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WAVE CLIMATE AT MOFFAT BEACH

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WAVE CLIMATE AT MOFFAT BEACH

by

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LIST OF SYMBOLS

- A Elevation above mean water level of highest crest in a given wave recording.
- B Elevation above mean water level of second highest crest in a given wave recording.
- C Elevation below mean water level of lowest trough in a given wave recording.
- D Elevation below mean water level of second lowest trough in a given wave recording.
- $E(\omega)$ Energy density of spectrum, i.e. energy contained in the frequency band $(\omega, \omega + d\omega)$.
- H Wave height, i.e. maximum crest to trough distance between any two consecutive upward zero crossings.

 $H_1 = A + C$

 $H_2 = B + D$

- $H_{1/3}$ Significant wave height, i.e. the average of the one third highest waves in a given recording.
- $H_{1/10}$ Average of the one tenth highest waves in a given recording.

 $H_{1/3,50}$. The magnitude of $H_{1/3}$, $H_{1/10}$, etc., which is equalled or

- $H_{1/10,50}$ exceeded 50% of the time.
- H The maximum wave height in a given recording or during a given period.
- H' Same as H , but pressure correction made using T_{H} instead of T_{z} .

 H_i i th wave in the series 0 < i < N.

- H_m Mean height of all waves in a given wave recording.
- H Wave height which is equalled or exceeded n% of the time in a given wave recording.

H Deepwater wave height.

H_p Most probable wave height in a given wave recording.

LIST OF SYMBOLS (ctd)

H_(p) Average of the p highest waves in a given recording. Mean square wave height of a given wave recording (eqn 4.2-5). Hrms Hs K H' Surface wave height. нz H's Theoretical surface wave height. Wave height measured at a distance z below water surface. Hz Theoretical pressure response correction factor K н' (eqn 3.4-1). Hs Ka Additional empirical pressure response correction factor (figure 5). ĸ Shoaling coefficient. L Wave length. L_H Wave length of the highest wave in a given recording. max Deepwater wave length. L Ν Number of waves in a given recording or in a given period of time. N Number of wave crests in a given recording. Number of upward zero crossings in a given recording. N p(H)dH Probability that the wave height H_ is equalled or P(H) exceeded (cumulative probability). Т Wave period. Average period between wave crests in a given wave recording. T_c ^т_{Н1/3} Average period of the one third highest waves of a given wave recording. T H max Period of the highest wave in a given wave recording. $\frac{2\pi}{\omega_0}$ Period corresponding to the peak of the energy density spectrum. Т

LIST OF SYMBOLS (ctd)

- T_R Period of a sine wave with the same power as the whole spectrum.
- T Surface wave period.

$$T_z = \frac{1}{N_z}$$
 Average period between upward zero crossings in a given wave recording. Also in eqn 7.2.1-1 the average period at depth z below the water surface.

- a, Amplitude of the i th component of the wave spectrum.
- d Water depth.

œ

- e Energy per unit surface area of the i th component of the wave spectrum.
- g Gravitational acceleration.

$$m_n = \int_{0}^{\infty} \omega^n E(\omega) d\omega \quad \text{nth order moment of the energy density} \\ o \qquad \text{spectrum.}$$

m Zero order moment of energy density spectrum, i.e. area under spectrum.

1

Root mean square deviation of the water surface elevation y about the mean water level \overline{y} in a wave recording

$$\sqrt{m_0} = \left[\frac{1}{\tau} \int_0^{\tau} (y - \overline{y})^2 dt\right]^{\frac{1}{2}}$$

n an integer

p an integer

p(H) Probability density of H.

t Time.

y Instantaneous water surface elevation with respect to the mean water level of a given wave recording.

y Mean water level of a given wave recording.

y(t) Stationary random function of the water surface elevation y.

z Depth below mean water level.

LIST OF SYMBOLS (ctd)

α	A dimensionless constant of proportionality in theoretical wave spectra.
β	A dimensionless constant in theoretical wave spectra.
ε	Spectral width parameter (eqns $4.2-8$ and $4.6-5$).
θ =	log _e N _z .
ρ	Density of water.
τ	Duration of a given wave recording.
ϕ_{i}	Phase angle of the i th component of the wave spectrum.
ω i	Frequency of the i th component of the wave spectrum.
ω o	Frequency corresponding to the peak of the energy density spectrum.
E(H _{max})	Mean maximum wave height during a given period of time.
$\mu(H_{max})$	Most probable maximum wave height during a given period of time.

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ABSTRACT

The analysis of a series of wave recordings made at Moffat Beach, Queensland using an OSPOS wave recorder during 1963-1964 is described. This analysis included (i) the establishment of representative parameters for each wave recording; (ii) the determination of the frequency of occurrence of the representative parameters over the total recording period and (iii) the comparison of the recorded data with visual observations at Cape Moreton and the adjustment of the frequency curves to a longer more representative period of time.

The results of the analysis provide confirmation of the applicability of the Rayleigh distribution to the short term wave height distribution within a given wave recording. On the other hand no definite conclusions are made concerning which theoretical representation of the long term wave height frequency of occurrence is to be preferred.

The data analysed shows that at least four distinct types of wave conditions make up the overall wave climate in this area. The general wave height and period characteristics representative of these conditions together with seasonal variations are presented.

The Moffat Beach wave data is compared with that obtained off the Gold Coast using Wave Rider buoys. Generally similar values of significant wave height are found but the wave periods are appreciably different. An analysis of possible explanations for this difference suggests that the filter effect due to pressure attenuation almost completely removes local wind generated seas from the OSPOS records. The question of a representative wave period for a given wave recording is considered and it is concluded that the use of the zero crossing period T_z for the analysis of unfiltered surface wave recordings of simultaneous sea and swell such as generally occurs off the southern Queensland coast cannot be recommended.



Fig1 Locality Plan

1. INTRODUCTION

Some years ago when investigations were being made concerning the location of the boat harbour for the then proposed shore based pilot service for the port of Brisbane, the Queensland Department of Harbours and Marine obtained some recordings of wave conditions on the Caloundra-Mooloolaba section of the Queensland coast. (Figure 1). These recordings have been analysed by the author, working intermittently over several years and, while it is some years since they were obtained and while also the methods by which they were obtained are somewhat outdated, they are of particular interest for two reasons. Firstly, they contain records made during the occurrence of two cyclones and secondly, the records are located fairly close inshore. Present day more sophisticated equipment being used on the Gold Coast beach erosion survey is normally located in comparatively deep water and up to the time of commencement of writing had not recorded during cyclonic conditions (ref. 23)*.

2. WAVE RECORDER

2.1 Description

The wave recorder with which the records were obtained was an OSPOS manufactured by the firm of Van Essen N.V., of Delft, Netherlands. This instrument is an Off-Shore Pressure Operated Suspended wave recorder. It consists of a buoyant cylinder which is suspended at a certain elevation above the sea bottom (figure 2). The cylinder contains a pressure transducer which actuates a pen whose movement is recorded on waxed paper. The recording paper is driven by a small battery operated motor, a clock mechanism switching the motor on for a certain period at regular intervals. The whole system is self contained and designed to operate unattended for periods up to one month in duration. Because of the self contained nature of the recorder, the scale of the recording charts is of necessity rather small. The scale for wave height was 1 cm to 1 m (0.12 ins to 1 ft), while the time scale was normally 0.25 mm per second (0.49 ins per minute).

2.2 Location

The actual recorder used was obtained from Comalco who had previously used it at Weipa on the North Queensland gulf coast. It was installed by the Harbours and Marine Department first off Moffat Head and then later off Point Cartwright (figure 1). At both places its location

^{*}This latter deficiency has since been rectified with recordings made during cyclones in February 1971 and February-March 1972 (ref. 44).



was altered once. In every situation the recorder was located as close as possible to the seven fathom line (L.W. datum) and the actual instrument was 18 to 20 feet below mean sea level.

2.3 Operation

The recorder could be arranged to record for a given length of time at regular intervals. In this case no change was made to the settings used at Weipa and so records five minutes long were obtained every hour. On this basis the recorder could run for approximately one month without attention.

Table 1

Recorder Site	Overall Reco Start	rding Period Finish	Total Time hours	No of hourly records obtained	% Operation
Moffat Beach l	17 Jan 1963	21 Jan 1963	3816	1953	51%
Moffat Beach 2	20 Nov 1963	21 Apr 1964	3792	909	24%
Mooloolaba l	6 Nov 1964	* Apr 1965	3720?	415?	11%
Mooloolaba 2	20 Oct 1965	? Jan 1966	2136?	549	26%

OPERATING PERIODS OF OSPOS WAVE RECORDER

* The date of final removal of the recorder is unknown in these two cases.

The overall periods of operation of the recorder at each site are shown on table 1. It is evident that the percentage operation is not very satisfactory especially at the first Mooloolaba site. From the data in table 1 it can be deduced that the overall operating efficiency was 28%. Analysis was subsequently carried out only on the Moffat Beach records for which the operating efficiency was 38%. This is quite a low value, although it must be kept in mind that high operating efficiencies are not very likely with this apparatus. For instance, with a similar recorder off the coast of Guyana in South America, the Delft Hydraulics Laboratory obtained 153 days of record in a ten month period, i.e. 50% operation (ref. 21), which is the same as at the first Moffat Beach site.

The intermittent operation was the result of a variety of circumstances. Weather conditions sometimes prevented the recording chart being changed at the correct time, while difficulties in obtaining spare parts and lack of servicing facilities made efficient operation difficult to achieve. The principal faults in the instrument were associated with the batteries which drove the recording chart and the timing mechanism which started the recorder each hour.

3. WAVE RECORD ANALYSIS

3.1 General Plan

A considerable amount of trial and error occurred before the final plan of the analysis was developed. This was partly due to the fact that the work was carried out largely as a spare time activity between other more pressing tasks. The general outline of the analysis finally carried out was as follows. Essentially the wave characteristics of height and period were determined from each recording, corrected for pressure attenuation and exceedance curves for each of the principal directions, north east, east and south east, determined. Wave heights and periods were then correlated with visual observations made at Cape Moreton during the recording periods. Comparison with visual observations over the eight year period 1960-1967 then allowed the wave height and period exceedance curves to be adjusted to compensate for the shortness and seasonal bias of the recording period. Finally, the wave height exceedance curves based upon H____ (maximum height in a given record) were corrected to allow for intermittent recording and also converted to $H_{1/3}$ (significant wave height) using data obtained from detailed analysis of the wave height distribution of selected recordings.

3.2 Chart Reading and Correction

A consequence of the unsatisfactory operation of the recorder was that the time scale or length of each recording was variable. Furthermore, the recorder chart often failed to start and so there were gaps in the records. The recorder was set to record for five minutes every hour, but since there was no indication on the chart of the time at which a recording was made, the only means of dating the records was by counting the number of records which had been made since the recorder was started. The dating of records was thus at times extremely difficult and at times impossible.

The breaks in the recording sequence could generally be detected without much difficulty. However, it was not always an easy matter to estimate their length. Since the records also recorded the variation in mean water level due to tidal action, an independent time scale was available to check that deduced from the number of recordings. For short breaks in the record, i.e. less than 6 hours, all that was necessary was to check the interval between successive high and low water levels and insert the appropriate number of hours required to make this correct. For longer breaks the heights and times of high and low water were compared with predicted and/or measured tide levels at nearby locations. This was generally satisfactory when the diurnal inequality of the tide was large and the wave action not too great. At times of significant wave action, when it was difficult to obtain a mean tide level from the wave recording, comparison was made with visual sea-swell observations at Cape Moreton to establish the correct time of the records. However, since correlation of

Table 2

RECORDING PERIODS SUITABLE FOR ANALYSIS

Recorder Site	Nominal Reco Start	ording Period Finish	No. of usable hourly recordings	No.of breaks in record	General Weather Conditions
Moffat Beach l	17 Jan 1963	15 Feb 1963	704	1	Average
Moffat Beach l	19 Mar 1963	2 9 Apr 1963	700	7	Cyclone 20-26 Apr
Moffat Beach l	30 Apr 1963	23 May 1963	549	1	Multiple complex depresions 2-14 May
Moffat Beach 2	13 Mar 1964	21 Apr 1964	756	7	Cyclone "Henrietta" 29 Mar- 10 Apr
Mooloolaba 2	20 Oct 1965	19 Nov 1965	429	0	Average

wave records with visual observations was the principal reason for wanting to date the records, this last procedure was only resorted to when all other methods failed.

