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MODAL FEATURES AND DYNAMIC BEHAVIOR OF A NONLINEAR 3D GUYED MAST WITH UNCERTAIN GUYS PRETENSION

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Abstract. The study of the nonlinear dynamic characteristics and response of a guyed mast, considering the uncertainty of the guys pretension is reported in this work. A computational model is constructed with the mast represented by an equivalent beam-column and the three guys at one level by cables with an initial pretension and only having tensile capacity. Starting from the energy formulation of beams and nonlinear cables, the continuous equations are discretized using finite element techniques, considering Hermite elements for the mast (Bernoulli beam theory) and quadratic elements for the nonlinear guys. Also, the second order effect due to the axial loads on the mast is taken into account. An ad hoc software, developed by the first author, is employed here to explore natural frequencies and modes of the structure considering the uncertainty propagation of the stochastic guys pretension. Since the guys design value can be modified at the construction stage and more, during the service life, the pretension force is modeled as a random variable with a probability density function (PDF) derived from the Principle of Maximum Entropy (PME). The model herein presented contributes to attain a more realistic description of the structure, mainly regarding the three-dimensional representation and the sensibility to the variability of the guys pretensions. The results here presented (natural frequencies and modes) obtained through an uncertainty quantification analysis, improve the understanding of the real dynamic properties and behavior of slender and flexible guyed structures.

NOMENCLATURE

Latin symbols

E : Young modulus
I : second order moment of the area
A : area
m : mass per unit length
c : viscous damping coeficient
u, v, w : displacements
θ : rotation in the $\hat{i}$ direction
D : sag of cable
L_c : length of the cable
Y : initial configuration of the cable
n: number of elements
t: time
p, q : generic distributed forces
M_t : generic distributed torsional moment
N : axial force in the beam

Greek symbols

ε: lagrangian elongation of cables
ω: circular frequency

Subscripts

c: relative to cable
b: relative to beam
x,y,z: relative to Cartesian coordinates
u,v,w: relative to displacements in the $\hat{i}$, $\hat{j}$ and $\hat{k}$ directions

1 INTRODUCTION

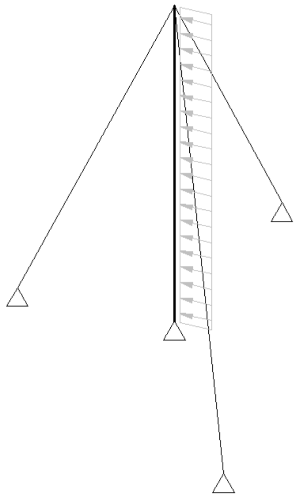
The increasing advance in telecommunications requires the installation of new devices to improve the quality of the services and to meet the consumer demands. In this context, guyed masts are a structural typology extensively employed to support devices such as antennas for radio, TV and other types of telecommunication equipment (Fig. 1 a). Their low cost offers clear advantages and nowadays they are commonly found in urban areas, besides the open country, where the anchors are more easily positioned.

Guyed masts are flexible structures. Then, their dynamic features and behavior are topics of interest since the structural behavior is affected and impacts on the quality of transmission. Despite this, the dynamic response is not studied in detail, with exception of special cases (Preidikman et al., 2006; Shi and Salim, 2015; de Oliveira et al., 2007; Saudi, 2014).

The present and other authors' works show that guyed structures have special sensitivity to the type and amplitude of the excitation (Lenci and Ruzziconi, 2009; Wei et al., 2011, 2016; Ballaben et al., 2017a), even avoiding the resonance effects. After the derivation of the equations of motion of a cable-stayed beam, the in-plane and out-of-plane eigenvalue problems are solved by Wang et al. (2014). Also, nonlinear modes are studied along with the contribution of the coupling term. A study on this regard and related to mechanical systems, is reported by Bellizzi and Sampaio (2012). The smooth decomposition method combined with the Petrov-Galerkin projection for the structure-preserving model reduction is used to analyze second-order discrete nonlinear structural systems under random excitation. Nonlinear mechanical systems under random excitation with homogeneous and non-homogeneous mass distribution were considered. In a recent work by the authors (Ballaben et al., 2017b), the nonlinear dynamic response of plane guyed structures is analyzed through reduced order models with consideration of uncertainties in structural parameters.

