

## An Efficient Approach to Voltage Stability Evaluation using Tellegen's Equations

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### ABSTRACT

In this paper, the adjoint networks based on Tellegen's Theorem are used to improve the PV curve assessment. PV curve is the most widely accepted method for determining the margin of the power system state to the voltage collapse point. The repetitive power flow and continuation power flow are used to access PV curves through tracing the power flow solutions for the change of loads. Since the Minimum Jacobi matrix eigenvalue is practically close to zero at neighbor voltage collapse point, there is a restriction in order to access the voltage stability level by repetitive power flow method. This problem solved by appropriate combination of adjoint networks based on Tellegen's Theorem and conventional equations. The proposed method was tested on SWCC test system. Comparison of three method results show advantages and efficiency of the proposed method.

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## 1. INTRODUCTION

The Power systems are deregulated electricity markets recently and cost reduction forces system to operates ever closer to load limits and maximum capacity of equipment's. The stressed power networks with reduced stability margins and reduced reactive-power reserves are faced several blackouts that occurred due to voltage instability. Voltage instability has become serious concern for system operators and is the one of the most important problems that an electrical network can face when the system is heavily loaded. Several researches have been carried out in this regard. well known method based on executing a large number of power flows using conventional equations and access PV or QV curves at buses [1]. The different approaches have been invented in order to compute the maximum load ability of power networks. Repetitive power flow (RPF) [1], continuation power flow (CPF) [2], [3], modal analysis method [4], and optimization method [5].

Conventional power flow suffers from the curse of nearing of min. eigenvalue of jacobian to zero, i.e., the numerical convergence and solution accuracy drastically deteriorates with problem ill-condition.

CPF is based on conventional power flow equations and continuous parameter. This method adds  $\lambda$  as extra parameter to system parameters and considers an extra equation respect to continuous parameter. the equations are solved with Newton's method. Any improper value that is choose by mistake for continuous parameter, makes it diverged. Also, in comparison with conventional method, CPF needs much more time to meet the results [1].

Power flow solutions based on Tellegen's equations and adjoin networks discovered first time by Ferreira and Jesus that is independent to conventional jacobian matrix. This power flow same as conventional power flow is fast and accurate method [7], [9].

In this paper, a combination of conventional power flow with Tellegen's equations and adjoint systems is proposed which access to PV curves by solving the constrain of conventional method efficiently.

## 2. REPETITIVE POWER FLOW

In this method by calculating the PV curve of network's buses and based on load variation behavior, the margin of instability voltage can be measured. The main equation in this approach is as follows [6]:

$$f(x, \lambda) = 0 \quad (1)$$

In the power network with the number of n buses Equation (1) can be showed as follows:

$$P_{0i} - \sum_{j=1}^n V_i V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) - \lambda (\Delta P_{Gi} - \Delta P_{Li}) = 0 \quad (2)$$

$$Q_{0i} - \sum_{j=1}^n V_i V_j (G_{ij} \sin \delta_{ij} + B_{ij} \cos \delta_{ij}) - \lambda (-\Delta Q_{Li}) = 0 \quad (3)$$

Where  $P_{0i}$  and  $Q_{0i}$  are the base-case real and reactive power injections at bus i respectively.  $V_i$  is the voltage magnitude at bus i.  $G_{ij}$  and  $B_{ij}$  are the real part and imaginary parts of the network admittance between buses i and j respectively.  $\delta_{ij}$  is the angle difference between buses i and j.  $\Delta P_{Gi}$  is the proposed real generation variation at bus i.  $\Delta P_{Li}$  and  $\Delta Q_{Li}$  are the proposed real and reactive load variations at bus i. Finally,  $\lambda$  is the load variation parameter. Equation (2) and (3) are the same equations of power network which are well known as conventional equations that be solved through Newton-Raphson method and network parameters such as voltage angle and magnitude are computed by them.

## 3. THE PROPOSED METHOD

### 3.1. Equations based on Tellegen's theorem

We nominate bus K parameters as  $V_k$ ,  $I_k$  and  $S_k$  at the specific work point, and load variation as disturbance  $\Delta S_k$  at bus K.

