# A new efficient way based on special stabilizer multiplier permutations to attack the hardness of the minimum weight search problem for large BCH codes 

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#### Abstract

BCH codes represent an important class of cyclic error-correcting codes; their minimum distances are known only for some cases and remains an open NP-Hard problem in coding theory especially for large lengths. This paper presents an efficient scheme ZSSMP (Zimmermann Special Stabilizer Multiplier Permutation) to find the true value of the minimum distance for many large BCH codes. The proposed method consists in searching a codeword having the minimum weight by Zimmermann algorithm in the sub codes fixed by special stabilizer multiplier permutations. These few sub codes had very small dimensions compared to the dimension of the considered code itself and therefore the search of a codeword of global minimum weight is simplified in terms of run time complexity. ZSSMP is validated on all BCH codes of length 255 for which it gives the exact value of the minimum distance. For BCH codes of length 511, the proposed technique passes considerably the famous known powerful scheme of Canteaut and Chabaud used to attack the public-key cryptosystems based on codes. ZSSMP is very rapid and allows catching the smallest weight codewords in few seconds. By exploiting the efficiency and the quickness of ZSSMP, the true minimum distances and consequently the error correcting capability of all the set of 165 BCH codes of length up to 1023 are determined except the two cases of the $\operatorname{BCH}(511,148)$ and $\operatorname{BCH}(511,259)$ codes. The comparison of ZSSMP with other powerful methods proves its quality for attacking the hardness of minimum weight search problem at least for the codes studied in this paper.


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## 1. INTRODUCTION

Reproducing at one point either exactly or approximately a message selected at another point is the fundamental problem of communication, as pointed out by Claude Shannon [1]. The Error-correcting codes are used to improve the reliability of such communication.

The error-correcting capability of a code C is directly related to its minimum distance. For Linear codes, the minimum distance is equal to its lowest non-zero weight codeword. The knowledge of the weights enumerator of a code is important and it permits to compute their analytical performances. Double weight codes are used to elevate the performance and cardinality of spectral amplitude coding (SAC) OCDMA (Optical Code-Division Multiple-Access) systems [2]. The problem of searching codewords of lowest weight in linear codes is NP-hard problem and it is equivalent to the problem of the minimum distance research [3].

BCH codes are used in many applications and many algorithms are developed for decoding them like in [4]. In [5], the employment of BCH codes in SC-FDM-IDMA scheme had yield to good improvement of the BER performance (Bit Error Rate). In [6], the authors propose to concatenate BCH and turbo codes with OSTBC system. This concatenation, called BCH-TURBO-OSTBC, had yield to good result in term of BER performance.

For BCH codes, only a lower bound is known and the minimum distance is known only for some lengths, special cases [7]-[10] and remains an open problem in coding theory. In this paper, our work will focuses on finding the minimum distance of large BCH codes.

The remainder of this paper is organized as follows: The next Section presents the main related works. The Section 3 presents the proposed scheme ZSSMP for BCH codes. The Section 4 presents the main results. The conclusion and the possible future directions of this research are outlined in Section 5.

## 2. RELATED WORKS

The determination of the minimum distance for primitive BCH codes is hard as pointed out by charpin in [11]. For this reason, many researchers have investigated several methods for finding this metric. This Section summarizes the most important ones.

In [12], Augot, Charpin, and Sendrier presented an algebraic system of Newton's identities. The existence of a solution for this system, prove the existence of words of a given weight in a code. The use of this method for both remaining unknown minimum distance BCH codes of length 255 prove that $\operatorname{BCH}(255,63,61)$ has minimum distance 63 , and $\operatorname{BCH}(255,71,59)$, has minimum distance 61 . The use of this method had yield also to new results for some BCH codes of length 511.

In [13], Chabaud made a comparison of the both probabilistic algorithms for finding minimumweight words in a linear code Leon's [14] and Stern's [15]. After that, Chabaud with Canteaut in [16], have developed a new probabilistic algorithm based on the best one. The application of this algorithm on narrowsense BCH Codes of length 511 had yield to some new results, however the minimum distance is still unknown for other codes.