In the present study and using the classical extended Hamilton's principle, the equations of motion that govern the vibrations of the system are obtained. The nonlinear model of the cable

(a)



(b)

Figure 1: Guyed mast. a) Typical guyed tower for mobile signal transmission; b) Model under study.

follows the approach reported by Gattulli and Lepidi (2007). Then, and after the statement of the weak form, the governing system is discretized by finite elements.

The FEM approximation of the complete model in a dynamic nonlinear analysis is computationally expensive. An ad hoc, nonlinear optimized code for guyed structures was developed by the authors within a finite element environment in order to reduce the analysis cost. With this tool, the time-expensive Monte Carlo simulations are possible within acceptable runtimes.

Although the guy pretension is determined at the design stage, it can change during the construction procedure and also along the structure service life with respect to the design value, affecting the system performance and even its stability (Margariti and Gantes, 2015). Since the guy pretension is a significant parameter of the structure, its variation is a relevant issue and the introduction of uncertainty appears adequate. In this work, the initial tension of the guys is considered stochastic. The probability density distribution (PDF) is selected by means of the Principle of Maximum Entropy (Shannon, 1948). All the values of pretension studied are within the ranges suggested by the standards. It was found that the consideration of uncertainty in the initial pretension (H) has an important impact in the determination of natural frequencies and modal shapes. Also, the dynamic response is very dissimilar and heavily influenced by H and the nonlinear behavior of the guys.

2 MODEL UNDER STUDY

In this section, the studied model is presented. First, the equations of motion for cables and beams are stated. Then, some comments about the nonlinear finite element discretization and solver methods are given. Finally, the specific details, constants, constraints and loads of the structure under study are listed.

2.1 Formulation and discretization of the equations of motion

Next, the equation of motion of nonlinear cables and beams with the addition of the second order effect are presented. Further details in the derivation of the equations can be found in Ballaben et al. (2016).

The following assumptions are made: a) both the cable and the beam-column are considered as homogeneous one-dimensional elastic continua obeying a linear stress-strain relationship; b) the equilibrium configuration for the inclined cable is described through a quadratic parabola under the assumptions of small sag to length ratio; c) axial extensions of the cable are described by the Lagrangian strain of the centerline; d) the flexural, torsional and shear stiffness of the cable are neglected; e) the shear strain of the beam-column is assumed negligible; f) the non-linearity of the problem arises from the cable formulation; g) a second order effect due of the axial load (assumed constant) is accounted for in the beam-column equation. Under these assumptions and using the classical extended Hamilton's principle, the general form of the weak formulation writes as

$$\mathrm{M}(\ddot{\mathrm{v}}, \phi) + \mathrm{C}(\dot{\mathrm{v}}, \phi) + \mathrm{K}_L(\mathrm{v}, \phi) + \mathrm{K}_{NL}(\mathrm{v}, \phi) + \mathrm{BC}(\mathrm{v}, \phi) = \mathrm{F}(\mathrm{v}, \phi), \tag{1}$$

where $\mathrm{M},\ \mathrm{C},\ \mathrm{F}$ are the mass, damping and external force operators, respectively. K_L and K_{NL} are the linear and nonlinear stiffness operators, respectively. BC is the boundary condition operator. ϕ denotes the admissible functions and v are solutions of Eqs. (1).

$$\mathrm{M}(\ddot{\mathrm{v}}, \phi) = \int_0^l m\ddot{v}\phi dx \tag{2a}$$

$$\mathrm{C}(\dot{v}, \phi) = \int_0^{l_b} c\dot{v}\phi dx \tag{2b}$$

$$\mathrm{F}(v, \phi) = \int_0^{l_b} (F_{u_b}\phi_b + F_{v_b}\phi_b)dx_b + \int_0^{l_c} (F_{v_c}\phi_c + F_{u_c}\phi_c)dx_c \tag{2c}$$

$$\begin{aligned}\mathrm{K}_L(v, \phi) = &\int_0^{l_b} (EI_b v_b''\phi_b'' - P_H v_b'\phi_b' + EA_b u_b'\phi_b')dx_b + \\ &+ \int_0^{l_c} (Hv_c'\phi_c' + EA_c u_c'\phi_c')dx_c\end{aligned} \tag{2d}$$