On completion of the correction of the chart time scale, it was found that there were five usable recording charts which could be dated with reasonable certainty and which were of sufficient length to be worth consideration. As table 2 indicates, all except one of these recordings were made at Moffat Beach. Refraction analysis indicated that wave conditions at the two Moffat Beach sites would not be greatly different from one another so it was decided to use the data from these two sites for analysis. The Mooloolaba record was set aside as sufficient hydrographic data was not available to check whether wave conditions were the same or different from those off Moffat Beach.

3.3 Data read from Recordings

The data obtained from the charts may be divided into two groups: (i) that read from all hourly recordings on the charts selected for analysis and (ii) further data read from selected recordings.

(i) Data read from all recordings

Maximum crest to trough height,	H max
Wave length of highest wave,	L _H max
Length of five minute record,	1
Number of upward zero crossings,	N _z

(ii) Additional data read from some recordings only

Crest to trough height of all waves defined by upward zero crossings,	н
Number of wave crests,	Nc
Tide level	
Height above mean level of highest and second highest waves,	A and B.
Depth below mean level of lowest and second lowest waves,	C and D

The various quantities referred to above are defined on figure 3.



..... Wave crests

O..... Zero crossings in an upward direction

Fig 3 Wave Record Parameters

3.4 Correction of Pressure Recordings to Surface Wave Heights

One of the major difficulties in using a subsurface wave recorder of the pressure type is that the recordings must be corrected using a pressure attenuation coefficient to obtain the surface wave heights, since the amplitude of the pressure fluctuation is attenuated with increasing depth below the water surface. The normally accepted pressure response factor is that given by small amplitude theory, i.e.,

$$K_{\rm P} = \frac{H_{\rm z}}{H_{\rm s}'} = \frac{\cosh \frac{2\pi (d-z)}{L}}{\cosh \frac{2\pi d}{L}} \qquad 3.4-1$$

where H is the apparent wave height on the recording measured at depth $^{\rm Z}$ z below the surface

 H'_s is the theoretical surface wave height.

A graph calculated from the above equation giving $\frac{Tz}{H}$ as a function of $\frac{z}{d}$ for wave period T and water depth d was supplied^S with the recorder (figure 4). The normal method of application is to determine an average period for each record, in this case T was used, and adopt a single correction factor for each recording. The alternative of correcting each individual wave using its own period is far too tedious.

A further simplification adopted was to assume a constant value of z relative to mean water level at each recorder site (18 ft at Moffat Beach I and 19.7 ft at Moffat Beach 2). Thisgave a constant value of $\frac{z}{d}$ for each site and so a simple calibration between K_p and T could be used. Since the error involved in this simplification had the same order of magnitude but the opposite sign for sea levels above and below mean sea level, its effect upon the statistical analysis of wave heights over a period of time has been assumed to be minimal. The maximum deviation of K_p from its true value at high or low tide level is ±6% for 7 second waves and decreases to ±2.5% for 10 second waves.



Fig 4 Pressure Attenuation Correction Graph

Various investigators (ref. 34) have, however, found that when the waves corrected in this manner are compared with simultaneous surface wave recordings, the pressure recorded waves are generally still too low. The reason for this is that the pressure attenuation process is one where the surface wave spectrum is passed through a low pass filter, the characteristics of which depend upon the water depth. Thus the higher frequencies (shorter wave periods) are attenuated to a greater extent than the lower frequencies (longer wave periods) and the apparent period of the pressure recording increases. Since K_p increases with T_z , the surface wave height H will be underestimated when T_z is increased. Thus in general pressure recorders underestimate the wave heights and overestimate the wave periods.

A considerable amount of literature (Appendix I) exists on the correction of bottom pressure records to surface wave characteristics. Much of it is difficult to compare owing to the different methods of analysis used involving different definitions of H and T. The one result that can be stated clearly is, however, that once the original data (surface waves) has been filtered (pressure attenuation), it is impossible to deduce the original data from the filtered record (pressure recording) unless some assumption is made about the nature of the original data.

The most satisfactory way of making this correction to date has been by direct calibration using simultaneous recordings from surface and pressure recorders. In this case such calibration of an OSPOS recorder has been made by Thabet (ref. 54) in connection with observations in the Mediterranean at Port Said. The calibration was made with the recorder at 5 m (16.4 ft) below the surface in 11 m (36 ft) depth of water with the surface recording obtained using a Neyrpic ultra-sonic recorder. The results of the calibration are shown on figure 5. They show that for wave periods greater than 8 seconds, the average periods of the subsurface and surface recordings tend to the same value, while for $H_{1/3} > 1 m$ (3 ft) the theoretical response factor K applies to waves of these periods.

A check on the Moffat Beach data indicated very few cases where $T_z < 8$ secs, while $H_{1/3}$ was generally greater than 2 feet. Figure 5 shows that the shortest and lowest waves recorded would have to be increased by an additional 10 to 20%. Since most of the waves recorded were well above this size, it was not deemed necessary to apply any correction factor other than the small amplitude one. Furthermore, the periods were assumed to be essentially correct also. The validity of this comparison rests on the assumption that the wave conditions are similar in both cases. In fact, at Port Said observed periods were generally between 5 and 10 seconds indicating waves directly generated by the wind, while at Moffat Beach observed periods were significantly higher and swell is known to occur frequently. The possibility thus arises that at Moffat Beach locally generated wind waves ride on top of the swell. The resulting wave spectrum could then be quite different to that at Port Said and the relation between the pressure recorded wave characteristics and the actual surface ones could also be different. In the absence of a direct comparison under the same conditions this point cannot be clarified. This question is however considered further in Section 7.2.1.







Fig 5 OSPOS Calibration Graphs (AFTER THABET - REF. 54)

3.5 Summary of Data Analysis

The analysis of the data may be divided into the following three parts:-

- the analysis of the variation of wave characteristics within a given recording and the determination of representative parameters for each recording;
- (ii) the determination of the frequency of occurrence of these representative parameters over the total recording period;
- (iii) the comparison of the instrumentally recorded data with visual observations at Cape Moreton with the aim of adjusting the frequency curves to a longer period of time to obtain more general data.

4. WAVE CHARACTERISTICS WITHIN A GIVEN RECORDING

4.1 Representation of a Wave Recording

The characteristics of a given wave recording may be represented in one or other of the following two forms:

- (i) a statistical distribution of the crest to trough heights and periods of the visible waves;
- (ii) an energy spectrum in which the recording is analysed in terms of the energy per unit area of water surface of a series of linear sinusoidal components whose frequencies are usually spaced at constant increments of frequency.

The choice as to which method of representation is adopted depends principally on the following two factors:

- (i) the use which is to be made of the data;
- (ii) the form in which the data was collected.

For a great many Engineering applications the designer requires a single representative value of wave height and wave period. This very often indicates that the calculation of an average value of H or T is all that is necessary. Experience indicates, however, that the average value of H is not the most representative one. Recourse is then made to the statistical distribution of wave heights to define some other more satisfactory representation of H such as the so called significant wave height $H_{1/3}$. When data is analysed as wave spectra very often the latter are subsequently reduced to a single representative wave height and period for the user's convenience. Consequently an elaborate representation of the characteristics of a wave recording is in many cases not the most useful. Furthermore, the calculation of wave spectra requires data which can be readily fed into a computer.

Since the recordings available were not suitable for spectral analysis owing to both their short length and the amount of time required to prepare the data, the wave spectrum type of analysis was not considered at any stage. Moreover, the large number of recordings made it impossible to prepare wave height distributions for each five minute recording. Consequently simplified methods based upon the theoretical wave height distribution were adopted.

As indicated earlier, it is usual to correct pressure recordings using a single attenuation factor for each recording based upon an average wave period. In this analysis the period adopted was the zero crossing period T_z which can be determined comparatively easily. It is not so easy to determine a representative wave height. Thus a commonly adopted method is to consider the wave recording as an amplitude modulated signal of constant frequency and to establish a general wave height distribution for the waves within a given recording. Once a general distribution is found to exist then it is possible to characterise any recording by a single representative wave height.

4.2 Theoretical Wave Height Distribution

If the water surface elevations are normally distributed with respect to time and if, further, there is only a narrow band of frequencies present, it can be shown theoretically (ref. 11) that the wave heights should follow a Rayleigh distribution which is defined by

$$-\frac{\pi}{4} \left(\frac{H}{H_{m}}\right)^{2}$$

p(H) = $\frac{\pi}{2} \frac{H}{H_{m}^{2}}$ e 4.2-1

where p(H) is the probability density and p(H)dH is the probability that the wave lies between H and H + dH

H_m is the mean wave height

тт

$$H_{m} = \frac{1}{N} \begin{array}{c} i=N\\ \Sigma\\ i=0 \end{array} H_{i} \qquad 4.2-2.$$

From equation 4.2-1 can be determined the wave height H which is the wave height equalled or exceeded n% of the time, i.e.

$$\int_{0}^{H} p(H) dH = \frac{100-n}{100}$$

whence

$$\frac{H_{n}}{H_{m}} = \sqrt{\frac{4}{\pi} \log_{e} \frac{100}{n}}$$
 4.2-3.

Furthermore for such a distribution the magnitudes of the various representative wave heights in common use such as the significant wave height $H_{1/3}$, $H_{1/10}$, etc, can be shown to have a constant relationship with each other and with the mean square wave height.

Thus according to Longuet-Higgins (ref. 42)

$$\frac{H}{H_{rms}} = (\log_e \frac{1}{p})^{\frac{1}{2}} + \frac{1}{p} \frac{\sqrt{\pi}}{2} \left[1 - erf \left\{ (\log_e \frac{1}{p})^{\frac{1}{2}} \right\} \right]$$
where erf is the error function $\left(erf x = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-x^2} dx \right)$

 $H_{(p)}$ is the average of the p highest waves (p = 1/3,1/10,etc) H_{rms} is the mean square wave height

$$H_{rms}^2 = \frac{1}{N} \sum_{i=1}^{1=N} H_i^2$$
 4.2-5

H is the height of a sine wave with the same amount of energy as the actual wave. It is related to H and to the most probable wave (mode) H by the following relationships

$$H_{rms} = \frac{2}{\sqrt{\pi}} H_m = \sqrt{2} H_p$$
 4.2-6.

An easily measured or calculated quantity is the root mean square deviation of the water surface elevation y about the mean water level \bar{y} . As this quantity forms a link with the wave spectrum presentation of wave characteristics and also to avoid confusion with H_{rms} , it will be

designated by \sqrt{m} , the square root of the zero moment, or area, of the energy density spectrum (see section 4.6).

$$\sqrt{\mathbf{m}}_{0} = \left[\frac{1}{\tau}\int_{0}^{\tau} (y - \overline{y})^{2} dt\right]^{\frac{1}{2}}$$

where τ is the duration of the recording. \sqrt{m} is related to H_{rms} as follows.*

^{*} It is important that the distinction between H and m be understood as there are conflicting usages in the literature (see ref. 27).

$$H_{\rm rms} = 2\sqrt{2m_{\rm o}}$$
 4.2-7.

The above relationships apply only when there is a narrow band of frequencies present. They are thus more likely to represent swell conditions correctly than sea conditions (storm waves). They can be assumed to apply to wave recordings for which the spectral width parameter ε defined below, is less than 0.4.

$$\varepsilon = \sqrt{1 - \left(\frac{N_z}{N_c}\right)^2} \qquad 4.2-8$$

where N_{τ} is the number of upward zero crossings

 N_{c} is the number of wave crests.

The theoretical numerical ratios between the various wave heights defined above are given in table 3.

In the normal situation where wave recordings are made at regular intervals it is desirable to be able to estimate the probable maximum wave height $\mu(H_{max})$ which might have occurred during the time interval between recordings. An approximate relation valid for large N_z can be deduced from either equation 4.2-3 or equation 4.2-4, e.g.,

$$\frac{\mu(H_{max})}{H_{m}} = \sqrt{\frac{4}{\pi} \log_{e} N_{z}} \quad \text{or} \quad \frac{\mu(H_{max})}{H_{rms}} = \sqrt{\log_{e} N_{z}} \quad 4.2-9.$$

 $\label{eq:table_3} \hline $$ THEORETICAL WAVE HEIGHT RATIOS FOR $$ $$ $$ $$ $$ $$ $$ $=0$ $$$

H _B H _A	H _{1/10}	H ₁ /3	Hrms	H _m	н р	√m _o
H _{1/10}	1.000	0.787	0.556	0.492	0.393	0.196
H _{1/3}	1.271	1.000	0.706	0.626	0.500	0.250
Hrms	1.800	1.416	1.000	0.886	0.707	0.354
Hm	2.032	1.596	1.128	1.000	0.7 9 8	0.399
н	2.546	2.000	1.414	1.253	1.000	0.500
√m _o	5.090	4.000	2.828	2.506	2.000	1.000

A comparison of values of $\frac{\mu(H_{max})}{H_{rms}}$ calculated from equation 4.2-9 and from a more exact formula of Longuet-Higgins (ref. 42) is given on figure 6a. For N > 100 the two values are essentially the same. Also shown is $\frac{E(H_{max})}{H_{rms}}$ where $E(H_{max})$ is the mean maximum wave height calculated by Longuet Higgins. $E(H_{max})$ is always greater than $\mu(H_{max})$.

