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Conflitos entre matemáticos profissionais e amadores: tentando resolver a quadratura do círculo e o ultimo teorema de Fermat

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Resumen

Resolver a quadratura do círculo constituiu durante muitos séculos um tema de pesquisa dos matemáticos. Quando não houve mais progressos significativos pelos profissionais, um outro grupo entrou no campo, sempre com novas tentativas de resolução - os amadores. Uma vez que eles não prestavam atenção na definição matemática do problema e recusavam explicações sobre a natureza desse problema, os matemáticos profissionais – em particular em instituições como Academias – não mais aceitaram analisar tais propostas e encerraram a comunicação. Há se apresentar o caso da Prussia no século XIX com várias etapas de lidar com amadores.

Já no caso da quadratura do círculo, os amadores foram instigados por um mítico prémio para quem resolvesse o problema, e ocuparam um novo espaço quando foi anunciado de verdade um prémio, em 1908 – do valor de uma fortuna: para encontrar a prova do ultimo teorema de Fermat: o prémio Wolfskehl. Os inúmeros memoriais submetidos excederam qualquer limite. Aí, estabeleceu-se a chamada "clínica Fermat", concebida por uma pessoa excepcional, um matemático que era tambem médico. A prática da clínica será apresentada.

Palabras clave: Quadratura do círculo, Crelle, Teorema de Fermat, Prémio Wolfskehl, Albert Fleck.

Introduction

In many cultures since Antiquity approximate values for the relation between the circumference and the diameter of a circle have been elaborated. The method for calculating the approximations was, since Euclid and in particular since Archimedes, was to inscribe polygons into the circle and to circumscribe them. Most spectacular was the result of Ludolph van Ceulen of 1590/1621, with 35 decimals. It arose the question whether its number of decimals would never remain a finite number. A decisive result was achieved by the German mathematician Heinrich Lambert (1718-1778) in 1767: π is not a rational number. And the French mathematician Adrien-Marie Legendre (1752-1833) conjectured in 1806 that π is a transcendental number, hence with infinitely many decimals and without periodicity.

Joseph Liouville proved in 1844 that there exist non-algebraic numbers, i.e. there exist numbers which are not solutions of algebraic equations – thus of equations with finite degrees of the exponents/indices and rational coefficients. Such numbers were now called transcendental numbers. It was Charles Hermite who showed in 1873 that *e* is such a transcendental number. Based on this methodology and procedures, Ferdinand Lindemann proved in 1882 that π is a transcendental number.

Regarding the construction of the quadrature of the circle, one of the three classical problems of Greek Antiquity, with ruler and compass, over centuries professional mathematicians tried in vain to resolve this Greek problem. It was the *Académie des Sciences* in Paris, which decided in 1775 to no longer accept submissions of pretended solutions of the quadrature problem and hence to no longer review such submissions. The deeper reason was that the Academy by its statutes had to assess projects submitted for their scientific value. This decision became emblematic for the treatment of amateurs by professionals. Given the observation that amateurs dedicating themselves so intensely to the futile research that they lose not only awareness of their capability but be financially disastrous to their families, the Academy declared it therefore as an act of humanity to destroy the widespread beliefs by an official declaration.

Dealing with amateurs in Prussia

The constructive phase and the role of Crelle, 1820 to 1858

Crelle is the founder of the famous *Journal für reine und angewandte Mathematik*, in 1826, the first permanent mathematical journal. He served from 1815 as senior building officer (Oberbaurat) in the department for constructions of the Prussian Ministry of the Interior. In 1828, he moved to the Prussian Education Ministry, as consultant for mathematics teaching (DZA I, fol. 52). Right from the first day of his service there, the Ministry asked him to review submissions with alleged solutions of the quadrature. Two voluminous files of the Ministry are preserved on the quadrature of the circle, from 1820 to 1916. Crelle showed a behaviour quite different from the Paris Academy.

He did not ostracise amateurs but instead tried to argue with them and – by showing them their errors – to make a "treatment", to lead them to an understanding of the mathematical exigencies. Crelle undertook it to study carefully the submissions and to detail the revealed problems and errors to the author. As he emphasised, not any "proof" can be a priori rejected since the demonstration that is not an algebraic number or is not geometrically constructible is still missing (on April, 5th, 1841; ibid., fol.s 121-124).

