International Journal of Electrical and Computer Engineering (IJECE)
Vol. 3, No. 6, December 2013, pp. 849~856
ISSN: 2088-8708

Symbol Error Rate Analysis of M-QAM with Equal Gain Combining over a Mobile Satellite Channel

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Article Info

Article history:

Received Jun 12, 2013
Revised Aug 20, 2013
Accepted Aug 26, 2013

Keyword:

Average symbol error rate
Equal gain combining
Moment generating function
Padé approximation
M-QAM

ABSTRACT

Mobile Satellite Communications (MSC) have become an essential part of the world telecommunication infrastructure. However, the systems suffer from multipath propagation effects. In this paper, error analysis of M-ary quadrature amplitude modulation (M-QAM) with Equal Gain Combiner (EGC) over mobile satellite channel was carried out. The satellite channel was modelled as the product of Rayleigh and Ricians. This was then used to develop a system model for the received signal which was simulated and evaluated in terms of Average Symbol Error Rate (ASER) using the exact closed-form expression derived from moment generating function (MGF) and Padé Approximants (PA) theory. The results showed that at 16dB, Rician factor ‘k’=0, ASER obtained are 41.83%, 18.56% and 10.81% for paths ‘L’ = 2, 3, 4 respectively. ASER values reduced as ‘k’ increased. The results are in agreement with the simulation.

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1. INTRODUCTION

Mobile satellite communication system as a wireless communication is a critical element in the overall telecommunications infrastructure. Multimedia services and products became a reality due to the emergence of modern communication and information technologies. However, the system suffers from multipath fading, scintillation, and shadowing [1-11]. Wireless systems are required to operate under increasingly hostile environment; this is because of the ever increasing demand and ubiquitous access of personal communication services. In mobile satellite communication the signal is degraded by scintillation and multipath fading [12-13].

Multipath propagation is a phenomenon that occurs when a transmitting signal propagates in multiples as a result of obstructions in the terrestrial environment which result in fading, while scintillations are variation in the amplitude level, phase, and angle of arrival of the received radio waves due to the total electron content (TEC) of the ionosphere. The net effect of these two phenomena is signal fading at the receiver. The terrestrial fading is modelled as Rayleigh distribution because there is usually no line of sight at the lower part of the troposphere due to obstruction along the signal path while scintillation can be modelled as Rician distribution because of the possibility of direct line of sight [1], [4], [5], [6], [13-15]. In order to provide tangible solution for foreseeable wireless video application, M-QAM has been used for transmission of video signals, as well as digital modulated radio frequency carrier [7], [16-19].

Diversity combining is the process of combining multiple cpoies of transmitted signals to enhance reliability of the transmission by minimizing the channel fluctuations due to fading. This is applied in wireless communications systems to reduce the degradation effects of fading and to improve signal strength. The chances that all these copies will be in deep fade are significantly minimal [7], [14]. Various combining techniques are used to combine the signals from multiple branches. The most common are selection combining (SC), equal gain combining (EGC) and maximal-ratio combining (MRC) [12], [15].

Most of the previous research work on mobile satellite channel was based on the development of fading models with Maximal Ratio Combiner and Selection Combiner as parts of receivers due the difficulty of finding useful expressions for Probability Density Function (PDF) of the EGC receiver output SNR in closed-form. To the best of the authors' knowledge much work has not been carried out on closed-form expression for the EGC receiver in MSC. In [13] a closed form expression was developed for MSC using MRC receiver in Rician by Rician channel. For the number of paths exceeding two, there is no closed-form expression for the PDF of output SNR for EGC until 1997 when [8], [11], [18] were able to derive a closed-form expression for the PDF of output SNR with three paths (L=3) over Rayleigh fading channel by using statistical decision theory and evaluating the probability of error directly from the characteristics functions (CHF) of the combiner.