The following equations are being used to calculate voltage at bus m:

$$\text{Real}(\Delta V_m) = \text{Real}(\sum_{k \in l} (\tilde{V}_{km}^* / (\Delta V_k + V_k)) \Delta S_k) - \sum (I_k^* \tilde{V}_{km}^* (\Delta V_k)^2) / (I_k^* \tilde{V}_{km}^* (\Delta V_k)^2) \quad (4)$$

$$\text{Im}(\Delta V_m) = -\text{Im}(\sum_{k \in l} \tilde{V}_{km}^* / (\Delta V_k + V_k) \Delta S_k) - \sum I_k^* \tilde{V}_{km}^* (\Delta V_k)^2 / (V_k (\Delta V_k + V_k)) \quad (5)$$

The power network parameters can be achieved with solving the above equation. These equations are the main equations of network and are completely different than equations mentioned in the paragraph II.

These equations are derived from famous Tellegen's equation:

$$\hat{i}' \Delta v - \hat{v}' \Delta i = 0 \quad (6)$$

With regard to adjoin circuit of power network under study [7].

$\Delta V$  and  $\Delta I$  represent the voltage and current variations of network's branches;  $\hat{i}$  and  $\hat{v}$  are the voltage and current of corresponding adjoint circuit of power network.

The program has been developed by MATLAB software using its matrix features to solve the network equations based on Tellegen's equations.

$\Delta V_m$  of any bus is calculated in separate stage [8]:

$$A X_m = e_{1m} \quad (7)$$

Where:

$$e_{1m}(i) = 1 \text{ if } i=m \text{ else } e_{1m}(i) = 0$$

$X_m$  is vector of parameters that are used in (4) and (5) equations.

Voltage of all buses can be calculated through one step with MATLAB features as follow:

$$E_m = [e_{1k} \dots e_{1m} \dots e_{1n}]$$

$$A[X_k \dots X_m \dots X_n] = E_m \quad (8)$$

Where k is number of slack buses in the power network plus 1

### 3.2. Combination of conventional and Tellegen's equations

In vicinity of the voltage collapse point, the RPF method diverges and the PV curves assessment is impossible. In order to solve this problem, RPF method calculates the last operation point and calculation will be continued by Tellegen's equations and adjoint systems. The voltage of buses will be calculated by Tellegen's equations and adjoint circuits up to voltage collapse point, thereafter by calculated system parameters, Jacobian matrix, its eigenvalues and eigenvectors will be calculated. As result, PV curves assessment will be completed.

Table 1. Loads and generations of the test system.

Bus no.	Bus type	Gen.MW	Max. Gen.MVA	Load MW	Load MVA
1	S	-	-	0	0
2	G	100	50	0	0
3	G	50	25	0	0
4	L	0	-	200	100
5	L	0	-	90	45
6	L	0	-	0	0
7	L	0	-	0	0
8	L	0	-	100	50
9	L	0	-	0	0

Table 2. Line parameters of the test system.

From bus	To bus	R>(*10 <sup>-3</sup> )	X(*10 <sup>-4</sup> )
1	4	0	576
2	7	0	625
3	9	0	586
4	5	100	850
5	7	320	1661
6	4	170	920
7	8	85	720
8	9	119	1008
9	6	390	1700

## 4. SIMULATION, RESULTS AND DISCUSSTIONS

To verify accuracy and efficiency of proposed method, simulations carry out on the SWCC network. Table 1 and 2 show parameters of the test system. The maximum limit reactive power in generators 2 and 3 are considered as 50 and 25 Mega Var.

$\lambda$  is set as below:

Min. eig. > 0.1 Then  $\lambda = 1/300$

0.1 > Min. eig. > 0.001 Then  $\lambda = 1/(3*10^5)$

Min. eig. < 0.001 Then  $\lambda = 1/(3*10^8)$

This means that loads of buses are increased by scale of  $\lambda$  in any step in relation to previous step.

Figure 1, 2 and 3 show PV curves that are accessed by the RPF method and Figure 4 shows the variations of minimum Jacobian matrix eigenvalue against total load supplied by network and at total load 431.9 Mw, the curve reduce very fast because the reactive powers of generators 2 and 3 receive to maximum capacity and type of them in simulations change from constant voltage bus to load bus.

As can be observed, the minimum Jacobian matrix eigenvalue and maximum load that are accessed by the RPF method, respectively are 0.0137 and 469.2 MW.

