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# Dynamic geometry and square of the circle in Thomas Hobbes

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**Abstract.** This paper aims to present the reconstruction of the arguments offered by Thomas Hobbes about the classic problem of the squareness of the circle in the seventeenth century, from the perspective of the work: *De Corpore* of the same author. Where research is based on the support offered by Dynamic Geometry in the 21<sup>st</sup> century in the classroom with the use of GeoGebra software. This to be able to address and overcome the problems of the history of mathematical science and philosophy. For this it was chosen, by the methodology of a design of a teaching experiment in the classroom carried out with future teachers of the bachelor's degree in mathematics of 8<sup>th</sup> semester. Among the most important results is the recognition of renowned philosophers in the development of the history of mathematics. This implied that prospective mathematics teachers had to integrate their mathematical knowledge into the construction of the quadrature of the circle, which at the time was a problem for Hobbes, but with the perspective of modeling mathematical concepts in dynamic geometry, today it is done efficiently.

## 1. Introduction

This investigation starts from the fact that within the history of mathematics many philosophers have been included who throughout history have contributed directly or indirectly to the different problems that are touched in the area of mathematical knowledge. However, some of them have gone unnoticed, as in the case of Thomas Hobbes. In his book *De Corpore* (DC) -1655- he tries to solve the problem of squaring the circle with a ruler and a compass. But due to the lack of adequate tools at the time, he could not solve these classic problems in mathematics and geometry until that moment. Suffice it to say that at the time addressing the problem of squaring the circle was a sample of the intellectual capacity that was had by the mathematicians of the beginnings of modernity. Some authors as John Wallis affirmed that for example in the case of Hobbes this one did not manage to advance by lack of knowledge of geometry, as it proposes it [1].

On the other hand, the squaring of the circle has been studied from different perspectives such as Archimedes, Hippocrates, Hobbes, da Vinci Leibniz, Lindemann, among others. But, the history of mathematics, until now, has not recognized the contribution made by the philosopher Thomas Hobbes in his book *De Corpore* regarding the subject. This research aims to present part of the reconstruction of the arguments he made at that time, but with the support of an innovative element such as dynamic geometry (GD), together with students in mathematics, within an experience in the didactics of geometry course in eighth semester of training, as future teachers of mathematics.

Deepening this sense, that the classic can contribute to education in modernity, mathematics, as another science, needs to be adapted to different scenarios, from any of its interpretations, and from its different uses, such is the case of the training of mathematics teachers. The inclusion of this type of environment, and new technologies involves a greater effort for those who belong to the time, because it implies the recognition of the tool and the identification of possible help by involving them in the



educational format that aims to promote. In addition to knowing the history of mathematics and advocating for it to collaborate in proposing new methodologies to address classical problems of mathematics. This is, "the new technologies have become rapidly a 'must-do' across higher education (HE) (...) as proposed in his article Acevedo-Rincón J P and Flórez-Pabón C E [2].

In this sense, new technologies and new pedagogies in the teaching of mathematics invite us to look at dynamic geometry. Which is presented as an option and novelty for the teaching of geometry and its classic problems, through the software that can be used in such work. For example, in this case, the use of GeoGebra (GG) modeling software allows the development of a classical construction of the language of geometry with the use of the ruler and the compass [3], which will facilitate the approach to the Hobbesian problem as we will see later.

## 2. Theoretical frameworks

The theoretical framework that this research takes into account is based on the approaches made by Thomas Hobbes in his book *De Corpore* on the classic mathematical problem of squaring the circle with a ruler and a compass [4]. However, the development of this research is framed within the concept of GD, and the modeling process developed through the use of GG as the inflection point of a modeling competence that future mathematics teachers should keep in mind. For this we will address three points, which we expose below.

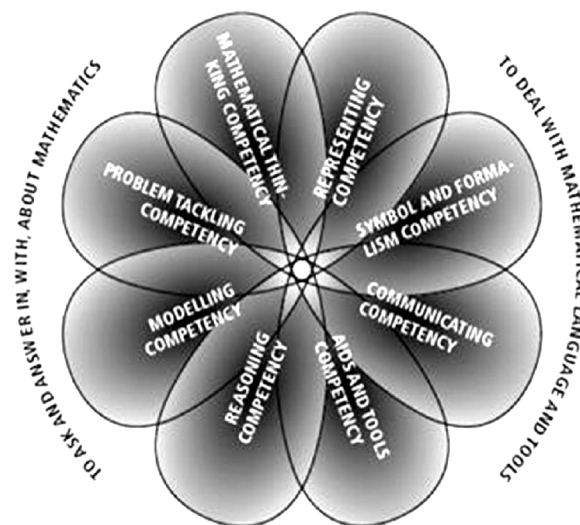
First, if we think of geometry as a process of interiorization and intellectual apprehension of spatial experiences, learning passes through a domain of the bases of construction of this branch of knowledge, and here abstraction plays a fundamental role. In this "mathematization"-reading the world through mathematics- the objects of the physical world are associated with abstract beings, which are defined and controlled by a body of assumptions, the axiom system of theory. In the transition to this world there are difficulties inherent to the process, coming from the confrontation between scientific and non-scientific concepts [5]. The questions that we will discuss here are two: the processes of formation of the concept of geometric object and the transition between the experimental and the abstract. We analyze the difficulties inherent to these processes and present the development of two sessions of work with the students of our course where it is evident how much software with resource of "moving drawings" (as for example Cabri-Géomètre (CG) o GG) can be ideal tools in the overcoming difficulties. We see a new way of teaching and learning Geometry emerge; from experimental exploration viable only in computerized environments, students conjecture and, with the constant feedback offered by the machine, refine or correct their conjecture, arriving at results that resist the "moving drawing", then moving on to the abstract phase of argumentation and mathematical demonstration [3,6].