This paper develops and analyses a closed-form expression for the error analysis of M-QAM in Rayleigh-Rician product fading channel using moments-based approach. With Padé approximant (PA), a closed-form expression for the MGF of the output SNR was derived with EGC to reduce the hardware complexity of MRC by weighing each signal branch equally (setting the weighing factor to unity) irrespective of the signal amplitude and co-phasing the entire signals to avoid signals cancellation. Numerical and computer simulations are presented to show to evaluate the performance in terms of Average Symbol Error Rate (ASER) in MSC.

2. CHANNEL FADING MODELLING

In [16] all transmitted and received signal are real while wireless channel, including satellite channel models generally assumed to be in complex form, this implies that the transmitted and received signals are represented as complex basedband representation of bandpass signal to facilitate analysis.

In this paper, a frequency flat-fading is considered, where delay spread is less than the duration of a transmitted symbol. The complex envelope of the transmitted signals is denoted as $s(t)$ and that of the received signal as $r(t)$. Then the received signal is $r(t) = s(t)\, p(t)$, where $p(t)$ is the complex low pass channel impulse response for the product of Rayleigh and Rician fading satellite channel. The magnitude of the complex fading envelope $X(t)$ through the satellite channel is therefore, $X(t) = |p(t)|$.

All frequency components are therefore subjected to the same channel gain and the multichannel satellite flat fading channel model is shown in Figure 1. The transmitted signal is assumed to be received as multiple copies through L independent fading channel. L-branch pre-detection EGC receiver was considered where 'L' antennae receive signals with statistically independent random amplitude. Each channel amplitude and phase at any given time or space are represented as random variable (RV). It was assumed that the channel amplitude, phase and delay associated with each channel are constant over the signalling interval. Therefore the received signal $r_l(t)$at l^{th} branch is given by

$$r_l(t) = P_l e^{-j\phi_l} s(t-\tau) + n_l(t) \tag{1}$$

where $s(t-\tau)$ is the delayed transmitted M-QAM signal, τ_l is the channel delay and $n_l(t)$ is Additive White Gaussian Noise (AWGN) with power spectral density 'N_o /2' and assumed to be uncorrelated among different branches.. AWGN with identical double-sided power spectral density N_o is added to each diversity branch signal indicating the statistically independent of the channel fading or complex fading envelope.

2.1. Statistics of the EGC Output SNR for the Product Fading Channel

The n^{th} order moment of the EGC output SNR per bit is given by [6], [7] and 11] as

$$E\left(\gamma_{egc}^{n}\right) = \frac{(2n)!}{L^n} \sum_{\substack{h_1, h_2, \ldots, h_L = 0 \\ h_1 + h_2 + \cdots + h_L = 2n}}^{2n} \prod_{j=1}^{L} \frac{E\left(\gamma_j^{\frac{h_j}{2}}\right)}{h_j!} \quad j = 1,2 \ldots L \tag{2}$$

where $E(.)$ is the expected value operator, the mean value of the SNR at the j^{th} input branch h_j is the channel gain and is related to n, as $h_j = 2n$. The instantaneous SNR per bit γ_l can be expressed as $\gamma_l = x_l^2 \frac{E_s}{N_o}$ where

x_l is the channel amplitude and E_s is the symbol energy. All channel gains are assumed to have the same average power i.e

$$E\left(\left|h_j\right|^2\right) = E(|h_l|^2) = \gamma_a = \Omega_o \frac{E_s}{N_o} \tag{3}$$

where $\Omega_o = x_l^2$ and γ_a is the average value of γ_l

The nth moment of the SNR per symbol of the l^{th} input channel in Rician fading is expressed by [15] as

$$E(\gamma_l^n) = \frac{\Gamma(1+n)}{(k_l+1)^n} {}_1F_1\,(-n; 1; k_l)\gamma_l^n \tag{4}$$

where${}_1F_1$ (.;.;.) is the confluent hyper geometric function of the first kind, $\Gamma(.)$ is the gamma function, $k_l = k$ is the Rician factor defined as the ratio of the power in specula components to the power in random components.

