

## A Study on the Suitability of Genetic Algorithm for Adaptive Channel Equalization

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### ABSTRACT

Adaptive algorithms such as Least-Mean-Square (LMS) based channel equalizer aim to minimize the Intersymbol Interference (ISI) present in the transmission channel. However the adaptive algorithms suffer from long training time and undesirable local minima during training mode. These disadvantages of the adaptive algorithms for channel equalization have been discussed in the literature. In this paper, we propose a new adaptive channel equalizer using Genetic Algorithm (GA) which is essentially a derivative free optimization tool. This algorithm is suitably used to update the weights of the equalizer. The performance of the proposed channel equalizer is evaluated in terms of mean square error (MSE) and convergence rate and is compared with its LMS and RLS counter parts. It is observed that the new adaptive equalizer based GA offer improved performance so far as the accuracy of reception is concerned.

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## 1. INTRODUCTION

In modern digital communication systems, the transmission of high-speed data through a channel is limited by intersymbol Interference (ISI) caused by distortion in the transmission channel. High-speed data transmission through channels with severe distortion can be achieved by designing an equalizer in the receiver that counteracts the channel distortion. In practice, the channel is time varying and is unknown in the design stage due to variations in the transmission medium. Thus, we need an adaptive equalizer that provides precise compensation over the time-varying channel and attempts to recover the transmitted symbols.

The most frequently used structure of equalizer is a transversal adaptive filter with an appropriate algorithm such as least mean square (LMS), recursive least squares (RLS), or QR-Decomposition-Based least squares lattice filter (QRD-LSL) [1]. The performances of the RLS and QRD-LSL algorithms are not dependent on the eigenvalue spread of covariance matrix, since the covariance matrix is inverted directly [1]. On the other hand, the LMS algorithm suffers from slow convergence in the case of large eigenvalue spread of the sample covariance matrix. However, these adaptive signal processing techniques employ large number of iterations to carry out channel equalization and thereby make their applications in real life prohibitive as they are computationally too expensive and are unsuitable for a fast dynamically changing channel as they require a latent time to collect the training data [2]. The convergence rate can be accelerated by use of the conjugate gradient (CG) method [3]. The goal of CG is to iteratively search for the optimum solution by choosing perpendicular paths for each new iteration. However, the above mentioned algorithms are based on the steepest descent algorithm, which is easy to implement but do not perform satisfactorily under high noise condition.

An online estimation of the channel and of the noise variance using a network of adaptive Kalman filters is presented in [4]. Other channel equalization approaches are based on nonlinear estimation using Neural networks [5]. However, most neural networks use the MSE as the cost function to be minimized by the network. The problems encountered by using neural networks in equalization are the slow rate of convergence and the possibility that the network does not reach the optimum MSE. On the other words, the network can get stuck in a local minimum. In this case, the network will not be able to optimize its parameters to the least MSE especially under high noise condition.

In this paper, a GA-based adaptive equalization is developed to solve these limitations. Genetic algorithm is based upon the process of natural selection and does not require gradient statistics. As a consequence, a GA is able to find a global error minimum [6-7]. Moreover, the GA with small population size and high mutation rates can find a good solution fast [8]. The organization of this paper is as follows. Section II, introduces the adaptive channel equalization system model and formalize the problem of adaptive algorithms. In Section III, a channel equalizer based on GA approach is presented. Simulation results are given in section IV and conclusions drawn in Section V.

## 2. BACKGROUND

The structure of the adaptive channel equalizer based on LMS algorithm is shown in Fig.1. As illustrated in figure, the received signal  $y(n)$  is different from the original signal  $x(n)$  because it was distorted by the overall channel transfer function  $C(z)$ , which includes the transmit filter, the transmission medium, and the receive filter. To recover the original signal  $x(n)$ , we need to process  $y(n)$  using the equalizer  $W(z)$ , which is the inverse of the channel's transfer function  $C(z)$  in order to compensate for the channel distortion. That is, we have to design the equalizer

$$W(z) = \frac{1}{C(z)}, \quad (1)$$

such that  $\hat{x}(n) = x(n)$ . As shown in Fig.1, an adaptive filter requires the desired signal  $d(n)$  for computing the error signal  $e(n)$  for the LMS adaptive algorithm.

