling signal increases and causes to saturate actuator. But in some systems it will be used, compulsorily. For example, in power systems which transient response is decisive, robust performance will be considered. 4) Operating limitation on controlling signal: increase of controlling signal causes saturation of actuators.  $H_2$  norm essence can be responsible for such target. 5) Minimizing disturbance effect: distortion can cause undesirable effect of transient response, so reducing the effect of disturbance, is one of the controlling targets. Mixed norm of  $H_2$  and  $H_\infty$  can be a useful strategy, to reach noted controlling targets. To date, number of studies has been performed on the mixed norm and multi-objective control. This paper attempts to present an implementation approach for controlling systems. One of the useful techniques used on controlling system is the robust state feedback. Prevalently, it is possible to

design a controller which regularly includes 5 noted targets above. In a more explicit description, controller includes two parts; the first one uses mixed  $H_2$  and  $H_\infty$  norm and the other uses  $\mu$  synthesis. These two parts, include some weights which have important roles in systems control, because each one of 2 and 3 targets, has its own definiteness and their combinations can create a new solution. Practically, we look for minimizing faults. If the available error function is not desirable, usage of a suitable weight function can lead to the target. So, designing weight function is extremely important. At first, a controlling problem will be changed to LFT standard form, considering uncertainty, then status equations will be written and constraint's weight function will be determined to reach the robust controlling targets. Then, using robust state feedback method, it will be is the time to select them again; using state feedback and weight functions repetition methods to supply the robust performance of system in an acceptable way. The first formulation of  $H_\infty$  control problem performed in 1981 by Zames. Next to Zames, Doyle, Zhou, and Glover were premiers of robust control. To date, a vast number of researches have performed studies in robust control,  $H_2$  control and  $H_\infty$  control. Doyle et al in [1] analyzed the state space with  $H_\infty$  and  $H_2$  standard form and its solving. The conditions of solving problem and its solution using Hamiltonian matrix introduction are of importance of this paper. This paper is a comprehensive reference that has been beneficial in many other research works. Doyle et al in [2] present a tutorial overview of linear fractional transformations (LFTs) and the role of the structured singular value,  $\mu$ , and linear matrix inequalities (LMIs) in solving LFT problems. Rotea et al in [3] combined  $H_2/H_\infty$ . Two important approaches are presented. 1) Minimal  $H_2$ -norm subject to an  $H_\infty$ -norm constraint. 2) Simultaneous  $H_2/H_\infty$  optimal control. In each step, problem formulation and controller were performed. Doyle et al in [4] are shown that different of a mixed  $H_2$  and  $H_\infty$  infinity norm arise from different assumptions on the input signals. Lanzon in his PHD thesis chooses the weight functions in  $\mu$  and  $H_\infty$  design [5]. Akbar et al in [6], study a mixed  $H_2/H_\infty$  control law is derived using auxiliary cost minimization approach for continuous time linear time invariant singularly Perturbed System (SPS). The time responses of closed-loop LQG, mixed  $H_2/H_\infty$  and  $H_\infty$  control system for a unit step input and their robustness measures such as Gain and Phase margin for the open-loop systems are analyzed. See for instance the references [7]–[9] for the mixed norm control. Tan et al in [10], Robust load frequency control for power systems is discussed. Keel et al in [11], show by examples that optimum and robust controllers, designed by using the  $L_1$ ,  $H_\infty$ ,  $H_2$ , and  $\mu$  formulations. The rest of this paper is as follows. Section 2 the discription of aircraft model. Section 3 establishes the problem to be addressed, the  $H_2/H_\infty$ ,  $\mu$  and  $H_2/H_\infty/\mu$  combination control will be demonstrated. Section 4, illustrates the approach and the results of simulations will be discussed. Section 5 presents the conclusions.