15.

A relationship between the most probable maximum wave height $\mu(H_{max})$ in a given recording and $H_{1/3}$ or H_{max} can be obtained using Figure 6a and the appropriate wave height ratio from table 3. The magnitude of $\frac{\mu(H_{max})}{H_{1/3}}$, etc, will consequently depend upon the number of waves in the recording.

The characteristics of the Rayleigh distribution are illustrated diagrammatically on figure 6b. Its general applicability to deep water sea waves has been demonstrated by many authors for both ocean and laboratory waves recorded by both pressure and surface type wave recorders (references 3,4,14,32,36 and 58). However both field and laboratory data (refs 52 and 33) also indicate that the wave height distribution becomes narrower in shallow water.



(b) CHARACTERISTICS of the RAYLEIGH DISTRIBUTION ($\varepsilon = 0$).

4.3 Observed Wave Height Distribution

This analysis was made using 63 recordings, 31 made during March-April 1963 and 32 during March-April 1964. The recordings chosen were intended to be those giving the daily maximum value of H from the two charts made during cyclones. However, the subsequent discovery of errors in dating, discussed in section 3.2, meant that this was no longer the case.

The length of the wave recordings was rather short in terms of the number of waves recorded (20 < N_Z < 45). Consequently, it was difficult to determine the wave height distribution for any individual recording with confidence. However, when the wave heights were expressed as dimensionless ratios of the mean wave height, i.e., as $\frac{H}{H}$, it was found that each recording gave a generally similar distribution except that there were discrepancies with regard to both the very high and very low waves within the recording. Since various checks indicated no significant variations in the main part of the wave height distribution with wave height or period or other parameters, the whole of the 63 recordings were combined together to give one overall distribution. In this way it was hoped that the limitation of the small sample size would be overcome.

The resulting combined wave height distribution is plotted in the form of an exceedance curve on figure 7. In this graph the ordinate represents the % of time that a given value of $\frac{H}{H}$ is equalled or exceeded and is plotted to a special scale calculated from the theoretical Rayleigh distribution (eqn 4.2-3). The resulting graph indicates that the wave height distribution can be reasonably approximated by the Rayleigh distribution. The latter can be generally assumed to apply to this phenomenon when the spectral width parameter $\varepsilon < 0.4$. For the data in question the average value of ε is 0.39 with extreme values of 0 and 0.69.

For each recording the following wave height measures were obtained:

H - highest wave in the recording

- $H_{1/10}$ average of the one tenth highest waves in the recording
- $H_{1/3}$ average of the one third highest waves in the recording commonly called the significant wave height

 H_m - average of all waves in the recording.

The average values of the various ratios between these quantities were determined and are given in table 4. The ratios $\frac{H_{1/10}}{H_{1/3}}$, $\frac{H_{1/10}}{H_m}$, $\frac{H_{1/3}}{H_m}$ are generally of the same order as the theoretical values which is consistent with figure 7. The ratio $\frac{H_{1/3}}{H_m}$ is, however, slightly lower than



Fig 7 Combined wave height distribution for 63 records at Moffat Beach

the theoretical value and there is some inconclusive evidence that this is due to the fact that $\varepsilon >> 0$ for some recordings.

AT MOFFAT BEACH							
H <u>max</u> H _{1/10}	H max H _{1/3}	H max H m	$\frac{H_1/10}{H_1/3}$	$\frac{H_1/_{10}}{H_m}$	$\frac{H_1/3}{H_m}$		
1.23	1.61	2.45	1.31	1.99	1.52		

		Table	4		
WAVE	HEIGHT	RATIOS	FOR	63	RECORDS
	AT	MOFFAT	BEACH		

The ratios involving H are all, however, significantly greater than the theoretical values which is a reflection of the fact that the recordings used in this analysis have values of H significantly greater than those of the overall recording period. The values of T_z are, however, generally similar both in magnitude and frequency of occurrence (fig. 8).



Fig 8 Wave Height and Period Frequency Distributions Comparison of 63 Selected Recordings and all Recordings.

4.4 Determination of Significant Wave Height

The usual representative wave height for characterising a wave recording is the significant wave height $H_1/_3$. This is both easier to determine directly as only the highest one third of the waves need be measured and since wave energy is proportional to H^2 , it is more representative of the severity of the wave action than H_m . However, when a large number of recordings have to be analysed short cut methods for determining $H_1/_3$ are usually adopted.

Originally it was intended to estimate $H_1/3$ for each recording by applying the experimentally determined value of the ratio $\frac{H_{max}}{H_1/3}$ in table 4 to measured values of H_{max} for all recordings. Unfortunately, the method of selection of the recordings for calculating $\frac{H_{max}}{H_1/3}$ resulted in a biased result and an unrepresentative value of this ratio. Values of $H_1/3$ estimated in this way would be too low. However, had the recordings been selected systematically, i.e. 9 a.m. recordings, or some other constant time interval, this method should have given a reliable estimate of $H_1/3$.

Since the wave height distribution approximates to a Rayleigh one the alternative is to apply the theoretical value of $\frac{H_{max}}{H_{1/3}}$ to observed values of H_{max} . In fact $\frac{H_{max}}{H_{1/3}} = f(N)$ where N is the number of waves (sec. 4.2). The theoretical value for the mean value of N (32) is 1.34, while the maximum and minimum values are about ±5% of this value. Application of this value of $\frac{H_{max}}{H_{1/3}}$, however, may give values of $H_{1/3}$ which are somewhat high, as it appears from figure 7 that the actual $H_{1/3}$ is relatively lower than the theoretical value. Thus referring to figure 7, if the average number of waves per record is 32, the maximum wave is equalled or exceeded 3.1% of the time and the average value of $\frac{H_{max}}{H_m}$ is 2.18, while the average value of $\frac{H_{1/3}}{H_m}$ is 1.52. Consequently $\frac{H_{max}}{H_{1/3}}$ is 1.43, and this value has been used to estimate $H_{1/3}$. For estimation of the probable maximum wave height, the theoretical ratio was used which gave $\frac{\mu(H_{max})}{H_{max}}$ equal to 1.29.

Another method of estimating $H_{1/3}$ has been proposed by Tucker (ref. 57), and has been described and applied by Draper (refs 25 and 26). This method is based upon the statistical theory developed by various authors (refs 10, 11 and 50) for the occurrence of the maxima of a random signal. In this case the height of the highest and second highest crests (A & B) and the depth of the lowest and second lowest troughs (C & D), in all cases measured as positive numbers relative to the mean water level, are determined for each recording. From these quantities the wave heights $H_1 = A + C$ and $H_2 = B + D$ are determined. The above mentioned theory shows that these two quantities are a function of the root mean square deviation of the water surface \sqrt{m} and the number of waves, i.e. N_2 .

$$\frac{H_1}{\sqrt{m_o}} = 2 (2\theta)^{\frac{1}{2}} (1 + 0.289\theta^{-1} - 0.247\theta^{-2}) \qquad 4.4-1$$

$$\frac{H_2}{\sqrt{m_o}} = 2 (2\theta)^{\frac{1}{2}} (1 - 0.211\theta^{-1} - 0.103\theta^{-2}) \qquad 4.4-2$$
where $\theta = \log_e N_z$.

The ratios
$$\frac{H_1}{\sqrt{m}}_{O}$$
 and $\frac{H_2}{\sqrt{m}}_{O}$ are graphed as a function of N_z on

figure 9 which was presented by Tucker (ref. 57).



 H_1 and H_2 and N_z can all be evaluated by simple measurement from the wave recording and \sqrt{m} can be obtained from figure 9. Moreover, it can be deduced from the work of Cartwright and Longuet-Higgins (ref.11) that

$$\frac{H_{m}}{\sqrt{m_{o}}} = \sqrt{2\pi(1 - \varepsilon^{2})}$$
$$= \sqrt{2\pi} \frac{N_{z}}{N_{c}}$$
4.4-3

Thus the mean wave height H can be determined for a wave record of any value of $\epsilon.$

If other wave heights are required, it is necessary to use theory based upon a narrow range of frequencies, i.e. $\varepsilon \rightarrow 0$. In this case the theoretical relations given in Section 4.2 apply and so the significant wave height $H_{1/3}$ is given by the relation

$$H_{1/3} = 4.00 \sqrt{m}$$
 4.4-4.

This analysis was carried out on the 63 recordings for which $H_{1/3}$ had been determined by measurement of the individual waves. The results (fig. 10) indicate that estimates of $H_{1/3}$, H_m , etc, based upon H_1 are generally too high, while those based on H_2 are relatively good. The overestimation of $H_{1/3}$, H_m when calculated from H_1 can be attributed to the fact that the higher waves are relatively more frequent than normal in these recordings which fact is evident on figures 7 and 8. The theoretical error in \sqrt{m} estimated from H_1 is, however, greater than when \sqrt{m} is estimated from H_2 (ref. 57).

It can be concluded that the wave height distribution can be estimated reasonably reliably from H_2 and N_z provided that ε is sufficiently small for the wave height distribution to approximate the Rayleigh one. Since this particular analysis was not made until after the recordings had all been read in terms of H_{max} this method was not used to estimate $H_{1/3}$. However, it is evident that the slightly greater effort involved is worthwhile to produce a more reliable result.

^{*} It should be pointed out that the definition of $H_{1/3}$ used here is in fact different to that defined previously in section 4.3. That they generally give similar results is very convenient, but not inevitable since they are theoretically only the same when $\varepsilon = 0$.





4.5 Wave Period Distribution

No detailed analysis was made of the wave period distribution. However, the period of the highest wave in each recording T_H was measured and compared with the zero crossing period T_z . For the 63 selected recordings it was found that the value of T_H was normally less than T_z , T_H max the average value of the ratio $\frac{max}{T_z}$ being 0.86. Moreover, further analysis indicated that $\frac{T_H}{T_z}$ was a function of the spectral width parameter ε . (fig.11). As ε increases $\frac{max}{T_z}$ decreases from 1.0 for $\varepsilon = 0$ to 0.7 for $\varepsilon = 0.65$.



The reason for this is presumably associated with the greater variability of wave period which can be expected to occur with large values of ε where a wide range of frequencies are present. A large wave will result when several of the spectral components are in phase at a given point and time. The period of the resulting visible wave will be influenced most by the higher frequencies present and so will be less than the average period T_{τ} .

For the smaller waves in the record the various spectral components will tend to be out of phase and the pressure recorder will reduce the relative height of the higher frequencies with a consequent increase in the period of the visible waves.

A consequence of this phenomenon is that, if accurate estimates of H_{max} are required, then the pressure attenuation factor to be used must be based upon the period T_{H} and not T_{z} . For the records analysed H'_{max} and not T_{H}_{max} value of l.61 for $\frac{H_{max}}{H_{1/3}}$ calculated using T_{H}_{max} was l.81 which is ll-l2% greater than the max value of l.61 for $\frac{H_{max}}{H_{1/3}}$ calculated using T_{z} (table 4). These are average values and the error will be greater for storm waves (ε large) than for swell (ε small). All wave heights quoted in this report are H_{max} and not H'_{max} .
4.6 Wave Spectra

While the spectral representation was not used in this investigation for reasons given in section 4.1, an understanding of it is necessary for the discussion in section 7.2 of this bulletin. The following summary is mainly based upon that given by Bonnefille (ref. 3).