Crelle had to experience, too, that some authors were not receptively for arguments and continued to send new, likewise erroneous versions. Being a consultant for mathematics teaching, he recommended in 1833 to the Ministry that it should be a subject in schools what the Greeks meant and what are basic requirements to tackle such problems (ibid., fol. s 86-89.).

As already seen in France, also the submitters in Prussia were convinced that they would win a prize for a solution. At first, the submitters were Prussian citizens, but somehow it must have spent the rumour internationally that it was in Prussia one would win this prize. The first foreign submission occurred in 1846, from the United States. In 1858, a voluminous submission reached from Malta.

Crelle retired in 1850, and the Ministry continued with its policy to take submissions seriously. Now, it sent submissions to the Berlin University professor Ernst Eduard Kummer (1810-1893), a specialist in number theory. Kummer continued to read the texts and to answer, but in a short manner, just getting rid of it and not constructive as had been Crelle.

The second phase in Prussia: critical neutrality

A second phase in handling submissions began late in 1858, after the retirement of Johannes Schulze, the decisive under-secretary in the Prussian *Kultusministerium* since 1818. The officials now did no longer pay much attention to quadrature submissions. The general manner became to no longer ask a mathematician for a report – the petition was simply put "ad acta" or sent back. A particularly besetting petitioner was a retired military officer from the dukedom Nassau who many times renewed his request for an assessment of his elaboration. In one of these submissions, of 1861, he had commented his result in this way: there remains a difference between the circumference and the polygon – but the humanity otherwise uses to be satisfied "with erroneous calculations if the error has met a general acknowledgement". Actually, his result was to give as difference: 3,14 – 2,07 = 0,17 (DZA III, fol.s 34-43; quotation on fol. 42).

The third phase, from 1882: Abnormity

In 1883, thus after Lindemann's proof of the transcendence of π in 1882, there emerged a new pattern. Friedrich Althoff, now under-secretary and one of Schulze's successors, annotated in the files regarding a submission sent by an Italian count: "Mr. Professor Kronecker to whom I had exhibited the annexes characterises them as 'completely meaningless' and its author as 'mathematically irresponsible" (ibid., fol. 159).

Leopold Kronecker's term: *unzurechnungsfähig*, which can also be understood as 'insane' gave the keyword for administering further submissions. Their authors were now suspect to be mentally ill. This occurred in 1887 when a text from a person resident in Berlin was submitted: the Ministry asked the police for information about this person. The answer said that nothing detrimental is known about the author, a mason, now retired, and in particular not that he might be insane (ibid., fol.s 180-182). There were more such inquiries in the following years. In 1896, for instance, the Berlin police reported about another author: he makes the impression to be a mentally ill person (ibid., fol. 235).

An Evaluation of the Submissions

Were there typical manners of "solutions"? There are, in fact - the most common ones were:

- it was claimed that the relation between the radius and the diameter is rational;
- a mechanical or a graphical manner of measuring was realised;
- it was "shown" that the value of π is another one as that affirmed by the science; such values were, among others: $\sqrt{3} + \sqrt{2}$; 3,125; 3,25.

And now, a more statistical manner of evaluating:

- between 1820 and 1890, there were 40 authors of submissions – some submitted various times; the maximum being five times;
- of these 40, 28 were from Prussia, two were from other German countries and 10 from non-German countries;

- the social class or profession was visible in 29 cases – in brackets the number of foreigners:
  - 5 (0) craftsmen
  - 9 (3) technicians or military officers
  - 9 (5) middle class (functionaries or working in education)

Fermat's Last Theorem : The Wolfskehl Prize

As is well known, the so-called last theorem of Fermat claims : there exists no set of positive integers *x*, *y*, *z* and a number *n* , with *n* greater than 2 which would satisfy the equation:

$$x^n + y^n = z^n.$$

The note, which Fermat scribbled at the margin, is likewise well known:

> "*Cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet*":

the margin being too small to allow writing the wonderful demonstration he claimed to have found. This problem to find Fermat's alleged proof was tackled for a long time by professional mathematicians and did not attract the attention of amateurs. The cases for *n* being 3 and 4 were solved in the 18^{th} century and the case *n* = 5 was solved in 1825 by Dirichlet and Legendre. One mathematician particularly active to generalise these results was, in the middle of the 19^{th} century, Kummer, one of the mathematics professors at Berlin University.