## 2. AIRCRAFT MODEL

In an airplane, five main sections could be listed as: motor, body section, landing system and wheels, wing, and tail. The pitch angle of an airplane is controlled by adjusting the angle (and therefore the lift force) of the rear elevator. The aerodynamic forces (lift and drag) as well as the airplane's inertia are taken into account. The X-29 aircraft is a recent example of a control configured vehicle that was designed with a high degree of longitudinal static instability (up to 35 percent at low subsonic speeds). The vehicle is stabilized by a full-authority, fly-by-wire flight control system. Linear models were used extensively prior to flight to determine the close loop stability, controllability, and handling qualities with the various control system modes through the flight envelope. This section describes the commercials aircraft models which is implemented. In [12] which is a comprehensive report of NASA, it has been researched over X-29 state equation and model. In [13] has been designed only the  $H_\infty$  controller over X-29. The X-29 airplane is a relatively small, single seat, high-performance aircraft powered by a single F404-GE-400 engine (General Electric, Lynn, Massachusetts). Empty weight is 6350 kg. The vehicle incorporates a forward-swept wing with close-coupled canards to provide a low-drag configuration. The airplane physical characteristics are presented in table 1. The aircraft model is obtained by linearizing the nonlinear equations of motion about a 280 ft/sec (307Km/hr) landing configuration [12]. The three input three output model which describes the longitudinal dynamics is given as follows [12]-[13].

Table 1. X-29 physical characteristics

Maximum thrust force	8130 Nm	Weight	6350 kg	Wing span	8.29 m
Angel attack	$\alpha$	Canard area	3.437 m <sup>2</sup>	Wing area	17.185 m <sup>2</sup>
Strake flap position	$\delta_{stf}$	Symmetric flap position	$\delta_{sf}$	Canard position	$\delta_c$
Pitch Euler angel	$\theta$	Pitch rate	$\dot{\theta}$	Horizontal speed	$v$

$$\dot{x} = Ax + Bu, y = Cx, x = \begin{bmatrix} v - (\text{ft/sec}) \\ \alpha - (\text{rad}) \\ \dot{\theta} - (\text{rad/sec}) \\ \theta - (\text{rad}) \end{bmatrix}, u = \begin{bmatrix} \delta_c - (\text{deg}) \\ \delta_{sf} - (\text{deg}) \\ \delta_{stf} - (\text{deg}) \end{bmatrix}, y = \begin{bmatrix} \theta - (\text{rad}) \\ v - (\text{ft/sec}) \\ \alpha - (\text{rad}) \end{bmatrix}$$

$$A = \begin{bmatrix} 0.0427 & -8.5410 & 0.4451 & -32.16 \\ 0.0008 & 0.5291 & 0.9896 & 0 \\ 0.0004 & 3.5420 & 0.2228 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0.0338 & 0.0939 & 0.0049 \\ 0.001 & 0.0013 & 0.0004 \\ 0.0272 & 0.0057 & 0.0135 \\ 0 & 0 & 0 \end{bmatrix} \tag{1}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 57.3 \\ 1 & 0 & 0 & 0 \\ 0 & 57.3 & 0 & 0 \end{bmatrix}$$

**3. PROBLEM STATEMENT**

**3.1. H<sub>2</sub>/H<sub>∞</sub> Controller**

Existence of uncertainty created due to an uncertain and erratic input (noise and disturbance) and Un-modeled dynamics cannot be described completely and precisely as a true system by a mathematical modeling. On the other hand, a true system should contain the following important objects: robust stability, robust and nominal performance, settling time, maximum over shoot and etc., which try to gain these objectives about the controlling problem [14]-[17]. The type of uncertainty is another important factor in the system analysis.

**Theorem 1**(small gain Theorem) supposes  $M \in RH_{\infty}$  and let  $\gamma > 0$ , then the interconnected system shown in Figure 1 is well-posed and internally stable for all  $\Delta(s) \in RH_{\infty}$  with  $\|\Delta\|_{\infty} < \gamma^{-1}$  if and only if  $\|M\|_{\infty} < \gamma$  [19].