During the training stage, the adaptive equalizer coefficients are adjusted by transmitting a short training sequence. This known transmitted sequence is also generated in the receiver and is used as the desired signal  $d(n)$  for the LMS algorithm. After the short training period, the transmitter begins to transmit the data sequence. In the data mode, the output of the equalizer  $\hat{x}(n)$  is used by a decision device to produce binary data. Assuming that the output of the decision device is correct, the binary sequence can be used as the desired signal  $d(n)$  to generate the error signal  $e(n)$  for the LMS algorithm. The signal samples at the equalizer input are of the form:

$$y(n) = \sum_{j=0}^{N-1} h(j)x(n-j) + v(n) \quad (2)$$

where  $x(n)$  denotes the data sample at time index  $n$ ,  $v(n)$  is the additive noise with the variance  $\sigma_v^2$ , and  $h(j)$  is the channel impulse response. The data samples take on values of  $x(n) = \pm 1$ , and the noise is assumed to be independent.

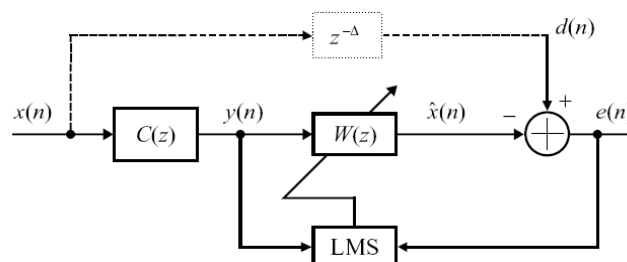


Figure. 1 Cascade of channel with LMS channel equalizer

The equalizer output is:

$$\hat{x}(n) = \mathbf{w}^T(n)\mathbf{x}(n) \quad (3)$$

where  $\mathbf{x}(n) = [x(n), x(n-1), x(n-2), \dots, x(n-N+1)]^T$  is the vector of data sample at the equalizer input, and  $\mathbf{w}(n) = [w(n), w(n-1), w(n-2), \dots, w(n-N+1)]^T$  is the vector of weighting coefficients of the adaptive filter.

The output  $\hat{x}(n)$  is used in estimating the transmitted data symbol  $x(n-K)$ , with  $K$  denoting the delay. The  $n$ -th output error sample is:

$$e(n) = \hat{x}(n) - x(n-K) \quad (4)$$

The weighting coefficients in the LMS algorithm are updated according to the following expression [1]:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e^H(n)\mathbf{x}(n) \quad (5)$$

Here,  $\mu$  is the step size which controls the rate of convergence of the LMS algorithm. The output mean square error (MSE) is:

$$\varepsilon(n) = E[e^2(n)] = \mathbf{w}^T(n)\mathbf{R}(n)\mathbf{w}(n) + E[x^2(n)] - 2\mathbf{w}^T(n)E[\hat{x}(n)x(n-K)] \quad (6)$$

where  $\mathbf{R} = \frac{1}{N} \sum_{n=1}^N \mathbf{x}(n)\mathbf{x}(n)$ . The average output MSE after  $n$ -th iteration can be expressed as:

$$\varepsilon_{avg}(n) = \varepsilon(n) + E[\mathbf{V}^T(n)\mathbf{R}\mathbf{V}(n)] \quad (7)$$

where  $\varepsilon(n)$  is the minimum MSE as given by (6) for optimal weighting coefficients vector  $\mathbf{w}_{opt}(n)$ , i.e. Wiener vector and  $\mathbf{V}(n) = \mathbf{w}(n) - \mathbf{w}_{opt}(n)$  is the weighting coefficient error vector. In the steady state, the MSE above  $\varepsilon(n)$  in (7) is known as the excess MSE. The weights  $\mathbf{w}(n)$  do not reach to their optimum values due to the mean square error (MSE) being trapped to local minimum. In other words true Weiner solution is not achieved because of gradient based training. The bit-error-rate (BER) performance of the equalizer further degrades when data transmission takes place through channels.

One of the main drawbacks of the adaptive algorithms is that the algorithms must go through many iterations before satisfactory convergence is achieved. This means they suffer from long training time and undesirable excess MSE during training. The excess MSE can increase significantly under high noise condition which means that the adaptive algorithms based on steepest descent can get stuck in a local minimum and therefore there is possibility that during training of the equalizers, its weights do not reach to their optimum values due to the excess MSE. To prevent this problem, a GA is proposed which is essentially does not require gradient based training algorithm as shown in the following section.