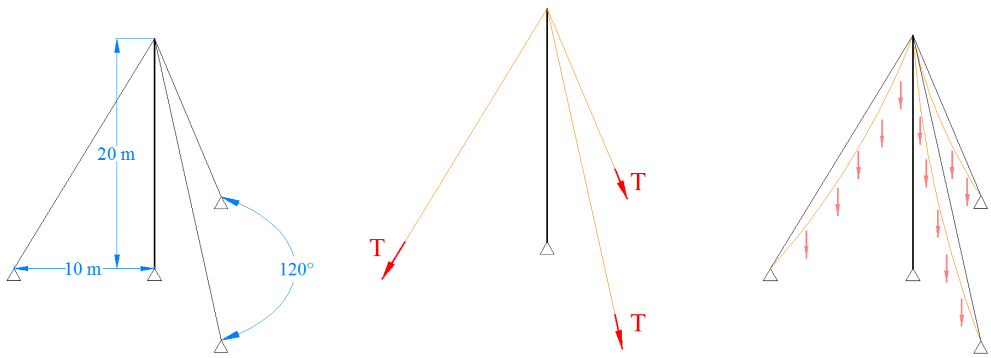
$$\mathrm{K}_{NL}(v, \phi) = \int_0^{l_c} EA_c[(Y_c' + v_c')(u_c' + Y_c'v_c' + \frac{v_c'^2}{2})\phi_c' + (Y_c'v_c' + \frac{v_c'^2}{2})\phi_c']dx_c, \tag{2e}$$

$$\begin{aligned}\mathrm{BC}(v, \phi) = &\left[Hv_c' + EA_c(Y_c' + v_c')(u_c' + Y_c'v_c' + v_c'^2/2)\right]\phi_c|_0^{l_c} + \\ &+ EA_c(u_c' + Y_c'v_c' + v_c'^2/2)\phi_c|_0^{l_c} + EI_b v_b'''\phi_b|_0^{l_b} - EI_b v_b''\phi_b'|_0^{l_b} + \\ &+ P_H v_b'\phi_b|_0^{l_b} + EA_b u_b'\phi_b|_0^{l_b}\end{aligned} \tag{2f}$$

Here $(*)' = d(*)/dx_i,\ i = c, b$ and $\dot{(*)} = d(*)/dt$, H is the component along the chord of the mean static cable pretension T in the cable (due to the small slope, H and T are practically equal), Y is the initial configuration of the cable and, due the hypothesis of small sag (D) to span ratio, a parabolic function is assumed $Y(x_c) = 4D(x_c/L_c - (x_c/L_c)^2)$. Finally, $\varepsilon = u_c + Y'v_c' + 1/2v_c'^2 + 1/2w_c'^2$ is the elongation of the cable; ϕ_b and ϕ_c stand for the beam and cable admissible functions, respectively. P_H is the guy force component in the mast axis direction.

After stating the weak formulation, the system is discretized by means of an ad hoc non-linear finite element (NLFEM) formulation. The column is modeled using a two nodes, 6-DOF (transverse and axial displacements and slope at each node) beam elements (Hermite interpolation functions for the transverse displacements and their derivatives and linear interpolation functions for the axial displacements and the torsional rotations). The cable is represented using a three nodes, 6-DOF (axial and transverse displacements at each node) cable element (cuadratic interpolation functions). The nonlinear dynamic response is obtained using the Newton-Raphson method for the iterations and the Newmark method for the time integration.