Zimmermann algorithm [17] is a general algorithm for computing the minimum distance of a linear code. It is implemented in GAP (package Guava) [18] over fields $\mathrm{F}_{2}$ and $\mathrm{F}_{3}$. It is also implemented, in Magma over any finite field.Zimmermann's algorithm is explained in detail in [19].

Wallis and Houghten in [20] have implemented the genetic algorithm for computing the minimum distance for BCH codes. Simulations results show that the genetic algorithm outperforms other artificial intelligence techniques like the Tabu Search presented in [21] and hill-climbing. In [22], the authors have optimized the parameters of the genetic algorithmand consequently they obtained more accurate results.

By formulating ant colony optimization (ACO) to incorporate Tabu Search (TS), Blandin [23], continues to improve his tabu search technique presented in [21]. This hybrid technique, called ACOTS, had yield to more accurate results.

The artificial intelligence Simulated Annealing presented in [24] was shown to be useful in finding the minimum distance for linear codes. In [25], Ajitha has used the metropolis algorithm to attack the minimum weight code word problem. This close algorithm to the Simulated Annealing gives more accurate results in comparison to previous works presented in [20]-[24]. In [25], Aylaj and Belkasmi continue to improvethe Simulated Annealing presented in [24]. The proposed Simulated Annealing (PSA) had yield to a fast convergence by reducing the number of iterations of the classical Simulated Annealing approachas well as obtaining good results in comparison to the previous works presented in [20], [22], [22], [25].

In [27], Berrou has presented an efficient approach based on the notion of Error Impulse response of a Soft-In decoder. The proposed idea make a relation between the minimum distance and the level of noise added to all-zero codeword by considering the minimum distance as the smallest level of noise from which the decoder fails in correction. In [22], the authors have improved this idea and presented the Multiple Impulse Method (MIM). In [28], the authors presented an efficient local search technique called MIM-RSC, which consist in applying the MIM method on some Random Sub Codes of reduced dimensions. The proposed method had yield to good results compared to the previous works presented in [20], [22], [23], [25], [26] as well as finding the true minimum distance of some BCH codes of length 1023 and 2047.

## 3. THE PROPOSED SCHEME

Instead of searching in some random sub codes like in [28], our proposed scheme consists in searching in few determined sub codes fixed by special permutations from the automorphism group of BCH codes. It is well known that for $\mathrm{BCH}\left(\mathrm{n}=2^{\mathrm{m}}-1, \delta\right)$ codes, the multiplier permutations defined on $\{0,1, \ldots, \mathrm{n}-1\}$ by $\mu_{2^{k}}: i \rightarrow 2^{k} i(\bmod n)$ with $1 \leq k \leq m-1$ are stabilizers. From these stabilizers, we take only stabilizers with

[^0]different cycle structure because they fix different sub code and apply Zimmermann algorithm on these sub codes.

For finding the minimum distance of BCH codes. The proposed scheme works as follows:

```
Inputs:
-A generator matrix G of BCH(n=2 m}-1,k,\delta
-The permutations 阷剆
Step 1:
For k=1 to N do
Find the sub code fixed by }\mp@subsup{\boldsymbol{\mu}}{\mp@subsup{2}{}{\mp@subsup{j}{k}{}}}{
End for
Step 2:
For k=1 to N do
Find the estimated minimum distance d of the sub code fixed by }\mp@subsup{\mu}{\mp@subsup{2}{}{\mp@subsup{\mathbf{j}}{\mathbf{k}}{}}}{}\mathrm{ by using the Zimmermann algorithm.
End for
Output:
-d as estimated minimum distance of BCH(n,k,\delta)
```

Examples: Let's take the $\mathrm{BCH}(15,5,7)$ code. A systematic generator matrix of $\mathrm{BCH}(15,5,7)$ code is:

$$
G=\left(\begin{array}{lllllllllllllll}
1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1
\end{array}\right)
$$

By using a mathematical tool, we obtain the generator matrix GS1 of the sub code fixed by $\mu_{2}=(0)(1,2,4,8)(3,6,12,9)(5,10)(7,14,13,11)$ and the generator matrix GS2 of the sub code fixed by $\mu_{4}=(0)(1,4)(2,8)(3,12)(6,9)(5)(10)(7,13)(14,11):$

In the second step, by applying the Zimmermann algorithm on the first sub code, the minimum distance obtained is equal to 7 , which is the designed distance for the considered BCH code, and therefore the minimum distance is 7 .

