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# Optimized Soft-Core Processor Architecture for Noise Jamming 

## By

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Master of Electrical \& Electronic Engineering at the Faculty of Engineering and Built Environment

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## Publications

DETAILS OF CONTRIBUTION TO PUBLICATIONS that form part of this dissertation, or include research presented in it, are as follows:

## Publication 1:

Madodana Mfana, Ali N Hasan, and Ahmed Ali, "Odd/Even Order Sampling Soft-Core Architecture towards Mixed Signals Fourth Industrial Revolution(4IR) Applications", Energies MDPI Journal. Volume 12, Issue 23, 4567, 29 November 2019

Author contributions: M Mfana developed the mathematical model, simulation model, assisted with conducting experiments, analysed simulation and experimental results, and wrote the paper. The other authors assisted with reviewing the paper.


## Abstract

Noise jamming is a traditional electronic counter measure (ECM) that existed since the establishment of electronic warfare (EW). Traditional noise jamming techniques have been shown to be failing when interacting with intelligent Radar systems such as pulse Doppler radar. Hence there is a need to introduce new noise jamming techniques with digital architecture that will provide improved performance against smart pulse Doppler radar. The work is undertaken to investigate the feasibility of digitizing noise jamming. It focuses on analog-to-digital conversion optimization towards noise jamming architecture, as a result digitization will allow for an opportunity for adaptation of intelligent processing that previously didn't exist.

In this dissertation, certain contributions to the field of noise jamming were made by introducing state of the art odd/even order sampling architecture by proving four case studies. Case study 1 experimentally investigates sample frequency behaviour. Case study 2 uses simulation to investigate step-size and dynamic range behaviour. Case study 3 uses FPGA implementation and SNR to investigate quantization error behaviour. Case study 3 also uses SNR to investigate superiority of proposed odd/even order sampling. Lastly case study 4 uses field measurements, FPGA implementation and SNR to investigate practical implementation of digitized noise jamming.

The main contribution is concerned with an architecture that digitizes, reduces sample frequency, optimizes digital resource utilization while reducing noise jamming signal-to-noise ratio. The approach evaluates and empirically compares three sampling techniques from lecture Mod- $\Delta$, Mod- $\Delta$ (Gaussian) and Mod- $\Delta$ (Sinusoidal) with proposed novel odd/even order sampling. Sampling techniques are evaluated in terms of quantization error, mean square error and signal-to-noise ratio.

It was found that the proposed novel odd/even order sampling achieved most case SNR performance of 6 dB in comparison to 18 dB for Mod- $\Delta$. Sampling frequency findings indicated that the proposed novel odd/even order sampling had achieved sampling frequency of 2 kHz in comparison to 8 kHz from traditional 1st order sigma-delta. Dynamic range findings indicated that the proposed odd/even order sampling achieved a dynamic range of 1.088 volts $/ \mathrm{ms}$ in comparison to 1.185 volts $/ \mathrm{ms}$ from traditional 1st order sigma-delta. Findings have indicated that the proposed odd/even order sampling has superior SNR and sampling frequency
performances, while the dynamic range is reduced by $8 \%$. It was also found that signal-to-noise ratio results for the proposed noise jamming outperforms gaussian noise jamming by at least 4 dB. Efficiency of traditional noise jamming as characterized by simple architecture and operation, was conducted. Noise jamming has not been studied to this extent until now.

Keywords: Soft-Core Architecture; Odd Order Sampling; Even Order Sampling; Analog to Digital Converter (ADC).

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## Nomenclature

1. RAM Random Access Memory
2. FPGA Field Programmable Gate Array
3. LUT Look Up Table
4. DSP Digital Signal Processing
5. ILA Integrated Logic Analyzer
6. RMS Root Mean Square
7. ARMS Average Root Mean Square
8. PDF Probability Distribution Function
9. CDF Cumulative Distribution Function
10. PSD Power Spectral Density
11. AR-PSD Autoregressive Power Spectral Density
12. ADC Analog-to-Digital Converter
13. SNR Signal-to-Noise Ratio
14. SQNR Signal-to-Quantization-Noise Ratio
15. R Bit Rate
16. D Distortion
17. dB Decibels
18. dBm Decibel-milliwatts
19. dBFS Decibels Relative to Full Scale
20. dBTP Decibels True Peak
21. MSE Mean Square Error
22. DR Dynamic Range
23. Mod- $\Delta \quad$ Mod-Delta
24. REW Radar \& Electronic Warfare
25. SAR Synthetic Aperture Radar
26. ISAR Inverse Synthetic Aperture Radar
27. EW Electronic Warfare
28. ECM Electronic Counter Measure
29. ECCM Electronic Counter-Counter Measure
30. ESM Electronic Support Measure
31. RGPO Range Gate Pull Off
32. VGPO Velocity Gate Pull Off
33. AGPO Angle Gate Pull Off

| 34. RSP | Radar Signal Processor |
| :--- | :--- |
| 35. DRFM | Digital Radio Frequency Memory |
| 36. FMCW | Frequency Modulated Continuous Wave |
| 37. AI | Artificial Intelligence |
| 38. ML | Machine Learning |
| 39. ANN | Artificial Neural Network |
| 40. BPNN | Back-Propagation Neural Network |
| 41. FNN | Fuzzy Neural Network |
| 42. GA-NN | Generic Neural Network |
| 43. GA | Generic Algorithm |
| 44. Hz | Hertz |
| 45. kHz | Kilohertz |
| 46. MHz | Megahertz |
| 47. GHz | Gigahertz |
| 48. I | Current |
| $49 . \mathrm{V}$ | Voltage |
| $50 . \mathrm{DoE}$ | Design of Experiment |

## Chapter 1: Introduction

### 3.1.1.Electronic Warfare and Electronic Counter Measures Overview

Electronic Warfare (EW) is an umbrella field of practice for a set of electronic attack and electronic intelligence systems used as target information protection against Radar. Electronic attack systems are usually being referred to as Electronic Counter Measures (ECM) and electronic intelligence systems usually referred to as Electronic Support Measures (ESM). ECM are divided to passive jamming and active jamming, and well-known passive jamming techniques are noise and chaff jamming. Well known active jamming techniques are Range Gate Pull Off (RGPO), Velocity Gate Pull Off (VGPO) and Angle Gate Pull Off (AGPO).

Electronic Counter Measure (ECM) techniques have always been an information technology battle ever since their inception during World War 2 [1]. The outcomes of various information exploitation experiments are vital in protecting both manned and unmanned airborne platforms [1]. The development of jamming waveforms and ECM techniques are based upon data exploitation and circuit analysis of various threats hardware and firmware [1]. Almost all modern tracking Radars use range gates in the digital signal processing (DSP) circuitry to compute range to target [1]. The range gate is an electronic switch which opens after some time delay $t_{d}$ following the pulse transmission path and the gates open for a few milliseconds [1]. If a return pulse above the automatic gain control (AGC) is detected while the switch is open, then the range to target is given by:

$$
R_{T}=t_{d} \frac{c}{2}
$$

Where
$R_{T}=$ Range in Km to the target
$t_{d}=$ Time delay of the range gate

$$
c=\text { speed of light }
$$

A technique called Range Gate Pull Off (RGPO) is used by a stand-off jammer to try and confuse the enemy Radar into thinking the target is at different range [1]. When the jammer detects an enemy's radar signal, the deception jammer then memorizes and retransmits the
signal, but at a stronger signal-to-noise ratio (SNR) than the original radar return. This is depicted in Figure 1.


Figure 1. This portrays general form of jamming, showing parameters at play during the attack
The jammer also has the advantage of one-way transmission which required half the power of the enemy radar's two-way radar transmission. Once the enemy radar is fooled by this technique, then all the jammer must do is "walk" the radar return resulting in the enemy radar tracking a false target [1].


Figure 2. This portrays general form of RGPO, showing parameters at play during a RGPO attack
RGPO is less effective against modern radars whose fast microprocessors can run many DSP algorithms [1]. Nevertheless, RGPO is still in use. Its simplicity makes it an ideal test bed to prove that one can use a GA to derive electronic counter-counter measure (ECCM) techniques given various other parameters [1]. If successful, then GAs will be tried on more sophisticated ECCM techniques [1]. An ECM skin cover pulse is frequently synthesized to be coincident, or as closely coincident as possible, to the true skin return [1]. The primary reason for its use is that it is difficult to control the J/S seen by the radar: if the jamming signal pulses are uniform
and substantially higher power than the true skin return, then an amplitude discriminant may be used by the radar to help identify the skin return [1].

Note that this definition of a cover pulse differs from the one used in the 1995 Benchmark Model, where the cover pulse was a non-coherent pulse that did not compress in the radar receiver [2]. The cover pulse specified here is a replica of the transmitted pulse and will produce a coherent detection in the same detection cell as the skin return. A pulse diversity-based jammer cancellation processor is designed in this literature to cancel the deception electronic counter measures (ECM) signals repeated by a digital radio frequency memory (DRFM) for radar systems [3]. To remove the conventional stationary hypotheses and the rigorous application conditions, an orthogonal transmission block code set is achieved in one pulse repetition interval by exploiting the orthogonal frequency division multiplexing (OFDM) [3]. The processor does not require parameter estimation and compensation across a period of a whole block code [3].

### 2.1.1. Noise Jamming Overview

Shixiang and Xizhang [4] describe noise jamming as the use of randomicity of signal to protect target information. Li, Tai. Wang and Yuan[5] introduce a passive jamming method such as chaff jamming which is commonly used to jam Radar. In general terms chaff jamming doesn't need a special electronic system but uses the reflective properties of metallic pieces to deflect radar and confuse the enemy, the target echo is protected from radar by a cloud of tinny metals released by the target [5]. Noise jamming is a form of passive jamming having two major advantages to EW designs when compared to active jamming techniques, such advantages are:

- Simplified electronic system architecture
- Minimum cost expenditure for EW manufacturing

Xun, Cui and Hao[6] claim that noise frequency modulation (FM) is the main form of noise jamming. Xun, Cui and Hao [6] also concluded this claim after conducting an empirical study on parameters of noise FM selection. Literatures [7, 8, 9, 10, 11] present a combination of studies leading to a single goal of diffusing Synthetic Aperture Radar (SAR) target image information. Hong-xu, Yi-yu, Wen-li and Zhi-tao [7] use a modified time delay jamming to introduce point and line image error to SAR target image. It can be noted in their results that line errors are enhanced in the jammed image when compared to the noise jammed one. It should be also noted that the general quality of the SAR image becomes horrible after being
jammed, but only straight lines which are modified and made thicker with time delay methodology are presented in [7].

As a matter of background, SAR is a coherent airborne or space borne side positioned radar system. In operation SAR is configured in a similar way to that of phased array, in contrast to many parallel antenna elements in phased arrays, SAR uses one antenna in time-multiplex. H. Hong-xu, Yi-yu, Wen-li and Zhi-tao [8] also conducted a related study in the frequency domain. Hong-xu, Yi-yu, Wen-li and Zhi-tao [8] developed a new algorithm named modified shift frequency jamming which introduce noise to the SAR image, the resulting in a blurry image. It is important to note that studies in [7] and [8] are related in a sense that a time delay in the time domain is a frequency shift in the frequency domain, but the application of these two methods achieves totally unrelated results. Lv [9] explore a simulation study for the behaviour of noise convolution jamming for attacking SAR images, the simulation results are comparable with those obtained in [8]. Literatures [12, 13, 14] present a combination of studies leading to a single goal of diffusing ISAR (Inverse Synthetic Aperture Radar) target images information.

Diffusing of pulse Doppler radar is an important research topic in EW engineering practice. Although several studies have been conducted on EW, no satisfactory methods combining pulse Doppler processing and dual-port memory mapping with noise jamming have been developed to outperform pulse Doppler radar. This lack of knowledge has led to the development of active jamming techniques with computational challenges resulting in an expensive electronics system cost.

### 2.1.2. Soft-Core Architecture

As field programmable gate array (FPGA) technology has steadily improved, FPGAs have become viable alternatives to other technology implementations for high-speed classes of digital signal processing (DSP) applications. Radar front-end signal processing, an application formerly dominated by custom very large-scale integration (VLSI) chips, may now be a prime candidate for migration to FPGA technology [15]. As this literature demonstrates, current FPGA devices have the power and capacity to implement a FIR filter with the performance and specifications of an existing, in-system, front-end signal processing custom VLSI chip. A 512tap, 18-bit FIR filter was built that could achieve sample rates of 7 MHz (with a clock rate of 117 MHz ) using Xilinx Virtex FPGA technology and was demonstrated through simulation and hardware implementation.

In literature a Xilinx Virtex 5 FPGA platform-based signal processing algorithm has been developed and experimentally verified for use in a MEMS based tri-mode 77 GHz FMCW automotive radar to determine range and velocity of targets near a host vehicle. The Xilinx Virtex 5 FPGA based signal processing and control algorithm dynamically reconfigures a MEMS based FMCW radar to provide a short, medium, and long-range coverage using the same hardware. The MEMS radar comprises of MEMS SP3T RF switches, micro-fabricated Rotman lens and a micro strip antenna embedded with MEMS SPST switches, in additional to other microelectronic components.

Current radar signal processor architectures lack either performance or flexibility in terms of ease of modification and large design time overheads. Combinations of processors and FPGA are typically hard-wired together into a precisely timed and pipelined solution to achieve a desired level of functionality and performance. Such a fixed processing solution is clearly not feasible for new algorithm evaluation or quick changes during field tests. A more flexible solution based on high-performance soft-core processing is proposed. To develop such a processing architecture, data and signal-flow characteristics of common radar signal processing algorithms are analysed. The proposed architecture achieves high data throughput while providing the flexibility that a software-programmable device offers. The end user can thus write custom radar algorithms in software rather than going through a long and complex HDLbased design. The simplicity of this architecture enables high clock frequencies, deterministic response times, and makes it easy to understand.

In modern radar systems, the architecture of the radar signal processor (RSP) is one of the most important design choices. The amount of useful information that can be extracted from the radar echoes is highly dependent on the computational performance that the RSP can deliver. Current RSPs lack either performance or flexibility in terms of ease of modification and large design time overheads. Combinations of processors and field-programmable gate arrays (FPGAs) are typically hard-wired together into a precisely timed and pipelined solution to achieve a desired level of functionality and performance. To address this gap between performance and flexibility, a custom processor architecture is proposed, the proposed architecture performance has been compared to other manufactures.

### 3.1.2.Research Objectives

Noise jamming is the simplest traditional type of electronic counter measure (ECM) that was used to traditionally attack Radar systems. Improvement in computational intelligence has led to development of modern pulse Doppler radar that outperforms traditional noise jamming. There is a need to upgrade the architecture of noise jamming to a level it can compete with pulse Doppler radar while retaining its simple functionality, and use simple soft-core processing to introduce intelligence to noise jamming.

The work that was undertaken introduced state of the art odd/even order analog-to-digital sampling, soft-core architecture from existing building blocks, and signal-to-noise ratio study of the proposed and existing sampling schemes. The primary major objective of this dissertation is to illustrate how odd/even order analog-to-digital sampling can play a critical role in optimizing and simplifying the received Radar signal in noise jamming architecture. The secondary major objective is to a new ordered analog-to-digital sampling scheme with efficient step-size, dynamic range and sample frequency. The tertiary major objective is a study of the proposed sampling scheme in comparison to existing sampling schemes. If successful, ordered analog-to-digital sampling might lead to optimized electronic warfare (EW) architecture with efficient computational specifications.

### 3.1.3.Synopsis of the Dissertation

Chapter 1 introduces the study. Electronic warfare is introduced and the general problem of electronic attack architecture is also presented. A short summary of noise jamming and softcore architecture using support from evidence from literature, is outlined

Chapter 2 introduces certain aspects of widely used digital building blocks, such as I/Q demodulator, channel equalization, analog-to-digital sampling, amplitude detector, phase detector and frequency detector. Widely used methods and algorithms, such as Kalman Filter, Peak Detection and Modified-Costas are described.

Chapter 3 derives the mathematical model from digital building blocks, factors advanced noise jamming techniques such as modified shift frequency, and convolution noise jamming. It develops requirements for simulation and implementation.

Chapter 4 presents case study 1 with an experimental setup consisting of signal synthesizer producing quadrature signal, quadrature demodulator, oscilloscope and spectral analyser.

Quadrature signal design was conducted on both the signal synthesizer and the spectral analyser.

Chapter 5 presents case study 2 with an experimental setup consisting of Matlab Simulink with a developed delta-sigma model. The delta-sigma model was modified to cater for sparate odd and even order sampling model. Simulink tools such as the signal generator and an oscilloscope were used for signal analysis.

Chapter 6 presents case study 3 with an experimental setup consisting of a MiniZed FPGA development board and a Vivado design suite. The vivado measurement tool such as integrated logic analyser was used for signal analysis.

Chapter 7 presents case study 4 with an experimental setup consisting of a MiniZed development board and a Vivado design suite like case study 3. The vivado measurement tool is also used like case study 3.

Chapter 8 presents a summary of results of the study with recommendations and suggestions.

# Chapter 2: Current Soft-Core Architecture Techniques 

### 2.2. Introduction

This chapter starts by defining the term digitization in the context of this research, and further defines building blocks. Building blocks were largely gathered from literature and these include the I/Q demodulator, channel equalization, digital sampling, phase and frequency detection. Finally, an overview of available sampling schemes and techniques, points such as material and methodologies used and developed, were emphasized.

### 2.3. Digitization Definition

Digitization is the computational process preceding advanced digital techniques such as digital signal processing, Radar signal processing and machine learning. The heart of the computational process is a sampling approximation of the analogue signal through level-based quantization techniques. Digitization is at the centre of 4IR with previously analogue system being digitized through the analog-to-digital converter. For 4IR applications such as 5G Cellular Networks Technology and Cognitive Electronic Warfare (EW) at some point interface to digital through analog-to-digital converter. Efficient use of digital resources such as memory largely depends on the sample design of the analog-to-digital converter. Existing even order sampling has been found to perform better than traditional sampling techniques.

### 2.3.1. Analog-to-Digital Sampling and Soft-Core Architecture Building Blocks

### 2.3.1.1. Analog-to-Digital Sampling and Soft-Core Architecture Building Blocks

Prototype Integrated Circuit (IC) with a goal to achieve low power computation, a final design with specification 190 MHz I/Q demodulator with amplitude gain mismatch of 0.1 dB , phase mismatch of $1.25^{\circ}$ and non-specified frequency mismatch was achieved by the Marzuki, Shakaff and Sauli [16]. Resource utilization specified by the author are 2 mixers, local oscillator (LO) divider, negative resistance circuit, buffer amplifiers and bias circuits [16]. Slyusar and Serduk [17] look at cascading two I/Q demodulators and obtaining peak-frequency characteristic, this design approach allowed for noise immunity on the combined I/Q demodulators. A novel I/Q demodulator to extract amplitude and phase for Continuous Wave
(CW) ultrasonic, the design makes use of phase shift selection [18]. Design implementation and testing were done on AD 8333 and AD 8332 with state-of-the-art $0.8 \mathrm{nV} / \sqrt{\mathrm{Hz}}, 19 \mathrm{~dB}$ plus differential amplifier with +5 V single rail supply.

A digital I/Q demodulator that eliminates digital filters is used to overcome problems with amplitude and phase mismatch in I and Q channels [19] test results showing quantization noise were presented showing the performance of a digital I/Q demodulator. Literature [20] introduces the idea of ultrasound complex signal generation using CORDIC algorithm for transmission and mixing, the system design is for measuring and analysis of forced sinusoidal vibration with frequencies ranging from 100 to 400 KHz .

PCB design following FPGA design process with Xilinx XC3SD3400A Spartan 3A DSP falling under 90 nm silicon technology as the heart of the system was developed [20]. Implementation architecture focused on optimizing resource utilization on the FPGA as they are available at limited quantities [20]. A multi-channel digital I/Q demodulator is proposed in the literature but evidence of its experimental result and performance parameters have not been confirmed. Overall system optimization architecture was presented, and logical architecture design was presented but results are also non-existent [20]. Literature [20] present a laboratory experimental setup but do not explain to us how the multi-channel digital I/Q demodulator is implemented and tested so that we can use it as a benchmark in our proposed study.

An odd number Analogue to Digital Converter is selected as input to the digital I/Q demodulator to help relieve special voltage stress in the receiving segment of a radar systems [21]. Literature [21] presents a new class of I/Q demodulators with odd order derived from the even order I/Q demodulator which is characterized by linear phase-frequency relation for wide band signals. The odd order I/Q demodulator presented in [21] solves the signal syntheses problem. The mathematical representation of the odd order I/Q demodulator are given in equations 1 and 2 .

$$
\begin{align*}
& \mathrm{I}=u_{1}-3 u_{3} \text { where } u_{i} \text { is the ADC samples }  \tag{1}\\
& \mathrm{Q}=-\left(3 u_{2}-u_{4}\right) \text { where } u_{i} \text { is the ADC samples } \tag{2}
\end{align*}
$$

HMIC integrated circuit (IC) with structural components of silicon and glass for RFID design, the HMIC IC is used to create a 90 degree shift I/Q Modulator/Demodulator for binary coding of RFID tags [22]. CMOS 90 nm silicon design using transformer coupling is presented in
literature [23], the I/Q demodulator focus is on a mixer where there is a replacement of cascaded transistors with transformer coupling. Literature [21, 23] present two interesting design approaches to the I/Q demodulator. Literature [21] proposes a selection of odd ADC samples and eliminating even samples. Eliminating odd or even samples means that half the wave data is used without compromising complex signal parameters such as amplitude and phase, elimination of half of the wave data could improve processing requirements such as signal mixing, accumulation, memory and buffering. Literature [23] looks at something specific such as efficient complex signal mixing in the I/Q demodulator, they improve complex signal mixing design by replacing traditional cascaded transistors design with transformer coupling.

Literature [24] presents an IC design for 65 nm CMOS fabrication of 60 GHz I Q modulator/demodulator for short range wireless commutation with a characteristic range of 10 to 100 m . A $60-\mathrm{GHz}$ Direct Conversion I/Q Modulator/Demodulator CMOS architecture is presented with differential signal levels for both I and Q [24].

Table 1. Performance of different fabrication technologies

| Literature | Bandwidth | Technology | Ampl. <br> Imbalance | Phase <br> Imbalance |
| :--- | :--- | :--- | :--- | :--- |
| $[16]$ | 190 MHz | SiGe $0.35 \mu \mathrm{~m}$ <br> BiCMOS | 0.1 dB | $1.25^{\circ}$ |
| $[24]$ | 800 MHz | 65 nm CMOS | 0.7 dB | $4.5^{\circ}$ |
| $[25]$ | 100 MHz | 90 nm CMOS | 0.1 dB | $5^{\circ}$ |
| $[26]$ | 2 GHz | $0.35 \mu \mathrm{~m}$ <br> CMOS | - | $2.8^{\circ}$ |

An I/Q modulator/demodulator with $1.6 \mathrm{~Gb} / \mathrm{s}$ data rate for 16-QAM 90nm CMOS fabrication design is presented [25]; the focus is on an I/Q demodulator design for high data rates on the E-band with a sub-harmonic down-conversion resistive mixer. A complex phase shifter is designed to draw the I and Q channels from the complex signal for operation at 2 GHz , the design was fabricated on $0.35 \mu \mathrm{~m}$ CMOS with phase imbalance of $2.8^{\circ}$ [26].
[27] aims to design a front-end DSP I/Q modulator / demodulator for the IEEE 802.11a WLAN test-bed, resource utilization for both modulator and demodulator is 120K FPGA gates is $2 \%$ usage of Xilinx FPGA XC2V-6000. [28] introduces the use of tandem scheme of two stage Signal Processing for design of an I/Q demodulator, a tandem scheme has a serial number reference from Analogue to Digital Converter (ADC), the quadrature voltage components and weighted signal processing array. [29] improves the performance of an I/Q demodulator by using band pass and low pass filters while considering the approach of constructing a multilayer I/Q demodulator.

Kalman filter with least mean square (LMS) is used as an adaptive channel equalizer, while the LMS and recursive least square (RLS) are used for predicting channel coefficients [30]. Two new channel equalization algorithms are derived from Kalman filter predictive algorithm optimized for inter-symbol interference (ISI), these algorithms are named K-LMS and K-RLS which were studied through a simulation [30]. Phase gradient techniques were used to estimate the aberrant phase from resolution cell in the range-azimuth, phase correction was designed [31].

Authors in literature [32] present an IC design that implements a compensation technique for amplitude and phase imbalance, the heart of the compensation technique is a self-aligned tuner optimized for digital modulation. The self-aligned tuner is a mixed signal IC that automatically corrects the amplitude and phase and a simulation model was used to test the performance of the tuner. The simulation results showed an amplitude correction from 1 dB to 0.052 dB and phase correction of $5^{\circ}$ to $0.57^{\circ}$ [32].

Lu [33] evaluates impacts of amplitude and phase imbalances on quadrature phase shift key modulation, channel model simulation is used as a tool to evaluate the impacts some of the channel activities which were modelled were Inter-Channel Interference and I/Q cross-talk power ratio. Simulation model results in [33] showed that for 1 dB amplitude and $5^{\circ}$ imbalance the Bit Error Rate (BER) remained below 0.5 dB .

Introduction Body Area Networks (BAM) where performance is traded over lower power consumption [34], a simulation model that defines minimally required receiver specifications the model could also be used in gaining insight into demodulator power behaviour. Literature [35] looks at analogue signal imbalances between the In-phase and Quadrature branches of the I/Q modulator/demodulator and the authors develop a compensation technique for amplitude and phase. The compensation technique focusses on resolving impairments due to mixer and
filter imbalances. Carrier frequency offset estimation for I/Q imbalance is studied in [36], leaner least mean square estimation provided high accuracy estimation with lower complexity when it comes to implementation; on the other hand, the Kalman filter demonstrated good channel time variations tracking.

### 2.3.1.2. I/Q Digital Sampling

In Electronic Warfare (EW) mixed signal processing methods have become normal processing techniques where analogue Radar is sampled to digital using ADC. Pulse Doppler radar techniques such as Frequency Modulated Continuous Wave (FMCW) and Linear Frequency Modulation (FM) with variant frequency introduces the problem of digital formation of I/Q signal in the EW segment [21].

During digital formation process several factors such as sample frequency, the number of bits produced and mean squared error distortion between input and digitally reconstructed signal are evident. A novel even order sampling I/Q demodulator date back as far as [37], and was later followed by the development of odd order sampling I/Q demodulator as recently as 2015 [21]. The odd order I/Q demodulator which is derived from an even order I/Q demodulator, is characterized by linear phase/frequency relation. Such a signal attributes help relieve complexities in digital formation such as samples produced per second and distortion error of for wide bandwidth I/Q signals [21, 37].