At a given point on the sea surface, the water surface elevation y is a function of time and can be considered as the sum of an infinite number of sinusoidal waves, each with an angular frequency $\omega_i = \frac{2\pi}{T_i}$ and a phase angle ϕ_i , i.e.,

 $y(t) = \sum_{i=1}^{N} a_{i} \cos (\omega_{i}t + \phi_{i}) \qquad 4.6-1$

y(t) is a stationary random function with constant statistical parameters and it can be shown that the probability density of y(t) is a normal (Gaussian) distribution

$$p(y) = \frac{1}{\sqrt{2m_0}} e^{-\frac{y^2}{2m_0}}$$
 4.6-2.

If the amplitude characteristics, or heights of individual waves, are considered, we obtain when $\varepsilon = 0$ equation 4.2-1, the Rayleigh distribution. On the other hand, consideration of the energy of the component waves leads to the energy density spectrum; commonly called the wave spectrum.

The energy of each component is

$$\mathbf{e}_{\mathbf{i}} = \frac{1}{2} \rho \mathbf{g} \mathbf{a}_{\mathbf{i}}^2$$

e, varies with angular frequency ω depending upon the characteristics of the wave motion. The energy density of the spectrum $E(\omega)$ is the energy contained in the frequency band (ω , ω + d ω).

$$E(\omega)d\omega = \frac{1}{2} \sum_{\omega}^{\omega+d\omega} a_{i}^{2}.$$

The total energy contained in the whole spectrum is

$$m_{o} = \int_{0}^{\infty} E(\omega)d\omega = \sum_{1}^{N} \frac{1}{2} a_{1}^{2} \qquad 4.6-3.$$

m is thus the area of the wave spectrum diagram or its zero order moment. As stated in section 4.2, it is also the mean square deviation of y(t).

Spectral moments of order n are defined in a similar way to m_o,

i.e.,

$$m_n = \int_{0}^{\infty} \omega^n E(\omega) d\omega.$$

Statistical theory (ref. 11) can be used to show that the average zero crossing period T of the sea surface and the average crest period T can be determined from the wave spectrum by the following equations:

$$T_z = 2\pi \sqrt{\frac{m_o}{m_2}}$$
, $T_c = 2\pi \sqrt{\frac{m_2}{m_4}}$. 4.6-4

The spectral width parameter ε is given by

$$\varepsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}} \qquad 4.6-5$$

which is the same as equation 4.2-8.

The shape of the wave spectrum varies with the type of waves. Sea resulting from wind generated waves has a relatively wide spectrum. Swell has a narrower spectrum, while in many actual cases the wave spectrum has components from both sea and swell present. Wind generated waves for fully developed seas from a wide range of sources both laboratory and field were found by Hess, Hidy and Plate (ref. 36) to follow Phillips (ref. 47) theoretical relationship for the high frequency side of the spectrum fairly well.

i.e.,

$$E(\omega) = \alpha g^2 \omega^{-5} \qquad 4.6-6$$

The magnitude of ω at which $E(\omega)$ is a maximum depends upon the wind velocity and the fetch length over which it blows.

A number of different analytical expressions have been suggested for the shape of the wave spectrum (refs 6 and 48). A widely accepted one is that of Pierson and Moskowitz (ref. 48) which applies for fully developed seas.

$$E(\omega)d\omega = \alpha g^2 \omega^{-5} e^{-\beta \left(\frac{\omega_0}{\omega}\right)} d\omega \qquad 4.6-7$$

where α and β are dimensionless constants

$$\alpha = 8.10 \times 10^{-3}$$

$$\beta = 0.74$$

$$\omega_{o} = \frac{g}{U}$$
 where U is the wind speed measured at an elevation of 19.5 metres.

If the spectrum has a constant shape, then it may be characterised by a single wave height, $H_{1/3}$ for instance, which is as shown in section 4.2, directly related to the wave energy described by m, and a single wave period. The characteristic period adopted is often that corresponding to the frequency ω of the spectral peak. This period is defined as T in this report. Alternatively T may be used (see further discussion in section 7.3).

The characteristics of the wave energy density spectrum are shown diagrammatically in figure 12.



4.7 Summary

The nature of the recorded data indicated that the characteristics of a single recording could be most conveniently represented by the significant wave height H_{1/3} and the zero crossing period T_z. Analysis of selected wave recordings suggested that the wave height distribution was essentially similar to the theoretical Rayleigh distribution. The ratio between $\frac{H_{max}}{H_{1/3}}$ adopted to obtain H_{1/3} for this investigation was 1.43, while the appropriate ratio $\frac{\mu(H_{max})}{H_{max}}$ for determining the probable maximum wave height was deduced to be 1.29.

The Tucker-Draper method for estimating $H_1/_3$ from the maximum and minimum ordinates of the record was found to be reasonably reliable if H_2 were used rather than H_1 .

The period of the highest wave in each recording was found to be normally less than T_z. The decrease in T_H was greater for recordings with wide spectra, i.e. large values of ε .

5. WAVE CHARACTERISTICS DURING RECORDING PERIOD

5.1 Presentation of Data

Each wave recording was characterised by its H and T and the following charts and figures prepared. The max following charts are following charts and figures prepared.

- (i) A strip chart for each wave record chart showing wave height and period as a function of time, together with the concurrent wind and sea-swell observations at Cape Moreton. Portions of these charts are given in figure 13. To simplify presentation the charts reproduced show only the six hourly average values of $H_{1/3}$ and T at 0300, 0900, 1500 and 2100, together with the corresponding visual observations. The maximum value of H observed during each six hour period is also shown.
- Joint wave height and period frequency of occurrence for each swell direction (figure 15).
- (iii) Wave height exceedance graphs for both each swell direction and all directions combined (figure 16a).
- (iv) Wave period exceedance graphs for the same conditions as in(1ii) (figure 16b).

5.2 General Wave Climate

When strip charts of the recorded wave data and sea-swell observations at Cape Moreton were examined closely, it was possible to define four basic wave conditions.

5.2.1 Normal conditions

These occur most of the time and are characterised by light NE to SE winds with a velocity normally less than 15 knots. Slight moderate seas occur accompanied by a low short to average swell. A typical occurrence is shown on figure 13a for 28th January to 2nd February 1963. The significant wave height is generally not greater than 3.5 feet. The largest H recorded during the six day period was about 9 feet which appears to be an exceptional value for these conditions. Wave periods generally lie between 8 and 10 seconds. The wave climate under these conditions is essentially a low swell with small wind waves generated by local sea breezes superimposed upon the swell. The period of the waves is generally longer for an easterly swell than for north east or south east swells (cf 2nd February with preceding days on figure 13a).

5.2.2 Moderate to strong local winds

The occurrence of strong breezes or moderate to fresh gale winds of 25 to 35 knots generally from the south east causes moderate to



Fig 13 Wave Heights & Periods & Meteorological Observations as Functions of Time.

rough seas. Such situations were recorded on 20 and 21st January and 31st March and 1st April 1963. The latter event is shown on figure 13b together with conditions before and after its occurrence. In this case $H_{1/3}$ attained a value of about 5.5 feet. The maximum H recorded was 9.6 feet and the corresponding measured value of $H_{1/3}$ was 6.3 feet. Under these gale conditions the locally generated wave tends to dominate over the swell and the wave period shortens to 7 to 8 seconds.

5.2.3 Offshore winds

Offshore winds are normally most frequent in the third quarter of the year for which there are no recordings. However, their effect was observed several times, particularly after the passing of a cyclone, Figures 13c and 13d show such conditions on the 1st and 2nd May 1963 and 8th to 10th April 1964. Less spectacular events of this type occurred on other occasions during the recording periods. Offshore winds generally produce smooth seas inshore where the recordings were made when the wind velocity is less than 15 knots. However, if the wind is greater than this (e.g. on 8th April 1964, figure 13d), slight seas may be observed inshore. Their effect upon the wave conditions is very marked and if they persist for a day or two the waves will be reduced to a low swell of 11 to 12 seconds period. The significant wave height observed under these conditions was between 1.5 and 2 feet, while the maximum wave height observed was of the order of 3 to 3.5 feet. The general wave climate is a long low swell inshore along the beaches.

5.2.4 Cyclones

Really severe wave conditions only occur during the time that a cyclone is present offshore or in the general vicinity. Two definite cyclones occurred during the recording period, one which produced large waves at Moffat Beach between 24th and 28th April 1963 and the other ("Henrietta") whose influence was felt between 2nd and 9th April 1964. A series of complex depressions also produced abnormal wave conditions between 5th and 13th May 1963.

The tracks of these cyclones are shown on figure 14 (ref. 13). It is evident that neither of the two major cyclones approached the coast close enough for cyclonic storm waves to be recorded. In both cases the waves recorded were basically high swells.

5.3 Wave Characteristics during Cyclones

5.3.1 April 1963 cyclone

Considering the April 1963 cyclone it will be seen from figure 13c that the wave heights increased from values below normal during the



Fig 14 Cyclone Tracks During Recording Period.

day of 24th April to an $H_{1/3}$ of almost 11 feet at 3 p.m. on the next day. This height was maintained until 9 a.m. on 26th April after which it decreased to normal conditions on 29-30th April. An H of 20 feet was actually recorded at 2 p.m. on the 25th April. The measured value of $H_{1/3}$ from this recording was 14 feet.

Essentially the waves were swells resulting from waves generated by the cyclone on the preceding day. During the middle of 25th April very rough seas were observed when the local wind velocity reached 30 knots with the result that the wave period was about 11 seconds. However, once the local seas started to subside the period of the swell lengthened to over 12 seconds with values of 13 seconds being observed at times. During the period of maximum wave heights average length heavy swells were reported with long heavy swells at 3 p.m. on 26th April. Following this short heavy swell was reported, although the wave recordings give no indication of this condition. The swell was at all times from the east.

The cyclone actually filled during the early hours of 26th April at a location 350 nautical miles due east of Brisbane. Swells with periods of the order of 12 seconds and $H_{1/3}$ of about 7 feet persisted until 9 a.m. on 28th April, after which time they died out and normal conditions were re-established on 30th April. During this latter period an H of 14.7 feet was recorded with a measured $H_{1/3}$ of 9.3 feet. After the cyclone had filled, a period of light offshore winds occurred during which time the typical low swell characteristic of this weather developed (see section 5.2.3).

5.3.2 Complex depressions May 1963

Following the April 1963 cyclone, there was a period of 8 to 10 days during which a number of complex depressions were recorded. Of the five depressions shown on figure 14b, it appears that number 17 which moved southwards along the Great Dividing Range commencing near Maryborough at about midnight on 5-6th May caused moderate to strong south east winds of sufficient strength to give rise to wave conditions on 6th May somewhat more severe than those described in section 5.2.2. On this occasion the wave period began to shorten soon after the wind shifted to the south east quarter. On 3rd May the period was generally 11 seconds, while on the 5th May it had fallen to 8 seconds. As depression 17 moved south during the 6th May the wind shifted to the north east and the swell became easterly. The maximum wave conditions from this depression occurred around 3 p.m. on the 6th May when $H_{1/3}$ was about 7.5 feet.

The waves remained at this general level for the next 18 hours largely under the influence of the small depression 18 which formed off Fraser Island early on the 7th May and travelled south along the coast. After midday on the 7th May, wave heights rose rapidly as a depression 16, which formed earlier north of New Caledonia, started to affect the area as well. Strong gale force winds from the east north east were recorded at Cape Moreton during the evening of 7th May. An average significant wave height of 12.5 feet with a recorded maximum H of 21 feet was obtained at 9 p.m. on this day. This condition quickly subsided to about 8 to 9 feet during the next day when offshore winds occurred as depression 16 moved south of Brisbane. The swell swung round from the east to the north east as waves presumably generated offshore on the previous day by the depression reached the coast. At the same time, the wave period lengthened to about 11 seconds. The depression filled at about midnight on 8-9th May and south easterly winds of 20 to 25 knots resumed resulting in rough seas during the evening of 10th May. These conditions were influenced to some extent by depression 19 which formed about midday on the 8th May midway between Fraser Island and New Caledonia, but which moved away from the Queensland coast towards New Caledonia. At this time a strong high pressure region existed over the central Tasman Sea. All these factors contributed to the significant wave heights remaining over 7 feet and the period remaining at about 10 seconds until after 12th May.

Considerable erosion was reported to have occurred on Brisbane's near north and south coast beaches as a result of the heavy seas during this period.

5.3.3 Cyclone "Henrietta" 29th March-10th April 1964

This cyclone was preceded by a period of south easterly gale force winds on 26-27th March which produced significant waves of 9 to 10 feet between 3 p.m. on 26th March to 3 p.m. on 30th March. (figure 13d). Wave periods were between 11 and 12 seconds, while the swell, initially south easterly, turned to the east during the afternoon of 27th March. The maximum observed H was 18.2 feet with a measured $H_{1/3}$ of 9.5 feet.