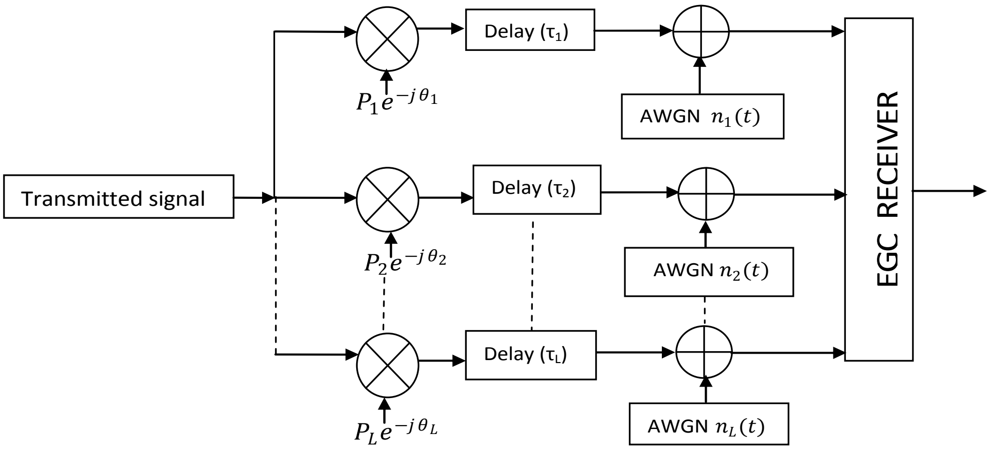

Figure 1. Multilink Satellite Flat-Fading channel model

For the Rayleigh fading equation (4) is expressed by [15] as

$$E(\gamma_l^n) = \Gamma(1+n)\gamma_l^n \tag{5}$$

The product of equations (4) and (5) gives

$$E(\gamma_l^n)_c = \frac{\Gamma^2(1+n)}{(k_l+1)^n} {}_1F_1\,(-n; 1; k_l)\gamma_l^{2n} \tag{6}$$

Equation (6) is the nth moment of the combined product fading channel. Replacing equation (6) in (2) the nth moment of the EGC output SNR per symbol over the product fading channel in closed-form is

$$E\left(\gamma_{egc}^n\right) = \frac{(2n)!}{L^n} \sum_{\substack{h_1, h_2, \dots, h_L = 0 \\ h_1 + h_2 + \cdots + h_L = 2n}}^{2n} \prod_{j=1}^{L} \frac{\Gamma^2(1+n)}{h_j!(k_l+1)^n} {}_1F_1\,(-n; 1; k_l)\gamma_a^{2n} \tag{7}$$

Equation (7) assumes an independent but not necessary identically distributed (non, iid), this implies different values of mean and variance. In this work, and with all the assumptions given an independent and identically distributed (iid) scenario is considered. The EGC receiver equally weighs and co-phases all input signals, then sum the received signals to produce the output signal. Therefore, for equally likely transmitted symbols and iid scenario then $E(\gamma^n)$ becomes

$$\mathrm{E}(\gamma^n) = (L\gamma_a)^n = L^n E(\gamma_a^n) \tag{8}$$

From equations (8) and (7), (9) is obtained

$$E(\gamma_l^n)_c = \frac{L^{2n}\Gamma^2(1+n)}{(k_l+1)^n}\,{}_1\mathrm{F}_1\,(-n;1;k_l)\gamma_{al}^{2n} \tag{9}$$

Therefore, for an iid the nth moment for the EGC in the product fading channel is

$$E\left(\gamma_{egc}^n\right) = \frac{L^{2n}\Gamma^2(1+n)}{(k_l+1)^n}\,{}_1\mathrm{F}_1\,(-n;1;k_l)\gamma_{al}^{2n} \tag{10}$$

Equation (10) shows that the statistical moments of the EGC receiver derived is dependent only on the statistical moments of the output SNR $E\left(\gamma_{egc}^n\right)$ per branch through the Rayleigh-Rician product fading channel. This is the closed-form expression for the EGC receiver output.