Moshkin, Nikolaev and Nikitin [38] study the complexity of the dynamic range related to the number of bits produced through quantization resolution while exploring phase. Several measurements error performances with different resolutions were conducted phase, 1-bit quantization showed the worst phase error performance. To reduce the number of bits produced by the ADC, dynamic range quantization encoder digital architecture design is presented in [39]. Number of bits produced introduce a sample storage capacity complexity, sample storage capacity is addressed by the design of sample and hold circuit with the purpose to extend the input signal range which allows for higher input voltages to be tolerated [40], [41] is also addressing similar issue but using the Multisim simulation tool.

Advances in solutions that address digital formation have led to the development of design and development of CMOS technology prototypes [42, 43, 44, 45, 46, 47]. To optimize the dynamic range Ordentlich, Tabak, Hanumolu, Singer, and Wornell [48] introduces the idea of modulo ADC which reduces the dynamic range of the input signal, this technique allows 5 volts signal to be processed as 2.5 volts for a modulo 2 and processed as 1.5 volts for a modulo

4 implementations. A second order delta-sigma implementation is introduced in [49]; the block architecture is like that of first order delta-sigma design with a difference of a second order integrator before the quantization step. A high-performance DAC approximated ADC architecture for Synthetic Aperture Radar (SAR), the only architecture advance is the switch mode which allows the switch between positive and negative quantitation [50, 51].

To address dynamic range quantization inefficiencies a low bit error encoder is designed and tested using CMOS technology [52]. The different approach to the design that addresses dynamic range resolution, [53] aims to increase the dynamic range by using the advantage of capacitance charging capabilities. The capacitance acts as a sample and hold stage after the quantizer thus allowing them to maintain high the resolution digital signal. [54] achieve the same designs but at low power. Different digital conversion technologies were evaluated towards the design of a PID controller [55]. For digital control the ADC stage is very important and maintaining a high-quality control feedback signal in most cases is vital. Just like in [53] and [54] in SAR imagery high resolution signal as always, the target and [56] is trying to achieve the same objective with $1 \mathrm{bit} /$ multi-level quantization stage. A 3 -stage neural network nonlinear ADC was designed and implemented in Matlab Simulink in [57], 16-bit quantization high resolution was demonstrated in the simulation results. A design like [57] is presented in [58] but it is not clear whether it's linear or non-linear. In EW processing the ADC stage is usually followed by enveloped detector stage [59] a mathematical model coherent envelope detector for ambient backscatter is presented. Dynamic range quantization bit resolution complexities are corrected by using the envelope detector stage [60]. Glow plasma is used to detect electromagnetic wave, full digital RF chain is also implemented [61].

### 2.3.1.3. Envelope Detector

After the envelope detector a phase or frequency detector stage usually follows. Phase detector with feed forward noise cancellation architecture, which uses sub-sampling for phase noise detection [62]. To implement an XOR phase detector an FPGA architecture was design and implemented in [63]. Digital phase locked loop for Internet-of-Things application is presented [64]. Similar to [64] a sampling phase detector with ultra-low phase noise is design for the microwave synthesizer's applications [65]. Simple Bang Band phase detector with 1-bit thirdorder single feedback loop delta-sigma ADC pre-processing stage for FMCW wave synthesis [66]. Similar to [65] a pulse phase-frequency detector for high-speed frequency synthesizers
applications [67]. A capacitive power sensor is used to design a large dynamic range phase detector [68].

The authors in this article [69] designed an empirical estimation algorithm that takes advantage of prominent amplitude peaks and uses interpolation (iterative search of maxima) to extract the envelope from the peaks and the algorithm was tested on audio waveforms and the results were positive [69]. It is commonly agreed that the envelope is a signal that varies slowly and pass the prominent peaks of data smoothly and in classical perspectives. Fourier spectrum is a critical indicator for evaluating the varying rate of an envelope [69]. The most intuitionistic view on the envelope estimation problem is to draw a curve that outlines the signal contour and passes through the main peaks [69]. A peak detection algorithm combined with moving window was proposed in [70], the proposed empirical algorithm then uses a low pass filter to smooth the quantization noise produced while generating envelope. In simple terms the algorithm first eliminates the negative part of the receives digital signal by taking an absolute value of all the individual samples, the proposed empirical algorithm then defines a processing window and fit enough samples [70]. From each processing window a maximum sample is taken and marked as peak, and all sample elements in the window are replaced with the peak [70], the final processing stage is the filtering which smoothens the envelope.

Autoregressive power density (AR-PSD) was used for homomorphic filtering to achieve Frequency Alterable Homomorphic Filtering (FAHF) proposed in [71]. The proposed FAHF is applied to multichannel cardiovascular field, the performance of the proposed algorithm is compared to Normalized Shannon Energy (NASE) and the Hilbert Transform (HT) algorithm, and the results showed that FAHF produced smoother envelope curves [71]. Experimental setup proposed in [71] consisted of traditional stethoscopes, digital stethoscopes, computer, audio amplifier, BIOPAC physiological signal recorder and four auscultation chest pieces. Butterworth band pass filter and threshold shrinkage denoising which included normalizing the signal by setting the variance of the signal to 1.0 , these processing efforts were considered as a pre-processing stage [71]. Authors in [71] also discovered during the experimental stage that the proposed FAHF algorithm can adaptively denoise multiplicative noise. A new approach of envelope extraction algorithm based on wavelet for cardiac sound signal segmentation, the new approach is based on the Morlet wavelet to extract the energy envelope of heart sound [72]. The sound data was captured by $16-$ bit ADC sampling at 1 MHz , a normalizer shown in equation (3) was used as the secondary pre-processing stage following the Analogue to Digital Converter stage [72].

$$
\begin{equation*}
x_{\text {norm }}(t)=\left(\frac{x(t)}{\max (x(t)}\right)^{2}, \text { where } x \text { is the } t^{t h} \text { sample } \tag{3}
\end{equation*}
$$

Wavelet analysis uses time-frequency localization features to extract dilation and translation versions, the requirement of the wavelet must be oscillatory, have amplitudes that quickly decay to zero and have at least one vanishing moment to be analysed [72]. The idea of spectral envelope in proposed in [73], the proposed spectral envelope is applied to pitch shifting and envelope preservation. To realize spectral envelope, algorithms such as linear prediction (LPC) or real cepstrum are applied [73]. Discrete cepstrum and the true envelope are subject matters which were studied towards the implementation of the spectral envelope in [73], spectral envelope can be considered a smoother version amplitude spectrum. A tangentially constrained spline for envelope estimation without the undershoots problem, that can be made optimally smooth by the proposed tangential points optimization in [74]. The quadratic spline is constrained using first derivatives, and it was applied to two numerical experiments [74]. It goes through basic theories of processing envelope of sound waveforms such as analogue to digital converter, digital signal processing, windowing and spectral analysis [75]. Linear prediction coding (LPC), cepstrum and discrete cepstrum as the study subject matter for determining spectral envelope were used [75], and discrete cepstrum was improved in such a way it penalizes too that are too steep slopes of the spectral envelope by a regularization term.

### 2.3.1.4. Phase Detector

A novel fast phase detection digital phase locked loop is studied in literature [76], this is achieved by generating an analytic signal through the implementation of Hillbert Transform and 16-bit CORDIC algorithm is used for all complex mathematical implementation. Two platforms were used for developing and testing the digital phase detector, the model was built on MATLAB and then further tested on an FPGA [76]. The proposed All-Digital Phase Looked Loop (ADPLL) presented in literature [76] is a fully digital unit with every component digitized allowing a full implementation in an FPGA, this digital design is presented in Figure 3 below. The difference between ADPLL compared to conventional Digital PLL is that the latter has a charge pump which is analogue by design which translates phase difference into corresponding voltage [76]. $\mathrm{F}(\mathrm{z})$ is a digital filter that eliminates noise from the input signal $\mathrm{x}(\mathrm{n})$ and $\mathrm{Ha}(\mathrm{z})$ is a digital filter that does corrections on poles that introduces instability at $30^{\circ}, 90^{\circ}$ and $150^{\circ}$.


Figure 3.This portrays DSP stages for the ADPLL as presented in literature [76]
Literature [77] presents two digital phase detector algorithms that the Modified Costas Phase Detector and the CORDIC Phase Detector. The implementation of $\tan ^{-1}\left(\frac{Q[n]}{I[n]}\right)$ is replaceable by its mathematical equivalent $\sin ^{-1}\left(\frac{Q[n]}{\sqrt{\left(Q[n]^{2}+\left[[n]^{2}\right)\right.}}\right)$, considering small input phase $\sin ^{-1}\left(\frac{Q[n]}{\sqrt{\left(Q[n]^{2}+I[n]^{2}\right)}}\right)$ can be approximated by an expression $\frac{Q[n]}{\sqrt{\left(Q[n]^{2}+I[n]^{2}\right)}}$ [77]. The mathematical expression $\sqrt{\left(Q[n]^{2}+I[n]^{2}\right)}$ is the square root of the input signal power and can be removed using Automatic Gain Control (AGC), thus the Costas Phase Detector can be expressed as $\epsilon[n]=Q[n]$ [77]. The limitation of the Costas Phase Detector is that it has negative slope in the left-half of the I-Q plane, to rectify this limitation the sign of $I[n]$ is added as a correction measure making the final form of the Modified Costas Detector to be $\epsilon[n]=$ $\operatorname{sgn}(I[n]) Q[n][77]$. A $180^{\circ}$ phase-shift digital delay locked loop (DLL) study towards the design of LPDDR4 memory controllers is presented in literature [78]. Fast-locking is used as an automatic shutdown procedure to reduce power consumption [78].

A 1906 gates chip fabricated with TSMC 55 nm CMOS technology Digital Phase and Frequency Detector (DPFD), the adaptation approach is applied in the fractional feedback loop to achieve highly accurate target frequency of digitally control oscillator (DCO) [79].


Figure 4.DPFD DSP block architecture [79]
DPFD is a two stage DSP blocks with the first stage being the phase detector $V[n]$ and the second stage being the frequency detector, the DSP equation is shown in equation 4 below.

$$
\begin{equation*}
Y[n]=2 \mu *\left(x[n]-x[n] / z^{2}-2 K / z\right) \tag{4}
\end{equation*}
$$

Table 2. Simplified Computational Requirements for DPFD [79]

| Logic Slices | Quantity |
| :--- | :--- |
| Flip Flop | 6 |
| Multiplexer | 6 |
| LUT | 11 |

### 2.3.2. An Overview of Analog-To-Digital Sampling Schemes and Techniques Used in This Study

The analog-to-digital converter is the heart of digitization and it is the main requirement in 4IR applications. Odd/even order sampling was studied, using step-size signal-noise-ratio (SNR), dynamic range and sampling frequency as performance parameters. Even order sampling has been applied to multiple signal sampling while achieving SNR of 64 dBFS [37]. With the rise for the need of digitization over the years other ordered sampling techniques for multiple signal sampling have been introduced [21]. In an attempt to maintain received signal power, complexities in dynamic range resolution in relation to quantization and phase errors were studied [38]. The methodology to address complexities in dynamic range resolution was
developed; it used experimental measurements of MSE on different quantization resolutions to analyze bit stream performance. The results produced indicated that the 1-bit quantization resolution has the worst phase MSE performance. To optimize digital resolution of analog-todigital converter a quantization encoder architecture is designed [39]. Emerging attempts in addressing optimization of dynamic range resolution, has led to development of the modulo analog-to-digital converter [48] . Traditional delta-sigma sampling technique is modified in an attempt to reduce signal crosstalk in multi-channel digital converters [49]. Holistically the experiments in this dissertation were conducted using simulation tools, laboratory equipment and implementation as detailed in the points below:

Materials used in this study include:

- Laboratory equipment, Excel, simulation tool and FPGA implementation.
- Laboratory equipment such as an oscilloscope, spectrum analyzer, Radar signal synthesizer and Excel were used as preliminary efforts to develop research assumptions to a feasible study. A four channel Tektronix oscilloscope with maximum resolution of 10 million sampling points, and maximum operating frequency of 1 GHz was used to capture a sample for preliminary processing with Excel. The Tektronix spectrum analyzer with maximum operating frequency of 26 GHz was used to evaluate the quadrature signal in the time and frequency domain. The Anritsu signal generator with maximum operating frequency of 20 GHz was used to generate Radar signal. Simulation was conducted on Matlab Simulink and the delta-sigma model in [14] was used and modified to cater for the odd/even order sampling architecture. An ADL5380-EVALZ-ND I/Q demodulator was used for quadrature demodulator. Xilinx Vivado was for FPGA firmware design with MiniZed being the FPGA of choice.

Methods include:

- High level research assumptions were that odd/even order sampling will reduce total sample number and signal frequency without affecting dynamic range.
- An I/Q sine signal was designed on the Anritsu signal generator and was supplied as input to an I/Q demodulator.
- Outputs of the I/Q demodulator were connected to two channels of the oscilloscope.
- The two channels of the oscilloscope were captured and the signal was stored as comma-separated values (CSV) files for further analysis on Excel results that were plotted using Excel Figures.

Methods developed include:

- Derived mathematical model used for to process the CSV captured sine signal from the oscilloscope.
- Simulation on Matlab Simulink was used investigate the performance of step size while evaluating the effect on dynamic range.
- $1^{\text {st }}$ order delta-sigma model was used in all simulation investigations and all results were plotted using Matlab Figures.
- Four sampling schemes were implemented on Xilinx Vivado, one being the proposed novel odd/even order sampling scheme and the other three were from literature.
- Performance of these schemes were captured and exported to CSV using the Integrated Logic Analyzer (ILA) from Vivado.
- CSV were further processed for SNR and results were plotted using Excel Figures.


### 2.4. Summary

In this chapter an attempt was made to develop a block overview of functional requirements for noise jamming architecture. Processing techniques were studied and presented in an attempt to migrate the analogue noise jamming technique. The literature study indicate an I/Q demodulator would be suitable as the initial processing block and followed by channel equalization. Both these pre-processing stages are mainly analogue while sampling, phase and frequency detector are digital processing blocks.

# Chapter 3: Proposed Ordered Analog-ToDigital Sampling and Noise Sample Mixing <br> <br> Overview 

 <br> <br> Overview}

### 3.2. Introduction

This chapter develops a mathematical model for odd/even order sampling from the simple form of I/Q modulator. The mathematical model caters for both quadrature signals and memory usage with strobe used to control the switch between odd/even or even/odd order sampling. Digital building blocks from previous chapter are factored further in the mathematical model. Advanced Noise jamming strategies such as Modified Shift Frequency Noise jamming, and Convolution Noise jamming were also factored into the mathematical model.

### 3.3. Derivation Odd/Even Ordered Analog-To-Digital Sampling and Noise Sample Mixing

The design of soft-core noise jamming architecture depends largely on the I/Q demodulator and Analogue to Digital Converter (ADC). Traditionally the design of I/Q demodulator ADC interface for Radar and Electronic Warfare (REW) applications follows five different options as depicted in [37].

### 3.3.1. Traditional I/Q Demodulator and Existing Even Order Sampling

Eklund and Arvidsson [37] modify the traditional I/Q demodulator and ADC interface to include an odd/even selection switch and digital filter to smooth the digital ADC output [37], this modification kicks off by declaring the complex signal and I and Q as shown in equations 1, 2 and 3 below.

$$
\begin{align*}
S(t)=a_{M}(t) & * \cos \left(w_{I F} * t+\varphi_{M}(t)\right) \\
& =I_{\text {raw }}(t) * \cos \left(w_{I F} * t\right)-Q_{\text {raw }}(t) * \sin \left(w_{I F} * t\right) \tag{1}
\end{align*}
$$

Table 3. Variable description for equation 1

| Variable | Description |
| :--- | :--- |
| $\mathrm{s}(\mathrm{t})$ | Complex signal |
| $\mathrm{I}(\mathrm{t})$ | Inphase component |
| $\mathrm{Q}(\mathrm{t})$ | Quadrature <br> component |
| $\mathrm{W}_{\text {IF }}$ | Intermediate Freq |

$$
\begin{equation*}
I_{\text {raw }}(t)=a_{M}(t) * \cos \left(\varphi_{M}(t)\right) \tag{2}
\end{equation*}
$$

$$
\begin{equation*}
Q_{\text {raw }}(t)=a_{M}(t) * \sin \left(\varphi_{M}(t)\right) \tag{3}
\end{equation*}
$$

Table 4. Variable description for equations 2 and 3

| Variable | Description |
| :---: | :---: |
| $I_{\text {raw }}(\mathrm{t})$ | Demodulated I component |
| $Q_{r a w}(\mathrm{t})$ | Demodulated Q component |
| $a_{m}(\mathrm{t})$ | Demodulated amplitude |
| $\varphi_{\mathrm{m}}(\mathrm{t})$ | A |

Considering a firmware, I/ Q demodulator model with I and Q modelled by equations 2 and 3 the mixing operation will yield the results shown in Table 5. When no filtering is applied it should be noted that all the mixing coefficients results will have 1 as the maximum and -1 as the minimum.

Table 5. I/Q demodulator even 7th order digital mixing

| Sample no: (n) | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I-coefficients | 1 | 0 | -1 | 0 | 1 | 0 | -1 | 0 |
| Q-coefficients $^{1}$ | 0 | 1 | 0 | -1 | 0 | 1 | 0 | -1 |

Eklund and Arvidsson [37] and Slyusar [21] applied an FIR filter to the digital firmware mixer to obtain the final form of the presented I/Q digital mixer. It should be noted that [21] implements a 4 order FIR filter instead of a 7 order FIR filter.

Table 6. I/Q demodulator FIR filtered even 7th order digital mixing

| Sample no: (n) | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I-coefficients | 1 | 0 | -11 | 0 | 15 | 0 | -5 | 0 |
| Q-coefficients ${ }^{1}$ | 0 | 5 | 0 | -15 | 0 | 11 | 0 | -1 |

Eklund and Arvidsson et al [37] and Slyusar et al [21] applied an FIR filter to the digital firmware mixer to obtain the final form of the presented I/Q digital mixer, it should be noted that [21] implements a 3 order FIR filter instead of a 7 order FIR filter. Slyusar [21] also introduces us to the idea of odd order sampling which is developed from even order sampling in [37], although limited theory derivation of odd order sampling he could apply it to single channel ADC I/Q sampling which uses 3rd order FIR filtering.

$$
\begin{align*}
& I_{\text {even }}=\left.\cos \left(w_{I F} *\left(T_{0}+\Delta\right)+\varphi_{I F}\right)\right) * C_{1}-\cos \left(w_{I F} *\left(T_{2}+\Delta\right)+\varphi_{I F}\right) * C_{11} \\
&+\cos \left(w_{I F} *\right.\left.\left(T_{4}+\Delta\right)+\varphi_{I F}\right) * C_{15}-\cos \left(w_{I F} *\left(T_{6}+\Delta\right)+\varphi_{I F}\right) \\
& * C_{5}  \tag{4}\\
& Q_{\text {odd }}=\left.\sin \left(w_{I F} *\left(T_{1}+\Delta\right)+\varphi_{I F}\right)\right) * C_{5}-\sin \left(w_{I F} *\left(T_{3}+\Delta\right)+\varphi_{I F}\right) * C_{15} \\
&+\cos \left(w_{I F} *\right.\left.\left(T_{5}+\Delta\right)+\varphi_{I F}\right) * C_{11}-\cos \left(w_{I F} *\left(T_{7}+\Delta\right)+\varphi_{I F}\right) \\
& * C_{1} \tag{5}
\end{align*}
$$

Table 7. Variable description for equation 4 and 5

| Variable | Description |
| :---: | :---: |
| $I_{\text {odd }}(\mathrm{t})$ | Odd sampled I component |
| $Q_{\text {even }}(\mathrm{t})$ | Even sampled Q component |
| $\mathrm{T}_{1,3}(\mathrm{t})$ | Odd sampling time |
| $\mathrm{C}_{l, 3}$ | FIR coefficients |
| $\mathrm{T}_{2,4}(\mathrm{t})$ | Even sampling time |
| $\Delta$ | Quantization error |

### 3.3.2. Extension of Existing Odd Order Sampling

Before we dive down to the methodology and odd/even sampling which considers of memory usage, we need to develop the theory of odd sampling further by deriving the odd sampling expressions. When the 3 rd order FIR filter in Table 4 is applied to equation 4 and 5 we get the derived equations 6 and 7 as presented below.

Table 8. I/Q demodulator odd 4 th order digital mixing

| Sample no: (n) | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| I-coefficients | 0 | 1 | 0 | -1 | 0 |
| Q-coefficients ${ }^{1}$ | 0 | 0 | -1 | 0 | 1 |

$$
\begin{align*}
& \left.I_{\text {odd }}=\cos \left(w_{I F} *\left(T_{1}+\Delta\right)+\varphi_{I F}\right)\right) * C_{1}-\cos \left(w_{I F} *\left(T_{3}+\Delta\right)+\varphi_{I F}\right) \\
&  \tag{6}\\
& \quad * C_{3} \\
& \left.Q_{\text {even }}=-\sin \left(w_{I F} *\left(T_{2}+\Delta\right)+\varphi_{I F}\right)\right) * C_{3}+\sin \left(w_{I F} *\left(T_{4}+\Delta\right)+\varphi_{I F}\right)  \tag{7}\\
& \\
& \quad * C_{1}
\end{align*}
$$

The I/Q modulator of the odd order presented in [21] did not clearly derive the odd order 3rd odder characterization equation presented in the paper and equations 8 and 9 below. The derivations from 4 to 9 pave the way for the proposed odd/even algorithm that considers of memory.

$$
\begin{gather*}
\left.\mathrm{I}=u_{1}-3 u_{3} \text { where } u_{1}=\cos \left(w_{I F} *\left(T_{1}+\Delta\right)+\varphi_{I F}\right)\right) * C_{1} \text { is the ADC samples }  \tag{8}\\
\left.\mathrm{Q}=-\left(3 u_{2}-u_{4}\right) \text { where } u_{2}=\cos \left(w_{I F} *\left(T_{2}+\Delta\right)+\varphi_{I F}\right)\right) * C_{3} \text { is the ADC samples } \tag{9}
\end{gather*}
$$

Table 9. 4th order FIR filtering I/Q demodulator odd order digital mixing

| Sample no: (n) | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ |
| :---: | :--- | :--- | :---: | :---: | :---: |
| I-coefficients | 0 | 1 | 0 | -3 | 0 |
| Q-coefficients ${ }^{1}$ | 0 | 0 | -3 | 0 | 1 |

To recap the literature sources which introduced the idea of even and odd order sampling were studied and characterization equations.

### 3.3.3. Derivation of Odd/Even Order Sampling

Both the even and odd order sampling algorithms do even and odd selection before the ADC changes the wave to digital making the previous algorithms implement pre-mixing. The issue with mixing instead of sample selection is that mixing makes a filter stage a requirement after
its implementation; the other issue with mixing is that mixers are usually hardware components which are not easy to digitize even in FPGA's internal mixing is done with internal PLL. It should be noted that previous approaches do not consider even and odd sample management.

The proposed methodology aims to achieve the following:

- "Elimination of the odd and even hardware mixing stages which introduce filtering complexities"
- "Introduction of flexibility that allows a single I/Q demodulator to select between even and odd"
- "Introduction of the use of fast memory at rate of 4 to 5 Megabits/s to the design of I/Q demodulators and to manage the flexibility between even and odd"

Let's consider the design of the I/Q demodulator that flexibly selects between even and odd modes at will. Further consideration is that, signal complications must also be assumed as nonexistent. This assumption is valid as the I/Q hardware has improved drastically over the years. This improvement in I/Q demodulator hardware has reached a level where such devices can be bought with operational specifications such as amplitude and phase imbalance of 0.07 dB and $0.2^{\circ}$ respectively.

Let's also consider the dual port ram modelling term $d R M_{w r_{s}}^{r d^{s}}$ where $r d^{s}$ is the read memory command, $w r_{s}$ is the write memory command and $s$ is the strobe command that selects between even or odd order. Let's further consider two terms that directly tune ADC sampling, such terms are odd_even_adc_tuning and even_odd_adc_tuning as shown in Figure 5 below.


Figure 5. The proposed ordered sampling model
Table 10. I/Q demodulator odd 7 th order digital mixing

| Sample no: <br> (n) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| I-coeff | 0 | 1 | 0 | -1 | 0 | 1 | 0 | -1 |
| Q-coeff $^{1}$ | 0 | 0 | -1 | 0 | 1 | 0 | 1 | 0 |

Since the proposed algorithm excludes the signal filtering stage, the filter phase $\Delta$ and coefficients $C_{n}$ terms in equation 4 to 7 fall away. When we apply the ADC tuning terms which are controlled by the memory strobe the form presented in equation 4 becomes the new equation 10 below:

$$
\left.\begin{array}{l}
I_{\text {odd }}=\{(\text { odd_even_adc_sampling } \\
\quad=1) \mid\left\{\cos \left(w_{I F} * T_{1}+\varphi_{I F}\right)-\cos \left(w_{I F} * T_{3}+\varphi_{I F}\right)\right. \\
\left.\left.\quad+\cos \left(w_{I F} * T_{5}+\varphi_{I F}\right)-\cos \left(w_{I F} * T_{7}+\varphi_{I F}\right)\right\}\right\}
\end{array}\right\}
$$

It should be noted that the algorithm presented in equations 10 and 11 selected odd or even sampling by using odd_even_adc_tuning which is controlled by the dual port memory.

$$
\left.\begin{array}{l}
Q_{\text {even }}=\{(\text { odd_even_adc_sampling } \\
\quad=1) \mid\left\{\sin \left(w_{I F} * T_{2}+\varphi_{I F}\right)-\sin \left(w_{I F} * T_{4}+\varphi_{I F}\right)\right. \\
\\
\left.\left.\quad+\sin \left(w_{I F} * T_{6}+\varphi_{I F}\right)\right\}\right\} \tag{13}
\end{array}\right\}
$$

Equations 10, 11, 12 and 13 built up the derivation to the final form of the proposed odd order sampling with memory consideration. The full form of the proposed odd order sampling is presented in equation 14. The full form of this algorithm selects odd sampling by using $s$ which controls the ADC sampling and dual port memory write location.

$$
\begin{align*}
& d R M_{w r_{s}}^{r d^{s}}=\sum_{s_{i}=0}^{N}\left\{\left\{\left(w r_{s}=1 \text { And } s=00\right) ?\right\} \mid\left\{\left\{\left(s_{i} \operatorname{Mod} 2\right.\right.\right.\right. \\
&\left.\left.\left.=1) ? \mid \sum_{i_{s}=0}^{255} I_{\text {odd }}\left(S_{i}\right),\right\} \mid\left\{\left(s_{i} \operatorname{Mod} 2=0\right) ? \mid \sum_{q_{s}=265}^{520} Q_{\text {even }}\left(S_{i}\right),\right\}\right\}\right\} \tag{14}
\end{align*}
$$

Table 11. Odd order sampling proposed memory mapping

| Input Addr | Dual-Port Mem | Output Addr |
| :---: | :---: | :---: |
| $0 \times 00$ | $\mathrm{I}(0)$ | $0 \times 00$ |
| $\ldots$ | $\ldots$ | $\ldots$ |
| $0 \times \mathrm{FF}$ | $\mathrm{I}(\mathrm{i})$ | $0 \times \mathrm{FF}$ |
| Reserved to guard data leak + 2-bit strobe |  |  |
| $0 \times 109$ | $\mathrm{Q}(0)$ | $0 \times 109$ |
| $\ldots$ | $\ldots$ | $\ldots$ |
| $0 \times 208$ | $\mathrm{Q}(\mathrm{i})$ | $0 \times 208$ |

Assuming sampling window of N samples as shown in equation 13 , to store such a window a 2-bit strobe shown in Table 12 is used to tune the ADC in such a way that either the combination $I_{\text {odd }}, Q_{\text {even }}$ or $I_{\text {even }}, Q_{\text {odd }}$ as shown in Tables 11 and 14 . The memory is mapped in such a way that 8 bits addressing is achieved, furthermore the memory data leak has been
accounted for by introducing guard gaps. 8 bits addressing, storage of $I_{\text {odd }}$ and $Q_{\text {even }}$ for odd order sampling and guard gaps are presented in Table 11.