### 3. GA-BASED CHANNEL EQUALIZATION

The LMS, and RLS based channel equalizers aim to minimize the ISI present in the linear dispersive communication channel. These are gradient based learning algorithms and therefore there is possibility that during training mode of the channel equalizer, its weights do not reach to their optimum values due to the mean square error (MSE) being trapped to local minimum. In this section we propose a new adaptive channel equalizer using GA optimization technique which is essentially a derivative free optimization tool. This algorithm is used to update the weights of the equalizer as explained in the following steps:

1. Simulate the signals as illustrated in Fig.2. In this figure, the random-number generator 1 provides the test signal  $x(n)$  used for probing the channel, whereas random-number generator 2 serves as the source of additive white noise  $v(n)$  that corrupts the channel output. The GA based adaptive equalizer has the task of correcting for the distortion produced by the channel in the presence of the additive white noise.

Random-number generator 1, after suitable delay, also supplies the desired response  $d(n)$  applied to the GA based equalizer in the form of a training sequence. This system is simulated as follows:

- Simulate some useful signal to be transmitted by using random bipolar (-1,1) sequence, i.e.,
 

```
%generate the input sequence x(n).
x=rand(1,data_length);
index1=find(x>0.5);
index2=find(x<=0.5);
x(index1)=1;
x(index2)=-1;
```
- Each of the input data samples is passed through the channel and then contaminated with the additive noise of known variance  $\sigma_v^2$  (where its variance is determined by the desired signal-to-noise ratio). The resultant signal is passed through the equalizer. In this way N numbers of desired signals are produced by feeding all the N input samples.
 

```
%generate noise v(n)
v=sqrt(0.001)*randn(1,data_length);
%input signal
u=filter(channel,1,x)+v;
%desired input, d(k). The filter will result in a delay of 7 samples.
d=filter([zeros(1,7) 1],1,x);
```
- The impulse response of the channel is described by the raised cosine [1]
 

```
%filter length
M=11;
%channel parameter W
W=3.5; %corresponds to high channel distortion
% create a 5 tap channel impulse response.
channel=[0 0.5*(1+cos(2*pi*(-1:1)/W)) 0];
```

(8)

where the parameter W controls the amount of amplitude distortion by the channel, with the distortion increasing with W.

2. Let the structure of the equalizer is a finite impulse response digital filter whose coefficients are initially chosen from a population of M chromosomes. Each chromosome constitutes NL number of random binary bits, each sequential group of L-bits represent one coefficient of the adaptive model, where N is the number of parameters of the model. The GA is an iterative update algorithm and each chromosome requires its fitness to be evaluated individually. Therefore, N separate solutions need to be assessed upon the same training set in each training iteration.
3. Each of the desired output is compared with corresponding channel output and K errors are produced. The mean square error (MSE) for a given group of parameters (corresponding to nth chromosome) is determined by using the relation
 
$$MSE(n) = \frac{1}{K} \sum_{i=1}^K e_k^2$$
 . This is repeated for N times. The MSE(n) is minimized such that the adaptive filter based GA approximates the inverse of channel.
4. Since the objective is to minimize MSE (n), n=1 to N, the GA based optimization is used. The GA operates on the basis that a population of possible solutions (chromosomes) is used to assess the cost surface of the problem. The GA evolutionary process creates a new generation of solutions by crossing two chromosomes. The solution variables or genes that provide a positive contribution to the population will multiply and be passed through each subsequent generation until an optimal combination is obtained. The population is updated after each learning cycle through three evolutionary processes: *selection*, *crossover* and *mutation*. These create a new generation of solution variables. The *selection* function creates a mating pool of parent solution strings based upon the "survival of the fittest" criterion. From the mating pool the *crossover* operator exchanges gene information. This essentially crosses the more productive genes from within the solution population to create an improved, more productive, generation. *Mutation* randomly alters selected genes, which helps prevent premature convergence by pulling the population into unexplored areas of the solution surface and adds new gene information into the population [6].
5. In each generation the minimum MSE is stored which shows the learning behavior of the adaptive model from generation to generation.
6. When the minimum MSE has reached a pre-specified level the optimization is stopped.
7. At this step all the chromosomes attend almost identical genes, which represent the desired filter coefficients of the equalizer.