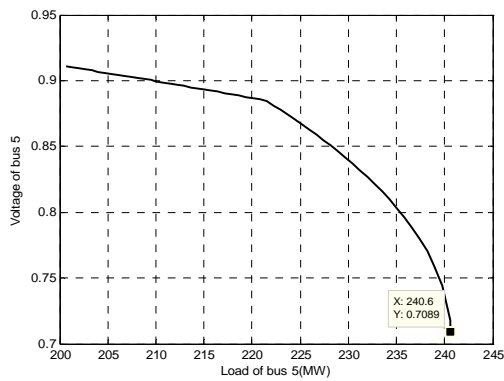


Figure 1. bus5 voltage-load curve

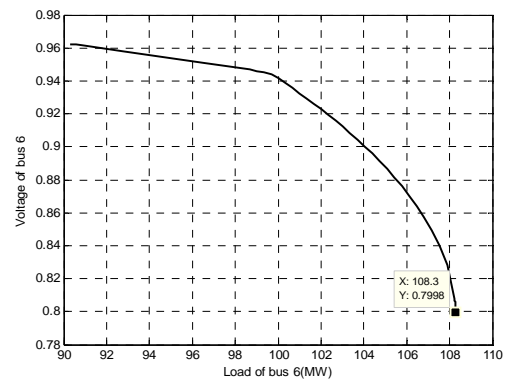


Figure 2. bus6 voltage-load curve

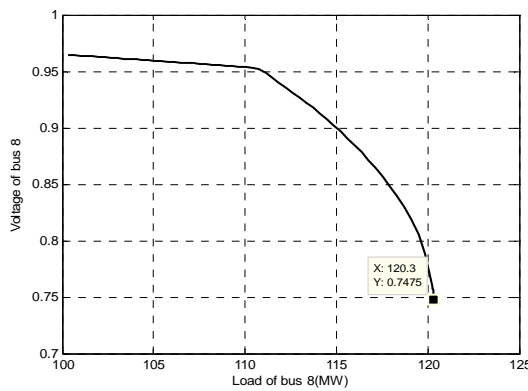


Figure 3. bus8 voltage-load curve

Figure 5, 6 and 7 show PV curves are accessed by the proposed method and Figure 8 shows the variations of minimum Jacobian matrix eigenvalue against total load supplied by network and Figure 9 zooms the variations of minimum Jacobian matrix eigenvalue neighbour of the collapse point that accessed by proposed methods. Dashed line shows same results of two methods and continues line is accessed by proposed method. The RPF is diverged and stopped at the end of dashed line and proposed method same as RPF method, starts with conventional equations up to 469.2 MW and continues calculation by Tellegen's equations and adjoint systems and this verifies that proposed method access voltage collapse point effectively. The advantage of Tellegen's equations is that, at first the buses voltage neighbour of the collapse point are calculated and by the results, the Jacobian matrix and its minimum eigenvalue will be achieved.