Table 12. Bit arrangement at memory located 0x104

| Bit No | 0 |  | 1 | 2 | 3 | 4 | 5 | 6 |  | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bit Function |  | Strobe |  | Reserved | Reserved | Reserved | Reserved |  | Flag |  |

$$
\begin{align*}
& I_{\text {even }}=\left\{(\text { odd_even_adc_sampling }=0) \mid\left\{\cos \left(w_{I F} * T_{0}+\varphi_{I F}\right)-\cos \left(w_{I F} * T_{2}+\varphi_{I F}\right)\right.\right. \\
& \left.\left.+\cos \left(w_{I F} * T_{4}+\varphi_{I F}\right)-\cos \left(w_{I F} * T_{6}+\varphi_{I F}\right)\right\}\right\}  \tag{15}\\
& I_{\text {even }}=\sum_{S_{i}=0}^{N}\left\{\left(o d d_{-} \text {even_adc_sampling }=0\right) \mid I_{\text {raw }}\left(S_{i}\right)\right\}  \tag{16}\\
& Q_{\text {odd }}=\left\{(\text { even_odd_adc_sampling }=0) \mid\left\{\sin \left(w_{I F} * T_{1}+\varphi_{I F}\right)-\sin \left(w_{I F} * T_{3}+\varphi_{I F}\right)\right.\right. \\
& \left.\left.+\sin \left(w_{I F} * T_{5}+\varphi_{I F}\right)-\sin \left(w_{I F} * T_{7}+\varphi_{I F}\right)\right\}\right\}  \tag{17}\\
& Q_{o d d}=\sum_{S_{i}=0}^{N}\left\{(\text { odd_even_adc_sampling }=0) \mid Q_{\text {raw }}\left(S_{i}\right)\right\}  \tag{18}\\
& d R M_{w r_{s}}^{r d^{s}}=\sum_{s_{i}=0}^{N}\left\{\left\{\left(w r_{s}=1 \text { And } s=01\right) ?\right\} \mid\left\{\left\{\left(s_{i} \operatorname{Mod} 2\right.\right.\right.\right. \\
& \left.\left.\left.=0) ? \mid \sum_{i_{s}=274}^{871} I_{\text {even }}\left(S_{i}\right),\right\} \mid\left\{\left(s_{i} \operatorname{Mod} 2=1\right) ? \mid \sum_{q_{s}=3}^{258} Q_{o d d}\left(S_{i}\right),\right\}\right\}\right\} \tag{19}
\end{align*}
$$

Table 13. Variable description for equations 14 and 19

| Variable | Description |
| :---: | :---: |
| $d R M$ | Dual-port memory |
| $w r_{s}$ | Memory write |
| $r d^{s}$ | Memory read |
| $S_{i}$ | Sample number |
| $s$ | Memory strobe |

### 3.3.4. Derivation of Odd/Even Order Sampling Memory Management and Envelope Detection

Equations 15, 16, 17 and 18 built up the derivation to the final form of the proposed even order sampling with memory consideration. The full form of the proposed even order sampling is presented in equation 19. The full form of this algorithm selects odd sampling by using $s$ which controls the ADC sampling and dual port memory write location.

Table 14. Even order sampling proposed memory mapping.

| Input Addr | Dual-Port Mem | Output Addr |
| :---: | :---: | :---: |
| $0 \times 03$ | $\mathrm{Q}(0)$ | $0 \times 03$ |
| $\ldots$ | $\ldots$ | $\ldots$ |
| $0 \times 102$ | Q (i) | $0 \times 102$ |
| Reserved to guard data leak + 2-bit strobe |  |  |
| $0 \times 112$ | $\mathrm{I}(0)$ | $0 \times 112$ |
| $\ldots$ | $\ldots$ | $\ldots$ |
| $0 \times 367$ | I (i) | $0 \times 367$ |

8 bits addressing, storage of $I_{\text {even }}$ and $Q_{\text {odd }}$ for even order sampling and guard gaps are presented in in Table 14. Dual-port RAM memory has unique properties that allow the stored data to be read in consecutive clock cycles. This means theoretically an extremely fast processing throughput can be achieved. The proposed envelope detection algorithm in [70] divides the signal $S(t)$ to into $k$ bunches of $N$ samples.

$$
\begin{align*}
& I_{o d d}=\sum_{c l k=2}^{130} \sum_{a d d r=0 \times 00}^{0 x 78}\left\{d R M_{w r_{s}=0}^{r d^{s}=1}(a d d r)\right\}  \tag{20}\\
& Q_{\text {even }}=\sum_{c l k=3}^{131} \sum_{\text {addr }=0 \times 633}^{0 x 75 B}\left\{d R M_{w r_{s}=0}^{r d^{s}=1}(\text { addr })\right\}  \tag{21}\\
& S_{256}=\sum_{i=1}^{256} S(i)=\sum_{i=1}^{128} I_{o d d}(i)+\sum_{i=1}^{128} Q_{\text {even }}(i)  \tag{22}\\
& S_{256}=\sum_{c l k=2}^{130} \sum_{a d d r=0 \times 00}^{0 x 78}\left\{d R M_{w r_{s}=0}^{r d^{s}=1}(a d d r)\right\}+\sum_{c l k=3}^{131} \sum_{a d d r=0 x 633}^{0 x 75 B}\left\{d R M_{w r_{s}=0}^{r s^{s}=1}(a d d r)\right\} \tag{23}
\end{align*}
$$

Equations 20, 21 and 22 built up the derivation to the final form of the proposed memory read algorithm. The full form of the memory read algorithm is presented in equation 23. The first
step in [70] takes the absolute value of $S(t)$, since we have modified $S(t)$ to 256 samples then we have taken the absolute value of $S_{256}$ as shown in equations 24 and 25 .

$$
\begin{align*}
& \left|S_{256}\right|=\sum_{i=1}^{256}|S(i)|=\sum_{i=1}^{128}\left|I_{o d d}(i)\right|+\sum_{i=1}^{128}\left|Q_{\text {even }}(i)\right|  \tag{24}\\
& \left|S_{256}\right|=\sum_{c l k=2}^{130} \sum_{a d d r=0 \times 00}^{0 x 78}\left|d R M_{w r_{s}=0}^{r s^{s}=1}(a d d r)\right|+\sum_{c l k=3}^{131} \sum_{a d d r=0 \times 633}^{0 x 75 B}\left|d R M_{w r_{s}=0}^{r d^{s}=1}(a d d r)\right| \tag{25}
\end{align*}
$$

The second step in [70] divides the signal $S(t)$ into $k$ bunches of $N$ samples, since we have modified $S(t)$ to 256 samples then we have divided $S_{256}$ in $k_{m}$ bunches of 256 samples. $k_{m}$ is a memory address with equivalent size to $k$ and the full $k_{m}$ memory sorting algorithm as presented in equations 26 and 27.

$$
\begin{equation*}
\left|S_{256}\right|=\sum_{i=1}^{k}|S(i)|=\sum_{i=1}^{k / 2}\left|I_{o d d}(i)\right|+\sum_{i=1}^{k / 2}\left|Q_{\text {even }}(i)\right| \tag{26}
\end{equation*}
$$

$\left|S_{256}\right|=\sum_{c l k=2}^{(k+2) / 2} \sum_{\text {addr }=0 \times 00}^{k_{m}}\left|d R M_{w r_{s}=0}^{r d^{s}=1}(a d d r)\right|+\sum_{c l k=3}^{(k+3) / 2} \sum_{\text {addr }=0 \times 633}^{k_{m}}\left|d R M_{w r_{s}=0}^{r d^{s}=1}(a d d r)\right|$
The third and the last step in [70] checks for maximum peaks and modifies each element in the $k_{m}$ set to the maximum peak as presented in equations 28 and 29.

$$
\begin{align*}
& I_{e n v}=\sum_{c l k=2}^{(k+2) / 2} \sum_{a d d r=0 x 00}^{k_{m}}\left\{\mid\left(d R M_{w r_{s}=0}^{r d^{s}=1}(a d d r) \mid\right.\right. \\
& \\
& \left.>\left|d R M_{w r_{s}=0}^{r d^{s}=1}(a d d r-1)\right|\right) ? \mid\left\{\left|d R M_{w r_{s}=1}^{r d^{s}=0}(a d d r-1)\right|\right.  \tag{28}\\
& \\
& \left.\left.=\left|d R M_{w r_{s}=0}^{r d^{s}=1}(a d d r)\right|\right\}\right\}
\end{aligned} \begin{aligned}
& Q_{e n v}=\sum_{c l k=3}^{(k+3) / 2} \sum_{a d d r=0 x 633}^{k_{m}}\left\{\mid\left(d R M_{w r_{s}=0}^{r d^{s}=1}(a d d r) \mid\right.\right. \\
&\left.>\left|d R M_{w r_{s}=0}^{r d^{s}=1}(a d d r-1)\right|\right) ? \mid\left\{\left|d R M_{w r_{s}=1}^{r d^{s}=0}(a d d r-1)\right|\right.  \tag{29}\\
&\left.\left.=\left|d R M_{w r_{s}=0}^{r d^{s}=1}(a d d r)\right|\right\}\right\}
\end{align*}
$$

Table 15. Variable description for equations 28 and 29

| Variable | Description |
| :---: | :---: |
| $a d d r$ | Dual-port memory address |
| $k$ | Peak detection bunch size |
| $K_{m}$ | Peak detection $\mathrm{m}^{\text {th }}$ element |
| $I_{e n v}$ | I component of envelope |
| $Q_{e n v}$ | Q component of envelope |

The odd/even sampling on I/Q signals. The signal $S(t)$ when applied to an I/Q demodulator signals $S_{I_{-} \text {odd }}$ and $S_{Q_{-} \text {even }}$ are produced. The introduction of $S_{I_{-} \text {odd }}(t)$ and $S_{Q_{-} \text {even }}(t)$ allow for $k$ bunches to be taken from $N$ samples and odd/even sampling reduces a $N$ I/Q to $N / 2$ I/Q signal samples. The memory reading procedure allows for both odd and even sample data to be read, from clock cycle 2 to 257 odd sample data is read as presented in equation 18 and 19, at 269 to 525 even sample data is read as presented in equations 20 to 21 . The memory reading procedure allows for both odd and even sample data to be read, from clock cycle 2 to 257 odd sample data is read as present.

### 3.3.5. Application of Modified Costas to Memory Samples to Achieve Angle Detection

Literature [77] in presents the idea of modified the Costas phase detector given by equation 30 with phase detector gain given by equation 31, the modified Costas phase detector can be improved to phase detect odd/even sampled I/Q signals stored in memory as shown in equation 32 and 33. The power term $\sqrt{P_{\text {in }}}$ in equation 34 can be replaced by the envelope $I_{e n v}$ or $Q_{e n v}$ to get equation 35 .

$$
\begin{align*}
& \epsilon[n] \quad=\operatorname{sgn}(I[n]) * Q[n]  \tag{30}\\
& k_{D}=\frac{\sqrt{P_{i n}} \sin (\epsilon[n])}{\epsilon[n]}  \tag{31}\\
& \epsilon_{128}=\sum_{i=1}^{128} \operatorname{sgn}\left(I_{o d d}(i)\right) * \sum_{i=1}^{128} Q_{\text {even }}(i)  \tag{32}\\
& \epsilon_{128}=\sum_{c l k=2}^{130} \sum_{a d d r=0 \times 00}^{0 x 78} \operatorname{sgn}\left(d R M_{w r_{s}=0}^{r d^{s}=1}(a d d r)\right) * \sum_{c l k=3}^{131} \sum_{a d d r=0 x 633}^{0 x 75 B} d R M_{w r_{s}=0}^{r d^{s}=1}(a d d r) \tag{33}
\end{align*}
$$

Equations 30, 31 and 32 built up the derivation to the final form of the proposed modified Costas phase detector algorithm with memory considerations. The full form of the modified phase detector algorithm with memory read is presented in equation 33 .

$$
\begin{align*}
& K_{D_{128}}=\sum_{i=1}^{128} \frac{\sqrt{P_{i n}(i)} \sin \left(\epsilon_{128}(i)\right)}{\epsilon_{128}}  \tag{34}\\
& K_{D_{128}}=\sum_{i=1}^{128} \frac{I_{e n v}(\mathrm{i}) * \sin \left(\epsilon_{128}(i)\right)}{\epsilon_{128}} \tag{35}
\end{align*}
$$

Subsection 3.3.3 introduced the idea of odd/even sampling on I/Q signals. The signal $S(t)$ when applied to an I/Q demodulator signals $S_{I_{-} \text {odd }}$ and $S_{Q_{-} \text {even }}$ are produced. The introduction of $S_{I_{-} \text {odd }}(t)$ and $S_{Q_{-} \text {even }}(t)$ allow for $k$ bunches to be taken from $N$ samples and odd/even sampling reduces a $N$ I/Q to $N / 2$ I/Q signal samples. The memory reading procedure allows for both odd and even sample data to be read, from clock cycle 2 to 257 odd sample data is read as presented in equation 18 and 19, at 269 to 515 even sample data is read as presented in equation 20 to 21 . The memory reading procedure allows for both odd and even sample data to be read, from clock cycle 2 to 257 odd sample data is read as present.

### 3.3.6. Application of Discrete Fourier Transforms(DFT) to Achieve Frequency Detection

The Discrete Fourier Transforms (DFT) a sequence of N complex samples in time domain $\boldsymbol{x}_{\boldsymbol{n}}=x_{0}, x_{1}, \ldots \ldots \ldots, x_{N-1}$ into another sequence of complex samples in frequency domain $\boldsymbol{X}_{\boldsymbol{k}}=X_{0}, X_{1}, \ldots \ldots \ldots, X_{N-1}$ as defined in equation 36.

$$
\begin{equation*}
X_{k}=\sum_{n=0}^{N-1} x_{n} * e^{-\frac{i 2 \pi}{N} k n}=\sum_{n=0}^{N-1}\left[\cos \left(\frac{2 \pi k n}{N}\right)-i * \sin \left(\frac{2 \pi k n}{N}\right)\right] \tag{36}
\end{equation*}
$$

Our goal is to conduct the spectral analysis of $S_{256}$ dual-port memory stored complex signal, then $x_{n}$ becomes the signal of interest which is $S_{256}$. Equation 37 describe the updated form of equation 36 :

$$
\begin{equation*}
X_{k}=\sum_{n=1}^{256} S_{256} * e^{-\frac{i 2 \pi}{256} k n}=\sum_{n=1}^{256}\left[\cos \left(\frac{2 \pi k n}{256}\right)-i * \sin \left(\frac{2 \pi k n}{256}\right)\right] \tag{37}
\end{equation*}
$$

$$
\begin{equation*}
X_{k}=\sum_{n=1}^{256} S_{256} * e^{-\frac{i 2 \pi}{256} k n}=\sum_{n=1}^{128} I_{\text {odd }} * e^{-\frac{i 2 \pi}{128} k n}+\sum_{n=1}^{128} Q_{\text {even }} * e^{-\frac{i 2 \pi}{128} k n} \tag{38}
\end{equation*}
$$

Equations 36, 37 and 38 built up the derivation to the final form of the proposed modified Discrete Fourier Transform (DFT) algorithm with memory considerations. The full form of the modified DFT algorithm with memory read is presented in equation 39.

$$
\begin{align*}
& X_{k}=\sum_{c l k=2}^{130} \sum_{a d d r=0 \times 00}^{0 x 78} \sum_{n=1}^{128}\left\{d R M_{w r_{s}=0}^{r d^{s}=1}(\text { addr }) * e^{-\frac{i 2 \pi}{128} k n}\right\} \\
&+\sum_{c l k=3}^{131} \sum_{a d d r=0 \times 633}^{0 x 75 B} \sum_{n=1}^{128}\left\{d R M_{w r_{s}=0}^{r d^{s}=1}(a d d r) * e^{-\frac{i 2 \pi}{128} k n}\right\} \tag{39}
\end{align*}
$$

### 3.3.7. Derivation of Noise Sampling Mixing through Modified Time Delay

A modified time delay presented by Xun, Cui and Hao [7] is concerned with time delay echo with inter-pulse jitter. Their research shows that time-delay echo jamming can only generate false information if it retains the signal inter-intra phase characteristics. The transmitted by a Linear Frequency Modulation (LFM) Radar can be shown by equation 1, where $a_{M}(t)$ is signal amplitude which is best described by the envelope detector equation 28 and $29 . w_{I F}$ is the frequency modulation frequency and $\varphi_{M}(t)$ is the inter-intra phase. The time-delay echo jammed signal is given by equation 40 :

$$
\begin{align*}
S(t) & =a_{M}\left(t-\tau_{d}\right) * \cos \left(w_{I F} *\left(t-\tau_{d}\right)+\varphi_{M}(t)\right)  \tag{40}\\
\tau_{d}(t) & =\tau_{d 0}+\tau_{\xi}(t) \tag{41}
\end{align*}
$$

The time-delay factor $\tau_{d}$ varies from pulse to pulse and the expression is given by equation 41 , where $\tau_{d 0}$ is a constant time delay which should not be less than the minimum echo time delay of the jammer. $\tau_{\xi}(t)$ is a random variable within a fast time range signed as $\left[\tau_{1}, \tau_{2}\right]$, if T is the transmitting pulse width then literature [7] has a constraint of $\tau_{2}-\tau_{1}<T$.

$$
\begin{equation*}
\tau_{a d d r}(t)=\tau_{a d d r 0}+\tau_{\xi}(t) \tag{42}
\end{equation*}
$$

$$
\begin{align*}
S_{256}\left(\tau_{d}\right)= & \sum_{c l k=2}^{130} \sum_{a d d r=0 \times 00}^{0 x 78}\left\{d R M_{w r_{s}=0}^{r r^{s}=1}\left(a d d r+\tau_{a d d r}\right)\right\} \\
& +\sum_{c l k=3}^{131} \sum_{a d d r=0 \times 633}^{0 \times 75 B}\left\{d R M_{w r_{s}=0}^{r d^{s}=1}\left(a d d r+\tau_{a d d r}\right)\right\} \tag{43}
\end{align*}
$$

Sample time information is locked on memory address, to make $\tau_{d}$ for memory stored discrete time samples then $\tau_{d}$ must be modified to memory delay $\tau_{\text {addr }} . \tau_{\xi}(t)$ is a modified random memory access address within range signed as $\left[\tau_{\text {addr1 } 1}, \tau_{a d d r 2}\right]$; if $T_{\text {addr }}$ is the stored pulse width and the modified operating constraint of $\tau_{a d d r 2}-\tau_{a d d r 1}<T_{a d d r}$. The final form for block-ram memory access model for noise generator obtained by modified time delay is presented in equation 43.

### 3.3.8. Derivation of Noise Sampling Mixing through Modified Shift Frequency

It is well known that shift frequency jamming derived from signal frequency characteristic is effective against Linear Frequency Modulation (LFM) Radar [8]. Few research outputs discuss the mathematical model that interface shift frequency with cyclic dual-port memory in pulsedoppler Radar, for this interface to be possible it important to modify the modified shift frequency equations in such a way 256 sample memory stored LFM Radar waveform can be used. The signal transmitted by a Linear Frequency Modulation (LFM) Radar can be shown by equation 1 , where $a_{M}(t)=1$ as signal amplitude, $w_{I F}$ as the frequency modulation frequency and $\varphi_{M}(t)$ is the inter-intra phase. The shift frequency jammed signal is given by equation 44:

$$
\begin{equation*}
s(t)=\cos \left(\left(w_{I F}+w_{d}\right) * t+\varphi_{M}(t)\right) \tag{44}
\end{equation*}
$$

To effectively modify the shift frequency mathematical model, it is necessary to transform equation 44 to frequency domain using the DFT introduced in equation 36 and when applied to equation 44 we get equation 45 below:

$$
\begin{align*}
& S_{k}=\sum_{n=0}^{N-1} s_{n} * e^{-\frac{i 2 \pi}{N} k n}  \tag{45}\\
& w_{d}=\xi * B_{r} \tag{46}
\end{align*}
$$

Sample frequency information is locked as DFT of memory address lock time information, to make $w_{d}$ for memory stored discrete frequency samples then $w_{d}$ must be modified to memory shift $w_{a d d r} . \xi$ is a modified random memory access address within range signed as $[a, b]$, if
$B_{r}$ is the LFM Radar bandwidth. The equation 47 for $w_{a d d r}$ has the sampled LFM Radar bandwidth $M_{r}$. The final form for block-ram memory access model for noise generator obtained by modified shift frequency is presented in equation 48.

$$
\begin{align*}
& w_{a d d r}=\xi * M_{r}  \tag{47}\\
& S_{k}=\sum_{c l k=2}^{130} \sum_{a d d r=0}^{0 x 78} \sum_{n=1}^{128}\left\{d R M_{w r_{s}=0}^{r d^{s}=1}\left(a d d r+w_{a d d r}\right) * e^{-\frac{i 2 \pi}{128} k n}\right\} \\
&  \tag{48}\\
& \quad+\sum_{c l k=3}^{131} \sum_{a d d r=0 x 633}^{0 x 75 B} \sum_{n=1}^{128}\left\{d R M_{w r_{s}=0}^{r d^{s}=1}\left(a d d r+w_{a d d r}\right) * e^{-\frac{i 2 \pi}{128^{2}} k n}\right\}
\end{align*}
$$

### 3.3.9. Derivation of Noise Sampling Mixing through Convolution

Lv [9] claims noise convolution jamming or smart noise jamming technique, this technique receive part of the LFM Radar process gains and this makes superior to existing noise blanketing techniques. The main advantage of noise convolution jamming is that it reduces jamming energy and improve jamming power utility greatly [9]. In operation noise convolution selects the initial and terminal time to control the position and width of jamming region to suppress distributed target [9]. J. Jiang, Y. Wu, H. Wang et al [80] reintroduces the ideas of RF noise jamming, noise AM jamming, noise FM jamming and noise convolution for jamming of ground targets. The results show that noise convolution jamming out performs other noncoherent jamming techniques [80]. Barrage noise jamming is by far the most important type of jamming for airborne phase array Radar [81]. [82]Few research outputs discuss the mathematical model that interface noise convolution with cyclic dual-port memory in pulsedoppler Radar. In operation scenario the jammer receives the radar's emission signal after time delay $\tau=R_{j}(s) / c$ [9]. $R_{j}(s)$ is the slant range between jammer and radar, it's a function of azimuth slow time $s$; while $t$ is the range fast time and $c=3 * 10^{8} \mathrm{~m} / \mathrm{s}$ is the velocity of propagation of electromagnetic waves in vacuum [9]. When jammer receives radar's emission signal it applies convolution with noise, afterwards amplifies and retransmits the jammed radar signal [9]

$$
\begin{equation*}
S_{c o n v}=f(t) * S(t) \tag{49}
\end{equation*}
$$

Where $S(t)$ in equation 49 is the radar signal that the jammer received, the radar is eventually odd/even sampled and get stored to dual-port cyclic memory as $S_{256}$. Equation 49 becomes equation 50 after ADC and memory stage.

$$
\begin{equation*}
S_{\text {conv }_{256}}=f(t) * S_{256} \tag{50}
\end{equation*}
$$

To conduct convolution on 256 sample radar wave it is necessary to understand the concept of single pulse convolution given by equation 51 in time domain and equation 52 in discrete time domain. Equation 51 is an impulse with amplitude $A_{1}$ and time delay $t_{1}$ in discrete time the discrete time delay become $i_{1}$. When jammer discovers radar signal it convolves the impulse with radar signal to get equation 53:

$$
\begin{align*}
& p(t)=A_{1} \delta\left(t-t_{1}\right)  \tag{51}\\
& p=\sum_{i=1}^{N} A_{1} \delta\left(i-i_{1}\right)  \tag{52}\\
& S_{\text {impulse }}=p(t) * S_{\text {conv }_{256}}=\sum_{i=1}^{246} A_{1} S_{256}\left(i-i_{1}\right) \tag{53}
\end{align*}
$$

Let suppose that the noise used for convolution is defined in equation 54,
$f(t)=U_{n}(t) e^{j \phi(t)}$
The noise envelope is defined by $U_{n}$ and it obeys various noise model distributions such as Rayleigh, Gaussian and uniform distributions, $\phi(t)$ is the phase function. The noise signal $f(t)$ has no deterministic expression and it is impossible to conduct mathematical derivation, but we can regard it as composed of finite number of pulses with different amplitudes and delays. We need to limit the pulse delay time in equation 51 and 52 , this is to set limit to pulse position according equation 55. [83, 84, 85, 86, 87] use noise models such as Rayleigh, Gaussian and uniform distribution to model the behaviour of digital systems in excessive noise environment. [88, 89] simulates and statistical analysis of Radio communication and Radar systems with radio noise on Radio communication systems, on Radar system convolution modulation.

$$
\begin{equation*}
0 \leq t_{1} \leq\left(2\left[R_{\max }-R_{j}(s)\right]\right) /{ }_{c} \tag{55}
\end{equation*}
$$

Where $R_{j}$ is the space between the jammer and Radar while $R_{\max }$ is the maximum range of the Radar. Solving for equation 53 and 54 we get equation 56 below with 56 being the final form for noise convolution algorithm:

$$
\begin{equation*}
S_{\text {impulse }}=\sum_{i=1}^{246} A_{1} S_{256}\left(i-i_{1}\right)=\sum_{i=1}^{256} U_{256}(i) e^{j \phi(i)} * S_{256}\left(i-i_{1}\right) \tag{56}
\end{equation*}
$$

### 3.4. Simulation and Experimental Requirements for Ordered Analog-to-Digital Sampling

EW components which are I/Q Demodulator, Channel Equalizer, Envelope Detector, Phase Detector and Frequency Detector as envisioned in the architecture breakdown. Derivation of the model on previous section covered each of these components, these are common EW building blocks and Soft-Core Architecture. Through derivation of the model a numerical issue of memory data rate constraint was deduced. Literature and numerical investigation on I/Q demodulators led to the development of odd/even order sampling which only stores into memory odd samples from the I channel and even samples from the Q channel. The proposed architecture has a challenging memory requirement of 8 to $10 \mathrm{Mb} / \mathrm{s}$ as depicted in Table 18 . With the development of odd/even order sampling an FPGA with 4 to $5 \mathrm{Mb} / \mathrm{s}$ is an organic choice for performance specification requirement. Although odd/even order sampling has been applied for digital communication applications, no satisfactory methods combining odd/even order sampling with soft-core noise jamming architecture have been developed so far.