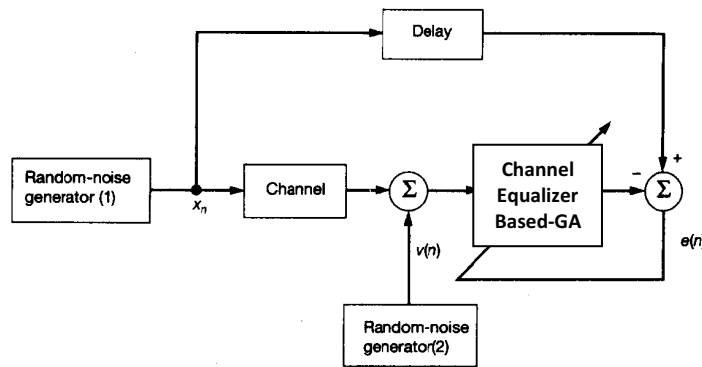


Figure 2. System model of adaptive channel equalizer based GA

#### 4. SIMULATION RESULTS

In this section, we study the performance of a simplified adaptive equalizer for an ITU V.29 modem under various channel conditions. This modem operates on the general switched telephone network lines. We use the LMS, RLS, and GA algorithms for the adaptive equalization of a linear dispersive communication channel. This channel produces unknown ISI (distortion) as was illustrated in section III. The block diagram of the system used in this work was depicted in Fig.2. The equalizer has 11 taps. The impulse response of the channel was defined by Eq. (8). In training mode the channel input after a delay of seven samples provides the desired response for the equalizer. For the LMS and RLS algorithms, we choose step-size  $\mu = 0.075$  and exponential weighting factor  $\lambda = 1$ . These values of  $\mu$  and  $\lambda$  assure the convergence of the adaptive equalizer in the mean square for both channel conditions (i.e. for both values of  $W=2.9$  and  $W=3.5$ ). While binary coded GA parameters include a population size ( $M$ ) of 40, the total number of bits used to represent each chromosome = 120 (i.e. 15 bits per variable),  $R_{min} = -2$ ;  $R_{max} = 2$  (where  $R_{min}$  and  $R_{max}$  represents the range or boundary values), a probability of crossover = 0.9 and a probability of mutation = 0.03. The tournament selection is used which is followed by two-point crossover.

The simulation result is in three parts: in part 1 the signal-to-noise ratio is high (SNR=30dB), in part 2 it is low (SNR=10dB), and in part 3 it is very low (SNR=0dB). In all parts of the simulation, the performance of the equalizer is tested under different channel conditions (channel with high distortion which corresponds to channel parameter  $W=3.5$  or low distortion corresponds to channel parameter  $W=2.9$ ).

Part 1: The simulation results for a fixed SNR=30dB (equivalently, variance  $\sigma_v^2 = 0.001$ ) and different values of channel parameter  $W$  are shown in Fig.3. This figure presents a comparison of the MSE performance of the GA to three other algorithms, the optimum Weiner Solution, the standard LMS algorithm and the recursive least-squares (RLS) algorithm. It can be seen that the LMS algorithm consistently behaves worst, in that it exhibits the slowest rate of convergence, the greatest sensitivity to variations in the parameter  $W$ , and the largest excess MSE. Also, note that the RLS algorithm consistently achieves the fastest rate of convergence and the smallest excess MSE, with the least sensitivity to variations in the channel parameter  $W$ . Most importantly, however, the MSE performance of the GA is closer to that advantage of the RLS algorithm than that disadvantage of the standard LMS algorithm. Note also, for low channel distortion ( $W=2.9$ ), the performance of the GA is very close to optimum solution.

Part 2: SNR=10dB (equivalently, variance  $\sigma_v^2 = 0.1$ ). Fig.4 shows the MSE performances for aforementioned algorithms for  $W=2.9$  and  $W=3.5$ . Insofar as the rate of convergence is concerned, we see that the GA and RLS algorithms perform in roughly the same manner, both requiring about 50 iterations to converge. The performance of the LMS algorithm is unsatisfactory especially for channel parameter  $W=3.5$ . See that increasing the channel parameter  $W$  has the effect of slowing down the rate of convergence of the adaptive equalizer and also increasing the steady-state value of the average squared error.