In the second moment with the purpose of making clear the inherent obstacles to learning we will work within the theory proposed by Fischbein, where the geometric object is treated as having two components, one conceptual and the other figural [3]. The conceptual component, through written or spoken language, with a greater or lesser degree of formalism depending on the level of axiomization with which one is working, expresses properties that characterize a certain class of objects. On the other hand, the figurative component corresponds to the mental image that we associate with the concept, and that in the case of geometry, has the characteristic of being "manipulated" through movements such as translation, rotation, and others, but maintaining certain invariant relations. The harmony between these two components is what determines the correct notion about the geometric object [7].

In the formation of the mental image, the drawing associated with the geometrical object performs key role. For the student it is not always clear that the design is only an instance. physical representation of the object. If, on the one hand, the drawing is a concrete support for expression and understanding of the geometric object - what is transparent in our attitude towards a problem: the first thing we do is to draw the situation, either on a sheet of paper or on the screen of a computer - on the other hand, it can be an obstacle to this understanding. And this is because it keeps particular characteristics which do not belong to the set of geometrical conditions that define the object. It is interesting to note that, depending on the stage of mental development, students work meticulously seeking the "perfection" of the drawing, as if it were "the object of the geometric", leaving the abstract properties, which give existence to the

object, in the background. They even confuse the physical characteristics of the drawing (thickness of the stroke, size of the point). with geometrical properties, when they say, for example, that "tangent circles intersect in infinite points" [3,8].

Finally, we will describe the programs built within the principles of "dynamic geometry". They are programs that oppose the computer assisted instruction (CAI) type. They are tools for construction: object drawings and geometric configurations are made from the properties that define them. By means of displacements applied to the elements that make up the drawing, it is transformed, maintaining the geometrical relations that characterize the situation [3]. Thus, for a given object or property, we have associated a collection of "moving drawings", and the invariants that appear there correspond to the geometrical properties intrinsic to the problem. And this is the important didactic resource offered: the variety of designs establishes harmony between conceptual and figurative aspects; classic geometrical configurations have multiple representations; geometric properties are discovered from the invariant in movement. The characteristics of the program with which we have been working, GG, offers the "ruler and electronic compass" feature, being the interface of construction menus in classic geometry language [9]. In addition, we must think that GG is collaborating in the development the modelling competency consists of the ability to analyze the foundations and properties of existing models and to assess their range and validity (As can be seen in Figure 1, it is part of other competencies for future math teachers to develop). It also involves being able to perform and utilize active modelling, including mathematising, which is what which is what the students did when transforming the extra-mathematical historical situation into a mathematical problem [10].



**Figure 1.** Definition of mathematical digital competency [10].

### 3. Method

In order to answer the research question: how can the arguments used in the demonstration of the squaring of the circle in Hobbes contribute to the teaching of concepts proper to the didactics of geometry in mathematics graduates? This research is of a qualitative type, which allows describing and interpreting classroom situations, based on design research [11]. A teaching experiment can be defined as a methodology specific to research in Mathematical Education in which different research approaches are contemplated. It adopts as its main characteristic the lack of differentiation between teacher and researcher, motivated by the purpose of researchers to experience firsthand the learning and reasoning of students [12]. In this way it is proposed to develop a teaching experiment with the participation of future teachers within the Geometry didactics course. This approach makes it possible to identify learning based on mathematical modeling, under the construction of applets in GG. Therefore, the instruments of information collection seek to take captures of what happens inside the classroom, for

this reason the video recording of student interactions, and records of them in the construction of applets as well as journal entries.

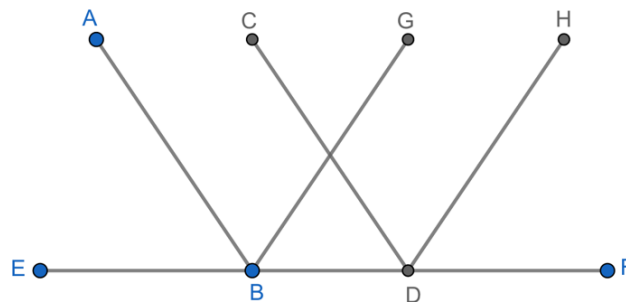
The participants in this research were 22 students for a mathematics teacher who were taking geometry didactics as part of their training. These students had little access to Information and Communication Technologies (ICT), so it was necessary to have some sessions in which they became familiar with the GG software, to recognize the instruments within it, and later to stage their disciplinary knowledge in mathematics.

