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Ergodic Capacity of MIMO Correlated Channels in Multipath Fading Environment with known Channel State Information

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ABSTRACT

In this paper we have evaluated the performance of Multiple Input Multiple Output (MIMO) channels infading environment. Both cases of correlated and uncorrelated MIMO channels are considered under the condition when Channel State Information (CSI) is not known at transmitter and CSI is known at receiver side. We have compared the capacity of 2x2, 3x3 and 4x4 MIMO channels and have shown that capacity increases linearly with increase in the number of antennas at transmitter and receiver side. Increase in the channel capacity is observed because of the uncorrelated channel paths Correlation among the signals is dependent on the antenna structure and properties and number of the scatterers in the environment. Antenna structure includes the number of elements, the inter-element distance, Angle of Arrival (AOA) and Direction of Arrival (DOA). Additionally, Correlation increases with the number of scatterers, their distribution, location and degree of movement. Signals with same spatial signature received are considered correlated which reduces the channel capacity. In sum, correlation among the sub-channels causes the degradation in the spectral efficiency of MIMO channels.

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1. INTRODUCTION

MIMO (Multiple-Input Multiple-Output) is technique which uses multiple antennas at the transmitter and receiver and exploits the multiple paths created by set of antenna array at the transmitter and receiver as shown in Figure 1. Time diversity and frequency diversity techniques were well-investigated and studied until new diversity techniques known as spatial diversity was proposed by Alamouti [6]. The concept of Multi-Element Antenna (MEA) arrays was first given in a seminal work by G. J Foschina and Gans [1]. In MIMO, mapping of data stream is performed over multiple parallel data streams into a single data stream and at the receiver side these data streams are de-mapped and decoded in a single stream. MIMO increases the signal quality i-e it's resistive to fading, coverage and capacity of the systems. Moreover, reduced power consumption and reduced cost of wireless networks can be considered as other advantages of the MIMO. Because of this MIMO is used in all the forthcoming indoor and outdoor wireless communication standards, for example WIMAX, WiFi, DVB-T2, HSPA+ and LTE-advanced. Alongside the inherent advantages, MIMO has to face certain challenges few to mention are non-linearity of power amplifier, costly Digital Signal Processing (DSP) algorithms and network planning and optimization.

One of the prime reasons for the attainment of the spectral efficiency of the MIMO is that it exploits the different spatial signature created by rich multipath scattering environment. The signal from the individual transmitter antennas appear highly uncorrelated at receiving antennas because of the different spatial signature of the signals. The rest of the paper is organized as follows: Section 2 includes the literature review or background study of the MIMO which includes bandwidth limitation issues for capacity of Single Input Single Output (SISO) channels and the limitation of the correlation on MIMO channel capacity. In section 3, we have discussed the discrete version of the fundamental MIMO system model. Section 4 contains the capacity expressions for the Single Input Multiple Output (SIMO) and Multiple Input Single Input (MISO) systems. We have discussed the analytical results of capacity of the deterministic and non-deterministic MIMO channels in section 5. Following this, simulations results and future works are discussed as part of section 6 and section 7 respectively.

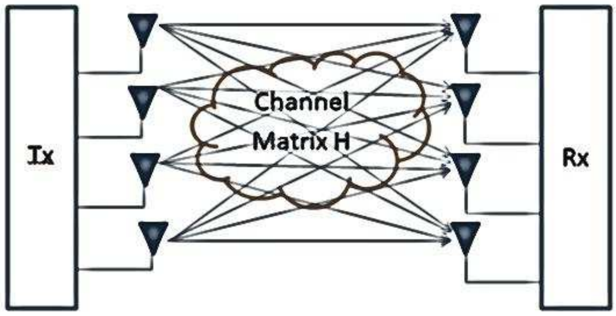

Figure 1. MIMO System Architecture

2. BACKGROUND STUDY

MIMO (Multiple-Input Multiple Output) communication architecture is very useful in wireless communication in rich multipath fading environment. It offers spectral efficiency far beyond what is offered by the conventional systems. The channel capacity of MIMO architecture increases in a linear fashion in case of independent Rayleigh fading channels.