As an initialization and before starting the dynamic analysis, the pretension of the guys is applied through deformation of the cables (step 1); then, the self-weight of the guys is activated (step 2). Steps 1 and 2 are depicted in Fig. 2 (b) and (c). The position of the anchors and the initial pretension are checked (step 3). If the error is less than 0.5%, the program uses this deformed/stressed state as the initial state of the dynamic analysis. Otherwise, the initial length of the cable is modified (step 4) and steps 1 to 4 are repeated until the error meets the prescribed tolerance.



(a) Geometry. (b) Step 1: pretension. (c) Step 2: Self weight.

Figure 2: Geometry of the studied guyed mast and first two steps to get the initial deformed/stressed configuration for the dynamic/uncertainty studies.

To reduce the runtime, the initial configurations as well as the rotation, mass and linear stiffness matrices were first solved and preallocated. Thus, the solver only needs to recalculate the linearized stiffness matrix of the cable elements and the residual vectors at each iteration. Also, a parallelization of the algorithm is implemented.

2.2 Numerical illustration

The studied case consists in a 20 m height guyed mast, as depicted in Fig. 2 (a), with one level of three cables connected to the mast at the top. The anchors of the cables are separated 10 m from the mast and 120^o of each other. The mast is fixed at the base and it is modeled using 5 beam elements with consideration of the second order effect. The cables are pinned at the anchor point and each cable is discretized with 5 three-node nonlinear cable elements. This article will focus on the influence that the uncertainty on the initial cables pretension value (H) have on the modal features of the structure. The assumed values of the constants for these example are detailed in Tab. 1.

3 UNCERTAINTY QUANTIFICATION

The uncertainty quantification is performed considering guys tension $\mathbb{H}$ (corresponding to the deterministic parameter H), as a random variable. The Principle of Maximum Entropy (PME) (Shannon, 1948; Cursi and Sampaio, 2015) allows to determine the best PDF that satisfies the imposed constraints, and introduces no unwarranted information, *i.e.* supplied data is equal to the removed uncertainty (Kapur and Kesavan, 1992).

Table 1: Numerical illustration. Values of constants and parameters of Eqs. (2).

Properties	Value
E (GPa)	209
$I_x = I_y = I_z$ (m^4)	3×10^{-5}
A_b (m^2)	1.5×10^{-3}
m_b (kg/m)	11.77
A_c (m^2)	7.85×10^{-5}
m_c (kg/m)	0.62
H (N)	500 - 13500

The PME states that, subjected to known constraints, the PDF which best represents the current state of knowledge is the one with largest entropy. PME addresses the problem in a statistically sound way. The approach is systematic and allows to handle data which is limited or coming from different sources.

The measure of uncertainty of a random variable X is defined by the following expression

$$S(f_X) = -\int_D f_X(X)\log(f_X(X))dX, \tag{3}$$

in which f_X stands for the PDF of X and D is its domain. The maximization problem is frequently solved using Lagrange's method, with a multiplier accounting for each constraint.

Assuming the constraints of positiveness and bounded second moment, the PME leads to a gamma PDF. The gamma distribution with parameters a and b, where $E(X) = ab$; $\sigma_X^2 = ab^2$, is given by the expression:

$$f(x|a,b) = \frac{1}{b^a\Gamma(a)}x^{a-1}e^{\frac{-x}{b}}. \tag{4}$$

Afterwards, Monte Carlo simulations are performed in order to find the influence of the guys tension $\mathbb{H}$ with the selected PDF, on the structural response. To achieve significant statistical results, a convergence study on the standard deviation was first performed to determine the necessary number of realizations of the Monte Carlo simulations.

The range of values of the initial pretension was chosen following the standard code CIRSOC 306 (1995). Once the realizations are finished, the PDF graphs are constructed using the *ksdensity* function of MATLAB, that estimates the PDF of a set of data using the *kernel* method. The bandwidth for the kernel function (here a normal function is used), is optimized in MATLAB, and it is useful when the target PDF is normal, but can give wrong results when that condition is not fulfilled. In this work, after several tests, and following an engineering criterion, the authors adopted a bandwidth of 0.05, that gives as result smooth PDFs and allows to observe the all the interesting statistical characteristics.