Envelope, phase and frequency detectors plus their literature fall under the sub-field of Digital Signal Processing (DSP). DSP mainly uses number crunching to implement equations and algorithms. Different equations and empirical algorithms to calculate envelope and phase of a complex signal were explored in the previous section, different strategies were explored and all of them as an algorithm. Discrete Time Fourier Transform (DTFT) has become the main DSP algorithm for converting discrete time sample to frequency domain. The specifications for most DSP equations and algorithms cannot be easily determined as the selected FPGA becomes a constraint to the processing requirements of any DSP algorithms, and the selection of FPGA for this study largely depends on the data rate for memory shown in Table 18.

Table 16. Component category form or purpose matrix for the proposed soft-core architecture with each component allocated to a form or purpose.

| Soft-Core Functionality | Firmware | Hardware | Measuring |
| :--- | :---: | :---: | :---: |
| Signal Synthesis |  |  | X |
| Demodulation |  | X |  |
| Digitization | X |  |  |
| Parameter Storage | X |  |  |
| Envelope Extraction | X |  |  |
| Phase Extraction | X | X |  |
| Freq Extraction | X |  |  |
| Noise Synthesis | X |  | X |
| Test \& Evaluation |  |  |  |

Table 17. Firmware components category matrix allocates the firmware components to either Signal Conditioning, Digital Signal Processing or Radar \& Electronic Warfare Signal Processing

| Firmware Component | Implementation | Detailed Design Functionality |
| :---: | :---: | :---: |
| Parameter Storage | Signal Conditioning | Odd samples of I channel are selected and store. Even samples of Q channel are selected. |
| Envelope(Env) | Digital Signal Processing | Recovers the pulse radar envelope for processing. |
|  |  | This processing step allows |
| Extraction |  | for the radar pulse shape to be reproduced, the beginning and end of the |

$\left.\begin{array}{lll}\hline & \begin{array}{l}\text { pulse to be marked. This } \\ \text { processing step also allows }\end{array} \\ & \text { for transmitted radar power } \\ \text { to be measured, reproduced }\end{array}\right]$

Different equations and empirical algorithms to calculate noise generation, frequency detection and noise FM to produce complex noise jamming signal were explored in section A, different strategies were explored and all of them are either an equation or an algorithm. Three different strategies for noise generation which are modified time delay, modified shift frequency and convolution were discovered during noise jamming background review. The specifications for RSP equations and algorithms cannot be easily determined as the selected FPGA becomes a constraint to the processing requirements of any RSP algorithms, and the selection of FPGA for this study largely depends on the data rate for memory shown in Table 16, 17 and 18.

| Hardware Component | Implementation | Specification Requirements |
| :---: | :---: | :---: |
| Demodulation | Signal Conditioning | Freq Range $400 \mathrm{MHz}-6$ <br> GHz , Amplitude Imbalance <br> 0.07 dB , Phase Imbalance <br> $0.2^{\circ}$, Demodulation <br> bandwidth 390 MHz |
| Digitization | Digital Signal Processing | Freq Range $1-5 \mathrm{GHz}$ with -3 dB at 9 GHz , Resolution 12Bit |
| Env, Phase, Freq Extraction | Radar \& EW Signal Processing | Memory Data Rate 8-10 $\mathrm{Mb} / \mathrm{s}(\mathrm{min}), 1-5 \mathrm{GHz}$ Analog-to-Digital Converter, <br> Integrated Logic Analyzer |
| Test \& Evaluation | Radar \& EW Signal Processing | (ILA) Data Rate $8-10$ <br> $\mathrm{Mb} / \mathrm{s}(\mathrm{min})$,  4 Mb <br> window(max)   |

### 3.5. Summary

A mathematical model was developed using digital building blocks from previous chapter and quadrature modulator equations. Advanced Noise jamming strategies such as Modified Shift Frequency Noise jamming, and Convolution Noise jamming were also factored into the mathematical model. Hardware and software requirements were developed for simulation and implementation purposes.

# Chapter 4: Case Study 1: Experimental Verification of Sample Frequency 

### 4.1. Introduction

This Chapter applies the derived mathematical model components derived in the previous section on potential hardware. Equipment such as signal generator was used to generate test signals. Oscilloscope was used to capture and digitize the test signals, spectral analyser was used to design and adjust signal parameters such as modulation angle, frequency and phase. The signal was analysed by applying the equations in Chapter 4 on Microsoft Excel to determine preliminary results.

### 4.2. Overview of Experimental Setup

This experimental setup is centred around generation of I/Q demodulator signals as the system under investigation. The experimental equipment available to conduct this experiment are presented in Table 19.

Table 19. Measurement equipment available for experimental setup

| Experimental Equipment | Equipment Range |
| :---: | :---: |
| Anritsu Signal Generator | 1 Hz to 20 GHz |
| Tektronix Oscilloscope | 1 Hz to 1 GHz |
| Tektronix Spectral Analyzer | 1 Hz to 26 GHz |

Flexible waveguides were selected for interconnection between the signal source, system under investigation and measurement equipment. The procured I/Q demodulator with dual p and n channels was used throughout measurement efforts. It is important to emphasize that the procured I/Q demodulator was supplied without n channel connected on the printed circuit board (PCB). The I/Q demodulator connection schedule is given in Table 20.

Table 20. ADL5380-EVALZ-ND I/Q Demodulator Electrical Connection Schedule

| Connector Schedule | Electrical Schedule |
| :---: | :---: |
| RF_p | SMA to Waveguide from Signal Gen |
| RF_n | No SMA connector PCB terminated |
| I_p | SMA to Waveguide to Oscilloscope |
| I_n | No SMA connector PCB terminated |
| Q_p | SMA to Waveguide to Oscilloscope |
| Q_n | No SMA connector PCB terminated |
| V_cc | Croc Clips to Power Supply |
| Ground | Croc Clips to Power Supply |

A proposed experimental setup to investigate research assumption for the new odd/even order sampling of I/Q signals was conducted in the laboratory environment. Experiment setup to acquire I/Q signals consists of Tektronix Oscilloscope with integrated spectral analyser and 12bit analog-to-digital converter, 5 GHz local oscillator (LO) driven by voltage-controlled oscillator (VCO) which can be configured to generate 1 Hz to 5 GHz and Anritsu Signal Generator. Anritsu signal generator was used as the signal source with $5 \Omega$ characteristic load and maximum signal power design to 0.0 dBm . The source was configured to produce a phase modulated complex signal with I and Q separated by a constant phase shift. To verify the design of a complex, signal the Tektronix spectral analyser was used in complex function. Other analysis such as spectrogram to confirm percentage pulse overlap between I and Q was also conducted and overlap of $19 \%$ was verified as shown in Table 21.

Table 21. Spectrogram measurements results

| Measurements |  | Settings |  |
| :---: | :---: | :---: | :---: |
| $\Delta$ Overlap | Freq | Time/Div | Span |
| $19 \%$ | 1.004649 GHz |  |  |
|  |  | 450 us | 24.40 MHz |

Table 22. High-end wave front frequency spectrum measurements results.

| Measurements |  | Settings |  |
| :---: | :---: | :---: | :---: |
| Amplitude | Freq | Time/Div | Span |
| -72.56 dBm | 1.004649 GHz |  |  |
|  |  | 10 dB | 24.40 MHz |

From the spectrogram window it was possible to place the marker at the signal wave front and to able to measure the high-end signal cut off in frequency domain as shown in measurement results in Table 22.

### 4.3. Experiment One - Collection of Signal Data

The next step in the experimental setup was to pass the confirmed complex signal to the I/Q demodulator hardware to recover complex I and Q components. I/Q demodulator operates at 5 volts and power supply was set to 5 volts and it was confirmed to draw 200 mA of current. Preliminary results on the performance of the procured I/Q demodulator are given in Figure 6.


Figure 6. This presents oscilloscope output performance of I/Q demodulator with I component connected to Channel I and Q connected to Channel 2.

A complex signal from the signal generator was passed through I/Q demodulator, and outputs of the I/Q demodulator were passed to the oscilloscope for analysis. Analysis results reveal that Q component lags I component by $63.90^{\circ}$ as shown in Table 23 and Figure 6.

Table 23. I/Q demodulator performance measurements results

|  | Measurements |  | Settings |  |
| :---: | :---: | :---: | :---: | :---: |
| Ch1 | Ch2 | Ch1 | Ch2 |  |
| Amplitude(P-P) | 4.92 V | 874 mV | $2 \mathrm{~V} / \mathrm{div}$ | $200 \mathrm{mV} / \mathrm{div}$ |
| Phase | $63.90^{\circ}$ | $0.00^{\circ}$ | $4 \mathrm{~ns} / \mathrm{div}$ | $4 \mathrm{~ns} / \mathrm{div}$ |

### 4.4. Experiment Two - Analysis of Signal Data

The initial step in approaching the analysis of the captured signal wave data is to verify that the exported CSV signal data is valid. The signal wave data acquired in Figure 6 was reproduced in Excel; the phase and amplitude of Excel reproduced data aligns with the one presented in Figure 8, Excel reproduced signal data is presented in Figure 7. It is important to note that the reproduced signal is limited to 256 samples, this is to ease processing on Excel.


Figure 7. This presents Excel reproduced acquired I/Q demodulator samples

The new odd/even order sampling mathematical model in equations 12 and 13 is applied afterward Fourier Transform Analysis was applied to obtain the sample frequency behaviour shown in Figure 8. Results in Figure 8 validates the research assumption that the new odd/even order sampling is half the sampling frequency.


Figure 8. This presents new odd/even order sample frequency behaviour

A separate investigation onto effects of the new odd/even sampling onto phase behaviour have indicated that it suffers from phase error with I sampling component affected more than the Q sampling component as shown in Figure 9. It is not clear what causes this behaviour but literature [1] have recommended the use of FIR filter to correct this issue.


Figure 9. This presents phase error behavior of the new odd/even order sampling

Peak detect algorithm derived in equations 28 and 29 was applied to waveform data obtained in the previous section. According to the proposed new algorithm for envelope detection with memory considerations the sample radar waveform is normalized to only positive values and
then segmented into K segments. All other elements in the K segments are replaced by the maximum peak to give the envelope in Figure 10. The product can be made smoother by introducing a low pass filters but this remains out of scope for this work.

Envelope Detector vs Sample Number


Figure 10. Envelope generated by the new peak detection envelope detector with memory consideration, in this test scenario $K=3$

Modified Costas phase detector derived in equations 32 and 33 was also applied to the same waveform data in Figure 6. The proposed new modified Costas phase detector with memory considerations, in sample manner calculates reliable phase error with limited processing power as shown in Figure 11.

Phase Detector vs Sample Number


Figure 11. Phase error generated by the new modified Costas phase detector with memory consideration
The next step in the experimental setup was to pass the confirmed complex signal to the I/Q demodulator to recover I and Q .

### 4.5. Summary

In this chapter the results of two experiments on signal data collection and analysis of signal data were presented and discussed. The first experiment was to develop a method of collecting digitized signal data. The second experiment used the collected signal data for processing further. Spectral results for experiment two has shown that sample frequency is reduced.

# Chapter 5: Case Study 2: Experimental Evaluation of Step-Size and Dynamic 

## Range

### 5.1. Introduction

This Chapter develops simulation model of odd/even order sampling. The model is used to evaluate the performance of state-of-the-art sampling method to traditional sampling methods. Analog-to-digital sampling parameters such as step-size and dynamic range were investigated.

### 5.2. Overview of Experimental Setup

In chapter 3 the mathematical model for odd/even order sampling with internal tuning of sampling in such a way a selection between odd and even sampling is achieved with ease. In this section a simulation model that aims to compare performance of the odd/even order sampling model to 1 st order delta-sigma model is outlined.

## Sigma-Detta AID Conversion



Figure 12. Simulink experimental setup design for $1^{\text {st }}$ order delta-sigma model

The delta-sigma model used in the proposed simulation experiment consists of a negative feedback loop (delta part of the design), first order integrator (sigma or summation part of the design), 1-bit quantization (first stage for digital formation), zero-order-hold (sampling in discrete-time as second stage digital formation), three stages 4th order digital filtering to eliminate sampling noise (digital filtering as the third and final stage of the ADC model).

### 5.3. Experiment One - Analysis of Step-Size Behavior

Simulation results for delta-sigma sampling are shown in Figures 13 and 15 with quantization step size clearly defined. Quantization step size measurement attributes for the same deltasigma sampling are given in Table 24. These results in Table 24 are as expected as they indicate that the sampling frequency is reduced from 8 kHz in delta-sigma sampling to 2 kHz in the new odd/even ordered sampling.


Figure 13. Simulated odd/even order sampling

Figure 13 above portrays the phase performance between 1st order delta-sigma and odd/even order sampling. The phase difference between 1st order delta-sigma and odd/even sampling was created on design by the transport delay distinguish between the signals and should be ignored for analysis. 1st delta-sigma is indicated by the yellow, odd order sampling by red and even order sampling by black. It should be noted there is ( $\mathrm{n}-1$ ) phase lag between even order sampling (black) and odd order sampling (red).

Ordered sampling adapted $4^{\text {th }}$ order designed in Figure 14, the design retains the negative feedback, first order integrator, 1 - bit quantization and decimation. Ordered samples selector is introduced as shown in Figure 14.


Figure 14. Simulink experimental setup design for ordered sampling derived from $1^{\text {st }}$ order delta-sigma model.

Table 24. 1st order delta-sigma and proposed ordered simulation measurements results.
$\left.\begin{array}{llll}\hline \begin{array}{ll}\text { Quantization } \\ \text { Step } \\ \text { Parameter }\end{array} & \text { Sim Results }(\Delta-\Sigma) & \text { Sim } & \text { Results } \\ \text { (Proposed) }\end{array}\right]$


Figure 15. This portrays the phase lag performance between odd and even order sampling. Odd order sampling marked by black and even order sampling by red. It should be noted the (n-1) phase lag between even order sampling (red) and odd order sampling (black) varies per period interval

### 5.4. Experiment Two - Analysis of Dynamic Range Behaviour

During the simulation investigation we also undertake mean square error (MSE) performance evaluation of the dynamic range for the proposed new odd/even ordered sampling. MSE performance between odd and even components of the ordered sampling was the system under investigation with 1st order delta-sigma being assumed to be the input signal with minimum distortion. To confirm the effects of ordered sampling on dynamic range distortion the even ordered sampling was made to be 4th order while the odd ordered sampling was made to be 1st order as shown in Table 25.

Table 25. Dynamic range behaviour simulation experimental setup parameters, with the order of even ordered sampling significantly increased in comparison with odd ordered sampling

| Ordered | Order |
| :---: | :---: |
| Sampling | $1^{\text {st }}$ |
| Odd | $4^{\text {th }}$ |

The results indicate that the dynamic range distortion increases by 0.1 Volts/ms for every increasing order as shown in MSE results in Figure 16 below.


Figure 16. Ordered sampling dynamic range investigation results with proposed new odd/even ordered sampling intercomponent investigation undertaken in Simulink

### 5.5. Summary

In this chapter the results of two experiments on analysis of step-size behaviour and analysis of dynamic range behaviour were presented and discussed. The first experiment was to investigate step-size quantization. The second experiment was to investigate dynamic range. Results have shown that step-size quantization error increases with increasing order as shown in Table 16, while the dynamic range is not significantly affected.

# Chapter 6: Case Study 3: Signal-To-Noise Ratio (SNR) Verification of Superiority of Ordered Analog-To-Digital Sampling 

### 6.1. Introduction

This chapter proposes new soft-core architecture for odd/even order sampling with memory considerations shown in Figure 17. The architecture uses digital signal processing (DSP) slices under the intelligent Processing sub block. Block-RAM store signal data, Registers interface with Scalar Processing sub block and Process Containers contain logic to configure both BlockRAM and Register configuration. Axi bus access is primary self-controlled but state transitions such as write, read and enable are externally controlled by Process Controllers. The derived equations 12 and 13 are implemented onto a DSP and signal data is loaded through the axi bus from Block-RAM. Different signal source data selection that switches between odd or even order through memory strobe and any other interesting application procedure is software programmed through the Scalar Processing sub-block. This section will implement the proposed architecture in Figure 17 and Table 26 and compare results to simulation for practical performance evaluation.


Figure 17. This presents the proposed new odd/even order sampling architecture block diagram

Table 26. Proposed new odd/even order sampling architecture breakdown

| Sub Block | Functionality |
| :---: | :---: |
| Scalar Processing | Access to Registers as Datatype for Software |
|  | Process Container access to Digital |
| Adaptable Hardware | Logic(LUT), Pin/Ports, Registers, Block RAM |
| Intelligent Processing | Access to a ground of DSP Slices for equation |
|  | manipulation |

### 6.2. Experiment One - Verification of Practical Application of New Odd/Even Order Sampling

Laboratory verified signals captured in Chapter 4 were loaded and stored into Xilinx FPGA flash memory to accommodate implementation into FPGA chipset. Resource utilization for the proposed new soft-core architecture for odd/even order sampling is presented in Table 27 shown below with total resource utilization sitting just below $50 \%$.

Table 27. FPGA resource utilization for the new odd/even order sampling

|  | Used | Available | Utilization |
| :---: | :---: | :---: | :---: |
| SliceUtilization |  |  |  |
| Slice LUTs | 2044 | 14400 | 14.19\% |
| LUT as Logic | 1828 | 14400 | 12.69\% |
| LUT as Memory | 216 | 6000 | 3.60\% |
|  |  |  |  |
| Reg as Latch | 0 | 28800 | 0.00\% |
| MultiplexerUtilization |  |  |  |
| F7 Muxes | 52 | 8800 | 0.59\% |
| F8 Muxes | 5 | 4400 | 0.11\% |
| Memory Utilization |  |  |  |
| Black RAM | 1.5 | 50 | 3.00\% |
| DSPUtilization |  |  |  |
| DSPs | 2 | 66 | 3.03\% |
| SpecificFeatureUtilization |  |  |  |
| XADC | 0 | 1 | 0.00\% |
| Total Utilization |  |  | 48.15\% |

Step size performance shown in Figures18(a) and 18(b) is comparable to the expected results derived from simulation presented in Chapter 4. The error performance in Figure 19 is also comparable with expected simulation results in chapter 4.


Figure 18. FPGA architecture implementation results: (a) Source signal which is in phase with I and odd/even order sampling results; (b) Odd and even order sampling results with Vivado integrated logic analyzer (ILA) window configured to 1 kilobytes


Figure 19. This figure presents error performance for odd/even order sampling

### 6.3. Experiment Two - Signal-To-Noise Ratio (Snr) Performance Comparison to Literature Available Sampling Schemes

Implementation from the previous subsection was followed by implementation of other sampling schemes from literature such as $\operatorname{Mod}-\Delta$, $\operatorname{Mod}-\Delta$ (Gaussian) and Mod- $\Delta$ (Sinusoidal). According to literature [13] Mod- $\Delta$ (Gaussian) is a gaussian process whose PSD is flat within the designed band and Mod- $\Delta$ (Sinusoidal) is a sinusoidal waveform whose frequency is chosen at random, uniformly on [0, B), and whose amplitude is square root of covariance.


Figure 20. This figure presents noise captured from Radar source on a field test.

Many field tests were conducted to investigate realistic noise behaviour of Radar systems. The field tests were conducted at OR Tambo International Airport. The best noise performance was stored, and noise extracted for both I and Q components as shown in Figure 20. In Mod- $\Delta$ (Sinusoidal) the sinusoidal can be replaced by noise in the Figure 20 to make Mod- $\Delta$ (Noise) as it is within the specification of random frequency.

Table 28. FPGA resource utilization for the existing Mod- $\Delta$ sampling

|  | Used | Available | Utilization |
| :---: | :---: | :---: | :---: |
| SliceUtilization |  |  |  |
| Slice LUTs | 2044 | 14400 | 14.19\% |
| LUT as Logic | 1828 | 14400 | 12.69\% |
| LUT as Memory | 216 | 6000 | 3.60\% |
| SliceRegUtilization |  |  |  |
| Reg as Flip Flop | 3158 | 28800 | 10.97\% |
| Reg as Latch | 0 | 28800 | 0.00\% |
| MultiplexerUtilization |  |  |  |
| F7 Muxes | 52 | 8800 | 0.59\% |
| F8 Muxes | 5 | 4400 | 0.11\% |
| MemoryUtilization |  |  |  |
| Black RAM | 1.5 | 50 | 3.00\% |
| DSPUtilization |  |  |  |
| DSPs | 20 | 66 | 3030\% |
| SpecificFeatureUtilization |  |  |  |
| XADC | 0 | ${ }_{1} 1$ | 0.00\% |
| Total Utilization |  |  | 75.45\% |

Mod- $\Delta$ sampling was implemented under Intelligent Processing sub block while other components of architectural design in Figure 17 and Table 26. The mathematical model equations of the new odd/even order sampling were replaced by that of Mod- $\Delta$ sampling. The modification is also evident resource utilization in Table 28 registering DSP usage of just over $30 \%$ with total resource utilization sitting just above $75 \%$.

Table 29. FPGA resource utilization for the existing Mod- $\Delta$ (Gaussian) sampling.

|  | Used | Available | Utilization |
| :---: | :---: | :---: | :---: |
| SliceUtilization |  |  |  |
| Slice LUTs | 2044 | 14400 | 14.19\% |
| LUT as Logic | 1828 | 14400 | 12.69\% |
| LUT as Memory | 216 | 6000 | 3.60\% |
| SliceRegUtilization |  |  |  |
| Reg as Flip Flop | 3158 | 28800 | 10.97\% |
| Reg as Latch | 0 | 28800 | 0.00\% |
| MultiplexerUtilization |  |  |  |
| F7 Muxes | 52 | 8800 | 0.59\% |
| F8 Muxes | 5 | 4400 | 0.11\% |
| MemoryUtilization |  |  |  |
| Black RAM | 1.5 | 50 | 3.00\% |
| DSPUtilization |  |  |  |
| DSPs | 22 | 66 | 33.33\% |
| SpecificFeatureUtilization |  |  |  |
| XADC | 0 | ${ }^{1} 10$ | 0.00\% |
| Total Utilization |  |  | 78.48\% |

Implementation Mod- $\Delta$ sampling was implemented under Intelligent Processing sub block was followed by that of Mod- $\Delta$ (Gaussian). The mathematical model equations of Mod- $\Delta$ sampling were replaced by that of Mod- $\Delta$ (Gaussian) sampling. The modification is also evident in resource utilization in Table 29 registering DSP usage of just over $33 \%$ with total resource utilization sitting just above 78\%.

Table 30. FPGA resource utilization for the existing Mod- $\Delta$ (Noise) sampling

|  | Used | Available | Utilization |
| :---: | :---: | :---: | :---: |
| SliceUtilization |  |  |  |
| Slice LUTs | 2044 | 14400 | 14.19\% |
| LUT as Logic | 1828 | 14400 | 12.69\% |
| LUT as Memory | 216 | 6000 | 3.60\% |
| SliceRegUtilization |  |  |  |
| Reg as Flip Flop | 3158 | 28800 | 10.97\% |
| Reg as Latch | 0 | 28800 | 0.00\% |
| MultiplexerUtilization |  |  |  |
| F7 Muxes | 52 | 8800 | 0.59\% |
| F8 Muxes | 5 | 4400 | 0.11\% |
| MemoryUtilization |  |  |  |
| Black RAM | 1.5 | 50 | 3.00\% |
| DSPUtilization |  |  |  |
| DSPs | 27 | 66 | 40.91\% |
| SpecificFeatureUtilization |  |  |  |
| XADC | 0 | 1 | 0.00\% |
| Total Utilization |  |  | 86.06\% |

Implementation Mod- $\Delta$ (Gaussian) sampling was implemented under the Intelligent Processing sub block was followed by that of Mod- $\Delta$ (Noise). The mathematical model equations of Mod$\Delta$ (Gaussian) sampling were replaced by that of Mod- $\Delta$ (Noise) sampling. The modification is also evident resource utilization in Table 30 registering DSP usage of just over $40 \%$ with total resource utilization sitting just above $86 \%$.


Figure 21. This figure presents $S N R$ performance between the new odd/even order, Mod- $\Delta$, Mod- $\Delta$ (Gaussian) and Mod$\Delta$ (Noise) sampling

After the four implementations were usable SNR investigation was conducted on Vivado ILA tool. The investigation was through experiments for quantization resolution from 2 to 12 bits and SNR results are presented in Figure 21. SNR show that below 3 bits resolution Mod- $\Delta$ has the worst performance registering 12 dB signal distortion, Mod- $\Delta$ (Noise) has worst overall performances for all types of quantization designs between 2 and 12 bits while the new odd/even has average overall performance. Above 8 bits the new odd/even order sampling and Mod- $\Delta$ (Gaussian) give the best performance with the new odd/even order sampling recording 6 dB

### 6.4. Summary

In this chapter the results of two experiments on verification of practical implementation of odd/even order sampling and signal-to-noise ratio comparison to sampling schemes from literature were presented and discussed. The first experiment was to investigate and verify the practical implementation of odd/even order sampling on the MiniZed development board. The second experiment was to investigate signal-to-noise ratio in relation to existing sampling schemes. Results have shown that odd/even order sampling has the best performance at above 11 bits resolution.

# Chapter 7: Case Study 4: Practical Implementation of Noise Jamming 

### 7.1. Introduction

This chapter investigates the signal-to-noise ratio performance trade-offs between field gathered noise from pulse Doppler radar, and that of simulation built from a gaussian modelled noise. Field experiments were conducted at OR Tambo International Airport and pulse Radar with pulse repetition interval of 176.81 us was selected as the device under investigation. The investigation is embodied in two experiments in the next sub chapters.

### 7.2. Experiment One - Field Based Noise Data Collection

This work was developed in two approaches, the first was an effort to acquire realistic noise data from industrial Radar emitters. The aim was to match existing levels in Radar devices, and the facility with world class Radar systems that operate daily is OR Tambo International. OR Tambo International monitors the airspace using such Radars for air control reasons. Field studies were conducted with a spectral analyzer and different waveforms from coherent and non-coherent Radars. Two good quality non-coherent Radar waveforms were used for noise extraction. During noise extraction, 32 samples of waveforms were separated and stored as new noise waveforms. The 32 sampled waveforms were stored in FPGA flash memory and read over a defined clock cycle to reproduce the noise.