3. M-QAM ERROR ANALYSIS

The statistical moment based error analysis for M-QAM has been carried out, for the product fading channel, based on [7, 12]. The method is the alternative method to calculate the M-QAM's average symbol error rates with EGC receiver named as MGF- based approach to the performance analysis of digital modulations over fading channels [15]. Amindavar and Ritcey in [2] proposed the use of Padé Approximants (PA) theory to approximate unknown complex probability density function PDF. Using the MGF and PA, useful expression for calculating the probability of error for higher order EGC combiner is obtained.

3.1. MGF of the Output SNR

The MGF of the EGC output SNR is by definition given by [5] as

$$M_\gamma(s) = E(\exp\,(s\gamma)) \tag{11}$$

This is represented as a formal power series (Taylor) as

$$M_\gamma(s) = \sum_{n=0}^{\infty}\frac{\mathrm{s}^{\mathrm{n}}}{n!}g_n \tag{12}$$

where $g_n = E(\gamma^n)$ is nth moment represented by equation (10).

All moments of $g_n$are finite and can be evaluated in closed-form as shown above, but might be divergent or convergent too slowly to be of any practical use [7]. Therefore, only a finite number N can be used.

$$M_\gamma(s) = \sum_{n=0}^{N}\frac{\mathrm{s}^{\mathrm{n}}}{n!}g_n + R(s^{N+1}) \tag{13}$$

where $R(s^{N+1})$ is the remainder after the truncation, with terms of order greater than N.

Putting (10) in (13) and neglecting the remainder, we have

$$M_\gamma(s) = \sum_{n=0}^{N}\frac{L^{2n}\Gamma^2(1+n)}{n!\,(k_l+1)^n}{}_1\mathrm{F}_1\,(-n;1;k_l)x^{2n}\quad \text{where } x = s\gamma_{al}^2 \tag{14}$$

Equation (14) cannot be said to have a positive radius of convergence or whether it is convergent. This finite power series is therefore not guaranteed to converge for all values of S. This however is possible, using PA techniques to obtain efficiently the limiting behaviour of the power series in compact rational form.

3.2. Padé Approximant (PA) to the MGF

PA is a rational function approximation of power series $M_\gamma(s)$ of a specified order B, for the denominator and A for the numerator, whose power series expansion agrees with the N=A+B order power expansion of $M_\gamma(s)$, where A and B are positive integers.

$$R(x) = \frac{\sum_{i=0}^{A}a_i x^i}{\sum_{j=0}^{B}b_j x^j} = M_\gamma(s) = \sum_{n=0}^{N}C_n x^n \tag{15}$$

where $R(x)$ is a power series in this case $M_\gamma(s)$ to be resolved to a rational function, C_n represents the coefficients obtained using MATLAB™ in equation (14), B= A+1, implies that order of denominator is greater than numerator by 1. a_j and b_j are real coefficients which are determined by solving the set of A+B+1 equations, with an assumption that $b_0 = 1$. The coefficients were obtained as described by [6], [11], and [12].The system of equations obtained forms a Hankel matrix and the determinant of which must not be zero for a solution to exist. According to [2] there exists a value of B above which Hankel matrix is rank deficient. The coefficients are determined using MATLAB application package and the Hankel matrix rank deficient above N=12. Therefore, the PA of equation (15) is given below

$$M_\gamma(s) = \frac{1-119.29e+4.98e^3x^2-8.75e^4x^3+6.05e^5x^4-1.13e^6x^5}{1-128.29x+6.02e^3x^2-1.29e^5x^3+1.29e^6x^4-5.33e^6x^5+6.28e^6x^6} \quad (e = 10) \tag{16}$$