After the above period of south easterly weather, $H_1/_3$ fell to 5.5 feet and the period shortened to 10 seconds on 1st April (figure 13d). The influence of cyclone "Henrietta" began to be felt with strengthening south east winds on 2nd April. South easterly swell of $H_1/_3$ about 7 to 8 feet and 10 seconds period was recorded on 2nd and 3rd April. During 3rd April the swell shifted first to the east and then to the north east during the evening when $H_1/_3$ reached a value of 11 feet with a maximum recorded H of over 21 feet. The period was between 11 and 12 seconds. The cyclone was then roughly halfway between Brisbane and New Caledonia (figure 14a) and at its maximum intensity. North easterly swell was again observed during the daylight hours of 4th April and, although the height reduced somewhat (average $H_1/3$ of 9 to 10 feet), the period lengthened to 12 seconds. It was during this time that the ship "Tungus" recorded hurricane winds and "waves phenomenally high, estimated to be over 50 feet high", as it drifted out of control through the path of the cyclone (ref. 9). During the night of 4-5th April the recorded values of $H_{1/3}$ were of the order of 13 feet or more with a recorded H_{max} of almost 21 feet.

Subsequently as the cyclone moved south east over Norfolk Island towards New Zealand, local winds and seas dropped (figure 13d). However, the swell height remained high with an $H_{1/3}$ of 9 to 10 feet up till 9 a.m. on 8th April, while at the same time the period steadily increased from 12 seconds at 9 a.m. on 5th April to 15 seconds at the same time on 8th April. This very definite effect is completely missed in the visual observations at Cape Moreton where an average swell of moderate height was recorded virtually all the time between the above two dates. During the afternoon of 8th April the swell started to decrease in height rather rapidly and to shorten in length as the cyclone moved further away and diminished in intensity. Moderate to fresh offshore breezes assisted in the calming of the waves and by the afternoon of 10th April typical offshore wind wave conditions were established.

5.4 Frequency of Occurrence of Waves of Various Heights and Periods

The joint wave height and period frequency graphs (figure 15) show a form which is consistent with the four general wave conditions previously described. The normal condition shows as the most frequently occurring waves. They have values of H between 4 and 5 feet ($2.8 < H_{1/3} < 3.5$ feet) and periods between 8 and 9 seconds for the north east and south east directions, while for the easterly direction they are somewhat longer ($8 < T_{2} < 10$ secs)



and lower (3 < H < 4 feet or 2.1 < $H_{1/3}$ < 2.8 feet). The offshore wind condition is evident in the tail on the right hand side of the graphs indicating wave heights (H) between 2 and 3 feet (1.4 < $H_{1/3}$ < 2.1 feet) with periods up to 13 seconds. The rough seas produced by the south easterly gales are evident by the relatively high frequency of waves from the south east with wave heights (H) about 8 feet ($H_{1/3}$ = 5.6 feet) and periods of 7 to 8 seconds. Finally the heavy swells from the cyclones and other large scale weather features show up as the cloud of points whose wave height generally increases with period.

When the wave height and period exceedance curves are considered, it is seen from log probability plots (figure 16) that, while a reasonable extrapolation may be made of the wave period graph, this is not possible for the wave height one. Moreover, the latter shows two distinct regions which appear to indicate that there are data from two distinct sources present. The high waves are apparently more frequent than would be expected if a log normal extrapolation is to be assumed (ref. 24). That this is so is not surprising since the recording period included two cyclones. Obviously the data had to be corrected in some way if it were to be used to give information concerning the general frequency of occurrence of various wave conditions.



Fig 16 Wave Height and Period Exceedance Graphs-Recording Period.

5.5 Summary

Consideration of the data obtained during the recording period suggested that four distinct types of wave condition exist. These are as follows:-

(i) Normal conditions

 $2 < H_1/_3 < 3$ ft or a little higher

 $8 < T_{\tau} < 10$ secs (See qualification in Section 7.6)

Waves from both the north east and south east are shorter than those from the east.

(ii) Moderate to strong local onshore winds

 $5 < H_{1/3} < 6$ ft 7 < T₂ < 8 secs.

(iii) Offshore winds

$$1.5 < H_{1/3} < 2 \text{ ft}$$

 $11 < T_z < 13 \text{ secs.}$

(iv) Cyclones

For the cases observed $H_{max} \ge 21$ ft

 $H_{1/3} \simeq 14$ ft 11 < T_z < 12 secs.

Long swells up to 15 secs may occur at times when cyclone is some distance away.

The exceedance curve for wave height clearly shows that this data is biassed towards the higher wave heights and consequently needs to be adjusted to give a true picture of the wave climate at Moffat Beach.

6. WAVE CHARACTERISTICS FOR PERIOD 1960-1967

6.1 Analysis of Sea-Swell Observations at Cape Moreton

To extend the general range of the recorded data, an analysis was made of visual observations of sea and swell state at Cape Moreton. This analysis was divided into two parts, one dealing with the overall period 1960-1967 and the other with the period during which the actual recordings were made.

For the analysis of the overall period, daily (9 a.m.) observations were used, while for the shorter recording period both daily and 3 hourly observations were considered.

The general percentage occurrence of the various sea and swell states is shown on figures 17a and 17b. The Bureau of Meteorology descriptions (ref. 8) are given in Appendix 2. The joint sea-swell frequencies are given on figure 17c for each of the three significant swell directions, North East, East and South East.

The general average conditions off Cape Moreton are seen to be smooth to slight seas (states 2 and 3) with a low swell (states 1 and 2). Sea states over 6 (> 20 ft high) have not been observed, although all swell states have been.

Seasonally northeasterly swells (8%) are more frequent during the October to December quarter and relatively infrequent during the April to June quarter. Medium and heavy swells from the north east are exceptional. South easterly swells (37%) on the other hand are more frequent during the April to September period and generally significantly less frequent during the October to March period. Easterly swells predominate (54%) and occur for just over half the time. They tend to be more frequent than usual early in the year and less frequent during the latter half.

Considering the periods during which the wave recordings were obtained, figure 17a shows that the seas were generally higher than usual (states 4 and particularly 5 more frequent). This is to be expected on account of the cyclonic weather. The swell states during the operating period also indicate a definite trend for long swells (states 2 and 5) to be less frequent than usual. This is particularly the case for state 2 (long low swell). On the other hand, short to average length swells are more frequent, there being a relatively large increase in state 6 (short heavy swell). These conditions are again consistent with the predominance of cyclonic weather and generally onshore winds.

The latter effect is shown on figure 18 giving the relative wind frequencies during the overall period (1957 to 1965) and the recording periods. Onshore winds predominated during the period over which the recordings were made. Thus shorter swells than normal can be generally expected during the recording period due to less than usual occurrence of offshore winds. The wave recordings would thus be expected to give periods somewhat shorter than average, although this factor could be outweighed by the occurrence of cyclonic



1960 - 1967 and Recording Period.



Fig 18 Wind Roses at Cape Moreton 1957-1965 and Recording Period.

conditions causing heavy swells whose length and hence period can be expected to be greater than normal conditions.

From the point of view of swell direction, north easterly swells are relatively more frequent than usual during the recording period. South easterly swells are relatively less frequent and easterly swells about the same frequency.

6.2 Correlation of Wave Characteristics with Sea-Swell States

The correlation between the wave characteristics and the Sea-Swell states was made separately for wave height and wave period for each swell direction. The three hourly average value of H or T was correlated with the simultaneous sea-swell condition and an average value of H or T calculated for each sea-swell condition and each swell direction. In making this comparison no correction was made for the possible differences in conditions due to the fact that the wave conditions observed at Cape Moreton might not reach Moffat Head until some time later. This procedure was justified on the basis that north easterly waves would arrive at the two points almost simultaneously and so needed no correction, while the correction for easterly and south easterly waves was only of the order of an hour or so over the distance of 25 miles. In view of the various other uncertainties of the analysis, it was considered that this factor could be ignored.

The two calibration graphs are shown on figure 19. In each case the wave characteristic of height or period is given as a function of sea and swell state for a given swell direction. The height and period contours have been drawn taking account of the number of observations relating to each point. The general result is that the wave height increases with both sea and swell state, but the rate of increase tends to be variable. Furthermore, for each direction the wave height H tends to an average maximum value of 12 to 14 feet above sea-swell state 5-5. Wave heights from the south east tend to be lower for a given sea-swell condition than from the east, while they tend to



Fig 19 Wave Height and Period as Function of Sea Swell State.

be higher from the north east. Both these trends are generally consistent with refraction effects (see section 7.2.3).

Wave periods on the other hand tend to decrease with increasing seastate for a given swell state and to increase with increasing swell state for a given sea state. The pattern of this variation shows a consistent variation with swell direction. For instance, for north easterly conditions, the period increases only slightly when both sea and swell states increase simultaneously. On the other hand, while this is the case for easterly conditions with sea states less than 4 and swell states less than 3, the wave period for swell states above 4 becomes essentially independent of swell state and decreases with increasing sea state. For south easterly waves this trend is reversed with respect to swell state with the wave period being independent of swell state for swell states up to 3, but decreasing with increasing sea state.

Comparison of exceedance curves for H and T determined using figure 19 and those actually obtained from the recorded data is shown on figure 20. The agreement is quite good especially for the wave period. There is, however, a tendency to overestimate the lower values and underestimate the higher values. This tendency is more marked for the wave height where the averaging of wave heights in compiling the correlation graph results in the elimination of the highest and lowest recorded values. An empirical correction was devised to account for this discrepancy and the subsequent data adjusted accordingly.



6.3 Wave Height and Period Frequencies 1960-1967

6.3.1 Normal Conditions

The calibration graphs were used to obtain wave height and period exceedance curves based upon the 9 a.m. sea swell observations for the period 1960-1967.

Wave height and period exceedance curves were obtained both for all directions and for the separate directions. These are shown on figure 21. The data from this figure is summarised in table 5.



Fig 21 Wave Height and Period Frequency of Occurrence 1960-1967.

Table 5

SUMMARY OF WAVE HEIGHTS AND PERIODS FROM VARIOUS DIRECTIONS 1960-1967

	Wave	Direction		Overall
	NE	E	SE	
%	8.1	53.6	35.7	97.5
H max50	5.6	5.7	4.5	5.1
H ₁ /3 50	3.9	4.0	3.1	3.6
μ(H _{max})	7.2	7.3	5.8	6.6
T ₂₅₀	9. 0	10.2	9.3	9.7

The trends of these values are generally consistent with known conditions. Thus wave heights tend to be lower than average from the south east, which is consistent with the significant refraction effects from this direction (section 7.2.3). Moreover, the periods are longer from the east which fact is consistent with the predominance of easterly swells. An interesting point is the indication that there is a minimum value below which the wave height hardly ever falls. It corresponds to H_{max} of 2 feet or $H_{1/3}$ of 1.4 feet. Thus there is always a low groundswell present. These recordings also indicate a minimum period of 7 seconds. This value is, however, not that associated with the residual swell. Figure 15 suggests that the latter value is more likely to be in the 10 to 12 seconds range.

6.3.2 Seasonal Variations

The variation of wave heights for each quarter for the period 1960-1967 is given on figure 22. The trends of the curves are summarised in table 6.

Table 6

H1/3 50	Wave	Dire	Direction	
	NE	E	SE	
Jan-March	3.9	4.3	3.6	4.0
Apr-June	3.9	4.4	3.4	3.8
July-Sept	3.9	3.9	3.0	3.4
Oct-Dec.	3.9	3.6	3.3	3.5

SEASONAL VARIATIONS IN WAVE HEIGHT 1960-1967

Table 6 shows that wave conditions are more severe in the first two quarters which include the cyclone season and least severe during the July-Sept quarter when offshore (westerly) winds are most common. Further, while the frequency of occurrence of north easterly waves varies significantly throughout the year, the size of the waves is very much the same. This probably reflects their essentially local nature.

The comparable seasonal data for wave period is given on figure 23. The trends are shown on table 7.



Fig 22 Wave Height Frequency of Occurrence Seasonal Variations.



Fig 23 Wave Period Frequency of Occurrence Seasonal Variations.