Part 3: In this case, the SNR measured at the channel output was 0dB. The MES performance of the GA and RLS algorithms are shown in Fig.5. Under this condition, the LMS algorithm exhibit very large fluctuations and become instable. The result presented in Fig.5 clearly shows the superior performance of the GA over the RLS algorithm. The mean squared error signal is minimized such that the GA approximates the inverse of channel.

Finally the performance of the equalizers is compared by plotting the Bit-error-rate (BER) graphs (see Fig. 6). It can be seen that, for less noisy channel conditions, the LMS and GA equalizers perform almost similarly. However, under high noise channel conditions, the GA equalizer outperforms its LMS and RLS counterparts.

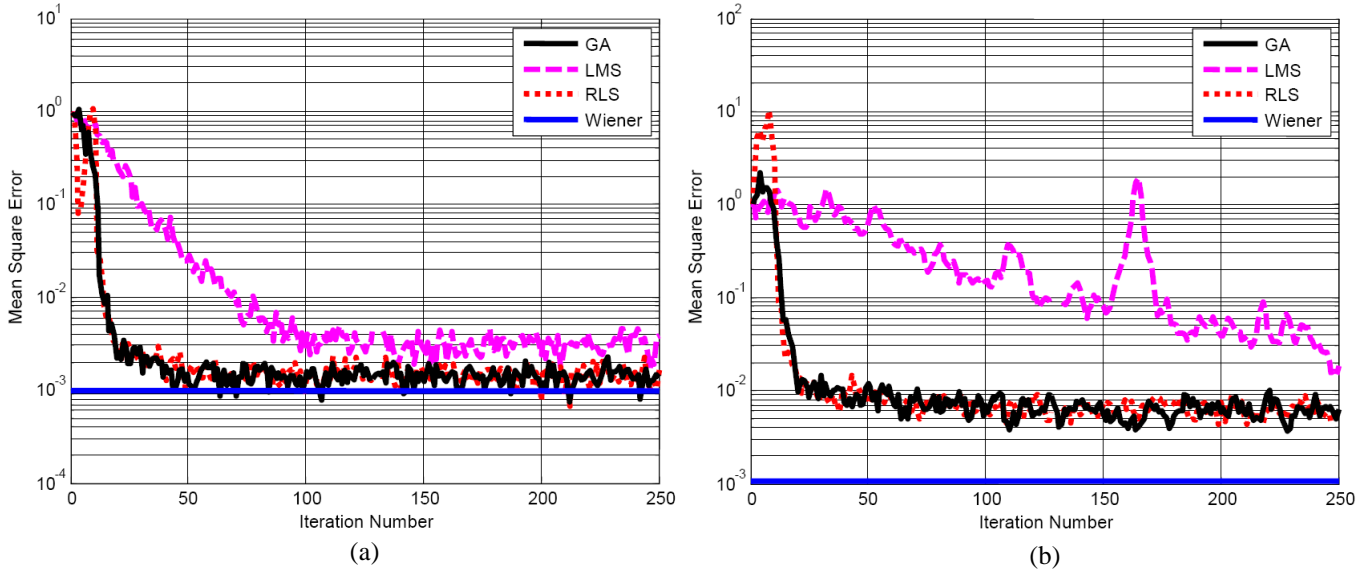


Figure 3. Training curves for the LMS, RLS, and GA algorithms for SNR=30dB and different values of W, (a) W=2.9, (b) W=3.5