The efficiency of the stochastic computational model is strongly dependent on the structural model and on the statistical tools. Regarding the first, the size and complexity conditions determine the time in which the deterministic model is solved. Without optimization, a dynamic study on a nonlinear finite element model of this size (102 degrees of freedom), is hardly feasible, due that, in top of this, the stochastic study requires thousands of realizations (*i.e.* solution of the deterministic model).

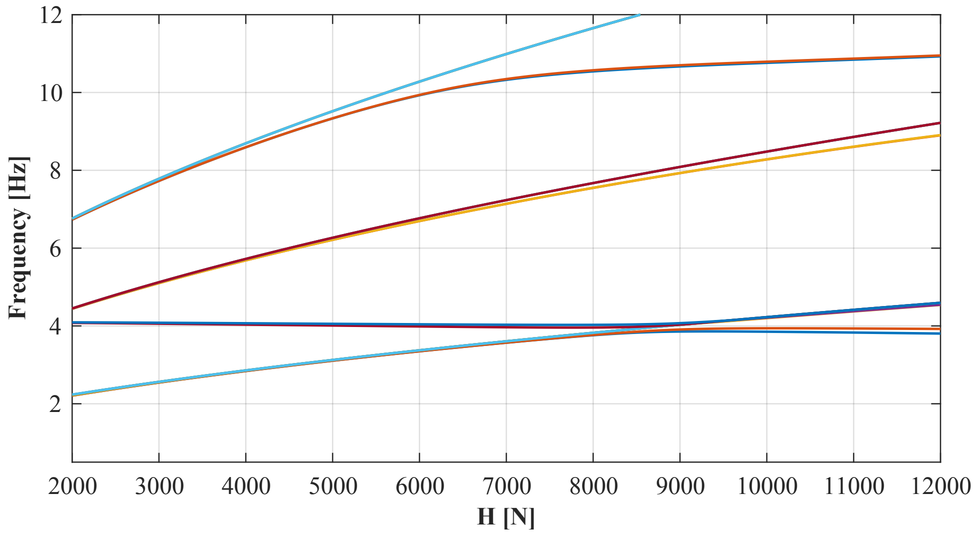


Figure 3: Deterministic eigenvalues evolution with the initial tension.

4 RESULTS

4.1 Natural frequency analysis

The stiffness matrix of the elements is computed using the initial deformed shape. The reduction of the stiffness of the column due the force components of the guys is also considered. In Fig. 3, the deterministic evolution of the first 20 natural frequencies with the tension is depicted. The nonlinear evolution of each eigenvalue (depicted as a single colored curve in Fig. 3) with $\mathbb{H}$ is a consequence of the nonlinear formulation of the cables. In some cases, the curves become parallel and close, though no crossings are observed.

Fig. 4 depictes the eigenvalue study results, considering uncertainty through $\mathbb{H}$. For a given value of E($\mathbb{H}$), rather than a single value of frequency associated to a modal shape, PDF of the natural frequencies is obtained. The evolution of the distributions with E($\mathbb{H}$) is depicted in Fig. 4, where the darkest colors denote a higher values of the PDF. In general, the wider frequency zones correspond to local cable modal shapes. The nonlinear influence of the cables is also apparent in the eigenvalue study considering a random $\mathbb{H}$, since the mode of the distribution of each eigenvalue varies in a nonlinear fashion with $\mathbb{H}$. Also, it can be seen that some of the closest eigenvalue curves are merged with a single PDF; these curves correspond to similar modal shapes and the stochastic modeling acts as a filter that allows a clearer representation of the reality by mean of PDFs. Since the guyed masts are flexible structures and the precise determination natural frequencies is an important issue, the relevance of the results depicted in Fig. 4 is apparent.

4.2 Modal shape analysis

Since the effective stiffness of each part (cables and column) depends on the initial tension, and given the wide range of $\mathbb{H}$ studied, each eigenvalue curve in Fig. 3 can not be, in general, associated to a single modal shape. At some values of the initial tension, rapid changes (with respect to the previous rate) in the derivative of the curves occur. In these zones, the veering phenomena is observed and the mode shape associated to a given eigenvalue starts to change gradually, in general, from a local (cable or column) shape to a global cable-column shape. Then, when the changes in the derivative slow down, the modal shape becomes local again.