Let's now take another example of the $\operatorname{BCH}(511,358,37)$ code. The permutations $\mu_{2}, \mu_{4}, \mu_{16}, \mu_{32}$, $\mu_{128}$, and $\mu_{256}$ have the same cycle structure and fix the same sub code. On the other hand, the permutations $\mu_{8}$ and $\mu_{64}$ fix also the same sub code. By using a mathematical tool, we obtain the generators matrix of the sub code fixed by $\mu_{2}$ and $\mu_{8}$.

In the second step, by applying the Zimmermann algorithm on the sub code fixed by $\mu_{2}$, the minimum distance obtained is equal to 39 , which is greater than the designed distance for the considered BCH code. Then we pass to the sub code fixed by the second permutation with different cycle structure, which is here $\mu_{8}$, by applying the Zimmermann algorithm on this sub code the minimum distance obtained is equal to 37 , which is equal to the designed distance for the considered BCH code, and therefore the minimum distance is 37 .

## 4. RESULTS AND DISCUSSIONS

This Section presents a validation of the proposed method on BCH codes of known minimum distance and its application for finding the minimum distance of BCH codes of unknown minimum distance. This Section presents also a comparison between the proposed scheme and previous work on minimum distance for BCH codes.

All results have been done using a simple configuration machine: Intel(R) Core(TM) i3-4005U CPU $@ 1.70 \mathrm{GHz}$ RAM 4GO. These results are made by running the cited algorithm in 1day for each code.

### 4.1. Validation of the proposed scheme

In order to validate the proposed method, it is applied on all BCH codes of known minimum distance presented in Table 1 All the narrow-sense primitive binary BCH codes of length 255 have their minimum distance equal to their designed distance except $\mathrm{BCH}(255,63,61)$, which has minimum distance 63 , and $\operatorname{BCH}(255,71,59)$, which has minimum distance 61 . The both last result have been proved in [12], by using the Newton's identities.

Table 1 summarizes the obtained results. It shows that the minimum weight found by the proposed method is equal to the true value of the minimum distance of all BCH codes of length 255 . Therefore, the proposed method is validated for length 255 .

Table 1. Validation of the proposed scheme

| BCH(n,k, $\delta$ ) | True value of minimum distance | d(ZSSMP) | $\mathrm{BCH}(\mathrm{n}, \mathrm{k}, \delta)$ | True value of minimum distance | d(ZSSMP) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| BCH $(255,247,3)$ | 3 | 3 | BCH(255,115,43) | 43 | 43 |
| BCH $(255,239,5)$ | 5 | 5 | BCH $(255,107,45)$ | 45 | 45 |
| BCH $(255,231,7)$ | 7 | 7 | BCH $(255,99,47)$ | 47 | 47 |
| BCH $(255,223,9)$ | 9 | 9 | BCH $(255,91,51)$ | 51 | 51 |
| BCH $(255,215,11)$ | 11 | 11 | BCH $(255,87,53)$ | 53 | 53 |
| BCH(255,207,13) | 13 | 14 | BCH $(255,79,55)$ | 55 | 55 |
| BCH(255,199,15) | 15 | 15 | BCH $(255,71,59)^{*}$ | 61 | 61 |
| BCH $(255,191,17)$ | 17 | 17 | BCH(255,63,61)* | 63 | 63 |
| BCH $(255,187,19)$ | 19 | 19 | BCH $(255,55,63)$ | 63 | 63 |
| BCH(255,179,21) | 21 | 21 | BCH $(255,47,85)$ | 85 | 85 |
| BCH(255, 171,23) | 23 | 23 | BCH $(255,45,87)$ | 87 | 87 |
| BCH(255, 163,25) | 25 | 25 | BCH $(255,37,91)$ | 91 | 91 |
| BCH $(255,155,27)$ | 27 | 27 | BCH $(255,29,95)$ | 95 | 95 |
| BCH(255,147,29) | 29 | 29 | BCH $(255,21,111)$ | 111 | 111 |
| BCH $(255,139,31)$ | 31 | 31 | BCH $(255,13,119)$ | 119 | 119 |
| BCH(255,131,37) | 37 | 37 | BCH(255,9,127) | 127 | 127 |
| BCH $(255,123,39)$ | 39 | 39 |  |  |  |

### 4.2. Comparison of the proposed scheme with Zimmermann algorithm

In order to compare the proposed scheme with Zimmermann algorithm, their applications on some BCH codes are made. Table 2 summarizes the obtained results. These results demonstrate that the proposed scheme made an efficient local search and consequently give accurate results in very short time. The obtained results show also the benefits of restricting the search space to sub codes with some properties related to the specified code.