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<u>Table 7</u>

T _{z50}	Wave	Direc	Overall	
	NE	Е	SE	
Jan-March	9.2	10.1	9.1	9.7
Apr-June	9.2	10.3	9.5	9.9
July-Sept	8.5	10.3	9.4	9.8
Oct-Dec.	8.9	10.3	9.6	9.6

SEASONAL VARIATIONS IN WAVE PERIOD 1960-1967

Generally it is seen that easterly waves are longer than north easterly or south easterly waves throughout the year, while north easterly waves are usually shorter than waves from the other directions. Seasonally north easterly waves vary the most in period, being significantly shorter during the second half of the year, particularly during the July-September quarter. South easterly waves tend to be shortest during the January-March quarter, while overall wave periods tend to be longer in the middle of the year.

6.3.3 Extreme Conditions

For the high waves it is necessary to extrapolate the exceedance curves of figure 21 to obtain estimates of their frequencies of occurrence or return period. Since the calibration procedure removed the highest observed waves, this extrapolation is necessary to complete the picture for the 8 year period 1960-1967. Any further extrapolation to a longer period must then be considered carefully.

Several methods may be used to extrapolate wave height data. Darbyshire (ref 15) and Draper (ref 24) recommended plotting the exceedance curve on logarithmic normal probability (log probability) graph paper, while Larras (refs 38, 39 and 40) has found that ordinary semi logarithmic paper using the logarithmic scale for frequency (exponential distribution) gives reasonable results, at least when there is not too much swell present. Thompson and Harris (ref 55) report that significant wave height data can be represented by both log normal and modified exponential distributions with a preference for the latter since semi logarithmic paper may be used and all wave heights are displayed to the same precision. Battjes (ref 2) has recently shown that significant wave data from the British Isles which does not plot as a straight line on log probability paper can be represented by a Weibull type distribution. The long term distribution of individual wave heights is however nearly exponential according to Battjes. Borgman (ref 5) has considered a very general theoretical model of the probability distribution function for individual maximum wave heights in a random number of random length storms each with

random intensities. His final formula appears to be a modified form of the Gumbel distribution commonly used in Hydrology and Meteorology (ref 59).

The equations for the logarithmic normal, exponential, modified exponential, Weibull and Gumbel distributions are given in Appendix 3. These show that the exponential and modified exponential distributions are essentially simplifications of the more general Weibull distribution.

The wave data for H at Moffat Beach has been plotted to test each of the four distributions mentioned in the preceding paragraph. The results are shown on figure 24. In each case it was possible to draw a reasonable straight line through the data points and thus extrapolate the data. There is little to choose between the log probability, Weibull and Gumbel plots except that the magnitude of the constant A (= 2.0 for H) in the Weibull plot results in the 2 foot wave height data point being lost. The Weibull plot is also not so convenient to use because of the zero shift in the wave height scale. The semi logarithmic plot is obviously not so good in this situation where there is swell present since the points for 4 feet and below do not lie on the straight line.

All four distributions indicate that there is a lower limit to the wave height but only the Weibull and exponential distributions can allow for this factor. In this case it is also evident that only the Weibull distribution gives a limiting minimum wave height which is physically credible. Moreover, only the Weibull distribution is compatible with the possibility of a physical limitation on the large wave heights.

The extrapolated values of the significant wave height $H_{1/3}$ and the most probable maximum wave height $\mu(H_{max})$ are shown for all four cases on figure 25. It can be seen that the Weibull extrapolation gives the lowest wave heights for a given return period, while the log probability extrapolation gives the highest. Both the Gumbel and semi logarithmic extrapolations give values which are very close to the mean value for all four methods.

It is obvious however from figure 25 that, if a choice is to be made between the various distributions for extrapolation of wave height data, the data must extend at least as far as the 99.9% level. In the present case, data is available only to the 99% level over which range there is little to choose between the various distributions on an empirical basis.

Adopting the mean curve on figure 25 we see that a significant wave height of about $16\frac{1}{2}$ feet is equalled or exceeded 0.01% of the time or once in 27 years, while an $H_{1/3}$ of about 13 feet is equalled or exceeded



Fig 24 Wave Height Exceedance Graphs-Various Extrapolations.

0.1% of the time or once in 2.7 years. The corresponding figures for the maximum probable wave height of an individual wave are 31 feet and $24\frac{1}{2}$ feet. These values appear to be generally consistent with the occurrence of cyclones off the southern Queensland coast. For instance, Coleman (figs 31 to 35 of ref 13) indicates that the average frequency of tropical cyclones crossing the 5° latitude-longitude square centred on Brisbane is 5.5 per decade or a little more than one every two years. The data from Moffat Beach indicates that significant wave heights from an average cyclone would be of the order of 12 to 14 feet, while the maximum individual wave heights would lie between 20 and 25 feet.

With regard to the wave periods, detailed extrapolation has not been attempted. Figure 26 shows that the log probability plot gives a reasonable straight line with a period of 15 seconds equalled or exceeded 0.01% of the time, and one of 14 seconds equalled or exceeded 0.1% of the time.

6.4 Summary

Analysis of sea-swell observations at Cape Moreton for the period 1960-1967 showed that waves during the recording period were higher than usual and that offshore winds during this period were less frequent than usual.

A reasonably satisfactory calibration of simultaneous sea-swell state for a given swell direction was obtained for both $H_{1/3}$ and T_{2} and the wave height and period exceedance curves adjusted for the period 1960-1967.

From the modified exceedance curves it is found that overall $H_{1/3}_{50}$ is 3.6 ft and T so is 9.7 seconds. Moreover, there is always a groundswell present, the significant height of which seldom drops below 1.5 ft. Seasonally, waves are highest in the January-March quarter and lowest in the July-September quarter.

Four methods were used to extrapolate the data for extreme conditions. The mean extrapolated wave heights appear to be more or less consistent with the frequency of occurrence of cyclones off the southern Queensland coast.

The meaningfulness of return periods for various values of a continuous variable such as $H_{1/3}$ can be questioned (see Battjes ref 2). A return period is only relevant when associated with discrete events. Thus, strictly speaking, one should only refer to the return period of $H_{1/3}$ if the sequence of the values is related to a series of events. For example, the maximum $H_{1/3}$ associated with a given storm event or the maximum annual $H_{1/3}$ could be selected, cf, flood frequency analysis. H_{max} , on the other hand, is a discrete event in itself and can be assigned a return period.



Fig 25(left) Comparison of Wave Height Exceedance Extrapolation. Fig 26(right) Wave Period Exceedance Graph-Log probability plot.

7. COMPARISON AND DISCUSSION OF WAVE DATA FOR SOUTHERN QUEENSLAND COAST

7.1 Comparison of Moffat Beach Data and Gold Coast Data

In recent years modern wave recording equipment using Wave Rider Buoys and an onshore recording station at North Burleigh has been installed as part of the Gold Coast Beach Erosion Investigation (ref. 46). Some results of wave characteristics observed for the period August 1968 to December 1969 were reported in reference 23. These are compared with the results of the present study on figure 27 and in table 8 below. Data for a more extended period (Aug. 1968 to Nov. 1971) has recently come available (ref 44) and is also included in table 8.

Table 8

COMPARISON OF MOFFAT BEACH AND GOLD COAST WAVE DATA

Location	Moffat Beach	Gold Coast			
Recorder Type	OSPOS	Wave Rider			
		Aug. 1968 to	Aug.1968 to		
Duration of 196 record ext	1960-1967 extrapolated	Hand Analysis (figs 14-25, 30-32 of ref.23)	Computer Analysis (tables 26- 34 of ref. 23)	Computer Analysis (ref.44)	
H _{1/3 50} - ft	3.6	3.5	3.8	4.0	
$\mu(H_{max})_{50} - ft$. 6.7	7.1*	7.8	7.9	
T _{z50} - secs	9.7	7.0	5.1	5.3	
L ₀₅₀ - ft	482	251	133	144	
= $5.12T_{z_{50}}^2$					
£	0-0.69**	Not available	0.49-0.95	Not available	

^{*}Value calculated by the author from $H_{1/3}$ ₅₀ using table 1 of Draper (ref.24) and a wave period of 7.0 secs. A similar calculation for the computer analysed data gives the figure 7.8 ft for $\mu(H_{max})$ which is identical to that in table 34 of reference 23.

Limited number of recordings only.



Fig 27 Comparison of Moffat Beach and Gold Coast Data (ref. 23).

The two sets of data are seen to give generally similar estimates of the significant wave height under normal conditions. The Moffat Beach data, however, indicate a somewhat lower frequency of occurrence for the lower waves and a higher frequency of occurrence for the higher waves. The latter effect can be explained by the absence of cyclonic waves from the original Gold Coast data. Subsequent to the publication of the data in reference 23, waves were recorded on the Gold Coast during a cyclone in February 1971 and a maximum wave height of 22 feet was observed, the corresponding significant wave height being 14.4 feet (ref 44). This is of the same order of magnitude as was recorded at Moffat Beach during the 1963 and 1964 cyclones. Even higher waves (H \approx 30 feet) were recorded off the Gold Coast one year later on 7th and 8th February 1972. The most recent Gold Coast wave height exceedance curve (ref 44) is compared with the Moffat Beach data on fig.28. The discrepancy in the low wave heights is evident on the log probability plot.

When wave periods are considered, however, there are significant differences between the two sets of data. The periods measured at Moffat Beach are generally 2.5 to 3 seconds longer than those obtained from hand analysis of the Gold Coast data. This is a serious discrepancy, the magnitude of which can perhaps be best appreciated by comparing the deep water wave lengths corresponding to the values of T in table 8. The Moffat Beach data indicate wave lengths almost twice as long as those from the Gold Coast data. The wave period discrepancy is even greater (4.6 secs) when the Gold Coast computer analysed results are considered.

A consequence of this difference in wave periods is the difference in the estimates of the probable maximum wave height from the two sets of data. Since $\mu(H_{max})$ increases with the number of waves (see section 4.2) it decreases as the wave period increases and hence the Moffat Beach data gives lower estimates of this quantity.

Another significant difference between the two sets of data is in the values of the spectral width parameter ε . For the Moffat Beach data $0 < \varepsilon < 0.69$ while for the Gold Coast data $0.49 < \varepsilon < 0.95$. The former values are generally characteristic of swell, the latter of sea.



Fig 28 Comparison of Moffat Beach and Gold Coast Data, (ref 44).

7.2 Possible Explanations of Wave Period Differences

Several possible explanations can be advanced for the significant difference between the wave periods obtained at Moffat Beach and at the Gold Coast.

7.2.1 The Characteristics of the Recorders

The OSPOS recorder measures the pressure fluctuation at a point below the water surface, while the Wave Rider measures the water surface fluctuation. Even though there is experimental evidence (see section 3.4) to suggest that the OSPOS recorder at Moffat Beach gave a reasonable representation of the surface waves, other evidence can be found to suggest that some filtering of the surface wave spectrum has occurred.

Assuming a constant value of $\frac{z}{D}$, as was done in the data analysis, the filter characteristic due to pressure attenuation can be calculated from equation 3.4-1. Figure 29a shows that wave periods of 2.7 seconds or less will be completely removed from the pressure recording while periods of 5.3 secs will have their amplitudes reduced to 50% of their surface values. Thus virtually all waves generated by local winds, sea breezes, etc., will not be recorded by the OSPOS recorder. The average zero crossing period of these records must then of necessity be greater than that determined by the Wave Rider buoys. The previously noted



difference in values for the spectral width parameter ϵ is further evidence of the removal of local seas from the OSPOS recordings.

The Wave Rider buoy on the other hand is specified (ref 46) to record waves between 1.25 sec. and 17 secs to within 3% and between 1 sec. and 33 secs to within 30%. Glukhovsky (ref 30) has produced empirical formulae for correcting the wave heights and periods measured by subsurface pressure recorders. These were derived from data obtained on the Caspian Sea and have been applied with reasonable success in the Atlantic Ocean by Tsypluklin (ref 56) and in the Baltic Sea by Cieslak and Kowalski (ref 12).

Glukhovsky's formula for calculating the surface wave period T is

 $T_{s} = \frac{T_{z}}{1 + \frac{0.13}{\sqrt{T_{z}}} + \frac{64}{(37+T_{z})^{2}}}$ 7.2.1-1

where T is the period at depth z below the water surface. (T $_z$ in seconds, z in metres).

Application of this formula to the Moffat Beach site for a measured wave period of 10 seconds gives a surface period of 9.0 seconds. Figures for other periods have been plotted on figure 29b. The applicability of this formula will depend upon the degree to which the wind generated seas for which it was derived approach conditions on the Queensland coast. Since the latter involve swell conditions as well as sea, the two situations are undoubtedly different. The fact that the corrected value of T_z is still considerably greater than the surface values measured with the Wave Rider buoys would tend to confirm that Queensland conditions are very much different to those for which the formula was derived. These conclusions are also almost certainly valid for Thabet's OSPOS calibration graphs (figure 5).