The second approach was conducted on the FPGA platform, gaussian noise was implemented using the DSP slices of the FPGA. Three waveforms were evaluated using Integrated Logic Analyser (ILA), one for gaussian noise, second for noise 1 and the third for noise 2. Additive noise (for gaussian, noise 1 and 2 respectively) function was applied to the input signal from Radar synthesizer to achieve noise jamming. Post analysis includes signal-to-noise ratio equations to conduct performance comparison between gaussian and field acquired noise. This subsection details the experimental investigations and results, experiment one dives straight to the experimental results of noise generation and experiment two built up on noise generation to achieve noise jamming. Field studies were conducted in OR Tambo International and several Radar performances were evaluated to determine the results using the spectral analyzer; the results are presented in Figures 22a to 22b.


Figure 22. Field test Pulse Radar (a) time behaviour of the Radar under test and (b) pulse repetition interval of the Radar
Noise from Figures 22a and 22b were separated and only 32 samples were recorded and saved towards the noise generation as shown in Figures 23a and 23b.


Figure 23. Separated noise behaviour from the Radar under test with (a) being noise 1 and (b) being noise 2

### 7.3. Experiment Two - Noise Jamming

Noise jamming of two schemes was implemented on the FPGA platform, one in Figure 24(a) is theoretical gaussian noise generated using the gaussian equation. The second scheme in Figure 24(b) is the proposed noise jamming from realistic noise from Radar sources.


Figure 24. Noise jamming results for (a) Gaussian noise jamming and (b) proposed noise jamming

FPGA resources were planned and optimized on the Xilinx Vivado platform. Our best efforts in implementing a noise jamming system capable of running hardware for both gaussian and proposed noise jamming is shown in Table 3. In should be noted that the major usage was on basic logic slices with only 1 DSP used for both addition operations of gaussian plus additive noise.

Table 31. FPGA resource planning and utilization

|  | Used | Available | Utilization |
| :---: | :---: | :---: | :---: |
| SliceUtilization |  |  |  |
| Slice LUTs | 2765 | 14400 | 19.20\% |
| LUT as Logic | 2522 | 14400 | 17.51\% |
| LUT as Memory | 243 | 6000 | 4.05\% |
| SliceRegUtilization |  |  |  |
| Reg as Flip Flop | 4518 | 28800 | 15.69\% |
| Reg as Latch | 0 | 28800 | 0.00\% |
| MultiplexerUtilization |  |  |  |
| F7 Muxes | 97 | 8800 | 1.10\% |
| F8 Muxes | 0 | 4400 | 0.00\% |
| MemoryUtilization |  |  |  |
| Black RAM | 3 | 50 | 6.00\% |
| DSPUtilization |  |  |  |
| DSPs | 1 | 66 | 6.00\% |
| SpecificFeatureUtilization |  |  |  |
| XADC | 0 | 1 | 0.00\% |
| Total Utilization |  |  | 69.55\% |

Performance evaluation of the two schemes was done using SNR; the goal of this study is to achieve noise jamming with minimum effective noise power. Figure 25 shows that the proposed noise jamming schemes outperforms and theoretical gaussian jamming while achieving effective jamming SNR of 11 dB .


Figure 25. Signal to noise-ratio (SNR) performance comparison of the realistic and theoretical schemes

### 7.4. Summary

In this chapter the results of two experiments on field-based noise data collection and noise jamming were presented and discussed. The first experiment was to investigate time based behaviour of the field-based pulse Radar, parameters such as pulse repetition period and noise level were measured. The second experiment was to investigate implementation tradeoffs between noise captured from field activities and noise model by Gaussian distribution. Results have shown that noise jamming from Gaussian distribution has superior signal-to-noise ratio when compared to field-gathered noise.

## Chapter 8: Conclusions and Recommendations

### 8.1. Conclusions

Chapter 3 presented numerical and experimental merits. Derived numerical demonstration of the proposed odd/even order sampling. Chapter 3 also used the numerical model to develop the simulation model to investigate MSE performance on odd/even order sampling. A clear description of numerical parameter that represents quantization error. Quantization error is very important in this study as it is largely influenced by the arrangements of samples. Figure 3 from simulation and Figure 10 from FPGA implementation seems to indicate that quantization error behaviour is consistent between simulation and implementation.

Chapter 4 develops an experimental setup to capture source signal data and used it to investigate the sample frequency performance using Excel for implementation of the numerical model. Spectral analysis on Excel in Figure 8 indicated that the sample frequency has reduced after odd/even order sampling; this validates the research assumption that initiated this study. Chapter 4 also implemented the proposed odd/even order sampling, Mod- $\Delta$, Mod- $\Delta$ (Gaussian) and Mod- $\Delta$ (Noise) on an FPGA platform to capture computational requirements. Chapter 4 went further to investigate SNR performance for the four sampling schemes to demonstrate merits that the new odd/even order sampling has.

It is important to note that the quantization error based on MSE calculation for both simulation and FPGA implementation is comparable as shown in Figure 16 for simulation and Figure 19 for implementation. A detailed step size and dynamic range investigation revealed that the step size reduced from 8 kHz to 2 kHz , while the dynamic range slightly reduced from 1.185 Volts/ms to 1.088 Volts/ms. Implementation investigations have revealed that FPGA resource utilization for the new odd/even order sampling, Mod- $\Delta$, Mod- $\Delta$ (Gaussian) and Mod- $\Delta$ (Noise) respectively is approximately $45 \%, 75 \%, 78 \%$ and $86 \%$. This is a clear indication that is very lightweight when it comes to computation requirements as it has the least resource utilization as compared to the studied schemes.

By reducing the sample frequency of digitized signal, the issue of complicated architecture and expensive FPGA selection can be reduced. This paper proposes a novel odd/even order
sampling architecture that eliminates odd and even sampling on quadrature signals to reduce sample frequency. Realistic case studies that investigate the behaviour of the proposed new novel odd/even order sampling architecture, using computational simulation, laboratory and implementation experimentation.

- Simulation investigated step-size, dynamic range and dynamic range error behaviour. Results verify that odd/even ordered sampling can significantly reduce the sample frequency from 8 kHz to 2 kHz while not adversely affecting the dynamic range.
- Laboratory experimentation investigated feasibility of research assumption that ordered sampling reduces sample frequency. Results verify this assumption using time and spectral analysis.
- Implementation experimentation investigated feasibility of implementing ordered sampling on FPGA platform in comparison to sampling architecture in literature. While on the other hand research investigated SNR behaviour of odd/even ordered sampling in comparison to Mod- $\Delta$, Mod- $\Delta$ (Gaussian) and Mod- $\Delta$ (Noise) literature. Results indicate that odd/even order sampling is the most economical in comparison to architectures evaluated with resource utilization at $45 \%$. SNR results were not conclusive for sampling resolution below 8 bits, for resolution between 8 bits and 11 bits odd/even ordered sampling is the second-best performer, while for sampling resolution above 11 bits performance is the best.


### 8.2. Recommendations for Further Work

Future studies should involve a detailed study of computation requirements of the different sampling schemes with numerical tracking of the cause for deviations in resource utilization. Mod- $\Delta$ (Sinusoidal) has two constraints, frequency domain and amplitude constraints defined as:

- Random frequency uniformly distributed (0, B)
- Amplitude is a square root of covariance

Mod- $\Delta$ (Sinusoidal) generally require the sinusoid to be noise to meet the frequency constraint while the amplitude of that noise must be a square root of covariance. The sinusoid used in this paper was a noise signal acquired realistic Radar equipment as shown in Figure 11, this acquisition was made without verification conducted against Mod- $\Delta$ (Sinusoidal) amplitude
constraint. Future work should also include amplitude investigation into the implementation of Mod- $\Delta$ (Noise) that validates if it satisfies the square root of covariance constraint.

Further study of Modified-Costas algorithm towards electronic warfare applications, and investigation into performance of quantization error should be undertaken as well as further study of peak detection algorithm towards electronic warfare, investigation into performance of dynamic range and signal-to-noise ratio as these usually affect amplitude performance. Digital resource utilization for both Modified-Costas and peak detection could also be investigated in comparison to existing phase and amplitude detection techniques.

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# Appendix A: Nove Odd/Even Order Sampling Data Analysis 

Table 32. Calculation of odd order sampling components up to $11^{\text {th }} \mathrm{order}$

| \# | I | Q | $\mathbf{I}(\mathbf{N}+1) 3$ | $\mathbf{I}(\mathrm{N}+1) 5$ | $\mathbf{I}(\mathbf{N}+\mathbf{1}) 7$ | $\mathbf{I}(\mathbf{N}+1) 9$ | $\mathbf{I}(\mathbf{N}+1) 11$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 3374 | 3432 | 3384 | 3374 | 3384 | 3406 | 3406 |
| 1 | 3366 | 3422 | 3384 | 3374 | 3384 | 3406 | 3406 |
| 2 | 3362 | 3410 | 3384 | 3374 | 3384 | 3406 | 3406 |
| 3 | 3360 | 3397 | 3362 | 3374 | 3384 | 3406 | 3406 |
| 4 | 3356 | 3386 | 3362 | 3374 | 3384 | 3406 | 3406 |
| 5 | 3348 | 3376 | 3362 | 3348 | 3384 | 3406 | 3406 |
| 6 | 3321 | 3367 | 3348 | 3348 | 3321 | 3321 | 3140 |
| 7 | 3256 | 3362 | 3348 | 3348 | 3321 | 3321 | 3140 |
| 8 | 3140 | 3360 | 3348 | 3348 | 3321 | 3321 | 3140 |
| 9 | 2963 | 3357 | 3140 | 3348 | 3321 | 3321 | 3140 |
| 10 | 2716 | 3351 | 3140 | 2716 | 3321 | 3321 | 3140 |
| 11 | 2404 | 3331 | 3140 | 2716 | 3321 | 3321 | 3140 |
| 12 | 2040 | 3276 | 2404 | 2716 | 3321 | 3321 | 3140 |
| 13 | 1638 | 3175 | 2404 | 2716 | 1632 | 3321 | 3140 |
| 14 | 1215 | 3014 | 2404 | 2716 | 1632 | 3321 | 3140 |
| 15 | 793 | 2784 | 1215 | 793 | 1632 | 793 | 3140 |
| 16 | 395 | 2487 | 1215 | 793 | 1632 | 793 | 3140 |
| 17 | 48 | 2135 | 1215 | -793 | 1632 | 793 | 3140 |
| 18 | 0 | 1742 | 48 | 793 | 1632 | 793 | 0 |
| 19 | 0 | 1322 | 48 | 793 | 1632 | 793 | 0 |
| 20 | 0 | 897 | 48 | 0 | 0 | 793 | 0 |
| 21 | 0 | 491 | 0 | 0 | 0 | 793 | 0 |
| 22 | 0 | 129 | 0 | 0 | 0 | 793 | 0 |
| 23 | 0 | 0 | 0 | 0 | 0 | 793 | 0 |
| 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |


| 33 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 34 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 36 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 37 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 38 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 39 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 40 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 41 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 42 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 43 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 44 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 45 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 46 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 47 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 48 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 49 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 50 | 0 | 0 | $0$ | 0 | 0 | 0 | 0 |
| 51 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 52 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 53 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 54 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 55 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 56 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 57 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 58 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 59 | 112 | 0 | 0 | 0 | 0 | 0 | 0 |
| 60 | 356 | 0 | 112 | 356 | 0 | 356 | 0 |
| 61 | 667 | 0 | 112 | 356 | 0 | 356 | 0 |
| 62 | 1036 | 0 | 112 | 356 | 1036 | 356 | 1855 |
| 63 | 1436 | 0 | 1036 | 356 | 1036 | 356 | 1855 |
| 64 | 1855 | 61 | 1036 | 356 | 1036 | 356 | 1855 |
| 65 | 2275 | 289 | 1036 | 2275 | 1036 | 356 | 1855 |
| 66 | 2671 | 583 | 2275 | 2275 | 1036 | 356 | 1855 |
| 67 | 3026 | 940 | 2275 | 2275 | 1036 | 356 | 1855 |
| 68 | 3321 | 1334 | 2275 | 2275 | 1036 | 356 | 1855 |
| 69 | 3552 | 1749 | 3321 | 2275 | 3552 | 3552 | 1855 |
| 70 | 3715 | 2172 | 3321 | 3715 | 3552 | 3552 | 1855 |
| 71 | 3812 | 2576 | 3321 | 3715 | 3552 | 3552 | 1855 |
| 72 | 3851 | 2942 | 3812 | 3715 | 3552 | 3552 | 1855 |
| 73 | 3838 | 3253 | 3812 | 3715 | 3552 | 3552 | 3705 |
| 74 | 3784 | 3501 | 3812 | 3715 | 3552 | 3552 | 3705 |
| 75 | 3705 | 3682 | 3784 | 3705 | 3552 | 3552 | 3705 |


| 76 | 3612 | 3794 | 3784 | 3705 | 3612 | 3552 | 3705 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 77 | 3515 | 3847 | 3784 | 3705 | 3612 | 3552 | 3705 |
| 78 | 3424 | 3846 | 3515 | 3705 | 3424 | 3424 | 3705 |
| 79 | 3345 | 3801 | 3515 | 3705 | 3424 | 3424 | 3705 |
| 80 | 3284 | 3727 | 3515 | 3284 | 3424 | 3424 | 3705 |
| 81 | 3243 | 3636 | 3284 | 3284 | 3424 | 3424 | 3705 |
| 82 | 3221 | 3539 | 3284 | 3284 | 3424 | 3424 | 3705 |
| 83 | 3219 | 3445 | 3284 | 3284 | 3424 | 3424 | 3705 |
| 84 | 3232 | 3363 | 3219 | 3284 | 3424 | 3424 | 3285 |
| 85 | 3254 | 3297 | 3219 | 3254 | 3254 | 3424 | 3285 |
| 86 | 3285 | 3251 | 3219 | 3254 | 3254 | 3424 | 3285 |
| 87 | 3319 | 3225 | 3285 | 3254 | 3254 | 3319 | 3285 |
| 88 | 3352 | 3218 | 3285 | 3254 | 3254 | 3319 | 3285 |
| 89 | 3381 | 3228 | 3285 | 3254 | 3254 | 3319 | 3285 |
| 90 | 3405 | 3248 | 3381 | 3405 | 3254 | 3319 | 3285 |
| 91 | 3423 | 3276 | 3381 | 3405 | 3254 | 3319 | 3285 |
| 92 | 3434 | 3310 | 3381 | 3405 | 3434 | 3319 | 3285 |
| 93 | 3438 | 3344 | 3434 | 3405 | 3434 | 3319 | 3285 |
| 94 | 3435 | 3374 | 3434 | 3405 | 3434 | 3319 | 3285 |
| 95 | 3429 | 3400 | 3434 | 3429 | 3434 | 3319 | 3407 |
| 96 | 3419 | 3419 | 3429 | 3429 | 3434 | 3419 | 3407 |
| 97 | 3407 | 3432 | 3429 | 3429 | 3434 | 3419 | 3407 |
| 98 | 3395 | 3437 | 3429 | 3429 | 3434 | 3419 | 3407 |
| 99 | 3383 | 3437 | 3395 | 3429 | 3383 | 3419 | 3407 |
| 100 | 3373 | 3432 | 3395 | 3373 | 3383 | 3419 | 3407 |
| 101 | 3367 | 3422 | 3395 | 3373 | 3383 | 3419 | 3407 |
| 102 | 3361 | 3410 | 3367 | 3373 | 3383 | 3419 | 3407 |
| 103 | 3359 | 3398 | 3367 | 3373 | 3383 | 3419 | 3407 |
| 104 | 3356 | 3386 | 3367 | 3373 | 3383 | 3419 | 3407 |
| 105 | 3349 | 3375 | 3356 | 3349 | 3383 | 3349 | 3407 |
| 106 | 3321 | 3368 | 3356 | 3349 | 3321 | 3349 | 3140 |
| 107 | 3256 | 3362 | 3356 | 3349 | 3321 | 3349 | 3140 |
| 108 | 3140 | 3359 | 3256 | 3349 | 3321 | 3349 | 3140 |
| 109 | 2962 | 3357 | 3256 | 3349 | 3321 | 3349 | 3140 |
| 110 | 2715 | 3351 | 3256 | 2715 | 3321 | 3349 | 3140 |
| 111 | 2404 | 3331 | 2715 | 2715 | 3321 | 3349 | 3140 |
| 112 | 2041 | 3276 | 2715 | 2715 | 3321 | 3349 | 3140 |
| 113 | 1640 | 3175 | 2715 | 2715 | 1640 | 3349 | 3140 |
| 114 | 1214 | 3013 | 1640 | 2715 | 1640 | 1214 | 3140 |
| 115 | 793 | 2783 | 1640 | 793 | 1640 | 1214 | 3140 |
| 116 | 395 | 2487 | 1640 | 793 | 1640 | 1214 | 3140 |
| 117 | 47 | 2136 | 395 | 793 | 1640 | 1214 | 0 |
| 118 | 0 | 1743 | 395 | 793 | 1640 | 1214 | 0 |


| 119 | 0 | 1322 | 395 | 793 | 1640 | 1214 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 120 | 0 | 897 | 0 | 0 | 0 | 1214 | 0 |
| 121 | 0 | 491 | 0 | 0 | 0 | 1214 | 0 |
| 122 | 0 | 128 | 0 | 0 | 0 | 1214 | 0 |
| 123 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 124 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 125 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 126 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 127 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 128 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 129 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 130 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 131 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 132 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 133 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 134 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 135 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 136 | 0 | 0 | $0$ | 0 | 0 | 0 | 0 |
| 137 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 138 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| 139 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 140 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 141 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 142 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 143 | 1 | 2 |  | $0$ | 0 | 0 | 0 |
| 144 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 145 | 3 | 0 | 1 | 3 | 0 | 0 | 0 |
| 146 | 0 | 0 | 1 | 3 | 0 | 0 | 0 |
| 147 | 0 | 0 | 0 | 3 | 0 | 0 | 0 |
| 148 | 0 | 1 | 0 | 3 | 0 | 0 | 0 |
| 149 | 0 | 0 | 0 | 3 | 0 | 0 | 0 |
| 150 | 0 | 2 | 0 | 0 | 0 | 0 | 0 |
| 151 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 152 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 153 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 154 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 155 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 156 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 157 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 158 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 159 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 160 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 161 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |


| 162 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 163 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 164 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 165 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 166 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 167 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 168 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 169 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 170 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 171 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 172 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 173 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 174 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 175 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 176 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 177 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 178 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 179 | 0 | 0 | 0 |  | 0 | 0 | 0 |
| 180 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 181 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 182 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 183 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 184 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 185 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 186 | 0 | 0 |  | $0$ | 0 | 0 | 0 |
| 187 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 188 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 189 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 190 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 191 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 192 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 193 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 194 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 195 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 196 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 197 | 99 | 0 | 0 | 0 | 99 | 0 | 0 |
| 198 | 351 | 0 | 99 | 0 | 99 | 0 | 0 |
| 199 | 672 | 0 | 99 | 0 | 99 | 0 | 0 |
| 200 | 1036 | 0 | 99 | 1036 | 99 | 0 | 0 |
| 201 | 1441 | 0 | 1036 | 1036 | 99 | 0 | 0 |
| 202 | 1864 | 50 | 1036 | 1036 | 99 | 0 | 0 |
| 203 | 2284 | 280 | 1036 | 1036 | 99 | 0 | 0 |
| 204 | 2680 | 587 | 2284 | 1036 | 2680 | 2680 | 0 |


| 205 | 3028 | 941 | 2284 | 3028 | 2680 | 2680 | 3555 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 206 | 3323 | 1337 | 2284 | 3028 | 2680 | 2680 | 3555 |
| 207 | 3555 | 1758 | 3323 | 3028 | 2680 | 2680 | 3555 |
| 208 | 3717 | 2181 | 3323 | 3028 | 2680 | 2680 | 3555 |
| 209 | 3813 | 2585 | 3323 | 3028 | 2680 | 2680 | 3555 |
| 210 | 3849 | 2946 | 3813 | 3849 | 2680 | 2680 | 3555 |
| 211 | 3835 | 3255 | 3813 | 3849 | 3835 | 2680 | 3555 |
| 212 | 3782 | 3504 | 3813 | 3849 | 3835 | 2680 | 3555 |
| 213 | 3704 | 3682 | 3782 | 3849 | 3835 | 3704 | 3555 |
| 214 | 3610 | 3795 | 3782 | 3849 | 3835 | 3704 | 3555 |
| 215 | 3512 | 3845 | 3782 | 3849 | 3835 | 3704 | 3555 |
| 216 | 3422 | 3842 | 3512 | 3849 | 3835 | 3704 | 3283 |
| 217 | 3344 | 3798 | 3512 | 3849 | 3835 | 3704 | 3283 |
| 218 | 3283 | 3725 | 3512 | 3849 | 3283 | 3704 | 3283 |
| 219 | 3242 | 3634 | 3283 | 3849 | 3283 | 3704 | 3283 |
| 220 | 3223 | 3536 | 3283 | 3223 | 3283 | 3704 | 3283 |
| 221 | 3220 | 3443 | 3283 | 3223 | 3283 | 3704 | 3283 |
| 222 | 3232 | 3362 | 3220 | 3223 | 3283 | 3232 | 3283 |
| 223 | 3255 | 3296 | 3220 | 3223 | 3283 | 3232 | 3283 |
| 224 | 3286 | 3250 | 3220 | 3223 | 3283 | 3232 | 3283 |
| 225 | 3319 | 3226 | 3286 | 3319 | 3319 | 3232 | 3283 |
| 226 | 3352 | 3219 | 3286 | 3319 | 3319 | 3232 | 3283 |
| 227 | 3382 | 3228 | 3286 | 3319 | 3319 | 3232 | 3424 |
| 228 | 3406 | 3248 | 3382 | 3319 | 3319 | 3232 | 3424 |
| 229 | 3424 | 3278 | 3382 | 3319 | 3319 | 3232 | 3424 |
| 230 | 3434 | 3311 | 3382 | 3434 | 3319 | 3232 | 3424 |
| 231 | 3437 | 3343 | 3434 | 3434 | 3319 | 3437 | 3424 |
| 232 | 3435 | 3375 | 3434 | 3434 | 3435 | 3437 | 3424 |
| 233 | 3428 | 3400 | 3434 | 3434 | 3435 | 3437 | 3424 |
| 234 | 3418 | 3420 | 3428 | 3434 | 3435 | 3437 | 3424 |
| 235 | 3407 | 3432 | 3428 | 3407 | 3435 | 3437 | 3424 |
| 236 | 3395 | 3437 | 3428 | 3407 | 3435 | 3437 | 3424 |
| 237 | 3384 | 3437 | 3395 | 3407 | 3435 | 3437 | 3424 |
| 238 | 3374 | 3430 | 3395 | 3407 | 3435 | 3437 | 3361 |
| 239 | 3366 | 3421 | 3395 | 3407 | 3366 | 3437 | 3361 |
| 240 | 3361 | 3410 | 3366 | 3361 | 3366 | 3361 | 3361 |
| 241 | 3358 | 3398 | 3366 | 3361 | 3366 | 3361 | 3361 |
| 242 | 3355 | 3387 | 3366 | 3361 | 3366 | 3361 | 3361 |
| 243 | 3349 | 3376 | 3355 | 3361 | 3366 | 3361 | 3361 |
| 244 | 3321 | 3367 | 3355 | 3361 | 3366 | 3361 | 3361 |
| 245 | 3255 | 3362 | 3355 | 3255 | 3366 | 3361 | 3361 |
| 246 | 3142 | 3359 | 3255 | 3255 | 3142 | 3361 | 3361 |
| 247 | 2963 | 3356 | 3255 | 3255 | 3142 | 3361 | 3361 |


| 248 | 2714 | 3352 | 3255 | 3255 | 3142 | 3361 | 3361 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 249 | 2404 | 3331 | 2714 | 3255 | 3142 | 2404 | 1638 |
| 250 | 2041 | 3275 | 2714 | 2041 | 3142 | 2404 | 1638 |
| 251 | 1638 | 3176 | 2714 | 2041 | 3142 | 2404 | 1638 |
| 252 | 1214 | 3015 | 1638 | 2041 | 3142 | 2404 | 1638 |
| 253 | 793 | 2782 | 1638 | 2041 | 793 | 2404 | 1638 |
| 254 | 395 | 2486 | 1638 | 2041 | 793 | 2404 | 1638 |