Table 2. Comparison between the Proposed Scheme and Zimmermann Algorithm for Some BCH Codes

| BCH $(\mathrm{n}, \mathrm{k}, \delta)$ | $\mathrm{d}($ Zimmermann $)$ | Run Time of <br> Zimmermann $(\mathrm{s})$ | $\mathrm{d}($ ZSSMP $)$ | Run Time of <br> Step1 $(\mathrm{s})$ | Run Time of <br> Step2 $(\mathrm{s})$ | Total Run Time <br> of ZSSMP $(\mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BCH $(511,277,57)$ | 71 | 46270.380 | 57 | 55 | 0.031 | 55.031 |
| BCH $(511,268,59)$ | 77 | 6536.488 | 59 | 52 | 1.996 | 53.996 |
| BCH $(511,250,63)$ | 83 | 64283.238 | 63 | 48 | 0.046 | 48.046 |
| BCH $(1023,573,101)$ | 159 | 62005.171 | 101 | 313 | 0.264 | 313.264 |
| BCH $(1023,553,105)$ | 167 | 39973.040 | 105 | 373 | 1.170 | 374.170 |

### 4.3. Comparison of the proposed scheme with canteaut-chabaud algorithm

Table 3 presents a comparison between the proposed scheme and Canteaut-Chabaud algorithm. The obtained results show that the proposed scheme give more accurate results than Canteaut-Chabaud algorithm as well as finding the true minimum distance for the 4 unknown minimum distance of BCH codes of designed distance $59,75,77$ and 85 . Therefore, the remaining codes of length 511 for which the minimum distance is still unknown are $\operatorname{BCH}(511,148,107)$ and $\operatorname{BCH}(511,259,61)$.

Table 3. Comparison between the Proposed Scheme and Canteaut-Chabaud Algorithm for BCH Codes of Length 511