7.2.2 Transformation of the Wave Spectrum in Shoaling Water

The Wave Rider buoys were located several miles (4 N.M. average) off the Gold Coast in comparatively deep water (about 150 feet). All three buoys gave similar wave characteristics and can be assumed to give a reasonable representation of conditions approaching those in deep water. The Moffat Beach records were obtained much closer inshore.

As the waves move into shoaling water, the wave spectrum is transformed and modified. At least two mechanisms operate to achieve this. Firstly the shoaling coefficient ($K_s = \frac{H}{H}$) varies with the wave period, such that provided $\frac{d}{L} < 0.15$ at a given location (given depth of water) K_s is larger for the longer periods than for the shorter ones. The energy density of the longer waves will thus tend to increase with respect to that of the shorter ones and the centre of gravity of the spectrum will shift towards the longer periods. The apparent wave period, i.e. zero crossing period T_z , or the period associated with the spectral peak, will thus become longer as the waves move into shallow water.

A second mechanism which may act to cause the same effect is an interaction between the long and short period components of the wave spectrum. Longuet-Higgins and Stewart (ref 43) have shown that the short waves riding on the longer ones will be both shortened and steepened when they are travelling on the crest of a long wave. This effect can result in the early breaking of the smaller waves, thus dissipating some of their energy.

An example of this type of spectral modification is given on figure 30a showing two spectra measured on a model beach in a wind-wave flume (ref 33). A shift of the main spectral peak towards the lower frequencies in shallow water is quite evident. The value of T_o increased from 1.11 secs in deepwater to 1.18 secs in shallow water.

The mechanism described above can be expected to occur in a relatively steep beach such as that formed in the laboratory wind wave flume. 0n the other hand, when relatively flat slopes are considered, the effects of bottom friction and wave breaking may result in an opposite result in that the long period components of the spectrum are dissipated in shoaling water, while the shorter period components are relatively unaffected in what is for them relatively deep water. The spectral peak in this case shifts towards the shorter periods. This type of behaviour has been reported by Rutkovskiy (ref 51) from observations made in the Baltic Sea. Figure 30b taken from Svasek (ref 53) shows three simultaneous spectra recorded in shallow water off the Dutch coast in which T remains constant, but T decreases in shoaling water. Larras (ref 41) has also noted a decrease in T in shallow water near the Loire estuary. Siefert (ref 52) found that the wave period distribution became wider in shallow water in the Elbe estuary, which result is consistent with the foregoing.

The decrease in T in shallow areas can also be attributed to the development of secondary wave crests which have been observed in



laboratory studies (refs 28 and 31) when the depth becomes relatively small in comparison with the wave length. The two or more secondary crests which occur under these conditions will obviously result in a decrease in T_{τ} .

On the other hand, Galvin (ref 29) reports a comparison between visual observations off both the Pacific and Atlantic coasts of the United States in which the periods of waves just prior to breaking are consistently longer than those observed offshore in nearby shipping lanes. The mean coastal wave period is 1.4 times the mean offshore wave period on the Pacific Coast and 1.2 times the mean offshore wave period on the Atlantic Coast. When offshore waves which would never reach the shore are eliminated, these ratios become 1.6 and 2.3 respectively. It should be noted that this comparison is based upon two independent sets of visual observations which may not be exactly comparable. Their reliability must of necessity be less than instrumental recordings.

7.2.3 Refraction of Wave Spectrum at Moffat Beach

Since the wave recorder locations were located quite close inshore (6 fathoms depth) the wave data obtained is affected by the processes of refraction and shoaling. The wave characteristics are thus a function of the meteorological conditions and the location of the recording instrument.

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To obtain an idea of the topographical effect refraction diagrams were plotted for both recorder sites for various wave directions. The wave period chosen was 10 seconds, the median period for easterly waves. The refraction coefficients relative to deep water conditions are given in table 9.

Table 9

REFRACTION COEFFICIENTS $\frac{H}{H}$ AT MOFFAT BEACH RECORDER SITES 10 second waves

Recorder Location		Deepwate	r Wave I	irection	
	NE	ENE	Е	ESE	SE
lst site (1963)	1.20	0.78	0.80	0.88	?
2nd site (1964)	1.19	0.92	0.88	0.93	?

From these figures it is seen that at the Moffat Beach recorder sites waves from the North East may be up to 20% higher than deep water waves from this direction, while waves from the ENE to ESE sector may be up to 20% lower than the deep water waves. No estimate could be made for south easterly waves since these waves were affected by the reefs to the north of Cape Moreton and estimation of a refraction coefficient at Moffat Beach was not possible.

Any conclusions deduced from the preceding must be taken as tentative since the actual average wave periods recorded ranged from 6 seconds to 16 seconds which variation alone can be expected to affect the refraction coefficients significantly. In addition to this the various components of the wave spectrum will be refracted independently, hence the wave spectrum will change in form as the waves travel shoreward. In certain cases the effect of topography can completely alter the inshore wave characteristics in comparison with those in deep water (pages 188 and 213f of ref 49). A common case is at locations in the lee of a headland where the wave height is naturally reduced and the wave spectrum narrowed due to differential refraction of the various spectral components. The longer periods are refracted more than the shorter ones and so the waves in such a location tend to be longer and more regular than on the exposed portion of the coastline.

Whether or not any such effect influences the wave characteristics at Moffat Beach is difficult to determine without extensive refraction analysis involving wave spectra and many wave periods. However, the site is essentially open to all directions from ESE to NE and major effects which would not occur at other exposed positions along the coast are thus unlikely. For the south east direction local topography is significant. However, the major features, the reefs to the north of Cape Moreton, are over 20 nautical miles distant from the site in question and their effects are difficult to assess since under strong winds significant wave generation can occur between Cape Moreton and Moffat Beach.

7.2.4 Difference in Wave Climate

An obvious cause for the difference in wave periods is simply that they result from different meteorological conditions at the two sites. Observations at three points off the Gold Coast show no significant differences in the offshore wave conditions. There are also no known factors which could be expected to change the offshore waves between the Gold Coast and Cape Moreton. However, it is possible that the wave climate inshore may at times be different to that offshore due to meteorological causes.

Westerly and other offshore winds of about 15 knots can generate over a fetch of 4 nautical miles waves with periods of the order of 2 seconds and significant wave heights of about 1 foot. When such waves are superimposed upon an incoming swell of 10 seconds period and 3 feet height, the average zero crossing period will be reduced to about 5.5 seconds (equation 4.6-4). The inshore wave period near the surf zone will, however, be of the order of 10 seconds assuming that the other factors mentioned previously do not come into play. Thus for conditions with offshore winds the Wave Rider buoys can be expected to record waves of a shorter period than would be recorded at an inshore location such as off Moffat Beach. Whether or not such conditions occur with sufficient frequency for the overall average wave period to be significantly reduced is difficult to determine without detailed analysis of actual wind and wave data. However, it can be stated that offshore winds (SSW to NNW) of 11 knots or greater occurred at Cape Moreton for about 19% of the period 1957-1965 inclusive (based upon 9 a.m. observations).

7.2.5 Conclusions concerning Wave Period Differences

From the preceding discussion it appears that while transformation of the wave spectrum due to shoaling and refraction, together with the occurrence at the Wave Rider sites of seas generated by offshore winds, can all have the effect of making the wave periods at Moffat Beach longer than those measured off the Gold Coast, the most probable cause is the filter effect due to pressure attenuation. The depth of immersion of the OSPOS recorder is such that local seas generated by sea breezes are virtually completely removed while the underlying swell is unaffected.

This explanation will also account for the relatively lower frequency of occurrence for waves of small height at Moffat Beach, since the longer periods recorded give too large a value of the pressure attenuation factor K_p which undercorrects the recorded wave heights.

The wave periods measured by the wave rider buoys are undoubtedly a reliable measure of T of the water surface, but it is not certain that they are the most physically significant measure of wave period in this situation.

7.3 The Choice of a Suitable Wave Period to Represent a Wave Recording

From the preceding sections it is evident that it would be desirable to obtain either some better measure of the wave period than the easily analysed zero crossing period T or to adopt procedures which would allow the latter to be modified to take account of the various factors mentioned in section 7.2.

Recently Harris (ref 35) has studied the question of representing a wave recording by a single wave height and wave period such as has been done in this study. His results show that various definitions of wave height are highly correlated with one another which confirms the assumption of a general wave height distribution such as the Rayleigh distribution. The various definitions of wave period for a given recording, however, show very little correlation with one another. The reason for this is not stated, but it could be that the wave spectra are very variable in shape and quite likely include percentages of local sea and swell. This would also be the situation along the Southern Queensland Coast.

For situations with wind waves only there appears to be a relation between some of the different wave period definitions. For instance J. Darbyshire and M. Darbyshire (ref 18) indicate that the ratio $\frac{T_c}{T_0}$ is inversely proportional to T_o, while in other papers J. Darbyshire (refs 16 and 17) shows that T_o = 1.14 T_{H 1/3} for the Atlantic Ocean and T_o = 1.08 T_{H 1/3} for the Irish Sea. A direct comparison is given by M. Darbyshire (ref 19) between T_o and T_z recorded by a shipbourne wave recorder at several locations with considerably different wave climates. A single graph relates T_o to T_z for all four locations and shows that for relatively sheltered places T_o T_z and that for exposed places T_o > T_z. The ratio $\frac{T_o}{T_c}$ thus tends to increase with the magnitude of T_z. A similar type of relationship is given by a formula derived by Scott (reported by Wiegel in ref 61).

 $\frac{1}{T_{0}} = \frac{0.501}{T_{z}} + \frac{1.43}{T_{z}^{2}}$ 7.3-1

On the other hand relationships between $T_{H^{-1/3}}$ and T_{Z} of the form $T_{H^{-1/3}} = A T_{Z} - B$ have been obtained by other investigators (refs 20 and 60) in situations were swell possibly predominated.

Since T can be calculated from equation 4.6-4 and T is known once the spectrum is known, the relationship between these two periods depends upon the form of the wave spectrum. The Bretschneider spectrum (ref 7) for instance gives T = 1.12 T and any of the other spectra proposed for wind generated seas, such as that of Pierson and Moskowitz (ref 48) quoted in section 4.6, will yield a similar relationship. The relationship will, however, change if a spectrum of different form occurs and may change markedly if two or more different spectra are superimposed, i.e. local sea riding on a ground swell, etc. An example of a change in spectral form is shown on figure 30b which shows three simultaneous wave spectra measured off the Dutch coast in shallow water (Svasek ref 53). In each case T is less than T the difference increasing as the higher frequencies (shorter periods) become relatively more prominent. In cases where double peaked spectra occur the choice of T , while not necessarily ambigious, may not be the most suitable one for engineering purposes. Indeed in such cases both values of T may be important since the two component spectra may both be relevant to design problems. For instance a local sea approaching the shore at an angle will be refracted less than the underlying swell, which breaks almost parallel to the beach (ref 45). The latter may, however, stir up additional quantities of sand which will increase the longshore transport of the smaller waves. Again, it is obvious in this type of situation that T is largely irrelevant and probably gives misleading information concerning the influence of the wave motion upon the bottom and upon sand transport rates. In situations where it is not desired to work with a complete wave spectrum, an alternative and more meaningful definition of wave period may be that given by Battjes (ref 1). He defines T_p as the period of a sine wave with the same power as the whole spectrum

$$\frac{T_R}{T_z} \approx 1.2 \text{ to } 1.3 .$$

and suggests that this should be used in refraction calculations. Depending upon the wave spectrum used, T_{R} is related to T_{z} by the following relation

In the case of superimposed spectra with double peaks two values of T_R would be required, one for each component spectrum.

From the above examples for wind waves, it seems that the appropriate spectral period T or T_R is of the order of 12.5% to 30% longer than the zero crossing period T_c^0 , the magnitude of this increase being greater for large T (eqn 7.3-1). Observations by the Delft Hydraulics Laboratory at Sekondi, Ghana (ref 22) using a bottom pressure recorder and a wave rider buoy simultaneously over a period of several months gave an average T from the pressure recorder of 10.9 secs and an average T from the wave rider of 12.2 secs for swell conditions. T was thus at least 11% greater than T since the surface value of the latter would be less than 10.9 secs. (Using Glukhovsky's formula (eqn 7.2.1-1) T at the surface is 9.6 and T = 1.29 T). The Atlantic City data of Harris (Table V of ref 35) on the other hand give individual values, some of which are very much higher than these ratios, i.e. T of the order of 100 to 200% greater than T for large T. This would tend to confirm that this data includes complex spectra where T_c^0 (see section 7.2.4) is very much influenced by local winds, while T refers to the underlying swell.