Consider additive uncertainty shown in Figure 2. In this case, robust stability task is:

$$q = (I + KG)^{-1} KP \Rightarrow \|(I + KG)^{-1} K\|_{H_{\infty}} < \gamma^{-1} \tag{2}$$

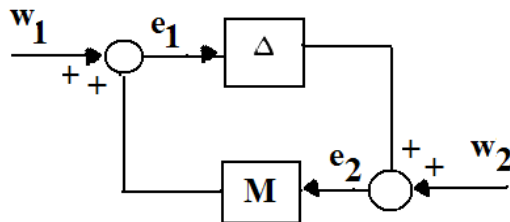


Figure 1. M – Δ Model

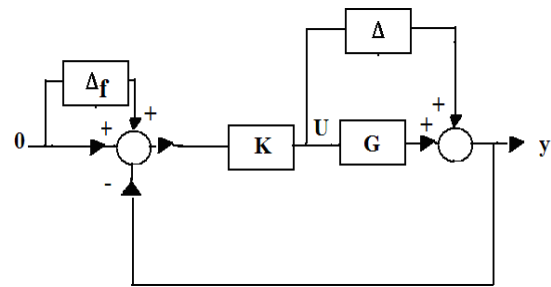


Figure 2. Additive uncertainty

The objective for the inner loop control is to design a state feedback law such that the closed loop system satisfies the following performance specifications,

**Objective 1:** if  $\Delta = 0$  then  $\|FS\|_{\infty} < 1$  (nominal performance).  $S = (I + GK)^{-1}$  (S is sensitivity function).

**Objective 2:** if  $\Delta \neq 0$  then the system is robust stability.  $M = (I + KG)^{-1} K$ ,  
 if  $\bar{\sigma}(\Delta(j\omega)) \leq \gamma(j\omega) \Rightarrow \|\gamma(S)M\|_{\infty} < 1$

**Objective 3:** n is white noise with one PSD (power spectral density). H<sub>2</sub> norm, caused due to a decrease in the controlling signal.  $\|RT_n U_1\|_{H_2} < 1$  (To minimize U<sub>1</sub> variance with noise input). F(s), R(s) and  $\gamma(s)$  is weighting function) from parseval equation and objective 3:

$$\|Y\|_2^2 = \int_0^\infty Y^T(t)Y(t)dt = \frac{1}{2\pi} \int_{-\infty}^\infty Y^*(j\omega)Y(j\omega)d\omega, Y(j\omega) = G(j\omega)U(j\omega) \Rightarrow \|Y\|_2^2 = \frac{1}{2\pi} \int_{-\infty}^\infty U^*(j\omega)G^*(j\omega)G(j\omega)U(j\omega)d\omega$$

$$\leq \frac{1}{2\pi} \sup \bar{\sigma}(G^*(j\omega)G(j\omega)) \int_{-\infty}^\infty U^*(j\omega)U(j\omega)d\omega = \|G(s)\|_\infty^2 \|U(s)\|_2^2 \Rightarrow \sup \frac{\|Y\|_2^2}{\|U\|_2^2} = \|G(s)\|_\infty^2 \rightarrow \frac{\|U_1\|_2}{\|n\|_2} \leq \|T_n U_1\|_\infty < 1$$

