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He's polynomials method for analytical solutions of telegraph equation

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Abstract. In this paper, He's polynomials solution method (HPSM) is fully utilized for solving telegraph equation. The proposed HPSM is technically presented and applied to homogeneous linear form the telegraph equation. The results are expressed in closed form with good agreement compared to those in literature thereby attesting to the efficiency and reliability of the method as proposed. The HPSM remarked to be less time consuming with high level of accuracy. As such, it can serve as alternative to other methods.

Keywords: Analytical solutions; He's polynomials; telegraph equation, Adomian decomposition

1. Introduction

In most physical and mathematical situations, modelling of physical phenomena leads to differential models in the form of equations which can be termed linear or nonlinear. Whichever way, the solutions of such are hard to obtain if they exist. Hence, the quest for effective and reliable methods of solution [1-12]. The model to be considered in this work is the generalized telegraph equation (TE) which is a linear partial differential equation (PDE) describing the current and voltage on an electrical transmission line with x and t as distance and time parameter respectively. The generalized form of the telegraph equation is as follows:

$$\begin{cases} \Omega_{tt} + (a+b)\Omega_t + ab\Omega = c^2\Omega_{xx} + g(x,t),\ \Omega = \Omega(x,t), \\ \Omega(x,0) = f_1(x), \\ \Omega_t(x,0) = f_2(x), \end{cases} \tag{1.1}$$

where the constants $a, b,$ and c are real numbers, while $g(x,t), f_1(x),$ and $f_2(x)$ are known functions. The unknown function, $\Omega = \Omega(x,t)$ to be determined denotes voltage or current through the wire at position, x with respect to time, t. The derivation of (1.1) is contained in [13].

Recently, a good number of solution analysts have deliberated on various techniques for the exact and/or approximate solution to (1.1) [14-24]. This work proposes He's polynomial method for the solution of (1.1) [8-12, 25, 26]. Related researches on communication system, circuit, wave

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transmission, networking and so on include those of [27-28]. It is worth noting that the method being introduced is entwined in terms of applications with numerical analysis, computational finance, stochastic or random differential equations, and so on [29-34].

2. Remark on the HPSM

Let θ be an operator (integral or differential), such that:

$$\theta(\nu)=0. \tag{2.1}$$

Suppose we defined a convex homotopy function, $H(c,f)$ by:

$$H(c,f)=f\theta(c)+(1-f)G(c), \tag{2.2}$$

such that $G(c)$ is referred to as a functional operator. Hence, we get:

$$H(c,0)=G(c) \text{ and } H(c,1)=\theta(c), \tag{2.3}$$

if $H(c,f)=0$ is satisfied and a given embedded parameter, $f\in(0,1]$ is considered. In HPM, $f=p$ is applied as an expanding parameter term to obtain:

$$\begin{cases} c=\nu=\lim\limits_{p\to 1}\left(\sum\limits_{j=0}^{\infty}p^j\nu_j\right), \\ N(\nu)=\sum\limits_{j=0}^{\infty}p^jH_j. \end{cases} \tag{2.4}$$

The approach takes the nonlinear term to be $N(\nu)$ whenever (2.1) is decomposed, such that $H_k\text{'}s$ are the so-called He's polynomials defined as:

$$H_k(\nu_0,\nu_1,\nu_2,\cdots,\nu_k)=\frac{1}{k!}\frac{\partial^k}{\partial p^k}\left(N\left(\sum_{j=0}^{k}p^j\nu_j\right)\right)_{p=0}, \; k\geq 0. \tag{2.5}$$

3. The Generalized Telegraph Model and the HPSM (He's Polynomials Solution Method)

The HPSM is applied to the Telegraph equation in (1.1) as follows. Let us re-write (1.1) in integral form, while the two-fold integral operator, $I_0^t(\cdot)$ is applied accordingly. Thus:

$$\begin{cases} \Omega=\Omega(x,0)+\Omega_t(x,0)t+I_0^t\left(c^2\Omega_{xx}+g(x,t)-\left((a+b)\Omega_t+ab\Omega\right)\right), \\ \Omega(x,0)=f_1(x),\Omega_t(x,0)=f_2(x),\; \Omega(x,t)=\Omega. \end{cases} \tag{3.1}$$

This implies that:

$$\Omega=\underbrace{f_1(x)+f_2(x)t+I_0^t\left(g(x,t)\right)}_{\text{initial-resultant}}+I_0^t\left(c^2\Omega_{xx}-\left((a+b)\Omega_t+ab\Omega\right)\right). \tag{3.2}$$