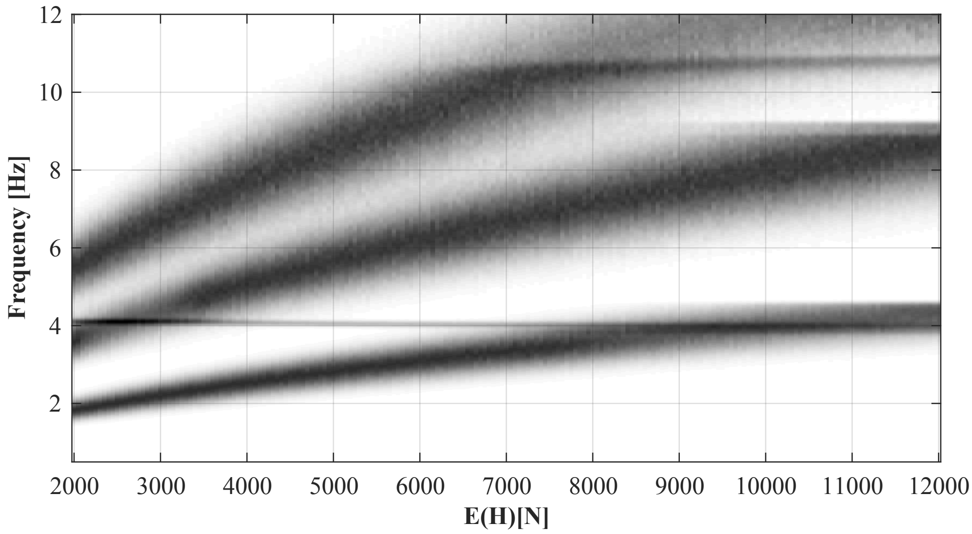


Figure 4: Eigenvalues evolution with the mean value of the stochastic initial tension.

An example of this process is illustrated in Fig. 5: the evolution of two eigenvalues with the cable initial tension where the veering phenomena is observed is shown at the top plot; the local and global modal shapes associated with certain ranges of H are depicted with different line patterns. The distinction between a local and a global modal shape is performed using the so-called localization factor Gattulli and Lepidi (2007):

$$\Lambda_{i,j} = \frac{(\mathbf{R}_j \phi_i)^T \mathbf{M} (\mathbf{R}_j \phi_i)}{\phi_i^T \mathbf{M} \phi_i}; \qquad \Lambda_{i,j} \in [0, 1] \tag{5}$$

where ϕ_i is the i*th* eigenvector and $\mathbf{M}$ is the mass matrix of the model while $\mathbf{R}_j$ is a diagonal matrix of ones and zeroes that allows a selection of the degrees of freedom along the cables. The $\Lambda_{i,j}$ factor expresses the localization level of the ith eigenvector in the jth cable domain. Clearly, $\Lambda_{i,j} = 0$ corresponds to a local cable modal shape and $\Lambda_{i,j} = 1$ to a local column modal shape. When $0 < \Lambda_{i,j} < 1$, a global (hybrid) cable-column modal shape is present. In Fig. 5 (bottom) the different modal shapes associated with the corresponding eigenvalue and range of H are depicted. Here, a smooth but rapid transition between a local (i.e. cable or column) mode to other local shape (i.e. column or cable, respectively) corresponds to the hybridization of the mode shapes. Also, the hybridization regions correspond to the veering zones, where an exchange of local modal shapes occur between the eigenvalues (as can be seen in Fig. 5 top for $H \approx 9500$ N). The global shapes are rare (are apparent in small ranges of H) probably due the difference in the stiffness between the cables and the column.

Additionally, a decrease in the frequency is observed in particular ranges of H (i.e. mode 1, H=11000:13500 N or mode 7, H=3000:9000 N), for increasingly higher values of H. This effect always happens in local column modes and it is a consequence of the second order effect, which leads to a reduction of the column stiffness, proportional to the initial tension of the guys. If the second order effect is neglected, this particularity is lost.