3.3. MGF- based Approach to Average Symbol Error Rate (ASER)

Having derived the PA as above, average symbol error rate (ASER) of M-QAM is obtained as follows. According to [15] the MGF based approach gives the ASER of M-QAM modulation scheme as

$$P_{se} = \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\left[\int_0^{\pi/2} M_\gamma(s)d\emptyset - \left(1-\frac{1}{\sqrt{M}}\right)\int_0^{\pi/4} M_\gamma(s)d\emptyset\right] \tag{17}$$

$$P_{se} = \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\left[\int_0^{\pi/2} M_\gamma\left(\frac{-g}{(\sin\emptyset)^2}\right)d\emptyset - \left(1-\frac{1}{\sqrt{M}}\right)\int_0^{\pi/4} M_\gamma\left(\frac{-g}{(\sin\emptyset)^2}\right)d\emptyset\right] \tag{18}$$

where $s = -\frac{g}{(\sin\emptyset)^2}$, and according to [3] $g = \frac{3}{2(M-1)}$

Using equation (18) the error performance of M-QAM with EGC diversity in MSC Rayleigh and Rician product fading channel is evaluated via numerical integration. Equation (18) consists of elementary functions, and single integrals which are quite easy to evaluate.

4. NUMERICAL RESULTS

The performance of M-QAM modulation over combined ionospheric scintillation and terrestrial flat fading channel has been evaluated by numerical computation using MATLAB package. The scintillation was modelled as Rician fading with k > 0 implies the existence of a line of sight, while Rayleigh fading, k = 0 implies no line of sight component in the propagation path was used to model the terrestrial multipath. Numerical results are presented to illustrate the ASER performance of M-QAM with EGC receiver over the mobile satellite channel, modelled as the product of Rayleigh- Rician fading.

Figure 2 shows the ASER of 4-QAM 16-QAM, and 256-QAM with EGC receiver operating over independent and identically distributed, product fading channel (k = 0) which is equivalent to Rayleigh × Rayleigh. This explains why the ASER was very high compared to ASER when k > 0, while Figure 3 depicts the ASER for the squared M-QAM considered for k = 5 which symbolises the presence of a line of sight component. From these figures, it is apparent that the higher the order of modulation the more the error rate, that is lower order M-QAM performs better than higher order M-QAM.

Figure 4 shows the ASER of 4-QAM, 16-QAM, and 256-QAM over iid with Rician factor k=10, while Figure 5 depicts the ASER of 64-QAM, over iid in a Rayleigh×Rician product fading channel with different values of k. Comparing the results obtained it is clearly evident as would be expected that ASER performance is better in channel where a strong line-of-sight exists (higher values of k). The ASER for 64-QAM at k = 0 and L=2, was observed to be 42.57% and 17.64% with L=4 at SNR of 12dB. With k=10, and at L=2 ASER was found to be 33.70% while at L=4 it was 12.19%. At 20dB the ASER obtained with k=0 and at L=2 was 2.20%, while at L=4 0.47% ASER was obtained. The ASER obtained with k=10 was 1.42% for L=2, while at L=4 it was 0.35%. It was observed from the results that at lower modulation level, the ASER was low compared to higher modulation level.

This reveals that the performance of 4-QAM is the same as Quadrature Phase Shift Keying, (QPSK) in mobile satellite communication. As observed from the Figures 2 to 5, the relative advantage of EGC diversity is obvious as the ASER improves with the increase in number of paths (L), line of sight (k) and SNR. This work is verified by computer simulation results, circle signs, which are compared to the mathematical analysis, (lines). Over 10000 samples are used for each SNR to generate the fading envelopes for the product fading channel. In the situation each bunch of multipath components and the phases are random and have similar delay times, meaning that a frequency flat-fading is considered.

The results presented obviously reveal a good agreement between analytical and computer simulations approaches. Therefore, the results are in agreement with [13], where MRC was used in combined frequency flat fading channel for the product fading channel of Rician and Rician and with [7] where the approach was used for different fading channels. It is clear from the Figures presented that excellent agreements exist between analytical (lines) and computer simulations (circles) results are observed.