In standard HPSM, the series solution is conveyed as:

$$\begin{cases} \Omega=\sum\limits_{i=0}^{\infty}p^i\Omega_i \\ \Omega=\Omega(x,t),p\to 1. \end{cases} \tag{3.3}$$

Hence, by homotopy convexity [9, 10] in line with (3.3), we have:

$$\begin{cases} \sum_{n=0}^{\infty} p^n \Omega_n = F(x) + I_0^t \left(c^2 \sum_{n=0}^{\infty} p^{n+1} (\Omega_n)_{xx} - \left((a+b) \sum_{n=0}^{\infty} p^{n+1} (\Omega_n)_t + ab \sum_{n=0}^{\infty} p^{n+1} (\Omega_n) \right) \right), \\ F(x,t) = f_1(x) + f_2(x)t + I_0^t \left(g(x,t) \right). \end{cases} \tag{3.4}$$

Thus, comparing the exponents (powers) of the $p's$ in (3.4), we have:

$$\begin{cases} \Omega_0 = F(x,t) \\ \Omega_1 = I_0^t \left\{ c^2 (\Omega_0)_{xx} - \left((a+b)(\Omega_0)_t + ab\Omega_0 \right) \right\} \\ \Omega_2 = I_0^t \left\{ c^2 (\Omega_1)_{xx} - \left((a+b)(\Omega_1)_t + ab\Omega_1 \right) \right\} \end{cases} \tag{3.5}$$

$$\begin{cases} \Omega_3 = I_0^t \left\{ c^2 (\Omega_2)_{xx} - \left((a+b)(\Omega_2)_t + ab(\Omega_2) \right) \right\} \\ \Omega_4 = I_0^t \left\{ c^2 (\Omega_3)_{xx} - \left((a+b)(\Omega_3)_t + ab(\Omega_3) \right) \right\} \end{cases}$$

$$\vdots$$

$$\Omega_{j+1} = I_0^t \left\{ c^2 \left(\Omega_j\right)_{xx} - \left((a+b)\left(\Omega_j\right)_t + ab\left(\Omega_j\right) \right) \right\}, j \geq 0. \tag{3.6}$$

So, the required solution is:

$$\Omega(x,t) = \lim_{p \to 1} \left(\sum_{n=0}^{\infty} p^n \Omega_n \right). \tag{3.7}$$

4. Applications

Let the following linear telegraph equation be considered [15, 24]:

$$\begin{cases} \Omega_{tt} + \Omega_t + \Omega = \Omega_{xx}, \ \Omega = \Omega(x,t), \\ \Omega(x,0) = e^x, \\ \Omega_t(x,0) = -e^x, \end{cases} \tag{4.1}$$

with an exact solution of the form:

$$\Omega(x,t) = e^{x-t}. \tag{4.2}$$

If (4.1) is compared with (1.1), then we have:

$$(a+b) = ab = c = 1, \ g(x,t) = 0, f_1(x) = e^x, \text{ and } f_2(x) = -e^x. \tag{4.3}$$

So, the recursive relation based on (3.5) and (3.6) is:

$$\begin{cases} \Omega_0 = (1-t)e^x, \\ \Omega_{j+1} = I_0^t \left\{ \left(\Omega_j\right)_{xx} - \left(\left(\Omega_j\right)_t + \left(\Omega_j\right) \right) \right\}, j \geq 0. \end{cases} \tag{4.4}$$

Hence,

$$\Omega_0 = (1-t)e^x,$$

$$\Omega_1 = I_0^t \left\{ (\Omega_0)_{xx} - \left((\Omega_0)_t + (\Omega_0) \right) \right\} = \frac{t^2 e^x}{2!},$$

$$\Omega_2 = I_0^t\left\{(\Omega_1)_{xx} - \left((\Omega_1)_t + (\Omega_1)\right)\right\} = -\frac{t^3 e^x}{3!},$$
$$\Omega_3 = I_0^t\left\{(\Omega_2)_{xx} - \left((\Omega_2)_t + (\Omega_2)\right)\right\} = \frac{t^4 e^x}{4!},$$
$$\Omega_4 = I_0^t\left\{(\Omega_3)_{xx} - \left((\Omega_3)_t + (\Omega_3)\right)\right\} = -\frac{t^5 e^x}{5!},$$
$$\vdots$$

Thus,

$$\begin{aligned}\Omega(x,t) &= (1-t)e^x + \frac{t^2e^x}{2!} - \frac{t^3e^x}{3!} + \frac{t^4e^x}{4!} - \frac{t^5e^x}{5!} + \cdots \\ &= \left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!} - \frac{t^5}{5!} + \cdots\right)e^x \\ &= e^{x-t}.\end{aligned} \tag{4.5}$$

The solution in (4.5) corresponds to those obtained in [14, 19]. Though, the approach contained herein appears simpler and straight forward. The approximate and the exact solutions are graphically displayed in Figure 1 and Figure 2 respectively.