Table 33. Quantization error noise generated by $11^{\text {th }}$ odd order sampling components

| Noise( $\mathrm{N}+1$ ) 3 | Noise( $\mathrm{N}+1$ ) 5 | Noise( $\mathrm{N}+1)^{7}$ | Noise( $\mathrm{N}+1$ ) 9 | Noise(N+1)11 |
| :---: | :---: | :---: | :---: | :---: |
| -10 | 0 | -10 | -32 | -32 |
| -18 | -8 | -18 | -40 | -40 |
| -22 | -12 | -22 | -44 | -44 |
| -2 | -14 | -24 | -46 | -46 |
| -6 | -18 | -28 | -50 | -50 |
| -14 | 0 | -36 | -58 | -58 |
| -27 | -27 | 0 | 0 | 181 |
| -92 | -92 | -65 | -65 | 116 |
| -208 | -208 | -181 | -181 | 0 |
| -177 | -385 | -358 | -358 | -177 |
| -424 | 0 | -605 | -605 | -424 |
| -736 | -312 | -917 | -917 | -736 |
| -364 | -676 | -1281 | -1281 | -1100 |
| -766 | -1078 | 6 | -1683 | -1502 |
| -1189 | -1501 | -417 | -2106 | -1925 |
| -422 | 0 | -839 | 0 | -2347 |
| -820 | -398 | -1237 | -398 | -2745 |
| -1167 | -745 | -1584 | -745 | -3092 |
| -48 | -793 | -1632 | -793 | 0 |
| -48 | -793 | -1632 | -793 | 0 |
| -48 | 0 | 0 | -793 | 0 |
| 0 | 0 | 0 | -793 | 0 |
| 0 | 0 | 0 | -793 | 0 |
| 0 | 0 | 0 | -793 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |


| 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 |  | 0 | 0 |
| 0 |  |  | 0 | 0 |
| 112 | 112 | 112 | 112 | 112 |
| 244 | 0 | 356 | 0 | 356 |
| 555 | 311 | 667 | 311 | 667 |
| 924 | 680 | 0 | 680 | -819 |
| 400 | 1080 | 400 | 1080 | -419 |
| 819 | 1499 | 819 | 1499 | 0 |
| 1239 | 0 | 1239 | 1919 | 420 |
| 396 | 396 | 1635 | 2315 | 816 |
| 751 | 751 | 1990 | 2670 | 1171 |
| 1046 | 1046 | 2285 | 2965 | 1466 |
| 231 | 1277 | 0 | 0 | 1697 |
| 394 | 0 | 163 | 163 | 1860 |
| 491 | 97 | 260 | 260 | 1957 |
| 39 | 136 | 299 | 299 | 1996 |
| 26 | 123 | 286 | 286 | 133 |
| -28 | 69 | 232 | 232 | 79 |
| -79 | 0 | 153 | 153 | 0 |


| -172 | -93 | 0 | 60 | -93 |
| :---: | :---: | :---: | :---: | :---: |
| -269 | -190 | -97 | -37 | -190 |
| -91 | -281 | 0 | 0 | -281 |
| -170 | -360 | -79 | -79 | -360 |
| -231 | 0 | -140 | -140 | -421 |
| -41 | -41 | -181 | -181 | -462 |
| -63 | -63 | -203 | -203 | -484 |
| -65 | -65 | -205 | -205 | -486 |
| 13 | -52 | -192 | -192 | -53 |
| 35 | 0 | 0 | -170 | -31 |
| 66 | 31 | 31 | -139 | 0 |
| 34 | 65 | 65 | 0 | 34 |
| 67 | 98 | 98 | 33 | 67 |
| 96 | 127 | 127 | 62 | 96 |
| 24 | 0 | 151 | 86 | 120 |
| 42 | 18 | 169 | 104 | 138 |
| 53 | 29 | 0 | 115 | 149 |
| 4 | 33 | 4 | 119 | 153 |
| 1 | 30 | 1 | 116 | 150 |
| -5 | 0 | -5 | 110 | 22 |
| -10 | -10 | -15 | 0 | 12 |
| -22 | -22 | -27 | -12 | 0 |
| -34 | -34 | -39 | -24 | -12 |
| -12 | -46 | 0 | -36 | -24 |
| -22 | 0 | -10 | -46 | -34 |
| -28 | -6 | -16 | -52 | -40 |
| -6 | -12 | -22 | -58 | -46 |
| -8 | -14 | -24 | -60 | -48 |
| -11 | -17 | -27 | -63 | -51 |
| -7 | 0 | -34 | 0 | -58 |
| -35 | -28 | 0 | -28 | 181 |
| -100 | -93 | -65 | -93 | 116 |
| -116 | -209 | -181 | -209 | 0 |
| -294 | -387 | -359 | -387 | -178 |
| -541 | 0 | -606 | -634 | -425 |
| -311 | -311 | -917 | -945 | -736 |
| -674 | -674 | -1280 | -1308 | -1099 |
| -1075 | -1075 | 0 | -1709 | -1500 |
| -426 | -1501 | -426 | 0 | -1926 |
| -847 | 0 | -847 | -421 | -2347 |
| -1245 | -398 | -1245 | -819 | -2745 |
| -348 | -746 | -1593 | -1167 | 47 |
| -395 | -793 | -1640 | -1214 | 0 |
| -395 | -793 | -1640 | -1214 | 0 |



| 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 |  |  | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 |  |  | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 99 | 99 | 0 | 99 | 99 |
| 252 | 351 | 252 | 351 | 351 |
| 573 | 672 | 573 | 672 | 672 |
| 937 | 0 | 937 | 1036 | 1036 |
| 405 | 405 | 1342 | 1441 | 1441 |
| 828 | 828 | 1765 | 1864 | 1864 |
| 1248 | 1248 | 2185 | 2284 | 2284 |
| 396 | 1644 | 0 | 0 | 2680 |
| 744 | 0 | 348 | 348 | -527 |
| 1039 | 295 | 643 | 643 | -232 |
| 232 | 527 | 875 | 875 | 0 |

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| 394 | 689 | 1037 | 1037 | 162 |
| :---: | :---: | :---: | :---: | :---: |
| 490 | 785 | 1133 | 1133 | 258 |
| 36 | 0 | 1169 | 1169 | 294 |
| 22 | -14 | 0 | 1155 | 280 |
| -31 | -67 | -53 | 1102 | 227 |
| -78 | -145 | -131 | 0 | 149 |
| -172 | -239 | -225 | -94 | 55 |
| -270 | -337 | -323 | -192 | -43 |
| -90 | -427 | -413 | -282 | 139 |
| -168 | -505 | -491 | -360 | 61 |
| -229 | -566 | 0 | -421 | 0 |
| -41 | -607 | -41 | -462 | -41 |
| -60 | 0 | -60 | -481 | -60 |
| -63 | -3 | -63 | -484 | -63 |
| 12 | 9 | -51 | 0 | -51 |
| 35 | 32 | -28 | 23 | -28 |
| 66 | 63 | 3 | 54 | 3 |
| 33 | 0 | 0 | 87 | 36 |
| 66 | 33 | 33 | 120 | 69 |
| 96 | 63 | 63 | 150 | -42 |
| 24 | 87 | 87 | 174 | -18 |
| 42 | 105 | 105 | 192 | 0 |
| 52 | 0 | 115 | 202 | 10 |
| 3 | 3 | 118 | 0 | 13 |
| 1 | 1 | 0 | -2 | 11 |
| -6 | -6 | -7 | -9 | 4 |
| -10 | -16 | -17 | -19 | -6 |
| -21 | 0 | -28 | -30 | -17 |
| -33 | -12 | -40 | -42 | -29 |
| -11 | -23 | -51 | -53 | -40 |
| -21 | -33 | -61 | -63 | 13 |
| -29 | -41 | 0 | -71 | 5 |
| -5 | 0 | -5 | 0 | 0 |
| -8 | -3 | -8 | -3 | -3 |
| -11 | -6 | -11 | -6 | -6 |
| -6 | -12 | -17 | -12 | -12 |
| -34 | -40 | -45 | -40 | -40 |
| -100 | 0 | -111 | -106 | -106 |
| -113 | -113 | 0 | -219 | -219 |
| -292 | -292 | -179 | -398 | -398 |
| -541 | -541 | -428 | -647 | -647 |
| -310 | -851 | -738 | 0 | 766 |
| -673 | 0 | -1101 | -363 | 403 |
| -1076 | -403 | -1504 | -766 | 0 |


| -424 | -827 | -1928 | -1190 | -424 |
| :---: | :---: | :---: | :---: | :---: |
| -845 | -1248 | 0 | -1611 | -845 |
| -1243 | -1646 | -398 | -2009 | -1243 |



| S | $\mathrm{N}(\mathrm{N}+1) 3$ | $\mathrm{N}(\mathrm{N}+1) 5$ | $\mathrm{N}(\mathrm{N}+1) 7$ | $\mathrm{N}(\mathrm{N}+1) 9$ | $\mathbf{N}(\mathbf{N}+1) 11$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 11383876 | 100 | 0 | 100 | 1024 | 1024 |
| 11329956 | 324 | 64 | 324 | 1600 | 1600 |
| 11303044 | 484 | 144 | 484 | 1936 | 1936 |
| 11289600 | 4 | 196 | 576 | 2116 | 2116 |
| 11262736 | 36 | 324 | 784 | 2500 | 2500 |
| 11209104 | 196 | 0 | 1296 | 3364 | 3364 |
| 11029041 | 729 | 729 | 0 | 0 | 32761 |
| 10601536 | 8464 | 8464 | 4225 | 4225 | 13456 |
| 9859600 | 43264 | 43264 | 32761 | 32761 | 0 |
| 8779369 | 31329 | 148225 | 128164 | 128164 | 31329 |
| 7376656 | 179776 | 0 | 366025 | 366025 | 179776 |
| 5779216 | 541696 | 97344 | 840889 | 840889 | 541696 |
| 4161600 | 132496 | 456976 | 1640961 | 1640961 | 1210000 |
| 2683044 | 586756 | 1162084 | 36 | 2832489 | 2256004 |
| 1476225 | 1413721 | 2253001 | 173889 | 4435236 | 3705625 |
| 628849 | 178084 | 0 | 703921 | 0 | 5508409 |
| 156025 | 672400 | 158404 | 1530169 | 158404 | 7535025 |
| 2304 | 1361889 | 555025 | 2509056 | 555025 | 9560464 |
| 0 | 2304 | 628849 | 2663424 | 628849 | 0 |
| 0 | 2304 | 628849 | 2663424 | 628849 | 0 |
| 0 | 2304 | 0 | 0 | 628849 | 0 |
| 0 | 0 | 0 | 0 | 628849 | 0 |
| 0 | 0 | 0 | 0 | 628849 | 0 |
| 0 | 0 | U 0 | 0 | 628849 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | $\bigcirc 0$ | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |


| 0 | 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 12544 | 12544 | 12544 | 12544 | 12544 | 12544 |
| 126736 | 59536 | 0 | 126736 | 0 | 126736 |
| 444889 | 308025 | 96721 | 444889 | 96721 | 444889 |
| 1073296 | 853776 | 462400 | 0 | 462400 | 670761 |
| 2062096 | 160000 | 1166400 | 160000 | 1166400 | 175561 |
| 3441025 | 670761 | 2247001 | 670761 | 2247001 | 0 |
| 5175625 | 1535121 | 0 | 1535121 | 3682561 | 176400 |
| 7134241 | 156816 | 156816 | 2673225 | 5359225 | 665856 |
| 9156676 | 564001 | 564001 | 3960100 | 7128900 | 1371241 |
| 11029041 | 1094116 | 1094116 | 5221225 | 8791225 | 2149156 |
| 12616704 | 53361 | 1630729 | 0 | 0 | 2879809 |
| 13801225 | 155236 | 0 | 26569 | 26569 | 3459600 |
| 14531344 | 241081 | 9409 | 67600 | 67600 | 3829849 |
| 14830201 | 1521 | 18496 | 89401 | 89401 | 3984016 |
| 14730244 | 676 | 15129 | 81796 | 81796 | 17689 |
| 14318656 | 784 | 4761 | 53824 | 53824 | 6241 |
| 13727025 | 6241 | 0 | 23409 | 23409 | 0 |
| 13046544 | 29584 | 8649 | 0 | 3600 | 8649 |
| 12355225 | 72361 | 36100 | 9409 | 1369 | 36100 |
| 11723776 | 8281 | 78961 | 0 | 0 | 78961 |
| 11189025 | 28900 | 129600 | 6241 | 6241 | 129600 |
| 10784656 | 53361 | 0 | 19600 | 19600 | 177241 |
| 10517049 | 1681 | 1681 | 32761 | 32761 | 213444 |
| 10374841 | 3969 | 3969 | 41209 | 41209 | 234256 |
| 10361961 | 4225 | 4225 | 42025 | 42025 | 236196 |
| 10445824 | 169 | 2704 | 36864 | 36864 | 2809 |
| 10588516 | 1225 | 0 | 0 | 28900 | 961 |


| 10791225 | 4356 | 961 | 961 | 19321 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 11015761 | 1156 | 4225 | 4225 | 0 | 1156 |
| 11235904 | 4489 | 9604 | 9604 | 1089 | 4489 |
| 11431161 | 9216 | 16129 | 16129 | 3844 | 9216 |
| 11594025 | 576 | 0 | 22801 | 7396 | 14400 |
| 11716929 | 1764 | 324 | 28561 | 10816 | 19044 |
| 11792356 | 2809 | 841 | 0 | 13225 | 22201 |
| 11819844 | 16 | 1089 | 16 | 14161 | 23409 |
| 11799225 | 1 | 900 | 1 | 13456 | 22500 |
| 11758041 | 25 | 0 | 25 | 12100 | 484 |
| 11689561 | 100 | 100 | 225 | 0 | 144 |
| 11607649 | 484 | 484 | 729 | 144 | 0 |
| 11526025 | 1156 | 1156 | 1521 | 576 | 144 |
| 11444689 | 144 | 2116 | 0 | 1296 | 576 |
| 11377129 | 484 | 0 | 100 | 2116 | 1156 |
| 11336689 | 784 | 36 | 256 | 2704 | 1600 |
| 11296321 | 36 | 144 | 484 | 3364 | 2116 |
| 11282881 | 64 | 196 | 576 | 3600 | 2304 |
| 11262736 | 121 | 289 | 729 | 3969 | 2601 |
| 11215801 | 49 | 0 | 1156 | 0 | 3364 |
| 11029041 | 1225 | 784 | 0 | 784 | 32761 |
| 10601536 | 10000 | 8649 | 4225 | 8649 | 13456 |
| 9859600 | 13456 | 43681 | 32761 | 43681 | 0 |
| 8773444 | 86436 | 149769 | 128881 | 149769 | 31684 |
| 7371225 | 292681 | 0 | 367236 | 401956 | 180625 |
| 5779216 | 96721 | 96721 | 840889 | 893025 | 541696 |
| 4165681 | 454276 | 454276 | 1638400 | 1710864 | 1207801 |
| 2689600 | 1155625 | 1155625 | 0 | 2920681 | 2250000 |
| 1473796 | 181476 | 2253001 | 181476 | 0 | 3709476 |
| 628849 | 717409 | 0 | 717409 | 177241 | 5508409 |
| 156025 | 1550025 | 158404 | 1550025 | 670761 | 7535025 |
| 2209 | 121104 | 556516 | 2537649 | 1361889 | 2209 |
| 0 | 156025 | 628849 | 2689600 | 1473796 | 0 |
| 0 | 156025 | 628849 | 2689600 | 1473796 | 0 |
| 0 | 0 | 0 | 0 | 1473796 | 0 |
| 0 | 0 | 0 | 0 | 1473796 | 0 |
| 0 | 0 | 0 | 0 | 1473796 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |


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| 0 | 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 9801 | 9801 | 9801 | 0 | 9801 | 9801 |
| 123201 | 63504 | 123201 | 63504 | 123201 | 123201 |
| 451584 | 328329 | 451584 | 328329 | 451584 | 451584 |
| 1073296 | 877969 | 0 | 877969 | 1073296 | 1073296 |
| 2076481 | 164025 | 164025 | 1800964 | 2076481 | 2076481 |
| 3474496 | 685584 | 685584 | 3115225 | 3474496 | 3474496 |
| 5216656 | 1557504 | 1557504 | 4774225 | 5216656 | 5216656 |
| 7182400 | 156816 | 2702736 | 0 | 0 | 7182400 |
| 9168784 | 553536 | 0 | 121104 | 121104 | 277729 |
| 11042329 | 1079521 | 87025 | 413449 | 413449 | 53824 |
| 12638025 | 53824 | 277729 | 765625 | 765625 | 0 |
| 13816089 | 155236 | 474721 | 1075369 | 1075369 | 26244 |
| 14538969 | 240100 | 616225 | 1283689 | 1283689 | 66564 |
| 14814801 | 1296 | 0 | 1366561 | 1366561 | 86436 |
| 14707225 | 484 | 196 | 0 | 1334025 | 78400 |
| 14303524 | 961 | 4489 | 2809 | 1214404 | 51529 |
| 13719616 | 6084 | 21025 | 17161 | 0 | 22201 |
| 13032100 | 29584 | 57121 | 50625 | 8836 | 3025 |
| 12334144 | 72900 | 113569 | 104329 | 36864 | 1849 |
| 11710084 | 8100 | 182329 | 170569 | 79524 | 19321 |
| 11182336 | 28224 | 255025 | 241081 | 129600 | 3721 |


| 10778089 | 52441 | 320356 | 0 | 177241 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 10510564 | 1681 | 368449 | 1681 | 213444 | 1681 |
| 10387729 | 3600 | 0 | 3600 | 231361 | 3600 |
| 10368400 | 3969 | 9 | 3969 | 234256 | 3969 |
| 10445824 | 144 | 81 | 2601 | 0 | 2601 |
| 10595025 | 1225 | 1024 | 784 | 529 | 784 |
| 10797796 | 4356 | 3969 | 9 | 2916 | 9 |
| 11015761 | 1089 | 0 | 0 | 7569 | 1296 |
| 11235904 | 4356 | 1089 | 1089 | 14400 | 4761 |
| 11437924 | 9216 | 3969 | 3969 | 22500 | 1764 |
| 11600836 | 576 | 7569 | 7569 | 30276 | 324 |
| 11723776 | 1764 | 11025 | 11025 | 36864 | 0 |
| 11792356 | 2704 | 0 | 13225 | 40804 | 100 |
| 11812969 | 9 | 9 | 13924 | 0 | 169 |
| 11799225 | 1 | 1 | 0 | 4 | 121 |
| 11751184 | 36 | 36 | 49 | 81 | 16 |
| 11682724 | 100 | 256 | 289 | 361 | 36 |
| 11607649 | 441 | 0 | 784 | 900 | 289 |
| 11526025 | 1089 | + 144 | 1600 | 1764 | 841 |
| 11451456 | 121 | 529 | 2601 | 2809 | 1600 |
| 11383876 | 441 | 1089 | 3721 | 3969 | 169 |
| 11329956 | 841 | 1681 | 0 | 5041 | 25 |
| 11296321 | 25 | 0 | 25 | 0 | 0 |
| 11276164 | 64 | 9 | 64 | 9 | 9 |
| 11256025 | 121 | 36 | 121 | 36 | 36 |
| 11215801 | 36 | $144$ | $289$ | 144 | 144 |
| 11029041 | 1156 |  |  | 1600 | 1600 |
| 10595025 | 10000 | 0 | 12321 | 11236 | 11236 |
| 9872164 | 12769 | 12769 | 0 | 47961 | 47961 |
| 8779369 | 85264 | 85264 | 32041 | 158404 | 158404 |
| 7365796 | 292681 | 292681 | 183184 | 418609 | 418609 |
| 5779216 | 96100 | 724201 | 544644 | 0 | 586756 |
| 4165681 | 452929 | 0 | 1212201 | 131769 | 162409 |
| 2683044 | 1157776 | 162409 | 2262016 | 586756 | 0 |
| 1473796 | 179776 | 683929 | 3717184 | 1416100 | 179776 |
| 628849 | 714025 | 1557504 | 0 | 2595321 | 714025 |
| 156025 | 1545049 | 2709316 | 158404 | 4036081 | 1545049 |
| 1212658261 | 26967691 | 34795150 | 66831677 | 90789110 | 96968638 |

Table 35. RMS calculation for fundamental and odd order harmonics

| $\mathbf{1 3 6}, 56$ | $\mathbf{2 0 , 3 6}$ | $\mathbf{2 3 , 1 3}$ | $\mathbf{3 2 , 0 6}$ | $\mathbf{3 7 , 3 7}$ | $\mathbf{3 8 , 6 2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |

Table 36. SNR calculations for all the sampling schemes investigated in this dissertation


## Appendix B: Mod- $\Delta$ Data Analysis

| \# | I | Q | $\mathrm{MD}(\mathrm{N}+1) 3$ |  | $\mathrm{MD}(\mathrm{N}+1) 5$ | $\mathbf{M D}(\mathbf{N}+1) 7$ | $\mathrm{MD}(\mathrm{N}+1) 9$ | $\mathbf{M D}(\mathrm{N}+1) 11$ | $\mathbf{M D}(\mathbf{N}+1) 13$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 3374 | 3432 | 66 | 44 | 0 | 2 | 0 | 56 | 78 |
| 1 | 3366 | 3422 | 56 | 37 | 4 | 8 | 28 | 46 | 66 |
| 2 | 3362 | 3410 | 46 | 31 | 4 | 14 | 20 | 38 | 56 |
| 3 | 3360 | 3397 | 38 | 25 | 1 | 6 | 25 | 34 | 46 |
| 4 | 3356 | 3386 | 34 | 23 | 0 | 2 | 24 | 32 | 38 |
| 5 | 3348 | 3376 | 32 | 21 | 4 | 0 | 4 | 28 | 34 |
| 6 | 3321 | 3367 | 28 | 19 | 3 | 12 | 19 | 20 | 32 |
| 7 | 3256 | 3362 | 20 | 13 | 4 | 4 | 28 | 57 | 28 |
| 8 | 3140 | 3360 | 121 | 81 | 4 | 9 | 4 | 56 | 20 |
| 9 | 2963 | 3357 | 56 | 37 | 0 | 8 | 24 | 4 | 121 |
| 10 | 2716 | 3351 | 68 | 45 | 6 | 4 | 6 | 19 | 56 |
| 11 | 2404 | 3331 | 19 | 13 | 7 | 3 | 31 | 28 | 68 |
| 12 | 2040 | 3276 | 28 | 19 | 1 | 12 | 25 | 36 | 19 |
| 13 | 1638 | 3175 | 100 | 67 | 3 | 4 | 11 | 56 | 28 |
| 14 | 1215 | 3014 | 120 | 80 | 0 | 8 | 16 | 38 | 100 |
| 15 | 793 | 2784 | 102 | 68 | 0 | 6 | 0 | 63 | 120 |
| 16 | 395 | 2487 | 63 | 42 | 0 | 15 | 0 | 25 | 102 |
| 17 | 48 | 2135 | 25 | 17 | 0 | 9 | 0 | 11 | 63 |
| 18 | 0 | 1742 | 11 | 7 | 0 | 11 | 0 | 48 | 25 |
| 19 | 0 | 1322 | 48 | 32 | $0$ | 0 | 0 | 0 | 11 |
| 20 | 0 | 897 | 0 | 0 | 0 | 0 | 0 | 0 | 48 |
| 21 | 0 | 491 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | 0 | 129 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 25 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 32 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 33 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 34 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 35 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 36 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 37 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |


| 38 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 39 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 40 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 41 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 42 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 43 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 44 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 45 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 46 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 47 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 48 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 49 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 50 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 51 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 52 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 53 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 54 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 55 | 0 | 0 | 0 | $0$ | 0 | 0 | 0 | 0 | 0 |
| 56 | 0 | 0 | 0 | 0 | 0 | 0 | 16 | 0 | 0 |
| 57 | 0 | 0 | 0 | 0 | 4 | 0 | 4 | 0 | 0 |
| 58 | 0 | 0 | 0 | 0 | 3 | 0 | 27 | 0 | 0 |
| 59 | 112 | 0 | 0 | 0 | 4 | 0 | 12 | 0 | 0 |
| 60 | 356 | 0 | 0 | 0 | 4 | 0 | 28 | 48 | 0 |
| 61 | 667 | 0 | 112 | 75 | 7 | 0 | 31 | 36 | 0 |
| 62 | 1036 | 0 | 100 | 67 | 3 | 4 | 3 | 27 | 112 |
| 63 | 1436 | 0 | 27 | 18 | 7 | 11 | 15 | 12 | 100 |
| 64 | 1855 | 61 | 12 | 8 | 2 | 12 | 18 | 28 | 27 |
| 65 | 2275 | 289 | 28 | 19 | 1 | 12 | 25 | 63 | 12 |
| 66 | 2671 | 583 | 63 | 42 | 0 | 15 | 0 | 35 | 28 |
| 67 | 3026 | 940 | 99 | 66 | 3 | 3 | 3 | 47 | 63 |
| 68 | 3321 | 1334 | 111 | 74 | 4 | 15 | 4 | 18 | 99 |
| 69 | 3552 | 1749 | 82 | 55 | 3 | 2 | 11 | 57 | 111 |
| 70 | 3715 | 2172 | 121 | 81 | 6 | 9 | 30 | 32 | 82 |
| 71 | 3812 | 2576 | 96 | 64 | 0 | 0 | 8 | 3 | 121 |
| 72 | 3851 | 2942 | 3 | 2 | 1 | 3 | 25 | 36 | 96 |
| 73 | 3838 | 3253 | 100 | 67 | 4 | 4 | 28 | 11 | 3 |
| 74 | 3784 | 3501 | 11 | 7 | 3 | 11 | 27 | 62 | 100 |
| 75 | 3705 | 3682 | 126 | 84 | 0 | 14 | 0 | 8 | 11 |
| 76 | 3612 | 3794 | 72 | 48 | 1 | 8 | 17 | 57 | 126 |
| 77 | 3515 | 3847 | 121 | 81 | 4 | 9 | 20 | 28 | 72 |
| 78 | 3424 | 3846 | 28 | 19 | 3 | 12 | 11 | 59 | 121 |
| 79 | 3345 | 3801 | 59 | 39 | 5 | 11 | 21 | 32 | 28 |
| 80 | 3284 | 3727 | 96 | 64 | 3 | 0 | 19 | 17 | 59 |


| 81 | 3243 | 3636 | 17 | 11 | 0 | 1 | 0 | 20 | 96 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 82 | 3221 | 3539 | 84 | 56 | 6 | 4 | 22 | 43 | 17 |
| 83 | 3219 | 3445 | 43 | 29 | 5 | 11 | 21 | 21 | 84 |
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| 89 | 3381 | 3228 | 119 | 79 | 2 | 7 | 10 | 24 | 85 |
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| 91 | 3423 | 3276 | 53 | 35 | 3 | 5 | 11 | 13 | 24 |
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| 93 | 3438 | 3344 | 95 | 63 | 3 | 15 | 27 | 42 | 77 |
| 94 | 3435 | 3374 | 106 | 71 | 7 | 10 | 15 | 46 | 95 |
| 95 | 3429 | 3400 | 110 | 73 | 3 | 14 | 3 | 43 | 106 |
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| 97 | 3407 | 3432 | 101 | 67 | 5 | 5 | 13 | 27 | 107 |
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| 112 | 2041 | 3276 | 27 | 18 | 1 | 11 | 25 | 36 | 18 |
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| 122 | 0 | 128 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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| 195 | 0 | 0 | 0 | 0 | 7 | 0 | 31 | 0 | 0 |
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| 197 | 99 | 0 | 0 | 0 | 4 | 0 | 12 | 0 | 0 |
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| 201 | 1441 | 0 | 32 | 21 | 0 | 0 | 24 | 12 | 95 |
| 202 | 1864 | 50 | 12 | 8 | 4 | 12 | 20 | 33 | 32 |
| 203 | 2284 | 280 | 33 | 22 | 3 | 1 | 27 | 8 | 12 |
| 204 | 2680 | 587 | 72 | 48 | 3 | 8 | 3 | 44 | 33 |
| 205 | 3028 | 941 | 108 | 72 | 5 | 12 | 5 | 56 | 72 |
| 206 | 3323 | 1337 | 120 | 80 | 5 | 8 | 5 | 20 | 108 |
| 207 | 3555 | 1758 | 84 | 56 | 1 | 4 | 9 | 59 | 120 |
| 208 | 3717 | 2181 | 123 | 82 | 3 | 11 | 27 | 35 | 84 |
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| 210 | 3849 | 2946 | 5 | 3 | 0 | 5 | 24 | 37 | 99 |
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| 211 | 3835 | 3255 | 101 | 67 | 2 | 5 | 26 | 9 | 5 |
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| 213 | 3704 | 3682 | 123 | 82 | 6 | 11 | 30 | 6 | 9 |
| 214 | 3610 | 3795 | 70 | 47 | 0 | 6 | 16 | 56 | 123 |
| 215 | 3512 | 3845 | 120 | 80 | 3 | 8 | 19 | 26 | 70 |
| 216 | 3422 | 3842 | 26 | 17 | 2 | 10 | 10 | 56 | 120 |
| 217 | 3344 | 3798 | 56 | 37 | 7 | 8 | 23 | 30 | 26 |
| 218 | 3283 | 3725 | 94 | 63 | 4 | 14 | 20 | 16 | 56 |
| 219 | 3242 | 3634 | 16 | 11 | 0 | 0 | 0 | 19 | 94 |
| 220 | 3223 | 3536 | 83 | 55 | 7 | 3 | 23 | 42 | 16 |
| 221 | 3220 | 3443 | 42 | 28 | 6 | 10 | 22 | 23 | 83 |
| 222 | 3232 | 3362 | 23 | 15 | 7 | 7 | 23 | 20 | 42 |
| 223 | 3255 | 3296 | 20 | 13 | 0 | 4 | 24 | 32 | 23 |
| 224 | 3286 | 3250 | 32 | 21 | 6 | 0 | 22 | 55 | 20 |
| 225 | 3319 | 3226 | 55 | 37 | 6 | 7 | 14 | 22 | 32 |
| 226 | 3352 | 3219 | 86 | 57 | 0 | 6 | 0 | 55 | 55 |
| 227 | 3382 | 3228 | 119 | 79 | 2 | 7 | 10 | 24 | 86 |
| 228 | 3406 | 3248 | 24 | 16 | 5 | 8 | 13 | 54 | 119 |
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| 231 | 3437 | 3343 | 96 | 64 | 2 | 0 | 26 | 42 | 78 |
| 232 | 3435 | 3375 | 106 | 71 | 7 | 10 | 15 | 45 | 96 |
| 233 | 3428 | 3400 | 109 | 73 | 3 | 13 | 3 | 43 | 106 |
| 234 | 3418 | 3420 | 107 | 71 | 0 | 11 | 24 | 36 | 109 |
| 235 | 3407 | 3432 | 100 | 67 | 6 | 4 | 14 | 26 | 107 |
| 236 | 3395 | 3437 | 90 | 60 | 6 | 10 | 6 | 15 | 100 |
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| 238 | 3374 | 3430 | 67 | 45 | 6 | 3 | 30 | 56 | 79 |
| 239 | 3366 | 3421 | 56 | 37 | 3 | 8 | 27 | 46 | 67 |
| 240 | 3361 | 3410 | 46 | 31 | 5 | 14 | 21 | 38 | 56 |
| 241 | 3358 | 3398 | 38 | 25 | 1 | 6 | 25 | 33 | 46 |
| 242 | 3355 | 3387 | 33 | 22 | 7 | 1 | 23 | 30 | 38 |
| 243 | 3349 | 3376 | 30 | 20 | 6 | 14 | 6 | 27 | 33 |
| 244 | 3321 | 3367 | 27 | 18 | 3 | 11 | 19 | 21 | 30 |
| 245 | 3255 | 3362 | 21 | 14 | 2 | 5 | 26 | 57 | 27 |
| 246 | 3142 | 3359 | 121 | 81 | 4 | 9 | 4 | 55 | 21 |
| 247 | 2963 | 3356 | 55 | 37 | 1 | 7 | 25 | 6 | 121 |
| 248 | 2714 | 3352 | 70 | 47 | 6 | 6 | 6 | 19 | 55 |
| 249 | 2404 | 3331 | 19 | 13 | 6 | 3 | 30 | 26 | 70 |
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| 251 | 1638 | 3176 | 100 | 67 | 3 | 4 | 11 | 57 | 26 |
| 252 | 1214 | 3015 | 121 | 81 | 7 | 9 | 15 | 38 | 100 |