| BCH(n,k, $\delta$ ) | Canteaut | d(ZSSMP) | $\mathrm{BCH}(\mathrm{n}, \mathrm{k}, \delta$ ) | Canteaut | d(ZSSMP) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| BCH( $511,502,3$ ) | 3 | 3 | $\operatorname{BCH}(511,241,73)$ | 73 | 73 |
| BCH $(511,493,5)$ | 5 | 5 | $\operatorname{BCH}(511,238,75)$ | $>=75$ | 75 |
| $\operatorname{BCH}(511,484,7)$ | 7 | 7 | BCH(511,229,77) | $>=77$ | 77 |
| BCH $(511,475,9)$ | 9 | 9 | BCH $(511,220,79)$ | 79 | 79 |
| BCH $(511,466,11)$ | 11 | 11 | $\operatorname{BCH}(511,211,83)$ | 83 | 84 |
| BCH $(511,457,13)$ | 13 | 13 | BCH $(511,202,85)$ | > $=85$ | 85 |
| BCH $(511,448,15)$ | 15 | 15 | BCH(511,193,87) | 87 | 87 |
| BCH $(511,439,17)$ | 17 | 18 | BCH $(511,184,91)$ | 91 | 91 |
| BCH $(511,430,19)$ | 19 | 19 | BCH $(511,175,93)$ | 95 | 95 |
| BCH $(511,421,21)$ | 21 | 21 | $\operatorname{BCH}(511,166,95)$ | 95 | 95 |
| BCH $(511,412,23)$ | 23 | 23 | $\operatorname{BCH}(511,157,103)$ | 103 | 103 |
| BCH $(511,403,25)$ | 25 | 25 | $\operatorname{BCH}(511,148,107)$ | >=107 | $>=107$ |
| BCH $(511,394,27)$ | 27 | 27 | BCH $(511,139,109)$ | 111 | 111 |
| BCH $(511,385,29)$ | 29 | 29 | $\operatorname{BCH}(511,130,111)$ | 111 | 111 |
| BCH $(511,376,31)$ | 31 | 31 | $\operatorname{BCH}(511,121,117)$ | 119 | 119 |
| BCH $(511,367,35)$ | 35 | 36 | $\operatorname{BCH}(511,112,119)$ | 119 | 119 |
| BCH $(511,358,37)$ | 37 | 37 | $\operatorname{BCH}(511,103,123)$ | 127 | 127 |
| BCH $(511,349,39)$ | 39 | 39 | $\operatorname{BCH}(511,94,125)$ | 127 | 127 |
| BCH $(511,340,41)$ | 41 | 41 | $\operatorname{BCH}(511,85,127)$ | 127 | 127 |
| BCH $(511,331,43)$ | 43 | 43 | BCH $(511,76,171)$ | 171 | 171 |
| BCH $(511,322,45)$ | 45 | 45 | $\operatorname{BCH}(511,67,175)$ | 175 | 175 |
| BCH $(511,313,47)$ | 47 | 48 | $\operatorname{BCH}(511,58,183)$ | 183 | 183 |
| BCH $(511,304,51)$ | 51 | 51 | BCH $(511,49,187)$ | 187 | 187 |
| BCH $(511,295,53)$ | 53 | 53 | $\operatorname{BCH}(511,40,191)$ | 191 | 191 |
| BCH $(511,286,55)$ | 55 | 55 | BCH $(511,31,219)$ | 219 | 219 |
| BCH $(511,277,57)$ | 57 | 57 | BCH $(511,28,223)$ | 223 | 223 |
| BCH $(511,268,59)$ | $>=59$ | 59 | $\operatorname{BCH}(511,19,239)$ | 239 | 239 |
| BCH(511,259,61) | > $=61$ | > $=61$ | BCH $(511,10,255)$ | 255 | 255 |
| BCH $(511,250,63)$ | 63 | 63 |  |  |  |

### 4.4. Comparison of the proposed scheme with Aylaj's SA algorithm

In order to compare the proposed scheme with Aylaj's SA algorithm presented in [26], their applications on some BCH codes are made. The Table 4 summarizes the obtained results. These results demonstrate that the proposed scheme outperforms Aylaj's SA algorithm in both time and result quality.

Table 4. Comparison between the PRoposed Scheme and Aylaj's SA Algorithm for Some BCH Codes of Length 511

| Some BCH Codes of Length 511 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BCH $(\mathrm{n}, \mathrm{k}, \delta)$ | $\mathrm{d}(\mathrm{SA})$ | Run Time of SA <br> $(\mathrm{s})$ | $\mathrm{d}(\mathrm{ZSSMP})$ | Run Time of Step1 <br> $(\mathrm{s})$ | Run Time of <br> Step2 $(\mathrm{s})$ | Total Run Time of <br> ZSSMP $(\mathrm{s})$ |
| $\mathrm{BCH}(511,304,51)$ | 68 | 21052 | 51 | 45 | 1.809 | 46.809 |
| $\mathrm{BCH}(511,286,55)$ | 73 | 40220 | 55 | 46 | 0.031 | 46.031 |
| $\operatorname{BCH}(511,250,63)$ | 91 | 29321 | 63 | 48 | 0.046 | 48.046 |

### 4.5. Comparison of the proposed scheme with MIM-RSC method

The Table 5 presents a comparison between the proposed scheme and MIM-RSC method presented in [26]. The obtained results show the efficiency of our scheme in giving accurate results in very short time.