For the development of this teaching experiment, they are considered as research phases: (i) preparation of the experiment, where in previous classes some constructions are made on paper using ruler, compass, conveyor, always on white sheets, to avoid the consolidation of prototypical forms. In addition to this, some sessions are carried out in which the students approach the use of dynamic geometry, as a support of dynamic images in the demonstrative processes. (ii) experimentation, in which simple demonstrations are carried out based on some corollaries and propositions of Hobbes [13].; and, (iii) retrospective analysis, in which it is expected to find the arguments and conclusions constructions of current geometry (GD), to which, due to lack of instruments, Hobbes himself did not arrive at that time.

#### 4. Results

The results obtained from the three phases of the teaching experiment will then be presented. First, methods of analytical dynamics geometry are ideally suited for the study of the motion of a rigid body, as in the case of the Hobbes circle square. The metrical structure of Euclidean space is homogeneous and isotropic, hence independent of the distribution of matter in the space and allowing to develop geometric concepts in terms of shape, from square to circle [14].

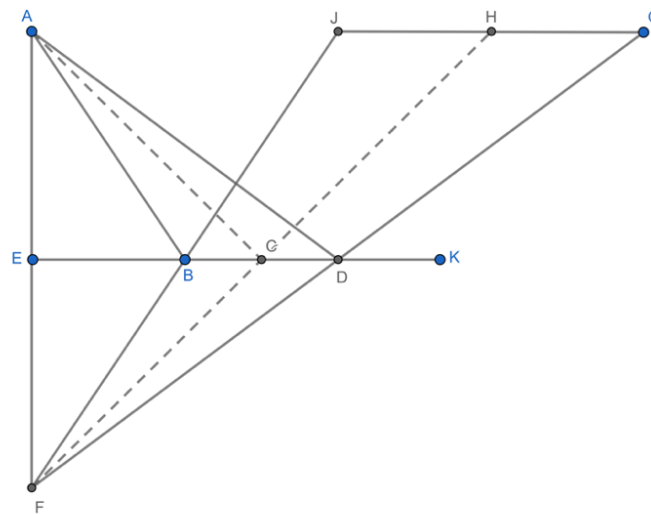
Second, the preparation of the experiment, in order to be able to reconstruct the Hobbesian argument in terms of knowing what squaring the circle is. This implied, on the part of the reading of the author's demonstration [13], written in chapter XIX De Corpore. In particular, Figure 2 shows the corollary "if two straight lines that affect the same straight line are parallel, their reflex lines will also be parallel".



**Figure 2.** Representation of the first proposal for the conservation of angles of incidence between two parallel lines.

This representation was modeling [14] was performed for the second phase of the experiment, together with students, using GG, to arrive at part of the demonstration of the seven corollaries exposed by the Malmesbury philosopher. The following proposition used was number 2: "if two straight lines starting from the same point collide on another straight line, their reflex lines prolonged towards the other side will concur at an angle equal to the angle formed by the incident straight lines", which is represented in Figure 3.

Subsequently, a joint discussion took place in which some of the needs of DG were proposed to arrive at the final construction of the squaring of the circle, and which became an impediment to start the first to demonstrate the squaring of the circle. This discussion allowed us to begin the third stage of the design, retrospective analysis.



**Figure 3.** Representation of proposition 2 on the construction of lines of concurrence.

Finally, this made it possible to distinguish some of the most representative problems in the history of mathematics, which are also dealt with by some philosophers, and which even in the 21st century become unsolvable as Lindemann stated in 1882: squaring the circle is impossible if, as the only tools, we have a rule and a compass and we can only use the norms established in ancient Greece. But that, according to the constructions made by the students, these have better approximations if the modeling with GD is incorporated in the mathematics classroom. That is to say, the use of GD, in particular modeling, allows future graduates to arrive at the arguments and conclusions that Hobbes himself did not arrive at that time, due to the lack of instruments to support his demonstration proposals.

## 5. Conclusions

The results of this mathematics teaching experiment allowed us to identify: (i) the students recognized the difficulties that the philosopher Thomas Hobbes went through at that time to complete the demonstration of the squaring of the circle; (ii) the basic arguments of Thomas Hobbes' demonstration were sufficient, but the chain of arguments needed an adequate visual interpretation of the geometric elements to conclude the demonstration; (iii) at present, the incorporation of technologies in teaching makes it possible to facilitate the paths traveled by mathematicians over long periods of time; (iv) mathematical modeling allows the recognition of mathematical elements in motion, that is, dynamic images facilitate the understanding of geometric constructions, as happened with the quadrature of the circle.

In this way, it is evident that the valuation of historical movements in the construction of current mathematics, allows us to identify strategies for addressing classical problems. That is, both the history of mathematics and philosophy are articulated to find elements that develop the spatial thinking and geometric systems of future mathematics teachers.

Finally, teaching experiments, such as research design methodology [15], allow identifying in different registers (graphs, writings, orals, etc.) the contributions of educational resources to the teaching processes of a mathematics based on Modeling of historical demonstration problems.

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