2.1 Bandwith Limitations

The concept of channel capacity and limitations on bandwidth was introduced by Claude Shannon in 1947 [2]. Capacity expression for the Single Input Single Output (SISO) system as given by Shannon's can be written as following:

$$C = \mathbf{W}\log_2(\mathbf{1} + \mathbf{SNR})\ ^{\mathbf{bit}}/_{\mathrm{sec}}/\mathbf{Hz} \tag{1}$$

Later on, many researchers worked on the capacity of coded channels until G.J. Foschini and M.J. Gans [1] investigated the limits of the bandwidth efficiency for wireless communication in fading environment. They exploited the Multi-Element Arrays (MEA) to increase the capacity of the wireless channel by using the spatial dimension. MEA promises great advantage in case of wireless LAN and building to building wireless communication links. The overall power was kept fixed and exploited the capacity offered by the MEA, the scale in the capacity was observed by increasing the SNR for the system having large number of antennas at the transmitter and the receiver.

The characteristic of the channel i-e Channel State Information (CSI) is not known at the transmitter but the receiver knows that the channel is subjected to Rayleigh fading. Having no knowledge of the CSI at the transmitter is considered to be practical which would otherwise need a fast feedback channel which causes additional network overheads and bandwidth. Actual goal was to find the capacity and Complementary Commulative Distributive Function (CCDF) which is compares the pro ability of error and SNR [1]. In [2] it is clear from the capacity formula that in case of single transmitter and receiver for the high SNR, 3 db increase in the SNR is considered to be an increase of 1 bit/cycle/Hz of capacity for single transmitting and receiving antennas. Now, one can imagine that how large capacity would be in case of the MIMO where there are multiple antennas at the transmitter and the receiver.

2.2 Correlation Limitations on MIMO Channel Capacity

In many cases signal correlation among antennas elements exists because of poor scattering environment. Therefore, it's necessary to study MIMO channel capacity in fading environment. Antenna located very close to each other limit the MIMO channel capacity because of the field coupling produces correlation. Therefore, a certain minimum distance between antennas must be ensured in order to avoid decrease in capacity. In case of ideal scattering environment channel correlation limits the minimum antenna spacing to half-wavelenght. In [11] S.Lokyasuggests optimal number of the antennas can be calculated by using following mathematical result.

$$N_T \approx \frac{2L}{\lambda} + 1 \quad (2)$$

Where L is aperture size of the linear antenna arrays and λ is the wavelength. Furthermore, minimum antenna spacing, as stated above, should be half-wave length.

$$d_{min} = \frac{\lambda}{2} \quad (3)$$

Moreover, correlation can be minimized either placing antenna with sufficient distance apart or using cross-polarized antennas.

3. MIMO SYSTEM MODEL

Fundamental mathematematical model for MIMO systems can be written as follows:

$$\mathbf{y} = H\mathbf{x} + \mathbf{z} \quad (4)$$

Where $\mathbf{x} = [x_1, x_2, x_3, \dots\dots, x_n] \in C^{N_T}$ is the transmitted vector of order $N_T \times 1$, $\mathbf{y} = [y_1, y_2, y_3, \dots\dots, y_n] \in C^{N_R}$ is the received MIMO vector of order $N_R \times 1$ and $\mathbf{z} = [z_1, z_2, z_3, \dots\dots, z_n] \in C^{N_R}$ is the noise vector of order $N_R \times 1$. The noise at the receiver is assumed to be Gaussian of equal power σ^2 and its components are independent. The channel matrix H is MIMO channel matrixof order $N_R \times N_T$ which contains the impulse responses of the individual path between N_T transmit and N_R receive antennas and it can be represented as follows:

$$H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1j} \\ h_{21} & h_{22} & \cdots & h_{2j} \\ \vdots & \vdots & \cdots & \vdots \\ h_{i1} & h_{i2} & \cdots & h_{ij} \end{bmatrix} \in C^{N_R \times N_T} \quad (5)$$