This situation changed completely when there was announced, in 1908, – this time really! - a prize for finding the lost proof: the Wolfskehl Prize. The prize was defined by Paul Wolfskehl in his last will. Wolfskehl (1856-1906), son of a rich banker, studied medicine and became an ophthalmologist. However, by 1880, he remarked the first symptoms of multiple sclerosis and had to abandon this profession. He decided to study mathematics, from 1880 and happened to become from 1881 a student of Kummer. Wolfskehl learned from him about the Last Theorem and of Kummer's attempts to prove it. From 1887 to 1890, when he was definitely tied to the wheelchair, he gave lessons on number theory at the *Technische Hochschule* Darmstadt. In his last will, Wolfskehl determined a prize of the almost incredible value of 100.000 *Reichsmark* for whom would prove Fermat's Theorem, within the next 100 years. The prize had an immense value; translated in nowadays value, it were 1 600 000 Euros! The evaluation of the submissions as well as the administration of the financial aspect was trusted to the Göttingen Academy of sciences. The donation entered into vigour on 27 June 1908. The Prize and its conditions were published in the German and foreign mathematical journals.

The effect of the prize announcing came as a complete surprise for the mathematical community. The Academy was overflown by submissions with alleged proofs. In the first year, already 621 submissions arrived in Göttingen (AAdW I). However, this number has to be differentiated: it contains effective submissions and inquiries. Unfortunately, the Göttingen Academy did not establish an inventory of the submissions; the total number of submissions is estimated to be 5000 (ibid., p. 6). The numbers of received papers is registered only for the first three years:

- 1907-1908: 621 “solutions” and “inquiries”; the solutions for this year are stored in two folders. Since the first one contains 48 submissions, one can estimate the number for the first year as about 100;[1]
- 1908-1909: 70 solutions;
- 1910: 63 submissions.

Besides these three years, a counting was again made only for 1939; then, the number was 35 (AAdW I). The last folder of solutions preserved in the archive is of 1943. The number of formally acceptable solutions in the 35 years between 1908 and 1943 is estimated to about 1.400. Actually, these figures have to be commented upon: they give just one of two types of received solutions. What is the second type, with about 3.600 submissions? The Academy had stipulated, in order not to be overthrown by amateur submissions:

it considers only those mathematical dissertations, which were published in periodical journals, as monographs or in book-form and purchasable in the book-trade,

and had categorically excluded to accept any manuscript submission. The Academy had been convinced to have thus excluded all amateur applications. However, the non-professionals developed a real ingeniousness in avoiding any reviewing and presenting notwithstanding printed elaborations. How this? Such authors presented as monographs booklets they had published themselves, privately – without a publisher and without professional revision. Some of these booklets has no more than 7 pages. Regarding journals, they bypassed the meaning of the restriction – scientific journal – and chose either not-specialised journal, like *Allgemeine Versicherungs-Presse*, *Deutsche Versicherungs-Zeitung*, *Mitteilungen des Vereins der Ingenieure der k.k. österreichischen Staatsbahn*, or *Natur und Kultur*, or even newspapers: *Beilage der Münchner Neuesten Nachrichten*. There was also the particular case of an Academy member who read an alleged proof in his class, which then had to be printed in the *mémoires* of the Academy: this was the case of Ferdinand Lindemann, member of the Munich Academy.

Albert Fleck and his “Fermat-Klinik”

How was this enormous and unexpected number of submissions to be evaluated? Apparently, the Göttingen mathematicians – although some of them were members of the Academy there – were not very eager to serve as reviewers. The Academy resolved to ask assistance outside, and foremost in Berlin. The 70 papers of the second year were sent to “Dr. Fränkel, Berlin” for reviewing. And the next year, 1910, the 63 submissions were sent to “Dr. Fleck in Berlin”. Hence, a highly telling and revealing person entered the scene for dealing with the “Fermatisten”, as the Germans now called this new type of amateurs.