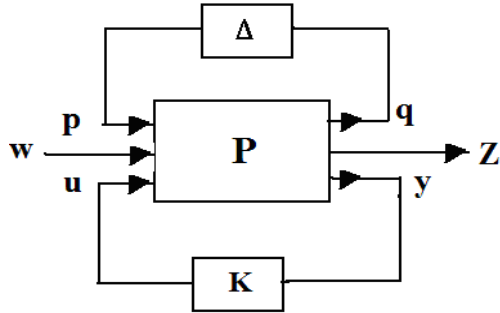


Figure 3. LFT Model

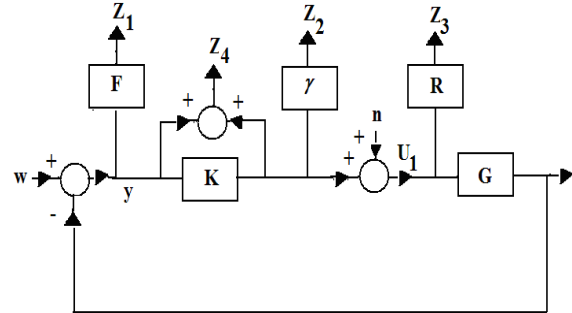


Figure 4. Graphical model of H2/H∞, μ combination problem with additive uncertainty

Then we have three tasks for controller design ( $\|FS\|_\infty < 1, \|\gamma(S)M\|_\infty < 1, \|T_n U_1\|_\infty < 1$ ), such that,

$$\left\| \begin{matrix} FS(K,G) \\ \gamma M(K,G) \\ RT((K,G)) \end{matrix} \right\|_\infty < 1 \quad (3).$$

A large class of systems with uncertainty can be treated as LFT (Linear Factorial

Transform). LFT model has been shown in Figure 3 [18] in which, W: the disturbance signals to the system which won't be a function of states of system, Z: the variable that will be controlled, P: the nominal open loop system, Y: the system measurable output.

**3.2. μ Controller**

Here we try to assess robust performance of this closed-loop system using μ -analysis. Robust performance condition is equivalent to the following structured singular value μ test [20].

$$\|T_{wz}(M, \Delta)\|_\infty < \gamma^{-1} \quad \forall \|\Delta\|_\infty < \gamma \Leftrightarrow \mu_{\Delta P}(M) < \gamma \quad \forall W \quad (4)$$

The complex structured singular value  $\mu_{\Delta(M)}$  is defined as  $\mu_{\Delta(M)} = \frac{1}{\min \{ \bar{\sigma}(\Delta) | \det(I - M\Delta) = 0 \}}$

Lower and upper bound of μ can be shown to be  $P(UM) \leq \mu_{\Delta}(M) < \min \bar{\sigma}(DMD^{-1})$ .

**3.3. New approach: The simultaneous H2/H∞/μ problem**

**Theorem 2:** consider a  $\bar{M}, \Delta_p$  system with  $\Delta_p = \begin{bmatrix} \Delta & 0 \\ 0 & \Delta_f \end{bmatrix}$ , let  $\beta > 0$  for all  $\Delta(s) \in M(\Delta)$  with  $\|\Delta\|_\infty < \frac{1}{\beta}$ , internally stable, and  $\|T_{wz}(\bar{M}, \Delta)\|_\infty < \beta$  if and only if  $\sup \mu_{\Delta_p}(\bar{M}) < \beta$  [20]. This can be shown as follows:

$$\mu_{\Delta_p} \leq \sqrt{\|\bar{M}_{11}(j\omega)\|^2 + \|\bar{M}_{22}(j\omega)\|^2 + 2\|\bar{M}_{12}(j\omega)\| \|\bar{M}_{21}(j\omega)\|} \quad (5)$$

At additive uncertainty  $\bar{M} = \begin{bmatrix} -M & M \\ -S & S \end{bmatrix}$  such that  $\|S\|_\infty + \|\bar{M}\|_\infty < 1$  and at multiplication uncertainty

$$\bar{M} = \begin{bmatrix} -T & T \\ -S & S \end{bmatrix} \text{ such that } \|S\|_\infty + \|T\|_\infty < 1.$$

Lemma 1: consider a system with additive uncertainty, Then H2/H∞, μ controller will be designed in a way

that  $\left\| \begin{bmatrix} FS(K, G) \\ \gamma M(K, G) \\ RT((K, G)) \end{bmatrix} \right\|_{\infty} < 1$  and simultaneously  $\|S\|_{\infty} + \|\overline{M}\|_{\infty} < 1$ . We have shown the noted mathematical

problem, in Figure.4. State space equation for Figure 4 will be (6).