Where h_{ij} determines the gain of the radio channel from ith receiving antenna to jth transmistting antenna. Channel matrix H has Singular Value Decomposition (SVD) i-e $H = U\Lambda^{\frac{1}{2}}V^H$ where $U \in C^{N_R \times N_R}$ and $V \in C^{N_T \times N_T}$ are unitary matrices and $\Lambda^{\frac{1}{2}} \in R^{N_R \times N_T}$ is a diagonal matrix of singular values of H. Number of positive singular values i-e rank of the channel matrix can be given by $rank(H) \leq \min(N_R, N_T)$

4. CAPACITY OF SIMO AND MISO CHANNELS

From Information-theoretic perspective capacity can be defined as the mutual information $I(X;Y)$ between two random variables X and Y which can mathematically be expressed as follows:

$$C = \max_{p(x)} \mathrm{I(X;\,Y)} \frac{\textbf{bits}}{\textbf{channel use}} \quad (6)$$

Where $p(X)$ is the probability mass function (pmf) of the transmitted signal.

Mutual information is measure of information that a one random variable X contains about another random variable Y which can also be given as:

$$I(X;Y) = H(Y) + H(Y|X) \quad (7)$$

Where H(Y) is entropy of the random variableY or measure of uncertainity of random variableor average information of the random variable Y. $H(Y) = -\sum_Y p(Y)\log(p(Y))$ and $H(Y|X) = \sum_X p(X)H(Y|X = x) = -\sum_X \sum_Y p(X,Y)\log[p(X,Y)]$ is conditional entropy of random variable X and Y and $p(X,Y)$ is joint probability mass function of random variable X and Y. $I(X;Y)=0$ if X and Y independent and $I(X,Y)=H(X)=H(Y)$ if X=Y.

5. MIMO CHANNEL CAPACITY

MIMO channel capacity can be evaluated for the following MIMO channel matrix cases:
1. Channel Matrix H is deterministic
2. Channel Matrix H is a random matrix

5.1Capacity of Deterministic MIMO Channels

For a MIMO system of N_T transmit and N_R receive antennas received vector **y** can be written as

$$\mathbf{y} = \sqrt{\frac{E_x}{N_T}}\,\mathbf{Hx} + \mathbf{z} \tag{8}$$

Where Ex is the energy of the transmitted signals and H is time-invariant channel matrix of N_R by N_T and $z = [z_1, z_2, \ldots, z_R]^T$ is a noise vector. Moreover, the autocorrelation of transmitted vector is defined as

$$R_{xx} = E[\mathbf{x}\mathbf{x}^H] \tag{9}$$

The autocorrelation of received matrix is defined as

$$R_{yy} = E[yy^H] = E\left(E\left[\sqrt{\frac{E_x}{N_T}}Hx + z\right]E\left[\sqrt{\frac{E_x}{N_T}}x^H H^H + z^H\right]\right) = E\left(\sqrt{\frac{E_x}{N_T}}Hxx^H H^H + zz^H\right) = \frac{E_x}{N_T}E[Hxx^H H^H + zz^H] = \frac{E_x}{N_T}HE[xx^H(\square)H]^H + E[zz^H] \tag{10}$$

Where H^H is the complex conjugate transpose (Hermitian) of the matrix H.Now, the capacity of SISO channel can be easily be extended to MIMO capacity in bps/Hz when Channel State Information (CSI) is not known at the transmitter which can be expressed as follows:

$$C = \log_2\left|\left(I_{N_R} + \frac{E_x}{N_T N_0}HH^H\right)\right| \tag{11}$$

Where $\frac{E_x}{N_0} = \gamma$ is average Signal-to-Noise ratio of the the receiving antenna and I_{NR} is identity matrix. For no CSI at the transmitter the covariance matrix Rxx is identity matrix i-e $R_{XX} = I$.
Now, the MIMO channel capacity when Channel State Information (CSI) is known at the transmitter in which case channel matrix H is a full rank matrix can be written as following

$$\mathbf{C} = \max_{Tr(R_{xx})=N_T} \log_2\left|\left(\mathbf{I}_{N_R} + \frac{\gamma}{N_T}\mathbf{HR}_{xx}\mathbf{H}^H\right)\right| \tag{12}$$