I learned first of the Fermat-Klink of a Dr. Fleck in reading a paper by Wilhelm Lorey (1873-1955), on the mathematical life in Berlin where he mentioned this clinic parenthetically, as generally known (Lorey 1951). Thanks to research by Manfred Stürzbecher (1997), a historian of medicine, it turned out that it had been the physician Albert Fleck and his clinic was just his desk where he analysed the alleged proofs of Fermat’s theorem. Why a physician was so highly competent in mathematics to detect errors even of a mathematician like Lindemann? Fleck presents one of the rare cases, analogously to Wolfskehl, to have studied mathematics and medicine – but in an inverse order!

[1] I am very grateful to Martin Blänkner, Archive of the Göttingen Academy, for this information and for assistance with this research.

Albert Fleck was born on December 6th, 1861 in Berlin; his father was a merchant. He attended the renowned *Graue Kloster* Gymnasium and passed there the *Abitur* exam in 1881. He continued to study at Berlin University and chose mathematics. No details are known so far but he might have studied with Kummer, too, and might have got to know Wolfskehl. It is not known why he - after eight semesters of mathematics - undertook to study another subject, medicine, in again eight semesters. He passed the *Staatsexamen*, the state exam in medicine, the necessary step for becoming admitted as physician. Strangely, he did not obtain the doctoral degree – as usual for physicians – in an immediate context, but only twelve years later, in 1915 at Leipzig University. The subject of the PhD thesis was professional diseases of painters, limers and varnishers. He was very active in publishing diagnostic literature for physicians and engaged in accident prevention for children.

Fleck was born as of Jewish religion, but he left the community in 1897. Nevertheless, the Nazi racists classified him as a Jew and forbade him to continue to practice his profession: They declared him a “Heilbehandler”, only permitted to deal with “Jewish” patients. Since they regarded his wife not as Jewish, he was considered to live in “Mischehe” – mixed marriage – and was not deported – at least not until 1943 when he died, still in Berlin.

Almost at the same time when he prepared his doctoral medical thesis, he was deeply immersed in analysing alleged Fermat proofs. And he was participating in the meetings of the Berlin Mathematical Society, founded by students of mathematics. In 1909 he presented there a paper on the Fermat problem (Fleck 1909). His efforts for analysing and commenting the submitted proofs were honoured in 1914 with the Leibniz medal in silver by the Berlin Academy of Sciences. The laudation emphasised the high merits of having detected the errors in countless attempts to find the proof.

The journal *Archiv für Mathematik und Physik* had apparently understood the challenge for the mathematical culture constituted by the enormous wave of submissions. As the reviews of alleged proofs published in the journal show, submissions were not only sent directly to the Academy – being “legal” ones or not – but there were also papers first sent to journals in order to get them published and obtain thus the status of a legal submission. No reports are known how other mathematical journals dealt with such submissions but the *Archiv* had decided to review them and had asked Fleck to do this work. It is evident from the published reviews that Fleck worked with two types of submissions: with direct submissions to the journal and with “legal” submissions sent to him by the Göttingen Academy. Those directly submitted are indicated as “manuscript” and those from Göttingen give the publication place and the number of pages.

Actually, there were two more reviewers who analysed also submissions: Oskar Perron (1880-1975), mathematics professor then at Heidelberg University, and Philipp Maennchen (1869-1945), mathematics teacher at Alzey. Of the total number of 111 published reviews, 11 were made by Perron and 3 by Maennchen. As already the number of 63 submissions sent to Fleck in 1910 shows, the number of submissions analysed by Fleck is much higher but unknown. Somehow, the journal must have thought to have complied with its self defined task to illuminate the public about required mathematical standards for attacking a proof for Fermat’s Last Theorem: In the first year, in 1909, the *Archiv* published 42 reviews. In 1910, it was 34 and in 1911 the number was 35. From 1912 on, no more reviews were published, without indicating a reason for this stop.