Lemma 2: consider a system with multiplication uncertainty, H2/H∞, μ controller will be designed in a way

that  $\left\| \begin{bmatrix} FS(K, G) \\ \gamma M(K, G) \\ RT((K, G)) \end{bmatrix} \right\|_{\infty} < 1$  and simultaneously  $\|S\|_{\infty} + \|T\|_{\infty} < 1$ . We draw the noted mathematical problem, in

Figure 6. State space equation for Figure 6 will be (7).

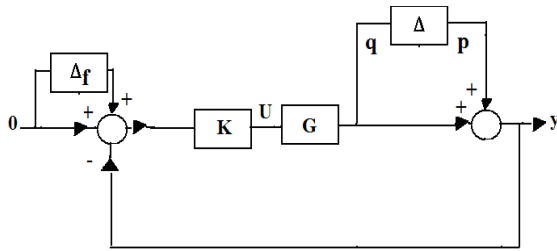


Figure 5. Multiplication uncertainty

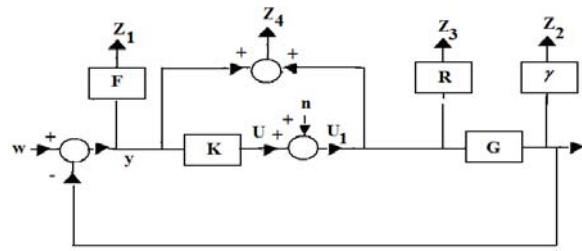


Figure 6. Graphical model of H2/H∞, μ combination problem With multiplication uncertainty

$$\begin{bmatrix} \dot{x} \\ \dot{x}_f \\ \dot{x}_\gamma \\ \dot{x}_R \end{bmatrix} = \begin{bmatrix} A & 0 & 0 & 0 \\ -B_f C & A_f & 0 & 0 \\ 0 & 0 & A_\gamma & 0 \\ 0 & 0 & 0 & A_R \end{bmatrix} \begin{bmatrix} x \\ x_f \\ x_\gamma \\ x_R \end{bmatrix} + \begin{bmatrix} 0 & B & B \\ B_f & -B_f D & -B_f D \\ 0 & 0 & B_\gamma \\ 0 & B_R & B_R \end{bmatrix} \begin{bmatrix} r \\ n \\ u \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} -D_f C & C_f & 0 & 0 \\ 0 & 0 & C_\gamma & 0 \\ 0 & 0 & 0 & C_R \\ -C & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ x_f \\ x_\gamma \\ x_R \end{bmatrix} + \begin{bmatrix} D_f & -D_f D & -D_f D \\ 0 & 0 & D_\gamma \\ 0 & D_R & D_R \\ I & -D & I-D \end{bmatrix} \begin{bmatrix} r \\ n \\ u \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} \dot{x} \\ \dot{x}_f \\ \dot{x}_\gamma \\ \dot{x}_R \end{bmatrix} = \begin{bmatrix} A & 0 & 0 & 0 \\ -B_f C & A_f & 0 & 0 \\ 0 & 0 & A_\gamma & 0 \\ 0 & 0 & 0 & A_R \end{bmatrix} \begin{bmatrix} x \\ x_f \\ x_\gamma \\ x_R \end{bmatrix} + \begin{bmatrix} 0 & B & B \\ B_f & -B_f D & -B_f D \\ 0 & B_\gamma D & B_\gamma D \\ 0 & B_R & B_R \end{bmatrix} \begin{bmatrix} r \\ n \\ u \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} -D_f C & C_f & 0 & 0 \\ D_f C & 0 & C_\gamma & 0 \\ 0 & 0 & 0 & C_R \\ -C & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ x_f \\ x_\gamma \\ x_R \end{bmatrix} + \begin{bmatrix} D_f & -D_f D & -D_f D \\ 0 & D_f D & D_f D \\ 0 & D_R & 0 \\ I & I-D & I-D \end{bmatrix} \begin{bmatrix} r \\ n \\ u \end{bmatrix} \quad (7)$$