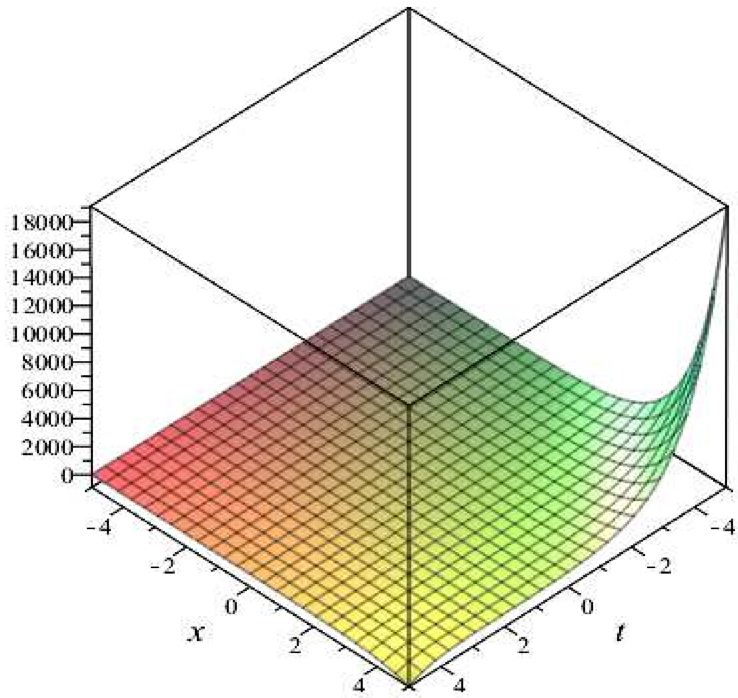

Figure 1: HPSM 6-term Approximate solution

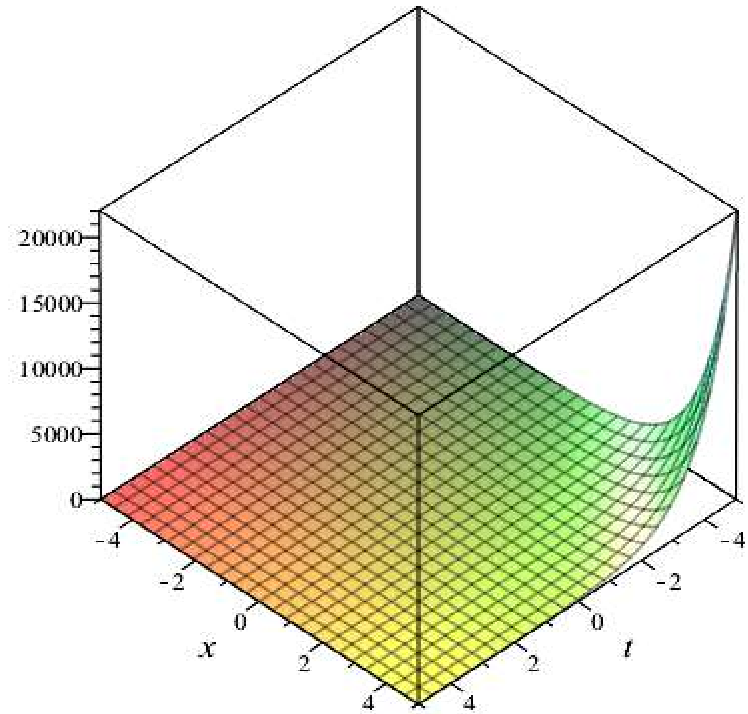

Figure 2: HPSM Exact solution

5. Conclusions

This work has successfully presented the application of the proposed solution method referred to as HPSM to the generalized telegraph equation in terms of approximate-analytical solutions. Closed form solutions of the solved problems were realized with ease, even with less computational time. Though, it may require coupling with other methods for highly nonlinear models. Hence, the HPSM is recommended for nonhomogeneous version of the generalized telegraph equation, and other highly nonlinear models.

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