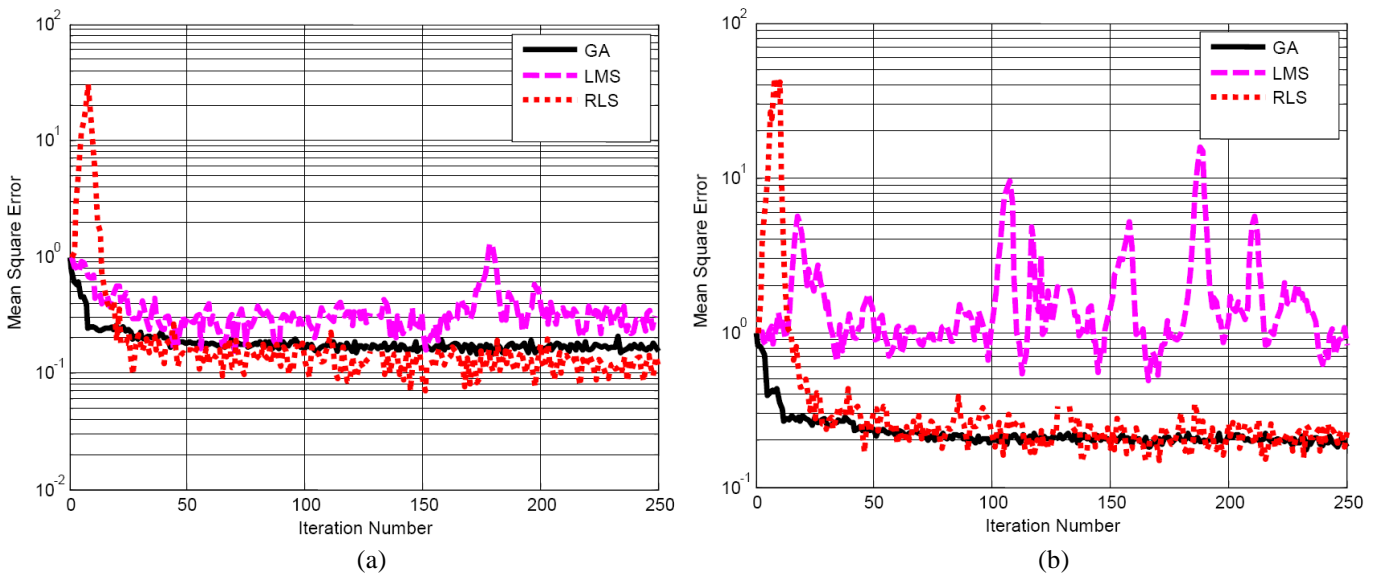


Figure 4. Training curves for the LMS, RLS, and GA algorithms for SNR=10dB and different values of W, (a) W=2.9, (b) W=3.5