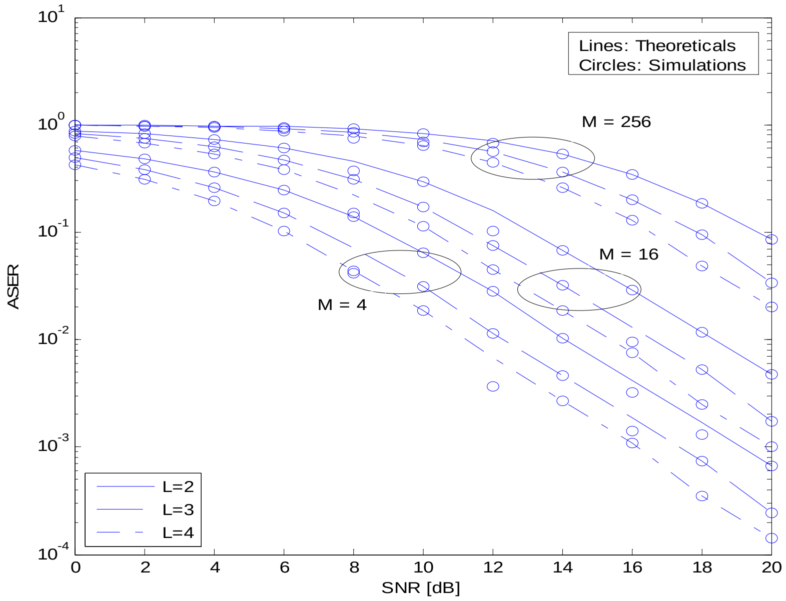

Figure 2. ASER of M-QAM with EGC receiver over combined Rayleigh × Rayleigh (k=0) Satellite fading channel

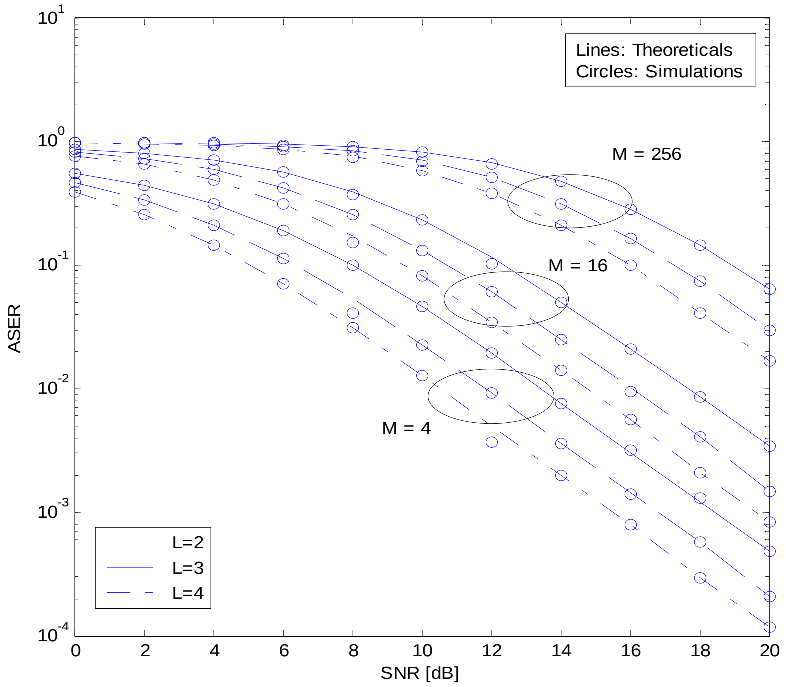

Figure 3. ASER of M-QAM with EGC receiver over combined Rayleigh × Rician (k=5) Satellite fading channel.