#### 4. SIMULATION RESULTS

These simulations were performed using robust multi-objective control toolbox [22] and μ-Analysis and Synthesis Toolbox [23] of MATLAB®. In this section, simulation result of X-29 aircraft model was drawn. At first the design weight functions were drawn in Figure 7-a-b. By considering the practical experiments, the weighting functions are selected. Then, step response of S, M and T function in Figure 8-b-c-d are shown. Note that, as regards, the system is multi input-output, the weight and sensitivity functions are

in matrix form. The maximum controlling signal contains 12 units. The sign of success is the combination of nominal and robust performance, together. Reaching the targets with the minimum controlling signal is of the gains of noted controller. The results shown in the figures indicate that the unstable system, but the closed-loop system has appropriate results whereas it has been considered three objects (provision) for itself. In comparison with previous control methodologies, we see that the proposed  $H_2/H_\infty/\mu$  combination is although simpler and perhaps of lower implementation cost, gives acceptable and proper robust performance.

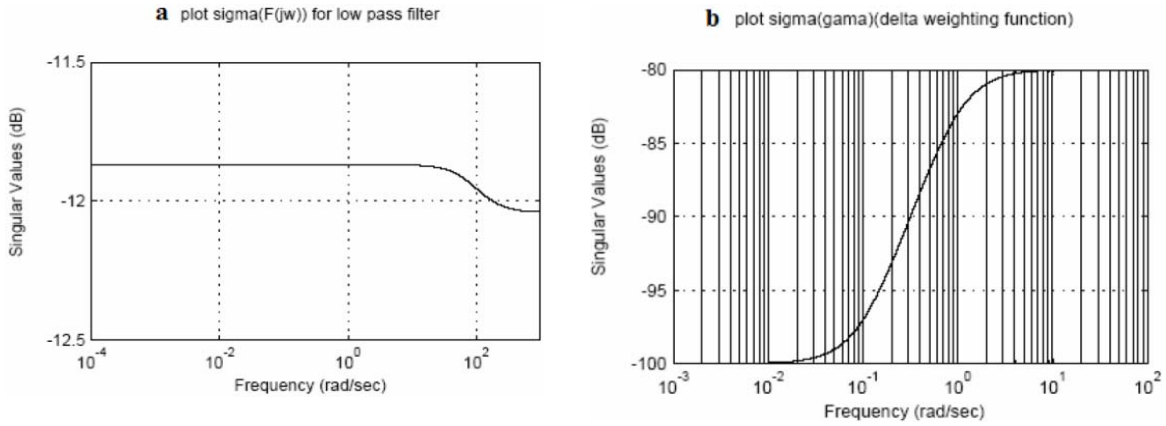


Figure 7. Singular value for weighting functions

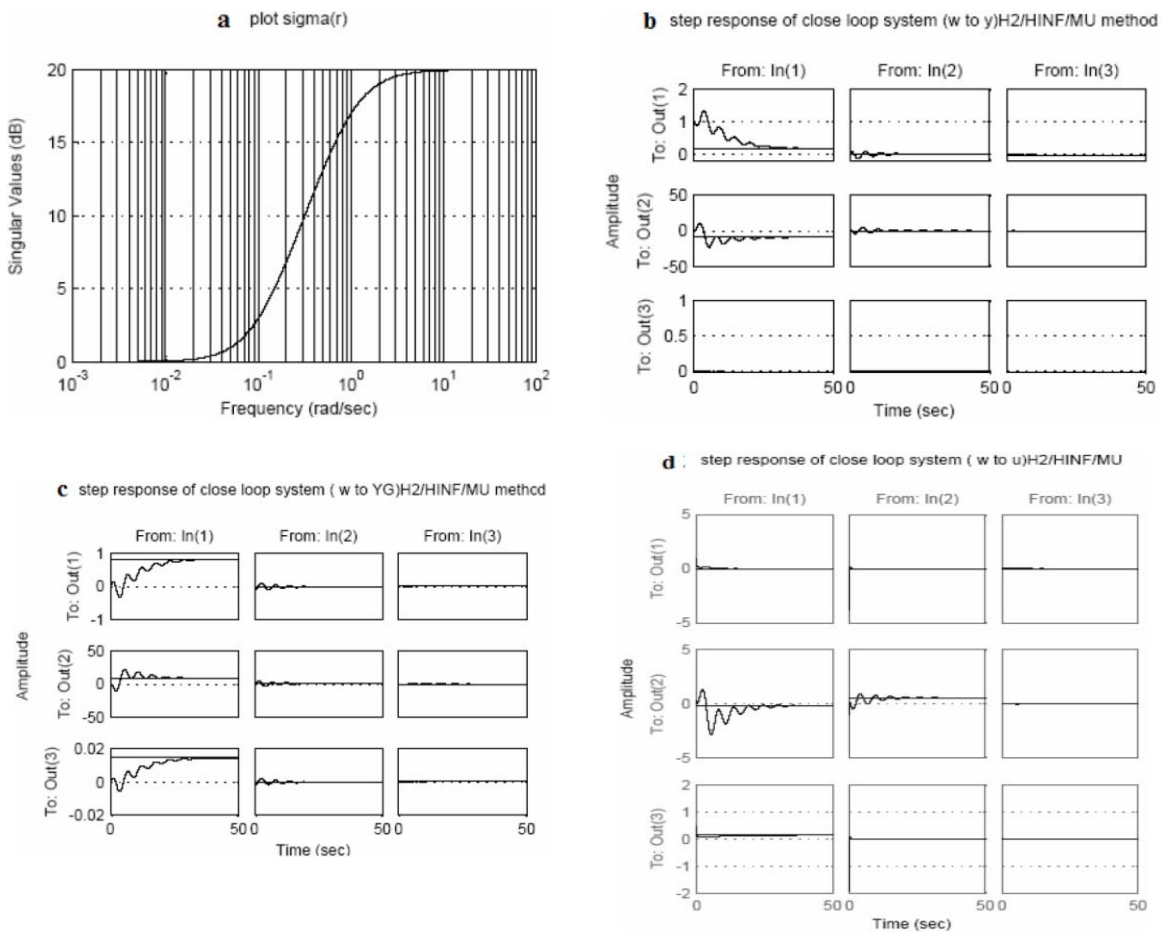


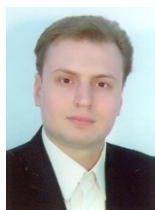
Figure 8. Singular value for weighting functions, Step response for S, M, T

## 5. CONCLUSION

Each of nominal and robust performance has their own strengths and weaknesses. Nominal performance means considering system operation without uncertainty, and has decisive effect on the operation of system. Robust performance means considering operation with uncertainty. It is obvious that whenever the singular values of controller are higher, systems performance is more desirable, but, also it provides higher chances of saturation occurrences. By using  $H_2$  norm, in conjunction with the design of appropriate filters or selection of other weighting functions, one can avoid over-increasing of the controlling signal which has an important role in determining the response of the close-loop system. This paper tends to reduce controlling signal, robust performance, and robust stability and to provide a new design for weight functions. The new approach presented in this paper utilizes a combination of two controllers of  $\mu$  and  $H_2/H_\infty$ . The controller for robust stability case, nominal performance, robust performance and noise rejection are designed simultaneously. Controller will be achieved from solving constraint optimization problem. Using two low pass filters and one high pass filter, we tended to optimize the solutions. This approach has been applied to X-29 air craft state space equations. The results shown in the figures indicate that the unstable system becomes stable in the presence of uncertainty using the proposed controller, and also show an appropriate desired performance.

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