| 253 | 793 | 2782 | 102 | 68 | 0 | 6 | 0 | 62 | 121 |
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| 254 | 395 | 2486 | 62 | 41 | 0 | 14 | 0 | 25 | 102 |


| Noise(N+1)3 | Noise( $\mathrm{N}+1$ ) 5 | Noise(N+1)7 | Noise( $\mathrm{N}+1$ ) 9 | Noise(N+1)11 | Noise( $\mathrm{N}+1$ ) 13 |
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| 3302 | 3318 | 3309 | 3302 | 3301 | 3289 |
| 3243 | 3252 | 3252 | 3228 | 3199 | 3228 |
| 3059 | 3136 | 3131 | 3136 | 3084 | 3120 |
| 2926 | 2963 | 2955 | 2939 | 2959 | 2842 |
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| 2021 | 2039 | 2028 | 2015 | 2004 | 2021 |
| 1571 | 1635 | 1634 | 1627 | 1582 | 1610 |
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| 356 | 352 | 356 | 328 | 308 | 356 |
| 592 | 660 | 667 | 636 | 631 | 667 |
| 969 | 1033 | 1032 | 1033 | 1009 | 924 |
| 1418 | 1429 | 1425 | 1421 | 1424 | 1336 |
| 1847 | 1853 | 1843 | 1837 | 1827 | 1828 |
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| 2629 | 2671 | 2656 | 2671 | 2636 | 2643 |
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| 3247 | 3317 | 3306 | 3317 | 3303 | 3222 |
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| 3059 | 3136 | 3131 | 3136 | 3084 | 3119 |
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| 724 | 793 | 785 | 793 | 731 | 672 |
| 354 | 395 | 381 | 395 | 370 | 291 |
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| 3333 | 3357 | 3352 | 3333 | 3325 | 3312 |
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| 3329 | 3343 | 3335 | 3343 | 3322 | 3316 |
| 3303 | 3318 | 3310 | 3302 | 3300 | 3291 |
| 3241 | 3253 | 3250 | 3229 | 3198 | 3228 |
| 3061 | 3138 | 3133 | 3138 | 3087 | 3121 |
| 2926 | 2962 | 2956 | 2938 | 2957 | 2842 |
| 2667 | 2708 | 2708 | 2708 | 2695 | 2659 |
| 2391 | 2398 | 2401 | 2374 | 2378 | 2334 |
| 2024 | 2040 | 2031 | 2016 | 2005 | 2022 |
| 1571 | 1635 | 1634 | 1627 | 1581 | 1612 |
| 1133 | 1207 | 1205 | 1199 | 1176 | 1114 |
| 725 | 793 | 787 | 793 | 731 | 672 |
| 354 | 395 | 381 | 395 | 370 | 293 |


| S | $\mathbf{N}(\mathbf{N}+1) 3$ | $\mathrm{N}(\mathrm{N}+1) 5$ | $\mathrm{N}(\mathrm{N}+1) 7$ | $\mathbf{N}(\mathbf{N}+1) 9$ | $\mathrm{N}(\mathrm{N}+1) 11$ | $\mathrm{N}(\mathrm{N}+1) 13$ |
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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| 0 | 981,7777778 | 0 | 225 | 0 | 0 | 121 |
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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| 7182400 | 6927424 | 7166329 | 7139584 | 7166329 | 6948496 | 7006609 |
| 9168784 | 8737936 | 9138529 | 9096256 | 9138529 | 8832784 | 8737936 |
| 11042329 | 10517049 | 11009124 | 10989225 | 11009124 | 10909809 | 10336225 |
| 12638025 | 12243001 | 12630916 | 12609601 | 12574116 | 12222016 | 11799225 |
| 13816089 | 13213225 | 13793796 | 13734436 | 13616100 | 13557124 | 13198689 |
| 14538969 | 14040009 | 14493249 | 14516100 | 14493249 | 14500864 | 13616100 |
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| 13032100 | 12697344,44 | 13032100 | 12988816 | 12916836 | 12630916 | 12159169 |
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| 11812969 | 11377129 | 11799225 | 11812969 | 11634921 | 11526025 | 11282881 |
| 11799225 | 11318738,78 | 11751184 | 11730625 | 11696400 | 11492100 | 11148921 |
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| 1473796 | 1284444,444 | 1456849 | 1452025 | 1437601 | 1382976 | 1240996 |
| 628849 | 525625 | 628849 | 619369 | 628849 | 534361 | 451584 |
| 156025 | 125080,1111 | 156025 | 145161 | 156025 | 136900 | 85849 |
| 1212658261 | 1179902826 | 1210081113 | 1207007930 | 1200160281 | 1187208913 | 1,164E+09 |


| Srms | $\operatorname{Nrms}(\mathbf{N}+\mathbf{1}) 3$ | $\operatorname{Nrms}(\mathbf{N}+\mathbf{1}) 5$ | $\operatorname{Nrms}(\mathbf{N}+\mathbf{1}) 7$ | $\operatorname{Nrms}(\mathbf{N}+\mathbf{1}) 9$ | $\operatorname{Nrms}(\mathbf{N}+\mathbf{1}) 11$ | $\operatorname{Nrms}(\mathrm{~N}+\mathbf{1}) \mathbf{1 3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 3 6 , 5 6}$ | $\mathbf{1 3 4 , 7 0}$ | $\mathbf{1 3 6 , 4 2}$ | $\mathbf{1 3 6 , 2 4}$ | $\mathbf{1 3 5 , 8 6}$ | $\mathbf{1 3 5 , 1 2}$ | $\mathbf{1 3 3 , 7 9}$ |


| BIT's | SNR | dB |
| :---: | :---: | :---: |
| $\mathbf{2}$ | $\mathbf{1 , 0 3}$ | $\mathbf{1 1 , 8 9}$ |
| 4 | $\mathbf{1 , 0 0}$ | $\mathbf{0 , 9 2}$ |
| $\mathbf{6}$ | $\mathbf{1 , 0 0}$ | $\mathbf{2 , 0 3}$ |
| $\mathbf{8}$ | $\mathbf{1 , 0 1}$ | $\mathbf{4 , 5 0}$ |
| $\mathbf{1 0}$ | $\mathbf{1 , 0 2}$ | $\mathbf{9 , 2 1}$ |
| $\mathbf{1 2}$ | $\mathbf{1 , 0 4}$ | $\mathbf{1 7 , 8 0}$ |



## Appendix C: Mod- $\Delta$ (Gaussian) Data Analysis

| \# | I | Q | $\mathbf{M D}(\mathbf{N}+1) 3$ |  | $\mathrm{MD}(\mathrm{N}+1) 5$ | $\mathbf{M D}(\mathrm{N}+1) 7$ | $\mathrm{MD}(\mathrm{N}+1) 9$ | $\mathbf{M D}(\mathbf{N}+1) 11$ | $\mathbf{M D}(\mathrm{N}+1) 13$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 3374 | 3432 | 2 | 2 | 6 | 14 | 24 | 44 | 34 |
| 1 | 3366 | 3422 | 2 | 2 | 3 | 10 | 10 | 12 | 42 |
| 2 | 3362 | 3410 | 2 | 2 | 0 | 10 | 26 | 36 | 16 |
| 3 | 3360 | 3397 | 2 | 2 | 4 | 5 | 28 | 20 | 48 |
| 4 | 3356 | 3386 | 0 | 0 | 5 | 12 | 31 | 16 | 125 |
| 5 | 3348 | 3376 | 1 | 1 | 4 | 2 | 18 | 28 | 22371 |
| 6 | 3321 | 3367 | 0 | 0 | 5 | 11 | 1 | 22 | 6 |
| 7 | 3256 | 3362 | 3 | 3 | 7 | 0 | 12 | 12 | 15 |
| 8 | 3140 | 3360 | 1 | 1 | 1 | 8 | 26 | 7 | 65 |
| 9 | 2963 | 3357 | 0 | 0 | 3 | 3 | 23 | 50 | 69 |
| 10 | 2716 | 3351 | 0 | 0 | 6 | - 6 | 1 | 40 | 59 |
| 11 | 2404 | 3331 | 2 | 2 | 3 | 0 | 12 | 61 | 80 |
| 12 | 2040 | 3276 | 24578 | 24578 | 4 | 6 | 27 | 46 | 84 |
| 13 | 1638 | 3175 | 28419 | 28419 | 3 | 10 | 18 | 34 | 89 |
| 14 | 1215 | 3014 | 1 | 1 | 2941 | 0 | 23 | 37 | 61 |
| 15 | 793 | 2784 | 2 | 2 | 1 | 1 | 6 | 35 | 64 |
| 16 | 395 | 2487 | 0 | 0 | 3 | 14 | 25 | 29 | 108 |
| 17 | 48 | 2135 | 0 | $\bigcirc 0$ | 6 | 16390 | 12 | 14 | 126 |
| 18 | 0 | 1742 | 1 | 1 | ) 2 | 15 | 9 | 28 | 124 |
| 19 | 0 | 1322 | 2 | - 2 | 4 | $\bigcirc 15$ | 0 | 41 | 9 |
| 20 | 0 | 897 | 0 | 0 | 0 | 5 | 28 | 32 | 37 |
| 21 | 0 | 491 | 1 | 1 | 4 | 4 | 15 | 60 | 44 |
| 22 | 0 | 129 | 0 | 0 | 3 | 8 | 29 | 47 | 22 |
| 23 | 0 | 0 | 0 | 0 | 6 | 12 | 8 | 29 | 0 |
| 24 | 0 | 0 | 3 | 3 | 0 | 3 | 18 | 8 | 30 |
| 25 | 0 | 0 | 1 | 1 | 4 | 14 | 7 | 18 | 7 |
| 26 | 0 | 0 | 0 | 0 | 6 | 8 | 4 | 7 | 51 |
| 27 | 0 | 0 | 2 | 2 | 4 | 4 | 13 | 4 | 22 |
| 28 | 0 | 0 | 3 | 3 | 5 | 6 | 6 | 13 | 101 |
| 29 | 0 | 0 | 0 | 0 | 0 | 4 | 0 | 6 | 6 |
| 30 | 0 | 0 | 1 | 1 | 3 | 4 | 0 | 32 | 118 |
| 31 | 0 | 0 | 2 | 2 | 4 | 8 | 6 | 32 | 124 |
| 32 | 0 | 0 | 0 | 0 | 0 | 3 | 10 | 6 | 106 |
| 33 | 0 | 0 | 0 | 0 | 5 | 12 | 4 | 42 | 124 |
| 34 | 0 | 0 | 2 | 2 | 6 | 8 | 24 | 4 | 106 |


| 35 | 0 | 0 | 2 | 2 | 4 | 13 | 28 | 56 | 14 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 36 | 0 | 0 | 0 | 0 | 1 | 14 | 3 | 60 | 93 |
| 37 | 0 | 0 | 0 | 0 | 0 | 12 | 30 | 35 | 71 |
| 38 | 0 | 0 | 0 | 0 | 4 | 9 | 8 | 30 | 114 |
| 39 | 0 | 0 | 3 | 3 | 7 | 0 | 20 | 8 | 22 |
| 40 | 0 | 0 | 2 | 2 | 5 | 12 | 22 | 52 | 9 |
| 41 | 0 | 0 | 0 | 0 | 0 | 15 | 4 | 22 | 1 |
| 42 | 0 | 0 | 0 | 0 | 2 | 13 | 5 | 4 | 40 |
| 43 | 0 | 0 | 2 | 2 | 7 | 8 | 8 | 4 | 52 |
| 44 | 0 | 0 | 0 | 0 | 4 | 2 | 3 | 8 | 112 |
| 45 | 0 | 0 | 1 | 1 | 5 | 7 | 12 | 35 | 97 |
| 46 | 0 | 0 | 0 | 0 | 6 | 4 | 24 | 12 | 87 |
| 47 | 0 | 0 | 3 | 3 | 0 | 13 | 13 | 24 | 1 |
| 48 | 0 | 0 | 0 | 0 | 0 | 6 | 14 | 45 | 83 |
| 49 | 0 | 0 | 0 | 0 | 6 | 0 | 28 | 14 | 115 |
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| 51 | 0 | 0 | 2 | 2 | 4 | 6 | 0 | 41 | 9 |
| 52 | 0 | 0 | 0 | 0 | 0 | 10 | 28 | 32 | 37 |
| 53 | 0 | 0 | 1 | 1 | 4 | 4 | 15 | 60 | 44 |
| 54 | 0 | 0 | 0 | 0 | 3 | 8 | 29 | 47 | 22 |
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| 56 | 0 | 0 | 3 | 3 | 0 | 3 | 18 | 8 | 30 |
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| 59 | 112 | 0 | 2 | 2 | 4 | 4 | 29 | 40 | 22 |
| 60 | 356 | 0 | 2 | 2 | 1 | 6 | 10 | 40 | 85 |
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| 63 | 1436 | 0 | 1 | 1 | 0 | 8 | 2 | 31 | 8 |
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| 72 | 3851 | 2942 | 0 | 0 | 32752 | 5 | 1 | 50 | 22253 |
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| 74 | 3784 | 3501 | 1 | 1 | 2 | 0 | 16396 | 61 | 38 |
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| 78 | 3424 | 3846 | 1 | 1 | 6 | 8540 | 24 | 29 | 18 |
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| 85 | 3254 | 3297 | 2 | 2 | 2 | 15 | 5 | 17 | 204 |
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| 133 | 0 | 0 | 0 | 0 | 0 | 12 | 30 | 35 | 71 |
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| 136 | 0 | 0 | 2 | 2 | 5 | 12 | 22 | 52 | 9 |
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| Noise( $\mathrm{N}+1$ ) 3 | Noise( $\mathrm{N}+1$ )5 | Noise( $\mathrm{N}+1)^{7}$ | Noise( $\mathbf{N}+1$ ) 9 | Noise( $\mathrm{N}+1$ )11 | Noise( $\mathrm{N}+1$ ) 13 |
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| 3319 | 3315 | 3319 | 3292 | 3317 | 3237 |
| 3352 | 3346 | 3344 | 3336 | 3294 | -13513 |
| -13002 | 3380 | 3378 | 3364 | 3376 | 3344 |
| 3406 | 3399 | 3402 | 3389 | 3378 | 3387 |
| 3422 | 3424 | 3421 | 3394 | 3411 | 2219 |
| 3433 | 3428 | 3433 | 3416 | 3423 | 3352 |
| 3435 | 3433 | 3431 | 3436 | 3386 | 3437 |
| 3433 | 3435 | 3425 | 3434 | 3411 | 3317 |
| 3426 | 3424 | 3413 | 3420 | 3380 | 3320 |
| 3415 | 3414 | 3411 | 3387 | 3399 | 3406 |
| 3406 | 3401 | 3402 | 3384 | 3400 | 3393 |
| 3395 | 3388 | 3382 | 3389 | 3395 | 3332 |
| 3381 | 3379 | 3373 | 3380 | 3367 | 3348 |
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| 3139 | 3141 | 3139 | 3139 | 3140 | 3065 |
| 2961 | 2962 | 2962 | 2936 | 2908 | 2893 |
| 731 | 2712 | 2702 | 2702 | 2670 | 2665 |
| 2402 | 2404 | 2399 | 2393 | 2393 | 2348 |
| 2039 | 2034 | 2027 | 2012 | 1996 | 2018 |
| -31002 | 1636 | 1631 | 1619 | 1636 | 1623 |
| -26690 | 1211 | 1214 | -15174 | 1176 | 1139 |


| 790 | 792 | 785 | 768 | 776 | 725 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 395 | 389 | 381 | 384 | 380 | 380 |


| S | $\mathbf{N}(\mathbf{N}+1) 3$ | N(N+1)5 | $\mathbf{N}(\mathbf{N}+1) 7$ | $\mathrm{N}(\mathrm{N}+1) 9$ | $\mathrm{N}(\mathrm{N}+1) 11$ | $\mathrm{N}(\mathrm{N}+1) 13$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
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| 11303044 | 11289600 | 11303044 | 11235904 | 11128896 | 11062276 | 11195716 |
| 11289600 | 11276164 | 11262736 | 11256025 | 11102224 | 11155600 | 10969344 |
| 11262736 | 11262736 | 11229201 | 11182336 | 11055625 | 11155600 | 10439361 |
| 11209104 | 11202409 | 11182336 | 11195716 | 11088900 | 11022400 | 361874529 |
| 11029041 | 11029041 | 10995856 | 10956100 | 11022400 | 10883401 | 10989225 |
| 10601536 | 10582009 | 10556001 | 10601536 | 10523536 | 10523536 | 10504081 |
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| 2683044 | 717221961 | 2673225 | 2650384 | 2624400 | 2572816 | 2399401 |
| 1476225 | 1473796 | 2979076 | 1476225 | 1420864 | 1387684 | 1331716 |
| 628849 | 625681 | 627264 | 627264 | 619369 | 574564 | 531441 |
| 156025 | 156025 | 153664 | 145161 | 136900 | 133956 | 82369 |
| 2304 | 2304 | 1764 | 267060964 | 1296 | 1156 | 6084 |
| 0 | 1 | 4 | 225 | 81 | 784 | 15376 |
| 0 | 4 | 16 | 225 | 0 | 1681 | 81 |
| 0 | 0 |  | 25 | 784 | 1024 | 1369 |
| 0 | 1 | 16 | 16 | 225 | 3600 | 1936 |
| 0 | 0 |  | 64 | 841 | 2209 | 484 |
| 0 | 0 | 36 | 144 | 64 | 841 | 0 |
| 0 | 9 | 0 | 9 | 324 | 64 | 900 |
| 0 | 1 | 16 | 196 | 49 | 324 | 49 |
| 0 | 0 | 36 | 64 | 16 | 49 | 2601 |
| 0 | 4 | 16 | 16 | 169 | 16 | 484 |
| 0 | 9 | 25 | 36 | 36 | 169 | 10201 |
| 0 | 0 | 0 | 16 | 0 | 36 | 36 |
| 0 | 1 | 9 | 16 | 0 | 1024 | 13924 |
| 0 | 4 | 16 | 64 | 36 | 1024 | 15376 |
| 0 | 0 | 0 | 9 | 100 | 36 | 11236 |
| 0 | 0 | 25 | 144 | 16 | 1764 | 15376 |
| 0 | 4 | 36 | 64 | 576 | 16 | 11236 |
| 0 | 4 | 16 | 169 | 784 | 3136 | 196 |
| 0 | 0 | 1 | 196 | 9 | 3600 | 8649 |
| 0 | 0 | 0 | 144 | 900 | 1225 | 5041 |


| 0 | 0 | 16 | 81 | 64 | 900 | 12996 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 9 | 49 | 0 | 400 | 64 | 484 |
| 0 | 4 | 25 | 144 | 484 | 2704 | 81 |
| 0 | 0 | 0 | 225 | 16 | 484 | 1 |
| 0 | 0 | 4 | 169 | 25 | 16 | 1600 |
| 0 | 4 | 49 | 64 | 64 | 16 | 2704 |
| 0 | 0 | 16 | 4 | 9 | 64 | 12544 |
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| 0 | 9 | 0 | 169 | 169 | 576 | 1 |
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| 0 | 0 | 36 | 0 | 784 | 196 | 13225 |
| 0 | 1 | 4 | 0 | 81 | 784 | 5776 |
| 0 | 4 | 16 | 36 | 0 | 1681 | 81 |
| 0 | 0 | 0 | 100 | 784 | 1024 | 1369 |
| 0 | 1 | 16 | 16 | 225 | 3600 | 1936 |
| 0 | 0 | 9 | 64 | 841 | 2209 | 484 |
| 0 | 0 | 36 | 144 | 64 | 841 | 0 |
| 0 | 9 | 0 | 9 | 324 | 64 | 900 |
| 0 | 1 | 16 | 196 | 49 | 324 | 900 |
| 0 | 0 | 36 | 64 | 16 | 3025 | 2601 |
| 12544 | 12100 | 11664 | 11664 | 6889 | 5184 | 8100 |
| 126736 | 125316 | 126025 | 122500 | 119716 | 99856 | 73441 |
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| 11689561 | 11669056 | 11669056 | 11600836 | 11655396 | 11546404 | 11155600 |
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| 11029041 | 11029041 | 11009124 | 10936249 | 10830681 | 10634121 | 10627600 |
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| 4165681 | 216225 | 4161600 | 4124961 | 4137156 | 4080400 | 3944196 |
| 2689600 | 2683044 | 2669956 | 2683044 | 2676496 | 2650384 | 2347024 |
| 1473796 | 1468944 | 1473796 | 1447209 | 1456849 | 1347921 | 1350244 |
| 628849 | 243110464 | 620944 | 613089 | 589824 | 549081 | 521284 |
| 156025 | 153664 | 153664 | 153664 | 150544 | 144400 | 110889 |
| 2209 | 2116 | 1936 | 1225 | 289 | 169 | 64 |
| 0 | 0 | 9 | 36 | 841 | 2209 | 4761 |
| 0 | 0 | 36 | 25 | 64 | 841 | 0 |
| 0 | 9 | 0 | 196 | 324 | 64 | 900 |
| 0 | 1 | 16 | 169 | 49 | 324 | 49 |
| 0 | 0 | 36 | 64 | 16 | 49 | 2601 |
| 0 | 4 | 16 | 16 | 169 | 16 | 484 |
| 0 | 9 | 25 | 36 | 36 | 169 | 10201 |
| 0 | 0 | 0 | 16 | 0 | 36 | 36 |


| 0 | 1 | 9 | 16 | 0 | 1024 | 13924 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
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| 0 | 0 | 0 | 9 | 100 | 36 | 11236 |
| 0 | 0 | 25 | 144 | 16 | 1764 | 15376 |
| 0 | 4 | 36 | 64 | 576 | 16 | 11236 |
| 0 | 4 | 16 | 169 | 784 | 3136 | 196 |
| 0 | 0 | 1 | 196 | 9 | 3600 | 8649 |
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| 0 | 0 | 16 | 81 | 64 | 900 | 12996 |
| 0 | 9 | 49 | 0 | 400 | 64 | 484 |
| 0 | 4 | 25 | 144 | 484 | 2704 | 81 |
| 0 | 4 | 0 | 225 | 16 | 576 | 1 |
| 4 | 4 | 4 | 121 | 25 | 4 | 1444 |
| 0 | 4 | 49 | 64 | 64 | 16 | 2916 |
| 0 | 0 | 16 | 4 | 9 | 64 | 12544 |
| 0 | 4 | 25 | 49 | 144 | 1296 | 9409 |
| 1 | 0 | 36 | 25 | 576 | 144 | 7396 |
| 1 | 4 | 0 | 144 | 169 | 529 | 1 |
| 0 | 9 | 0 | 36 | 196 | 2304 | 7056 |
| 9 | 9 | 4 | 9 | 784 | 121 | 12544 |
| 0 | 1 | 4 | 1 | 81 | 784 | 6241 |
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| 0 | 0 | 0 | 100 | 784 | 1024 | 1369 |
| 0 | 1 | 16 | 49 | 225 | 3600 | 1936 |
| 0 | 0 | 9 | 64 | 841 | 2209 | 484 |
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| 0 | 9 | 0 | 9 | 324 | 64 | 900 |
| 0 | 1 | 16 | 196 | 49 | 324 | 900 |
| 0 | 0 | 36 | 64 | 16 | 49 | 2601 |
| 0 | 4 | 16 | 16 | 169 | 16 | 484 |
| 0 | 9 | 25 | 36 | 36 | 169 | 10201 |
| 0 | 0 | 0 | 16 | 0 | 36 | 36 |
| 0 | 1 | 9 | 25 | 0 | 1024 | 13924 |
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| 0 | 4 | 16 | 169 | 784 | 3136 | 196 |
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| 0 | 0 | 16 | 81 | 64 | 900 | 12996 |
| 0 | 9 | 49 | 0 | 400 | 64 | 484 |
| 0 | 4 | 25 | 144 | 484 | 2704 | 81 |
| 0 | 0 | 0 | 225 | 16 | 484 | 1 |