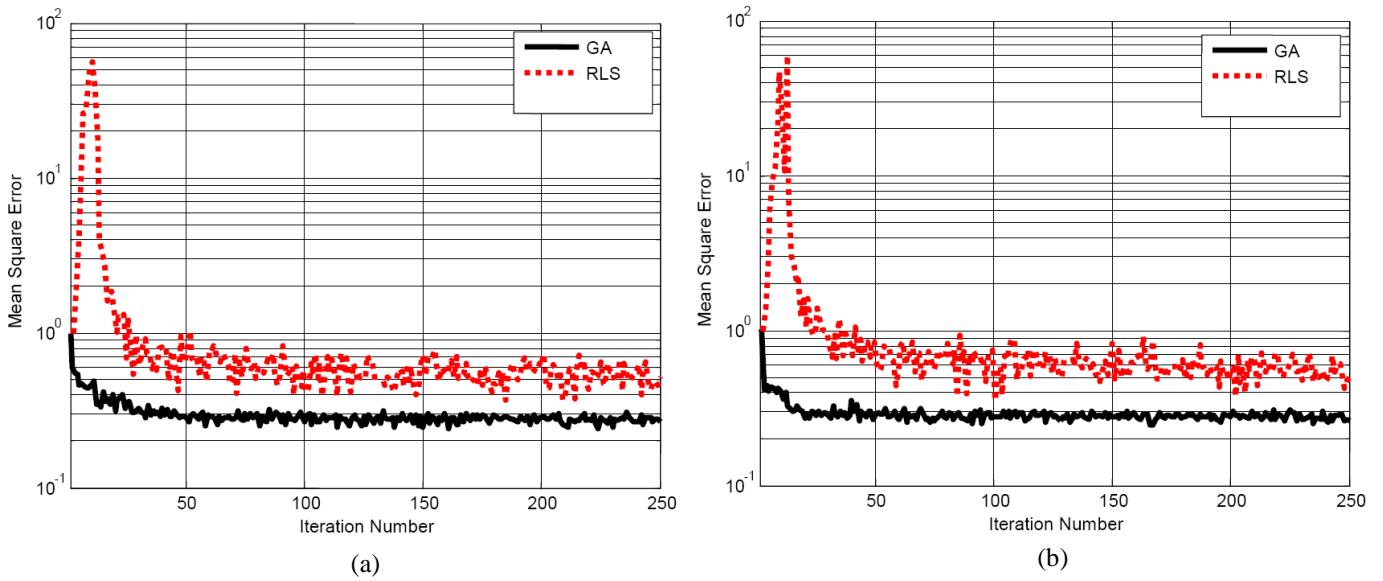


Figure 5. Training curves for the LMS, RLS, and GA algorithms for SNR=0dB and different values of W, (a) W=2.9, (b) W=3.5

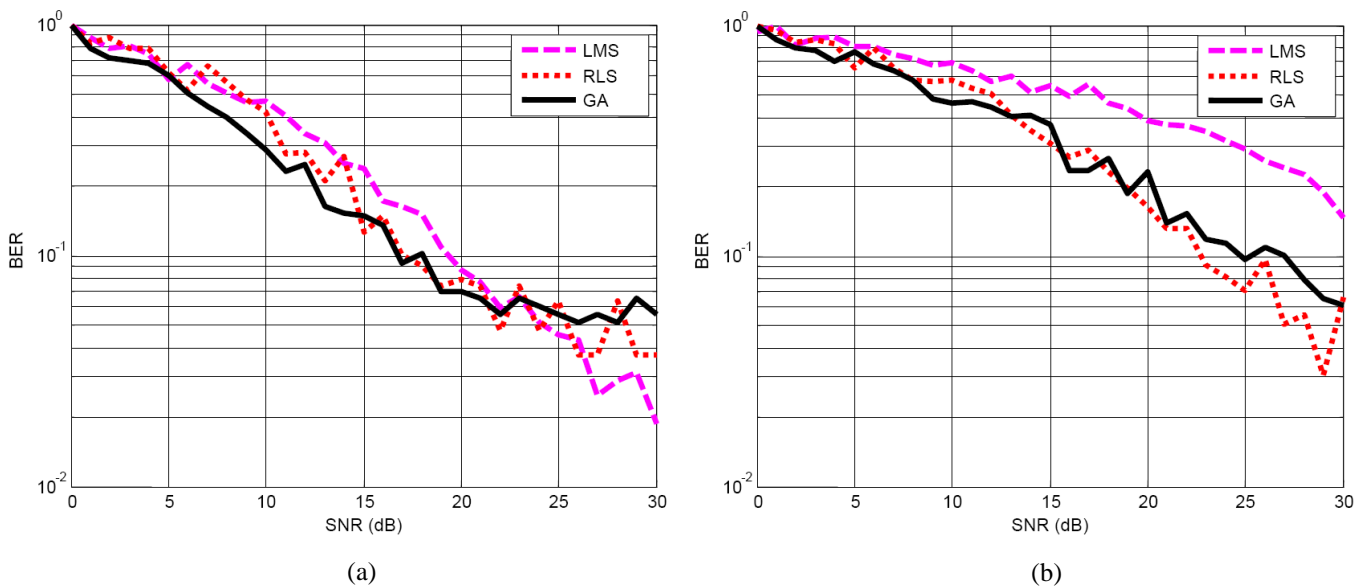


Figure 6. BER Performance for the LMS, RLS, and GA algorithms for channel parameters, (a) W=2.9, (b) W=3.5

## 5. CONCLUSIONS

The standard adaptive algorithms such as LMS, and RLS are associated with local minima problem when they are used to train the weights of the equalizers. The use of these algorithms in the design of adaptive equalizer at times fails to provide satisfactory performance. To alleviate these limitations, this paper proposes the use of derivative free optimization techniques such as Genetic Algorithm. It can also be used with Particle Swarm Optimization (PSO) technique. The performance of the GA-based channel equalizer is obtained and compared with standard adaptive algorithms. It is found that retaining the same BER performance, the GA-based channel equalizer takes lesser convergence rate (it requires about 50 iterations to

converge) as compared to the convergence rate offered by the standard LMS algorithm (it requires more than 100 iterations to converge).

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## BIOGRAPHY OF AUTHOR



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