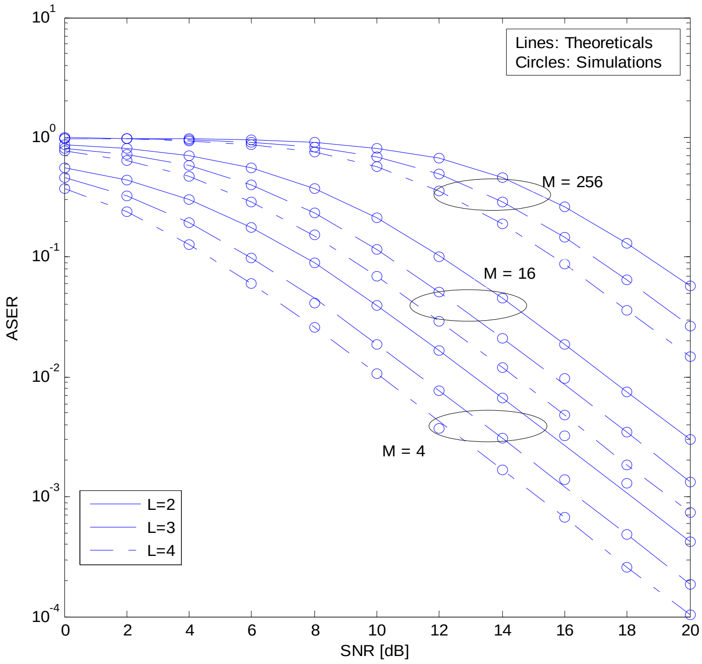

Figure 4. ASER of M-QAM with EGC receiver over combined Rayleigh × Rician (k=10) Satellite fading channel.

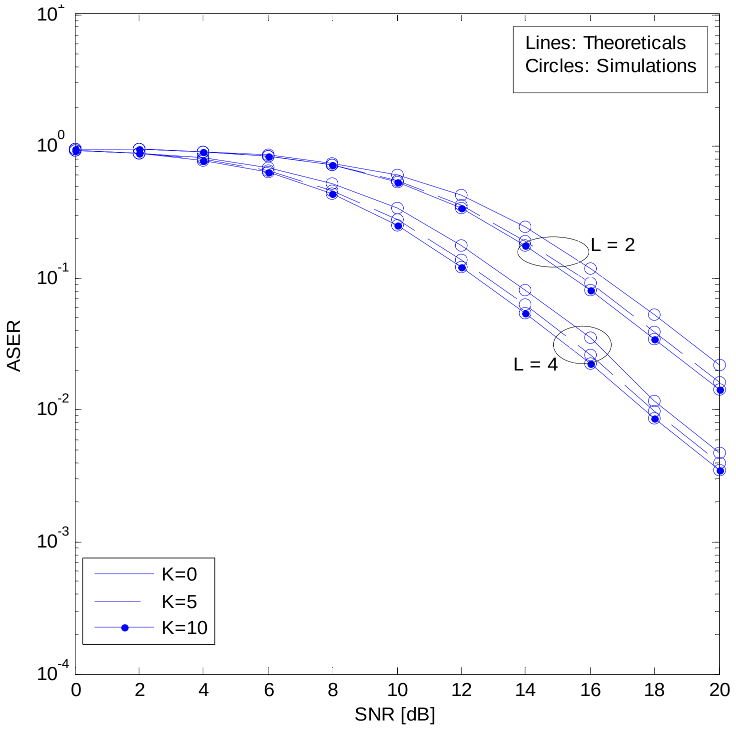

Figure 5. ASER of 64-QAM with EGC receiver over combined Rayleigh × Rician Satellite fading channel.

5. CONCLUSION

The symbol error rate of M-QAM with EGC receivers operating over a product fading channel (Rayleigh and Rician) with MGF have been developed and well presented. A closed-form expression was derived for the mobile satellite channel using the MGF-based approach. The MGF obtained was approximated with PA theory in order to accelerate its convergence. Simple to evaluate rational expression for the MGF was obtained. The numerical and computer simulation results are presented, there is an agreement between theoretical and simulation results. This paper has revealed that the degradation caused by scintillation through the ionosphere and multipath fading in the troposphere which include the terrestrial environment can be effectively reduced by employing efficient and robust modulation technique such as M-QAM with EGC.

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