Table 5. Comparison between the proposed Scheme and MIM-RSC method for some BCH codes of lengths 511 and 1023

| BCH $(\mathrm{n}, \mathrm{k}, \delta)$ | $\mathrm{d}($ MIM-RSC $)$ | Run Time of <br> MIM-RSC $(\mathrm{s})$ | $\mathrm{d}($ ZSSMP $)$ | Run Time of <br> Step1 $(\mathrm{s})$ | Run Time of <br> Step2 $(\mathrm{s})$ | Total Run Time of <br> ZSSMP $(\mathrm{s})$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| BCH $(511,277,57)$ | 77 | 3288 | 57 | 55 | 0.031 | 55.031 |
| BCH $(511,268,59)$ | 81 | 10667 | 59 | 52 | 1.996 | 53.996 |
| BCH $(1023,573,101)$ | 165 | 18295 | 101 | 313 | 0.264 | 313.264 |
| BCH $(1023,553,105)$ | 177 | 51756 | 105 | 373 | 1.170 | 374.170 |

### 4.6. Results of the proposed scheme for some large $\mathbf{B C H}$ codes

In order to find the minimum distance of some large BCH codes, the proposed schemeis applied by using a simple machine of the configuration given above. The obtained results are given in the Table 6 so that
$\mathrm{d}_{\mathrm{f}}$ represent the minimum distance found by our scheme. This table shows the height capacity of the proposed technique to find a minimum weight codeword in very short time.

Table 6. True Minimum Weights of Some BCH Codes of Length 1023 Found by the Proposed Scheme