5.2. Capacity of Non-Deterministic or Random MIMO Channels

The practical MIMO channels are essentially non-deterministic and time varying. Therefore, the channel matrix H is considered random and we assume random channel is ergodic process. MIMO channel capacity can be found by time average of the deterministic channel capacity in bps/Hz which can be given as follows:

$$\bar{\mathbf{C}} = \mathbf{E}\left[\left(Tr(R_{xx}) = N_T^{max}\right)\log_2\left|\mathbf{I}_{N_R} + \frac{\gamma}{N_T}\mathbf{HR}_{xx}\mathbf{H}^H\right|\right. \tag{13}$$

This is considered as ergodic channel capacity of the MIMO channels In general, MIMO channels are not independent and identically distributed (iid). The channel correlation is closely related to the capacity of MIMO channels. Here we consider the capacity of MIMO channel when the channel gain between transmitter and receiver are correlated. For high SNR, the deterministic channel can be approximated as follows:

$$\mathbf{C} \approx \left(T_{r(R_{xx})=N}^{max}\right. \log_2|\mathbf{R}_{xx}| + \log_2\left|\frac{\gamma}{N_T}\mathbf{H}_w\mathbf{H}_w^H\right| \tag{14}$$

Now, consider the following correlated channel model

$$H = \sqrt{R_r}H_w\sqrt{R_t} \tag{15}$$

Where Rt is the correlation matrix and reflects the correlation between transmit antennas. Rr is the correlation matrix reflecting the correlation between receive antennas and H_w denoted the iid Rayleigh fading channel gain matrix. The diagonal entries of Rr and Rt are constrained to be unity. Then, we can write MIMO channel as

$$C = \log_2 \left| I_{N_R} + \frac{\gamma}{N_T} \sqrt{R_r} H_w R_t H_H^w R_r^{\frac{H}{2}} \right| \tag{16}$$

If $N_T = N_R = N$, Rt and Rr are full rank, and SNR is high we can approximate above relation as

$$C \approx \log_2 \left| \left(\frac{\gamma}{N_T} H_w H_w^H \right) \right| + \log_2 \det(R_r) + \log_2 (R_t) \tag{17}$$

Above equation shows that MIMO channel capacity have been reduced due to correlation between transmit and receive antennas by an amount equal to $\log_2 \det(R_r) + \log_2 (R_t)$.
We can calculate the ergodic MIMO channel capacity when there exists correlation between transmit and receive antennas, with the following exponential channel correlation matrices:

$$R_r = I_4 \tag{18}$$

and

$$R_t = [(1 @ 0.75e^{(-j0.17)} @ (0.43e^{(-j0.35\pi)} @ 0.25e^{(-j0.53\pi)})) \& ((0.75e^{j0.17} @ 1 @ (0.75e^{j0.17} @ 0.43e^{(-j0.35\pi)})) \& ((0.43e^{j0.35\pi} @ \tag{19}$$

MIMO channel capacity increases linearly in case of independent fading channels. Increase in correlation usually causes a decrease in SNR. For example, with a correlation coe-efficicent of 0.7 there may be 3dB decrease in SNR.

6. SIMULATION RESULTS AND DISCUSSIONS

In this section, we shall discuss our simulation results that are obtained following the analytical results as discussed in section 5. The simulation results are very much consistent with analytical results.