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| 11723776 | 11710084 | 11723776 | 11703241 | 11519236 | 11634921 | 4923961 |
| 11792356 | 11785489 | 11751184 | 11785489 | 11669056 | 11716929 | 11235904 |
| 11812969 | 11799225 | 11785489 | 11771761 | 11806096 | 11464996 | 11812969 |
| 11799225 | 11785489 | 11799225 | 11730625 | 11792356 | 11634921 | 11002489 |
| 11751184 | 11737476 | 11723776 | 11648569 | 11696400 | 11424400 | 11022400 |
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| 11607649 | 11600836 | 11566801 | 11573604 | 11451456 | 11560000 | 11512449 |
| 11526025 | 11526025 | 11478544 | 11437924 | 11485321 | 11526025 | 11102224 |
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| 11383876 | 11370384 | 11356900 | 11289600 | 11343424 | 11048976 | 11282881 |
| 11329956 | 11329956 | 11289600 | 11249316 | 11202409 | 10949481 | 11015761 |
| 11296321 | 11282881 | 11289600 | 11235904 | 11195716 | 11222500 | 10497600 |
| 11276164 | 11256025 | 11249316 | 11222500 | 11102224 | 11002489 | 7001316 |
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| 11215801 | 11195716 | 11209104 | 11135569 | 11075584 | 10989225 | 723341025 |
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| 8779369 | 8767521 | 8773444 | 8773444 | 8620096 | 8456464 | 8369449 |
| 7365796 | 534361 | 7354944 | 7300804 | 7300804 | 7128900 | 7102225 |
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| 2683044 | 961124004 | 2676496 | 2660161 | 2621161 | 2676496 | 2634129 |
| 1473796 | 712356100 | 1466521 | 1473796 | 230250276 | 1382976 | 1297321 |
| 628849 | 624100 | 627264 | 616225 | 589824 | 602176 | 525625 |
| 156025 | 156025 | 151321 | 145161 | 147456 | 144400 | 144400 |
| 1212658261 | 12751739404 | 2946509190 | 5789411088 | 3849500082 | 1333070940 | 4,785E+09 |



## Appendix D: Mod- $\Delta$ (Noise) Data Analysis

|  |  |  | MD( $\mathrm{N}+1$ ) | $\mathbf{M D}(\mathrm{N}+1)$ | $\mathbf{M D}(\mathrm{N}+1)$ | $\mathbf{M D}(\mathrm{N}+1)$ | $\mathbf{M D}(\mathrm{N}+1) 1$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \# | I | Q | 3 | 5 | 7 | 9 | 1 | 3 |
| 0 | 3374 | 3432 | 120 | 3 | 15 | 22 | 4 | 34 |
| 1 | 3366 | 3422 | 38 | 27993 | 9 | 3 | 0 | 42 |
| 2 | 3362 | 3410 | 123 | 2 | 27845 | 23 | 19 | 16 |
| 3 | 3360 | 3397 | 120 | 4 | 22508 | 29 | 50 | 48 |
| 4 | 3356 | 3386 | 103 | 7 | 31109 | 7 | 57 | 125 |
| 5 | 3348 | 3376 | 120 | 3 | 9 | 12 | 63 | 22371 |
| 6 | 3321 | 3367 | 22371 | 0 | 32645 | 4 | 46 | 6 |
| 7 | 3256 | 3362 | 5 | 0 | 14 | 31117 | 0 | 15 |
| 8 | 3140 | 3360 | 84 | 4 | 4 | 24 | 61 | 65 |
| 9 | 2963 | 3357 | 59 | 2 | 1 | 8 | 24 | 69 |
| 10 | 2716 | 3351 | 16475 | 7 | 3 | 26 | 14 | 59 |
| 11 | 2404 | 3331 | 108 | 5 | 7 | 24 | 6 | 80 |
| 12 | 2040 | 3276 | 90 | 28422 | 14 | 4 | 3 | 84 |
| 13 | 1638 | 3175 | 61 | 7 | 11 | 29 | 6 | 89 |
| 14 | 1215 | 3014 | 60 | 16384 | 5 | 12 | 11 | 61 |
| 15 | 793 | 2784 | 114 | 1 | 15 | 1 | 33 | 64 |
| 16 | 395 | 2487 | 31 | 3 | 31111 | 21 | 57 | 108 |
| 17 | 48 | 2135 | 1 | 1699 | 10 | 6 | 1 | 126 |
| 18 | 0 | 1742 | 51 | 7 | 12 | 22 | 40 | 124 |
| 19 | 0 | 1322 | 14 | 3 | 10 | 28 | 52 | 9 |
| 20 | 0 | 897 | 81 | 518 | 14 | 10 | 48 | 37 |
| 21 | 0 | 491 | 115 | 7 | S 13 | 28 | 33 | 44 |
| 22 | 0 | 129 | 33 | 3 | 7 | 10 | 23 | 22 |
| 23 | 0 | 0 | 33 | 6 | 2 | 14 | 1 | 0 |
| 24 | 0 | 0 | 105 | 5 | 6 | 29 | 19 | 30 |
| 25 | 0 | 0 | 113 | 6 | 9 | 7 | 51 | 7 |
| 26 | 0 | 0 | 16 | 6 | 1 | 18 | 12 | 51 |
| 27 | 0 | 0 | 11 | 4 | 8 | 22 | 9 | 22 |
| 28 | 0 | 0 | 88 | 2 | 4 | 9 | 37 | 101 |
| 29 | 0 | 0 | 25 | 4 | 0 | 1 | 44 | 6 |
| 30 | 0 | 0 | 52 | 2 | 1 | 8 | 22 | 118 |
| 31 | 0 | 0 | 27 | 6 | 7 | 20 | 0 | 124 |
| 32 | 0 | 0 | 42 | 5 | 1 | 16 | 30 | 106 |
| 33 | 0 | 0 | 100 | 7 | 3 | 1 | 7 | 124 |
| 34 | 0 | 0 | 67 | 2 | 3 | 23 | 51 | 106 |
| 35 | 0 | 0 | 74 | 6 | 12 | 1 | 22 | 14 |
| 36 | 0 | 0 | 65 | 1 | 9 | 19 | 37 | 93 |


| 37 | 0 | 0 | 86 | 1 | 5 | 19 | 6 | 71 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 38 | 0 | 0 | 35 | 0 | 31116 | 12 | 54 | 114 |
| 39 | 0 | 0 | 105 | 4 | 6 | 9 | 60 | 22 |
| 40 | 0 | 0 | 64 | 0 | 0 | 5 | 42 | 9 |
| 41 | 0 | 0 | 66 | 1 | 14 | 12 | 60 | 1 |
| 42 | 0 | 0 | 35 | 7 | 7 | 22 | 42 | 40 |
| 43 | 0 | 0 | 40 | 1 | 3 | 0 | 14 | 52 |
| 44 | 0 | 0 | 71 | 3 | 6 | 30 | 29 | 112 |
| 45 | 0 | 0 | 33 | 3 | 5 | 7 | 7 | 97 |
| 46 | 0 | 0 | 88 | 4 | 6 | 19 | 50 | 87 |
| 47 | 0 | 0 | 122 | 1 | 6 | 22 | 22 | 1 |
| 48 | 0 | 0 | 57 | 5 | 12 | 5 | 9 | 83 |
| 49 | 0 | 0 | 66 | 516 | 10 | 6 | 1 | 115 |
| 50 | 0 | 0 | 26 | 6 | 12 | 22 | 40 | 76 |
| 51 | 0 | 0 | 3 | 0 | 10 | 28 | 52 | 9 |
| 52 | 0 | 0 | 33 | 6 | 14 | 10 | 48 | 37 |
| 53 | 0 | 0 | 115 | 6 | 13 | 28 | 33 | 44 |
| 54 | 0 | 0 | 33 | 3 | 7 | 10 | 23 | 22 |
| 55 | 0 | 0 | 120 | 6 | 2 | 14 | 1 | 0 |
| 56 | 0 | 0 | 105 | 5 | 6 | 29 | 19 | 30 |
| 57 | 0 | 0 | 113 | 6 | 9 | 7 | 51 | 30 |
| 58 | 0 | 0 | 16 | 6 | 1 | 2 | 60 | 51 |
| 59 | 112 | 0 | 11 | 4 | 8 | 26 | 45 | 22 |
| 60 | 356 | 0 | 88 | 2 | 8 | 4 | 0 | 85 |
| 61 | 667 | 0 | 25 | 4 | 11 | 13 | 56 | 106 |
| 62 | 1036 | 0 | 36 | 2 | 13 | 4 | 50 | 17 |
| 63 | 1436 | 0 | 127 | 16642 | 3 | 19 | 63 | 8 |
| 64 | 1855 | 61 | 69 | 0 | 32640 | 19 | 1 | 6 |
| 65 | 2275 | 289 | 112 | 3 | 6 | 16 | 54 | 59 |
| 66 | 2671 | 583 | 95 | 6 | 2 | 9 | 5 | 77 |
| 67 | 3026 | 940 | 9 | 5 | 14 | 26 | 15 | 125 |
| 68 | 3321 | 1334 | 36 | 4 | 1938 | 19 | 5 | 47 |
| 69 | 3552 | 1749 | 69 | 0 | 5 | 13846 | 9 | 22368 |
| 70 | 3715 | 2172 | 117 | 2 | 16655 | 16 | 26 | 16466 |
| 71 | 3812 | 2576 | 6002 | 5 | 26602 | 32660 | 32519 | 25 |
| 72 | 3851 | 2942 | 32 | 0 | 11 | 3 | 40 | 22253 |
| 73 | 3838 | 3253 | 69 | 4 | 5820 | 20 | 4 | 12 |
| 74 | 3784 | 3501 | 7 | 3 | 6 | 4095 | 35 | 38 |
| 75 | 3705 | 3682 | 51 | 6140 | 12 | 28 | 42 | 124 |
| 76 | 3612 | 3794 | 69 | 30361 | 2 | 16793 | 20248 | 6009 |
| 77 | 3515 | 3847 | 105 | 3 | 0 | 7 | 16423 | 31613 |
| 78 | 3424 | 3846 | 81 | 30357 | 6 | 4 | 16387 | 18 |
| 79 | 3345 | 3801 | 22 | 28421 | 7 | 10 | 42 | 97 |


| 80 | 3284 | 3727 | 116 | 0 | 0 | 16 | 52 | 100 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 81 | 3243 | 3636 | 34 | 6132 | 5 | 27 | 22 | 71 |
| 82 | 3221 | 3539 | 43 | 7 | 1 | 9 | 59 | 119 |
| 83 | 3219 | 3445 | 22391 | 4 | 22525 | 28 | 20 | 5822 |
| 84 | 3232 | 3363 | 204 | 1 | 18366 | 0 | 38 | 56 |
| 85 | 3254 | 3297 | 8 | 4 | 3 | 17 | 54 | 204 |
| 86 | 3285 | 3251 | 52 | 6 | 26364 | 13825 | 14 | 76 |
| 87 | 3319 | 3225 | 24 | 6 | 16393 | 6 | 25 | 85 |
| 88 | 3352 | 3218 | 6015 | 3 | 14 | 18 | 8 | 21 |
| 89 | 3381 | 3228 | 70 | 3 | 14 | 18932 | 0 | 31 |
| 90 | 3405 | 3248 | 7 | 5 | 14 | 16785 | 23339 | 104 |
| 91 | 3423 | 3276 | 22371 | 4 | 7 | 0 | 51 | 99 |
| 92 | 3434 | 3310 | 19725 | 7 | 14 | 23 | 19 | 68 |
| 93 | 3438 | 3344 | 102 | 1 | 14 | 12 | 16407 | 112 |
| 94 | 3435 | 3374 | 22323 | 513 | 12 | 13 | 59 | 100 |
| 95 | 3429 | 3400 | 5 | 32736 | 12 | 15 | 27 | 103 |
| 96 | 3419 | 3419 | 24 | 3 | 12 | 31 | 557 | 79 |
| 97 | 3407 | 3432 | 79 | 26330 | 2 | 4 | 33 | 87 |
| 98 | 3395 | 3437 | 40 | 24839 | 25702 | 14 | 42 | 57 |
| 99 | 3383 | 3437 | 37 | 1 | 26595 | 14 | 3 | 81 |
| 100 | 3373 | 3432 | 16 | 0 | 6 | 26 | 12 | 20 |
| 101 | 3367 | 3422 | 25 | 4 | 12 | 2036 | 39 | 21876 |
| 102 | 3361 | 3410 | 22394 | 7 | 31117 | 11 | 21 | 25 |
| 103 | 3359 | 3398 | 22 | 1 | 5 | 5 | 24 | 55 |
| 104 | 3356 | 3386 | 103 | 7 | 28620 | 26 | 63 | 40 |
| 105 | 3349 | 3375 | 99 | 2 | 3 | 1989 | 53 | 29 |
| 106 | 3321 | 3368 | 66 | 6 | 0 | 14 | 16674 | 61 |
| 107 | 3256 | 3362 | 68 | 5 | 11 | 4 | 18 | 45 |
| 108 | 3140 | 3359 | 92 | 0 | 10 | 16 | 47 | 40 |
| 109 | 2962 | 3357 | 26 | 4 | 7 | 25 | 34 | 37 |
| 110 | 2715 | 3351 | 5936 | 4 | 1 | 23 | 22 | 105 |
| 111 | 2404 | 3331 | 16446 | 26613 | 10 | 15 | 15 | 28 |
| 112 | 2041 | 3276 | 75 | 28671 | 5 | 13 | 49 | 55 |
| 113 | 1640 | 3175 | 93 | 7 | 2 | 4 | 63 | 108 |
| 114 | 1214 | 3013 | 126 | 2 | 1674 | 15 | 1 | 52 |
| 115 | 793 | 2783 | 124 | 1 | 32131 | 7 | 63 | 71 |
| 116 | 395 | 2487 | 9 | 16646 | 6073 | 25 | 31 | 62 |
| 117 | 47 | 2136 | 49 | 5 | 32524 | 28 | 33 | 55 |
| 118 | 0 | 1743 | 58 | 4 | 7 | 10 | 23 | 69 |
| 119 | 0 | 1322 | 44 | 1 | 2 | 14 | 1 | 0 |
| 120 | 0 | 897 | 24 | 4 | 6 | 29 | 19 | 30 |
| 121 | 0 | 491 | 113 | 6 | 9 | 7 | 51 | 7 |
| 122 | 0 | 128 | 16 | 6 | 1 | 18 | 12 | 51 |


| 123 | 0 | 0 | 11 | 4 | 8 | 22 | 9 | 22 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 124 | 0 | 0 | 88 | 2 | 4 | 9 | 37 | 101 |
| 125 | 0 | 0 | 25 | 4 | 0 | 1 | 44 | 6 |
| 126 | 0 | 0 | 52 | 2 | 1 | 8 | 22 | 118 |
| 127 | 0 | 0 | 27 | 6 | 7 | 20 | 0 | 124 |
| 128 | 0 | 0 | 42 | 5 | 1 | 16 | 30 | 106 |
| 129 | 0 | 0 | 100 | 7 | 3 | 1 | 7 | 124 |
| 130 | 0 | 0 | 67 | 2 | 3 | 23 | 51 | 106 |
| 131 | 0 | 0 | 74 | 6 | 12 | 1 | 22 | 14 |
| 132 | 0 | 0 | 65 | 1 | 9 | 19 | 37 | 93 |
| 133 | 0 | 0 | 86 | 1 | 5 | 19 | 6 | 71 |
| 134 | 0 | 0 | 35 | 0 | 31116 | 12 | 54 | 114 |
| 135 | 0 | 0 | 105 | 4 | 6 | 9 | 60 | 22 |
| 136 | 0 | 0 | 64 | 0 | 0 | 5 | 42 | 9 |
| 137 | 0 | 0 | 66 | 1 | 14 | 14 | 62 | 1 |
| 138 | 2 | 0 | 35 | 7 | 9 | 22 | 42 | 40 |
| 139 | 0 | 0 | 40 | 1 | 3 | 0 | 14 | 54 |
| 140 | 0 | 0 | 71 | 3 | 6 | 30 | 29 | 112 |
| 141 | 0 | 0 | 35 | 5 | 5 | 8 | 8 | 97 |
| 142 | 1 | 0 | 88 | 4 | 7 | 20 | 51 | 87 |
| 143 | 1 | 2 | 122 | 1 | 7 | 22 | 22 | 2 |
| 144 | 0 | 0 | 57 | 5 | 12 | 8 | 12 | 84 |
| 145 | 3 | 0 | 67 | 517 | 13 | 6 | 1 | 115 |
| 146 | 0 | 0 | 27 | 7 | 12 | 22 | 40 | 79 |
| 147 | 0 | 0 | 3 | 0 |  | 28 | 52 | 9 |
| 148 | 0 | 1 | 36 | 1 | 14 | 10 | 48 | 37 |
| 149 | 0 | 0 | 115 | 6 | 13 | 28 | 33 | 44 |
| 150 | 0 | 2 | 33 | 3 | 7 | 10 | 23 | 22 |
| 151 | 0 | 1 | 120 | 6 | 2 | 14 | 1 | 0 |
| 152 | 0 | 0 | 105 | 5 | 6 | 29 | 19 | 30 |
| 153 | 0 | 0 | 113 | 6 | 9 | 7 | 51 | 30 |
| 154 | 0 | 0 | 16 | 6 | 1 | 18 | 12 | 51 |
| 155 | 0 | 0 | 11 | 4 | 8 | 22 | 9 | 22 |
| 156 | 0 | 0 | 88 | 2 | 4 | 9 | 37 | 101 |
| 157 | 0 | 0 | 25 | 4 | 0 | 1 | 44 | 6 |
| 158 | 0 | 0 | 52 | 2 | 1 | 8 | 22 | 118 |
| 159 | 0 | 0 | 27 | 6 | 7 | 20 | 0 | 124 |
| 160 | 0 | 0 | 42 | 5 | 1 | 16 | 30 | 106 |
| 161 | 0 | 0 | 100 | 7 | 3 | 1 | 7 | 124 |
| 162 | 0 | 0 | 67 | 2 | 3 | 23 | 51 | 106 |
| 163 | 0 | 0 | 74 | 6 | 12 | 1 | 22 | 14 |
| 164 | 0 | 0 | 65 | 1 | 9 | 19 | 37 | 93 |
| 165 | 0 | 0 | 86 | 1 | 5 | 19 | 6 | 71 |


| 166 | 0 | 0 | 35 | 0 | 31116 | 12 | 54 | 114 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 167 | 0 | 0 | 105 | 4 | 6 | 9 | 60 | 22 |
| 168 | 0 | 0 | 64 | 0 | 0 | 5 | 42 | 9 |
| 169 | 0 | 0 | 66 | 1 | 14 | 12 | 60 | 1 |
| 170 | 0 | 0 | 35 | 7 | 14 | 22 | 42 | 40 |
| 171 | 0 | 0 | 40 | 1 | 3 | 0 | 14 | 52 |
| 172 | 0 | 0 | 71 | 3 | 6 | 30 | 29 | 112 |
| 173 | 0 | 0 | 33 | 3 | 5 | 7 | 7 | 97 |
| 174 | 0 | 0 | 88 | 4 | 6 | 19 | 50 | 87 |
| 175 | 0 | 0 | 122 | 1 | 6 | 22 | 22 | 1 |
| 176 | 0 | 0 | 57 | 5 | 12 | 5 | 9 | 83 |
| 177 | 0 | 0 | 66 | 516 | 10 | 6 | 1 | 115 |
| 178 | 0 | 0 | 26 | 6 | 12 | 22 | 40 | 76 |
| 179 | 0 | 0 | 3 | 0 | 10 | 28 | 52 | 9 |
| 180 | 0 | 0 | 33 | 6 | 14 | 10 | 48 | 37 |
| 181 | 0 | 0 | 115 | 7 | 13 | 28 | 33 | 44 |
| 182 | 0 | 0 | 33 | 3 | 7 | 10 | 23 | 22 |
| 183 | 0 | 0 | 120 | 6 | 2 | 14 | 1 | 0 |
| 184 | 0 | 0 | 105 | 5 |  | 29 | 19 | 30 |
| 185 | 0 | 0 | 113 | 6 | 9 | 7 | 51 | 7 |
| 186 | 0 | 0 | 16 | 6 | 1 | 18 | 12 | 51 |
| 187 | 0 | 0 | 11 | 4 | 8 | 22 | 9 | 22 |
| 188 | 0 | 0 | 88 | 2 | 4 | 9 | 37 | 101 |
| 189 | 0 | 0 | 25 | 4 | 0 | 1 | 44 | 6 |
| 190 | 0 | 0 | 52 | 2 |  | 8 | 22 | 118 |
| 191 | 0 | 0 | 27 | 6 | 7 | 20 | 0 | 124 |
| 192 | 0 | 0 | 42 | 5 | 1 | 16 | 30 | 106 |
| 193 | 0 | 0 | 100 | 7 | 3 | 1 | 30 | 124 |
| 194 | 0 | 0 | 67 | 2 | 3 | 23 | 51 | 106 |
| 195 | 0 | 0 | 74 | 6 | 12 | 1 | 22 | 14 |
| 196 | 0 | 0 | 65 | 1 | 9 | 22 | 8 | 93 |
| 197 | 99 | 0 | 86 | 1 | 8 | 18 | 37 | 71 |
| 198 | 351 | 0 | 35 | 0 | 16651 | 12 | 22 | 85 |
| 199 | 672 | 0 | 105 | 4 | 6 | 21 | 8 | 117 |
| 200 | 1036 | 0 | 35 | 3 | 12 | 6 | 11 | 41 |
| 201 | 1441 | 0 | 33 | 0 | 15 | 20 | 4 | 13 |
| 202 | 1864 | 50 | 67 | 7 | 15 | 2 | 22 | 73 |
| 203 | 2284 | 280 | 52 | 5 | 15 | 24 | 6 | 124 |
| 204 | 2680 | 587 | 104 | 4 | 14 | 18 | 49 | 92 |
| 205 | 3028 | 941 | 105 | 3 | 9 | 25 | 2 | 89 |
| 206 | 3323 | 1337 | 68 | 0 | 1 | 22 | 21 | 43 |
| 207 | 3555 | 1758 | 114 | 32513 | 9 | 27 | 27 | 124 |
| 208 | 3717 | 2181 | 13 | 1 | 1 | 18890 | 46 | 54 |


| 209 | 3813 | 2585 | 8189 | 7 | 18911 | 16399 | 10 | 22264 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 210 | 3849 | 2946 | 125 | 1 | 5 | 29073 | 35 | 49 |
| 211 | 3835 | 3255 | 584 | 6141 | 5 | 2 | 58 | 530 |
| 212 | 3782 | 3504 | 20230 | 32715 | 4 | 2 | 40 | 32 |
| 213 | 3704 | 3682 | 124 | 0 | 9989 | 32662 | 59 | 114 |
| 214 | 3610 | 3795 | 6012 | 6 | 6097 | 2 | 15 | 5950 |
| 215 | 3512 | 3845 | 103 | 4 | 18346 | 12 | 31 | 26 |
| 216 | 3422 | 3842 | 24881 | 5 | 4 | 13 | 35 | 86 |
| 217 | 3344 | 3798 | 20235 | 0 | 9 | 16858 | 6 | 101 |
| 218 | 3283 | 3725 | 6008 | 6 | 4 | 2044 | 54 | 67 |
| 219 | 3242 | 3634 | 105 | 2 | 2 | 13 | 16416 | 105 |
| 220 | 3223 | 3536 | 104 | 2 | 11 | 29 | 57 | 15 |
| 221 | 3220 | 3443 | 108 | 7 | 4 | 1 | 12 | 29 |
| 222 | 3232 | 3362 | 94 | 20444 | 1 | 31 | 13 | 63 |
| 223 | 3255 | 3296 | 6002 | 30637 | 14 | 10 | 22 | 36 |
| 224 | 3286 | 3250 | 62 | 1 | 7 | 7 | 21 | 15 |
| 225 | 3319 | 3226 | 4 | 30639 | 10 | 25 | 31 | 47 |
| 226 | 3352 | 3219 | 122 | 26609 | 28603 | 13 | 41 | 121 |
| 227 | 3382 | 3228 | 32 | 4 | 26594 | 15 | 36 | 6004 |
| 228 | 3406 | 3248 | 56 | 0 | 6135 | 19 | 5 | 106 |
| 229 | 3424 | 3278 | 110 | 1 | 5 | 18909 | 48 | 30244 |
| 230 | 3434 | 3311 | 22393 | 6 | 518 | 1977 | 35 | 58 |
| 231 | 3437 | 3343 | 55 | 2 | 3 | 20 | 39 | 16421 |
| 232 | 3435 | 3375 | 32 | 0 | 30715 | 9 | 14 | 77 |
| 233 | 3428 | 3400 | 44 | 3 | 18802 | 6134 | 22 | 70 |
| 234 | 3418 | 3420 | 16 | 4 | 1 | 5 | 16441 | 49 |
| 235 | 3407 | 3432 | 16403 | 4 | 2 | 18883 | 17 | 56 |
| 236 | 3395 | 3437 | 43 | 7 | 9 | 18006 | 21 | 23 |
| 237 | 3384 | 3437 | 123 | 32757 | 13 | 21 | 53 | 15 |
| 238 | 3374 | 3430 | 5863 | 30627 | 4 | 25 | 16408 | 75 |
| 239 | 3366 | 3421 | 61 | 4 | 12 | 23 | 55 | 68 |
| 240 | 3361 | 3410 | 113 | 5 | 13 | 3 | 39 | 15 |
| 241 | 3358 | 3398 | 112 | 30410 | 8 | 1 | 28 | 7 |
| 242 | 3355 | 3387 | 64 | 4 | 7 | 11 | 61 | 25 |
| 243 | 3349 | 3376 | 1636 | 30697 | 29967 | 21 | 45 | 124 |
| 244 | 3321 | 3367 | 16447 | 25580 | 18311 | 1 | 39 | 106 |
| 245 | 3255 | 3362 | 14 | 1 | 4 | 2 | 39 | 14 |
| 246 | 3142 | 3359 | 54 | 0 | 27901 | 29 | 42 | 93 |
| 247 | 2963 | 3356 | 113 | 7 | 15733 | 1992 | 27 | 71 |
| 248 | 2714 | 3352 | 8176 | 4 | 0 | 1 | 55 | 114 |
| 249 | 2404 | 3331 | 16695 | 4 | 13 | 0 | 44 | 22 |
| 250 | 2041 | 3275 | 35 | 1 | 10 | 24 | 50 | 9 |
| 251 | 1638 | 3176 | 37 | 16390 | 16398 | 20 | 7 | 1 |


| 252 | 1214 | 3015 | 60 | 6 | 2 | 2 | 62 | 40 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 253 | 793 | 2782 | 18 | 5 | 9 | 12 | 55 | 52 |
| 254 | 395 | 2486 | 26 | 0 | 16652 | 52 | 5 | 112 |