| BCH(n,k, $\delta$ ) | df | Run time of Step1 <br> (s) | Run time of Step2 <br> (s) | Total Run Time (s) | BCH(n,k, $\delta$ ) | df | Run time of Step1 <br> (s) | Run time of Step2 <br> (s) | Total Run Time (s) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BCH(1023,1013,3) | 3 | 185 | 0.078 | 185.078 | BCH(1023,638,85) | 85 | 368 | 0.733 | 368.733 |
| BCH $(1023,1003,5)$ | 5 | 192 | 0.093 | 192.093 | BCH $(1023,628,87)$ | 87 | 353 | 0.436 | 353.436 |
| BCH $(1023,993,7)$ | 7 | 190 | 0.078 | 190.078 | BCH $(1023,608,91)$ | 91 | 340 | 0.249 | 340.249 |
| ВСН $(1023,983,9)$ | 9 | 193 | 0.093 | 193.093 | BCH $(1023,598,93)$ | 93 | 358 | 0.218 | 358.218 |
| BCH( $1023,973,11$ ) | 11 | 197 | 0.109 | 197.109 | BCH $(1023,588,95)$ | 95 | 372 | 0.546 | 372.546 |
| BCH(1023,963,13) | 13 | 200 | 0.124 | 200.124 | BCH $(1023,573,101)$ | 101 | 313 | 0.264 | 313.264 |
| BCH $(1023,953,15)$ | 15 | 201 | 0.124 | 201.124 | BCH (1023,563,103) | 103 | 372 | 0.218 | 372.218 |
| BCH $(1023,943,17)$ | 17 | 208 | 0.124 | 208.124 | BCH (1023,553,105) | 105 | 373 | 1.170 | 374.170 |
| BCH $(1023,933,19)$ | 19 | 210 | 0.140 | 210.140 | BCH (1023,493,119) | 119 | 353 | 0.218 | 353.218 |
| BCH( $1023,923,21$ ) | 21 | 219 | 0.140 | 219.140 | BCH (1023,483,121) | 121 | 397 | 0.202 | 397.202 |
| BCH $(1023,913,23)$ | 23 | 212 | 0.139 | 212.139 | BCH (1023,473,123) | 123 | 390 | 0.187 | 390.187 |
| BCH( $1023,903,25)$ | 25 | 212 | 0.155 | 212.155 | BCH (1023,453,127) | 127 | 406 | 0.358 | 406.358 |
| BCH $(1023,893,27)$ | 27 | 217 | 0.140 | 217.140 | BCH (1023,443,147) | 147 | 358 | 14.039 | 372.039 |
| BCH( $1023,883,29$ ) | 29 | 222 | 0.186 | 222.186 | BCH (1023,423,151) | 151 | 356 | 0.217 | 356.217 |
| BCH $(1023,873,31)$ | 31 | 226 | 0.171 | 226.171 | BCH (1023,403,157) | 157 | 338 | 0.186 | 338.186 |
| BCH $(1023,863,33)$ | 33 | 229 | 0.171 | 229.171 | BCH (1023,393,159) | 159 | 340 | 1.466 | 341.466 |
| BCH $(1023,858,35)$ | 35 | 232 | 0.156 | 232.156 | BCH $(1023,383,165)$ | 165 | 401 | 0.904 | 401.904 |
| BCH $(1023,848,37)$ | 37 | 243 | 0.171 | 243.171 | BCH $(1023,368,171)$ | 171 | 401 | 0.420 | 401.420 |
| BCH $(1023,838,39)$ | 39 | 238 | 0.156 | 238.156 | BCH (1023,318,183) | 183 | 413 | 0.436 | 413.436 |
| BCH $(1023,828,41)$ | 41 | 232 | 0.171 | 232.171 | BCH (1023,288,191) | 191 | 405 | 0.186 | 405.186 |
| BCH $(1023,808,45)$ | 45 | 238 | 0.186 | 238.186 | BCH (1023,258,213) | 213 | 395 | 0.186 | 395.186 |
| BCH $(1023,798,47)$ | 47 | 255 | 0.171 | 255.171 | BCH (1023,208,231) | 231 | 409 | 0.109 | 409.109 |
| BCH $(1023,788,49)$ | 49 | 252 | 0.171 | 252.171 | BCH (1023,183,239) | 239 | 418 | 0.124 | 418.124 |
| BCH( $1023,778,51$ ) | 51 | 264 | 0.187 | 264.187 | BCH (1023,143,253) | 253 | 401 | 0.108 | 401.108 |
| BCH $(1023,768,53)$ | 53 | 254 | 0.217 | 254.217 | BCH (1023,133,255) | 255 | 395 | 0.093 | 395.093 |
| BCH $(1023,758,55)$ | 55 | 258 | 0.202 | 258.202 | BCH (1023,123,341) | 341 | 407 | 0.108 | 407.108 |
| BCH $(1023,748,57)$ | 57 | 263 | 0.187 | 263.187 | BCH $(1023,121,343)$ | 343 | 385 | 0.108 | 385.108 |
| BCH $(1023,738,59)$ | 59 | 265 | 0.186 | 265.186 | BCH $(1023,111,347)$ | 347 | 397 | 0.093 | 397.093 |
| BCH $(1023,728,61)$ | 61 | 280 | 0.218 | 280.218 | BCH $(1023,101,351)$ | 351 | 417 | 0.078 | 417.078 |
| BCH $(1023,718,63)$ | 63 | 270 | 0.217 | 270.217 | BCH (1023,91,363) | 363 | 339 | 0.077 | 339.077 |
| BCH(1023,708,69) | 69 | 270 | 0.421 | 270.421 | BCH(1023,76,375) | 375 | 355 | 0.078 | 355.078 |
| BCH(1023,698,71) | 71 | 282 | 0.187 | 282.187 | BCH (1023,56,383) | 383 | 373 | 0.077 | 373.077 |
| BCH(1023,688,73) | 73 | 282 | 0.186 | 282.186 | BCH( $1023,36,447)$ | 447 | 378 | 0.046 | 378.046 |
| BCH(1023,678,75) | 75 | 279 | 0.249 | 279.249 | BCH (1023,26,479) | 479 | 371 | 0.046 | 371.046 |
| BCH(1023,668,77) | 77 | 282 | 0.249 | 282.249 | BCH $(1023,16,495)$ | 495 | 372 | 0.031 | 372.031 |
| BCH(1023,658,79) | 79 | 279 | 0.264 | 279.264 | BCH(1023,11,511) | 511 | 348 | 0.030 | 348.030 |
| BCH(1023,648,83) | 83 | 352 | 0.342 | 352.342 |  |  |  |  |  |

## 5. CONCLUSIONAND PERSPECTIVES

In this paper, we have proposed a new efficient scheme to find the minimum distance for large BCH codes. The experimental results show that the proposed scheme outperforms several known powerful techniques. The true value of the minimum distances and consequently the error correcting capability of all the 165 BCH codes of length up to 1023 are determined except the two cases of the $\mathrm{BCH}(511,148)$ and $\mathrm{BCH}(511,259)$ codes. In the perspectives of this work, we will apply this powerful scheme to construct good large cyclic codes, and adapt this scheme to compute the minimum distance for other linear codes.

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A new efficient way based on special stabilizer multiplier permutations to ... (Issam Abderrahman Joundan)
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