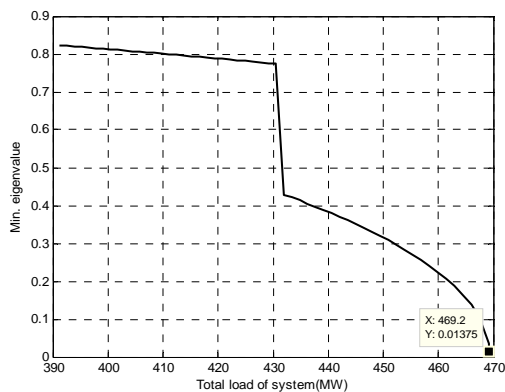


Figure 4. Minimum eigenvalue-total load curve

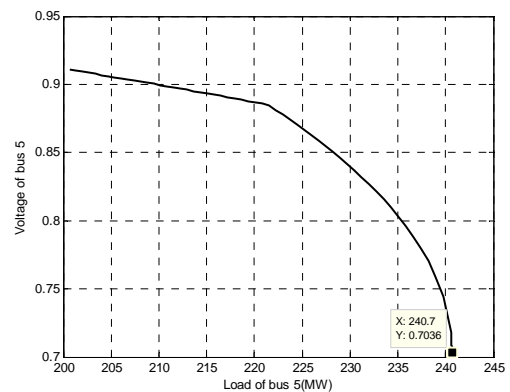


Figure 5. bus5 voltage-load curve

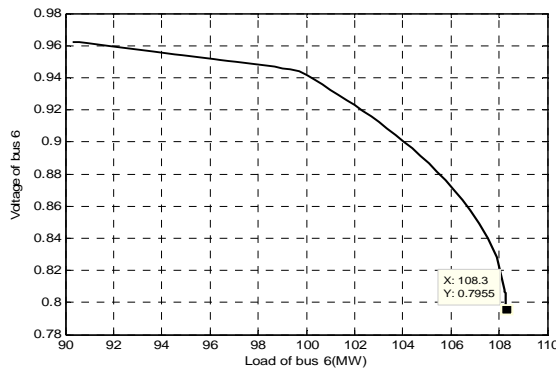


Figure 6. bus6 voltage-load curve

Results shown in Table 3, compare CPF and proposed method. Two methods access to 469.3 MW as total load ability of system. The computing elapsed time by proposed method is less than elapsed time by CPF. The min. eigenvalue that two methods are converged, is less than  $10^{-4}$ . Analysis shows that the proposed method is faster than CPF to find the voltage collapse point while its accuracy is same as CPF.

Table 3. CPF and proposed method results.

Method	Min. eigenvalue	Elapsed time (second)	Max. Load ability (MW)
Proposed method	$<10^{-4}$	2.9401	469.3
CPF	$<10^{-4}$	3.4606	469.3

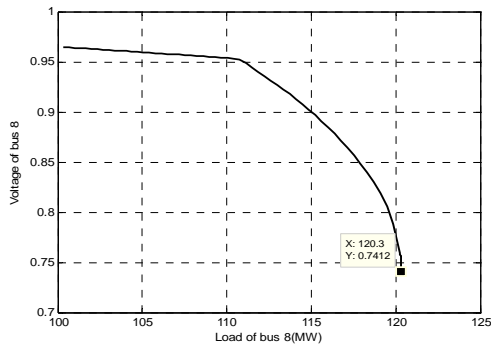


Figure 7. bus8 voltage-load curve

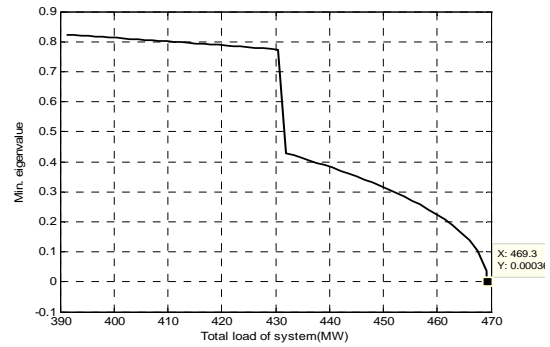


Figure 8. Minimum eigenvalue-total load curve

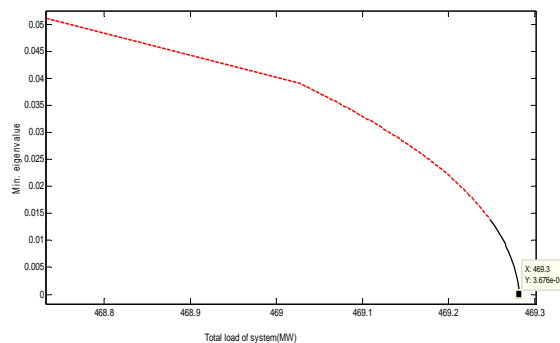


Figure 9. Minimum eigenvalue-load curve

## 5. CONCLUSION

This paper has presented efficient and new approach to voltage stability assessment with combination the Tellegen's power flow equations and the conventional power flow equations. RPF which is based on solving the conventional equations of network through Newton-Raphson method will not be converged near the voltage collapse point. CPF and proposed method solve the limitations of RPF.

The proposed method will be converged around voltage collapse point efficiently. On the contrary of CPF approach that removes the limitations by adding continuation parameter to conventional equations of network and two extra stages, prediction and correction, the proposed method has more computational speed and simplicity.

The adjoint quantities hold important information about the system information that becomes decisive when power system is close voltage collapse point.

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