In Fig. 6(a) the veering (and hibridization of modal shapes) zones are highlighted in darker color, for the first 20 eigenvalues. In Fig. 6(b) the zones where there is a 95% probability of presence of a global modal shape when H is considered a random variable $\mathbb{H}$ are plotted in darker colors (on the mean eigenvalue curves). It can be seen that the darker zones are far beyond the veering zones, and occupy the whole space where the local column modal shapes are

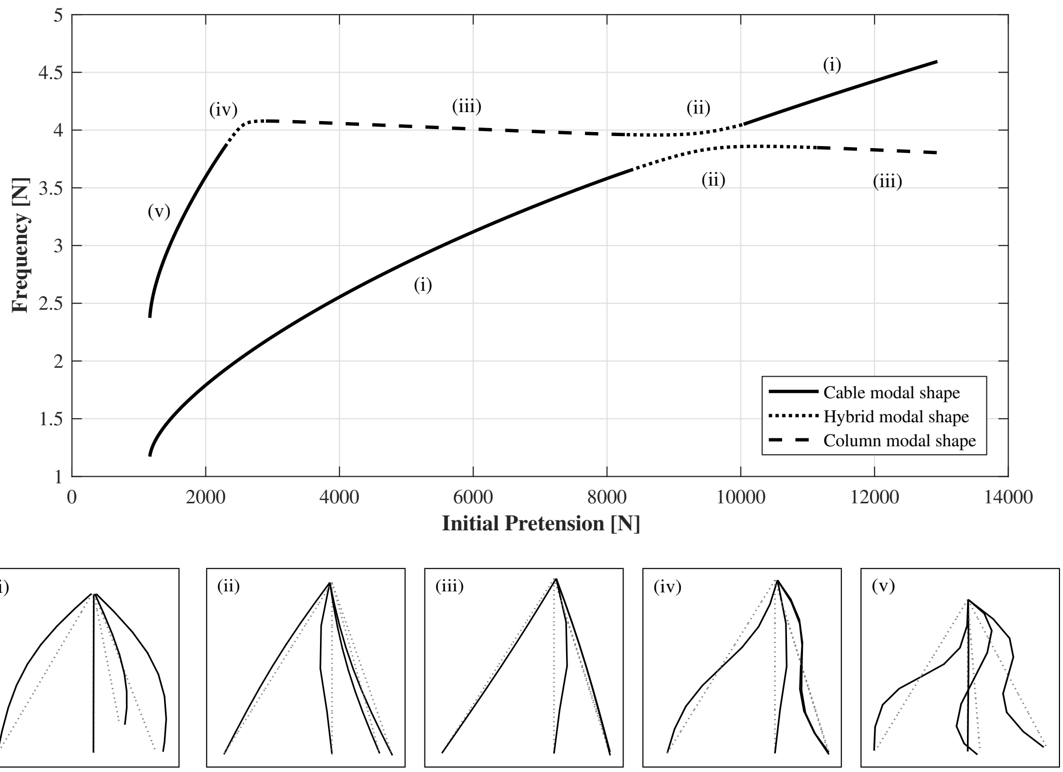


Figure 5: Hybridization of modal shapes and veering of eigenvalues: evolution with the initial tension and veering of eigenvalues (top) and modal shapes associated with the eigenvalues (bottom).

expected in a deterministic study. Then, the consideration of a stochastic $\mathbb{H}$ not only modifies the size of the zones of probable hybridization, but also suggests that a local column modal shape is highly improbable, for any given value of $E(\mathbb{H})$.