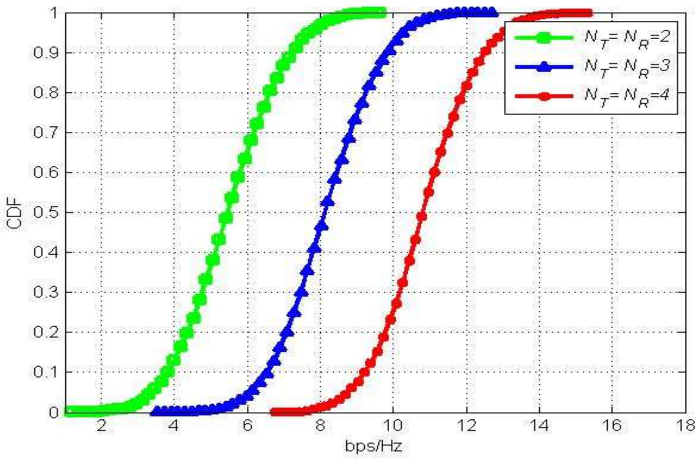

Figure 2. CDF for 2x2, 3x3 and 4x4 MIMO channels

Figure 2 shows the Cummulative Distribution Function (CDF) graph of the MIMO channels.From above graph we can see that capacity of 4x4 MIMO channel isgreater than capcity of 2x2 and 3x3 MIMO channel. We obtain gain of approximately 2bps/Hz with 4x4 MIMO channel over 2x2 MIMO channel at 10dB SNR. Similarly, if we increase SNR we can achieve more capacity gain. Channel capacity for the 2x2, 3x3 and 4x4 MIMO channel at SNR of 10dB is summarized in Table 1.

Table 1. Channel Capacity of 2x2, 3x3 and 4x4 MIMO channels at 10dB SNR.

MIMO Channel	Capacity(bits/sec/Hz)at10dB
2x2	4
3x3	6.5
4x4	9

Table 2. Capacity loss in 3x3 and 4x4 MIMO Correlated Channels at 10dB SNR

MIMO Channel	iid MIMO Capacity (bits/sec/Hz)	Correlated MIMO Capacity (bits/sec/Hz)	Capacity Loss (bits/sec/Hz)
3x3	7	6	1
4x4	11	9	2

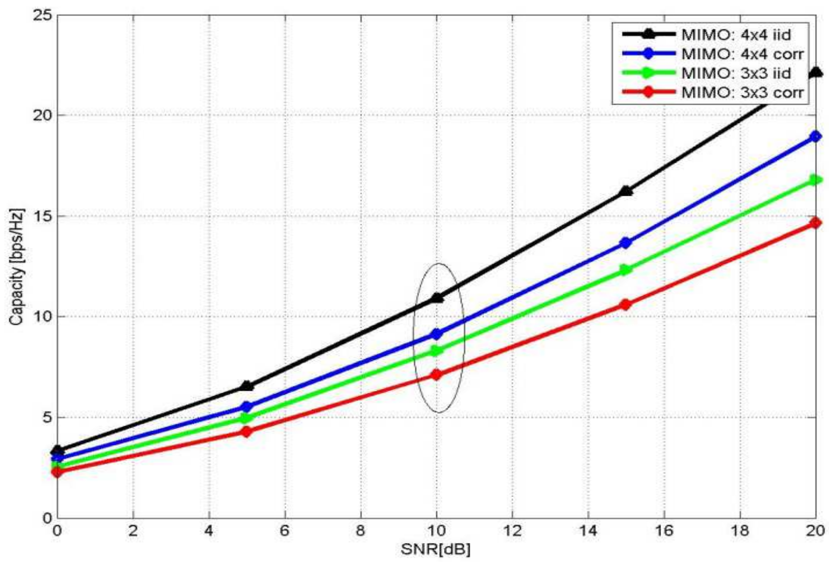

Figure 3. Comparison of 3x3 and 4x4 MIMO correlated channel capacity

Figure 3 shows the performance of ergodic MIMO correlated channel. The simulation results are consistent with the analytical expression. We can observe that there is capacity loss due to correlation and the capacity curves are shifted downward. A capacity loss of 1bps/Hz is recored at SNR of 10dB.Therefore, a capacity gain of 1bit/sec/Hz can be achieved on a penalty of 3dB SNR. Moreover, it can be seen that capacity increases linearly when N_T becomes equal to N_R.

7. CONCLUSION AND FUTURE WORK

MIMO channels promise high spectral efficiency. Moreover, the capacity depends upon the order of the receiver and transmitter antennas under certain limits. In practice, when there is correlation between channels, degradation in capacity can be observed. Finally, capacity of MIMO channel can be investigated for time-selective and frequency-selective channels when CSI at transmitter is not available or is imperfect which is in case when feedback channel from receiver to transmitter for training sequences imposes bandwidth constraints and is not feasible.

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