The social composition of the submitters was more mixed than in the case of the quadrature solvers. The submitters used to indicate their profession and this was informed, too, in the published reviews. When a submitter had not indicated his profession, the review told "profession unknown". Here, no craftsmen showed up. The relatively least profession was a merchant. Learned professions were well represented: many engineers, mathematics teachers, public officials, insurance mathematicians, military officers, judges, pastors, pharmacist and the more. There were even university teachers.

Some of Fleck's reviews should be mentioned as examples here. One of the first attempts reviewed by him is in fact one, which had been submitted to the Academy as a printed text: by a *Reallehrer*, i.e. a teacher at a modern secondary school. Fleck is analysing it in a neutral, purely mathematical style: the author, a Johann Kleiber from Munich, had made inadmissible substitutions.[2] The text had been published in a journal of scientific dissemination: *Natur und Kultur*, occupying only 3 pages. There, the author had made affirmations casting strong doubts about his intellectual and mathematical level:

> *The theorem to be proved [...] is so simple that every student of a higher grade of a Realschule, every student of a Sekunda of a Gymnasium could participate in the competition, being capable of some success.*

The author's alleged proof ended with the self-certitude: „Thus, Fermat's theorem is definitely settled on the whole line".[3]

Another example of contributing nothing to the real problem was one by Friedrich Pietzker, a well known mathematics Gymnasium teacher; he had published his solution simply in the journal for mathematics and science teachers co-edited by him: *Unterrichtsblätter für Mathematik und Naturwissenschaften*, in 1908. Fleck showed missing basic considerations.

But there were a very few more demanding submissions. For example, there was one by Benno Lind, with 45 pages, published in the renown and pertinent historical journal *Abhandlungen zur Geschichte der mathematischen Wissenschaften mit Einschluß ihrer Anwendungen*, issue XXVI, 1910. The review needed two and a half pages. In a first part, the author gave a valuable history of the problem and of its elementary proof attempts. Fleck put it as an error to have tried to give thereafter proper results without the necessary maximal diligence and self-criticism. While the first 26 equations of his text are correct, but already known, from equation 27 on there are perpetuating errors, due to wrong operating with variables modulo certain numbers. Here, Fleck cold not abstain from commenting that much printing ink has been wasted airily and that the author seems not yet to have acquired the forces and the self-discipline needed for overcoming big number theoretic difficulties.

The most prominent case of Fleck's reviews were attempts by Lindemann, who was confident after his transcendence proof to be able to solve also the Fermat problem. The first paper analysed by Fleck was the above mentioned one in the Munich Academy publications, of 1907 with 66 pages. There the author declared an earlier attempt of 1901 as wrong. But it was not difficult for Fleck to find numerous errors in applying number theory (nr. 25, 1909, pp. 108-110). Lindemann's third attempt was an 82-pages book, printed by a publisher in Leipzig. Fleck had no problems in detecting errors in operating with congruencies (nr. 33, 1909, p. 370).

Epilogue

There was the wide-spread rumour that the Wolfskehl prize had entirely lost its enormous value after World War I and the ensuing big inflation of the years 1922-23. Clearly, the

[2] Beweise des Fermatschen Satzes. Nr. 16. Albert Fleck, in: *Archiv für Mathematik und Physik*, 14 (1909), p. 371.
[3] I am grateful to Martin Blänkner, Archiv der Akademie Göttingen, who provided these quotations (my translation).

Göttingen Academy had been discrete about this. When the new currency *Reichsmark* was introduced after the inflation, in 1924, the value of the prize was determined to 20.000 *Reichsmark*. This accumulated to 75.000 Reichsmark until 1948 – was diminished again by the introduction of the *Deutsche Mark* in Western Germany to only 7.500 DM. In the forty years since then, this accumulated, however, again to 75.000 DM – hence, the not negligible value of the Wolfskehl Prize which Andrew Wiles received in 1997 when the Göttingen Academy conferred to him – eleven years before its expiration – the so long strived for Prize of the Wolfskehl will.

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