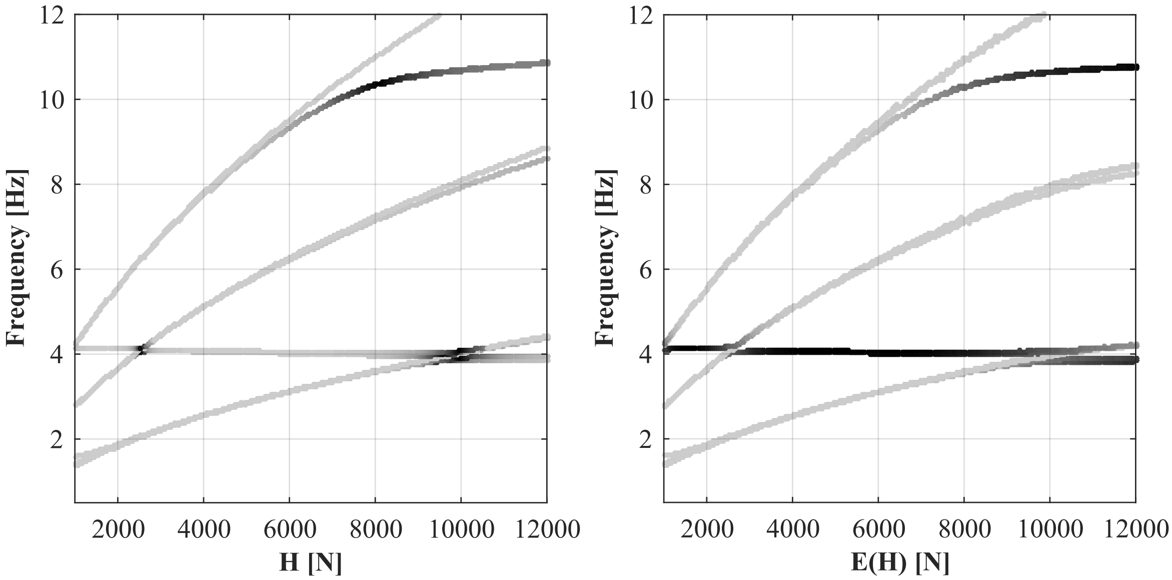
The results, considering a random $\mathbb{H}$, have important repercussions, in both the academic and professional fields, i.e. from an academic point of view, the use of the most probable modal shapes could lead, in the formulation of reduced order models, to a better predictions of the dynamic behavior. From the professional perspective, the statistic description the natural frequencies is a safer, simpler and realistic definition of an important design parameter. Moreover, the evaluation of the most probable modal shapes may guide to a better understanding of the dynamic behavior of the structure.

5 FINAL REMARKS

In this work, an optimized finite element formulation is used to study the nonlinear modal features of a guyed mast. The results are analyzed from both the classic deterministic and statistical points of view, considering the initial tension of the guys with a gamma distribution as the random structural parameter $\mathbb{H}$.

Regarding the deterministic study, the natural frequencies of the guyed mast show a nonlinear relationship with the initial tension. The veering phenomena is observed and the regions of veering corresponds to transition zones (which exhibit global or hybrid -column and cable-modes) between local -column or cable- modes. The so-called localization factor is employed to distinct local from hybrid modes. Also, due the influence of the second order effect, a reduction in the natural frequency occurs (for increasing values of $E(\mathbb{H})$) for local column modal shapes.

When the initial tension is considered stochastic, the probability density distribution of the



(a) Deterministic localization factor. (b) Stochastic localization factor.

Figure 6: Localization factor for each eigenvalue (darker colors indicate hybridization of cable and column modal shapes.)

eigenvalues were plotted. Again, it can be observed a nonlinear evolution of the mean value of the eigenvalue PDF with E($\mathbb{H}$). The veering phenomena is lost within the crossing of the PDFs path of different eigenvalues. The modal shapes also are affected by the randomness of H, since the hybridization occurs far beyond the points of veering, from the zones of local column modes to the point thta the probability of occurrence is almost negligible for any given combination of E($\mathbb{H}$) and eigenvalue.

The results, considering a random H, have important aftermaths, which can impact from reduced order models formulation to a realistic knowledge of the dynamic properties of a flexible structure important design parameter which are important design parameters. Moreover, the evaluation of the most probable modal shapes may guide to a better understanding of the dynamic behavior of the structure. Future works over this model should include dynamics analysis, with variation in amplitude, frequency and direction of the load, consideration of stochastic wind load, guys with different tension and/or stochastic tension or the study of the dynamics during the breakage of a guy.

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