From the foregoing it is concluded that to obtain a physically meaningful value of the characteristic wave period of a wave recording, analysis involving wave spectra is required so that T or T_R may be obtained. Furthermore, it appears that if this were done, then the average period values obtained off the Gold Coast could be of the same order as the T_z values measured by the OSPOS recorder at Moffat Beach. For example, Scott's formula (eqn 7.3-1) gives T = 10 secs when T_z = 7 secs, while Battjes' T_R is 8.4 to 9.1 secs. Both these figures apply to values to T_z measured at the surface. However, Rutkovskiy (ref 51) presents simultaneous wave spectra measured both at the surface and at several depths below the surface at two locations. Using the values of T_z given by Rutkovskiy and values of T_o deduced from his spectra, it is found that near the surface $\frac{T_o}{T_z}$ is of the order of 1.3 to 1.4. This ratio reduces with increasing depth approaching a value of unity near the bed.

Wave spectral analyses made of the Gold Coast data (ref 44) do in fact confirm these suppositions. $\frac{T_o}{T_c}$ was found to range from 1 to as high as 3 with values of the order of 1.5 persisting for some days. This information, which is as yet by no means complete, suggests that the Moffat Beach data obtained with the OSPOS recorder give values of T_z which are of the same order as T_o for the surface wave spectra.

7.4 <u>A Proposal for the Determination of $H_{1/3}$ and T_z for Sea and Swell Superimposed</u>

The use of T_t to characterise the period of a wave recording is not to be recommended in all cases. In the light of the work reported in this bulletin, it is suggested that T_t is only practically useful when a single spectrum of waves is present. In such situations the spectrum can be represented by a single generalised form and T_t can be fairly simply related to T_o or T_R or any other physically meaningful value associated with the wave spectrum. Locally generated wind waves from a single disturbance in the absence of swell should be meaningfully represented by T_z. Swell with no sea present or swell records which have been filtered by pressure attenuation can also be characterised by T_z. Mixed conditions, where sea is superimposed upon swell, or two or more seas or swells from different sources are superimposed, cannot be adequately represented by T_z.

In the case of the southern Queensland coast, local seas are normally superimposed upon swell. In this case analysis of records made with a surface wave recorder such as the Wave Rider in terms of T is not a very meaningful procedure. In this situation analysis must be made either, using a complete spectral analysis of each record and selecting a suitable representative period such as T , the period associated with the peak of the frequency spectrum, or T_R , the period of a sine wave with the same power as the whole spectrum, or by filtering the data so as to exclude the local sea from the record before T is calculated. An appropriate filter would be chosen after comparison of filtered T values and wave periods derived from spectral analysis of selected representative records.

By careful choice of filter characteristics it should be possible to separate the separate spectra for local sea and swell and to obtain representative T values for each type of wave condition. For instance the original data can be analysed to give $H_{1/3}$ and T directly and then passed through high and low pass filters with suitable characteristics to give $H_{1/3}$ and T for both the sea and swell. The validity of this operation could then be checked for each case by combining the component values of $H_{1/3}$ and T using the relations given below and comparing the result with the directly calculated overall values of $H_{1/3}$ and T. With experience it should be possible to devise means of adjusting the filter characteristics to resolve any discrepancies between the two estimates of the overall $H_{1/3}$ and T and T and T and thus obtain reasonably reliable separate values for the individual sea and swell
spectra.

The necessary relationships are the following:

$$H_{1/3}^{2} s+s = H_{1/3}^{2} sea + H_{1/3}^{2} swell$$

$$T_{z s+s} = T_{z sea} T_{z swell} \sqrt{\frac{(H_{1/3}^{2} sea + H_{1/3}^{2} swell)}{H_{1/3}^{2} sea T_{z swell} + H_{1/3}^{2} swell}}$$

$$7.4-1$$

$$7.4-1$$

$$7.4-1$$

$$7.4-2$$

Equation 7.4-2 is derived from equation 4.6-4 assuming that the spectral energy for each component spectrum is concentrated at the frequency corresponding to the average zero crossing period T_{z} of that spectrum.

Whether the procedure outlined above would be successful, and, if successful, could be economically applied to all recordings, is a matter which requires further extensive investigation and analysis of actual data. Obviously there will be cases where it will be difficult to separate the sea and swell, particularly when their heights are of comparable magnitude and when more than two spectra are superimposed. The occurrence of secondary spectral peaks with a frequency half that of the main spectral peak as has been observed in some laboratory investigations could also influence the result to some extent. The energy associated with these is, however, generally relatively small and should not be of any great importance.

7.5 Summary

When compared with wave data obtained by Wave Rider buoys off the Gold Coast, the Moffat Beach data gives comparable estimates of the frequency of occurrence of different significant wave heights with the exception that the high waves are more frequent and the lower waves less frequent at Moffat Beach. The differences can be attributed partly to the absence of the influence of cyclones on the Gold Coast data in reference 23 and partly to incomplete pressure attenuation correction of the Moffat Beach data.

The wave periods (T₂) measured by the OSPOS recorder at Moffat Beach are significantly longer than those obtained from the Wave Rider buoys. This difference in wave period affects the estimates of probable maximum wave height for the two sets of data.

Several possible causes of this discrepancy in wave periods have been considered and it is concluded that it is essentially caused by the filter effect due to pressure attenuation which almost completely removes local wind generated seas from the OSPOS record. This conclusion is supported by the values of ε which indicate that the Wave Rider records have the nature of sea, while the OSPOS records are characteristically swell.

For situations such as the southern Queensland coast where both sea and swell are present simultaneously, the use of T to give a characteristic wave period cannot be recommended. Spectral analysis is required to give more reliable representative periods such as T or T_R . Filtering of the recorded data from the Wave Rider equipment may allow more meaningful values of T to be obtained. At the present stage it is suggested that the wave periods measured at Moffat Beach are more physically meaningful for Engineering purposes than those published in reference 23. This conclusion would appear to be generally confirmed by the most recent Gold Coast spectral analysis (ref 44).

In particular, the wave period calibration of the Cape Moreton swell observations using Wave Rider observations, which is given in Table 17 of reference 23, should be treated with considerable reserve until other measures of wave period become available. At the same time, it should be noted that the wave periods given in section 5.5 of this bulletin for normal conditions represent swell periods and not sea periods.

8. CONCLUSIONS

Useful engineering data has been obtained from wave recordings made at Moffat Beach during 1963 and 1964. This includes data concerning two cyclonic events.

The wave height distributions analysed from Moffat Beach confirm the general applicability of the Rayleigh law for the short term wave height distribution.

Extrapolation of recordings of limited duration to an eight year period has been possible using a calibration based upon visual observations at Cape Moreton. The reliability of this procedure would be substantially improved if the sites of the recorded and visual observations were coincident.

Log-probability, Weibull and Gumbel plots all fit the long term wave height exceedance graphs quite well, only the semi logarithmic plot seems inferior. The extrapolated values derived from these plots are not in agreement with one another and no conclusions as to the most reliable means of extrapolating wave height data can be made from the empirical data presented in this bulletin. The mean of the four extrapolated values is, however, consistent with both the frequency of occurrence of cyclones in this area and with recorded data from the Gold Coast.

The long term significant wave height exceedance graphs measured at Moffat Beach using an OSPOS recorder are generally similar to those obtained off the Gold Coast with Wave Rider buoys. There is, however, a definite tendency to underestimate the wave height having a given exceedance frequency or return period for small waves.

There are appreciable discrepancies between the Moffat Beach data and the Gold Coast data with regard to zero crossing wave period and probable maximum wave height. It is considered that this discrepancy is primarily caused by the filtering action due to sub-surface pressure attenuation of the OSPOS recorder. The Moffat Beach data essentially represents the period of the swell, while the Wave Rider data is very greatly influenced by locally generated seas. The periods from the latter data are consequently appreciably shorter than those obtained at Moffat Beach.

The zero crossing period is the simplest, consistent measure of wave period that can be determined. This does not mean that it is the most relevant or indeed the most meaningful measure of wave period from the point of view of the Coastal Engineer. Indeed it can only be considered useful when waves of one kind only, sea or swell, but not both simultaneously, occur.

Spectral analysis of the Wave Rider data is required to obtain more satisfactory estimates of the wave period. Either the period T associated with the frequency of the peak of the energy density spectrum or the period T_R of a sine wave with the same power as the whole spectrum should be more useful than the zero crossing period T particularly in situations such as the southern Queensland Coast, where both sea and swell are present. The limited spectral data now available give values of T which are generally consistent with the values of T obtained with the OSPOS recorder.

An alternate method of dealing with mixed spectra (i.e. sea plus swell) could be to filter the recorded output from the Wave Rider buoys to separate the sea and swell waves. The significant wave height and zero crossing period would then be determined for both sea and swell separately as well as for the combined condition.

Refraction calculations using the zero crossing periods from the Wave Rider data cannot be expected to give the best estimates of inshore wave heights. If refraction of the full wave spectrum is not possible, then T or T_R are more likely to give reliable results. The Moffat Beach wave periods, being closer to the expected T or T_R off the southern Queensland coast are thus more likely to give correct inshore wave heights than the T values derived from the Wave Rider buoys. Sand transport calculations will also be affected by the wave period values adopted.

It is concluded that in general new instrumental methods will require new analysis techniques to obtain parameters of engineering significance. A more sophisticated wave recording system such as the Wave Rider will require procedures such as spectral analysis which take into account the more complete data obtained by that system. On the other hand, a simpler analysis is quite suitable for a wave recording system such as the OSPOS which incorporates its own built-in filtering system. Properly used and correctly interpreted, such a system will yield data which may be just as useful as that obtained from a more complicated system using too simple a system of analysis. It is therefore considered that the Tucker-Draper method of analysis for zero crossing wave period is inadequate for use with unfiltered Wave Rider recordings obtained off the exposed eastern Australian coast, but is generally suitable for the prefiltered data of a pressure recorder such as the OSPOS located not too far below the water surface.

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Appendix 2

Sea - Swell State Descriptions (From Reference 8)

State	Sea	Swell
0	Calm (glassy)	No swell
1	Calm (rippled)	Low swell of short or average length
2	Smooth (wavelets)	Long low swell
3	Slight	Short swell of moderate height
4	Moderate	Average swell of moderate height
5	Rough	Long swell of moderate height
б	Very rough	Short heavy swell
7	High	Average length heavy swell
8	Very high	Long heavy swell
9	Phenomenal	Confused swell.

Appendix 3

Equations for Long Term Wave Height Exceedance Graphs

P(H) is the probability that the wave height will equal or exceed a given value, i.e. it is the cumulative probability.

 $P(H) = \int p(H) dH$

where p(H) dH is the probability that the wave height lies between H and H + dH.

Exponential Distribution

$$P(H) = e^{-\frac{H}{B}}$$
where B is a scale factor determining the relative width of the distribution

or

$$H = -B \log_e P(H)$$

Modified Exponential Distribution

$$P(H) = e^{-\frac{H - H_o}{B}} \text{ for } H \ge H_o$$
$$= 1 \qquad \text{ for } H < H_o$$

where H_{o} is the minimum wave height that occurs.

or

$$H = H_0 - B \log_0 P(H)$$

Weibull Distribution

$$P(H) = e^{-\left(\frac{H - H_{o}}{B}\right)^{c}}$$
for $H \ge H_{o}$
= 1 for $H < H_{o}$

or

$$H = H_o - B(\log_e P(H))^{1/c}$$

Gumbel Distribution

$$P(H) = 1 - e^{-e} - \left(\frac{H - H_p}{B}\right)$$

where $\underset{p}{\text{H}}$ is the most probable wave height.

or

$$H = H_p - B \log_e [-\log_e (1 - P(H))]$$

Logarithmic Normal Distribution

$$P(H) = P(\log H)$$

= $\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\log H} e^{-\left(\frac{\log^2 \frac{H}{m}}{2\sigma^2}\right)} d (\log H)$

where \log m and σ are the mean and standard deviation of log H.

