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An Unsupervised Technique to Estimate λ^0-Fuzzy Measure Values and Its Application to Multi-criteria Decision Making

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***Abstract*—The use of Choquet integral as an aggregation operator in multi-criteria decision-making problems requires the prior estimation of fuzzy measure values. λ^0-measure is one form of fuzzy measure which was introduced to reduce the usual computational complexity associated with the estimation of fuzzy measure values. However, the existing techniques to estimate λ^0-measure require some amount of initial data from the decision-makers. This paper, therefore, aimed at proposing a completely unsupervised estimation technique, where the λ^0-measure values are directly derived based on the available decision matrix, without the need for any initial data from the decision-makers. The technique was developed by incorporating the CRITIC method into the original λ^0-measure estimation technique. The usage of the proposed technique was illustrated based on a university course evaluation problem. The same problem was also solved with a conventional additive operator for the comparison purpose.**

Keywords-* *Choquet integral; fuzzy measure; CRITIC; university course evaluation

I. Introduction

In the context of decision science, an evaluation involving a finite set of alternatives based on pre-determined criteria is usually treated as a multiple criteria decision making (MCDM) problem. The ultimate purpose of any standard MCDM analysis is to systematically compute the aggregated score of each available alternative so that they can be ranked accordingly from the most to the least preferred one. As such, aggregation is reported as one crucial phase in MCDM.

Suppose that $x_{i1}, x_{i2}, \dots, x_{in}$ denotes the performance scores of an alternative i with respect to n number of evaluation criteria, then aggregation can be defined as a procedure of synthesizing these scores into a single, aggregated score. Note that the mathematical function that combines these scores is usually referred as an aggregation operator. The procedure, of course, is repeated to compute the aggregated score of every available alternative, thus enable the decision-makers to select the finest possible alternatives with better confidence.

The available aggregation operators can be divided into two main categories, namely additive and non-additive operators. Needless to say, the additive operators such as simple weighted average, geometric mean, and ordered weighted average have received better attention for real application thanks to their computational simplicity. Sadly, these operators are unsuccessful in mathematically capturing the presence of interdependencies between the evaluation criteria, which in turn may result in misleading aggregated scores and ranking of alternatives. In other words, they are only appropriate to be applied in a problem where the criteria are assumed to be independent of each other. However, such an assumption may not be true in many situations as, in reality, most of the criteria are intertwined through different degrees of interrelations.

Fortunately, the Choquet integral [1], which is regarded as one of the non-additive operators, is free from such drawback; it has the ability to efficiently deal with the interrelated criteria when aggregating the performance scores [2]. Due to this interesting characteristic, of late, the application of Choquet integral is seen to be progressively extending across a wide array of MCDM problems, to name a few, site selection, benchmarking, and risk assessment. The integral equipped with such ability as it utilizes the concept of fuzzy measure. To be precise, the usage of Choquet integral as an aggregation operator requires the prior identification of fuzzy measure values. These values represent the weight of every possible subset of the criteria, including the individual weight of every criterion [3].

As a result, one will need to estimate 2^n values of fuzzy measure before applying the integral, where n denotes the number of criteria involved in the analysis. For instance, if a MCDM problem considers the following three criteria, A , B, and C, then the weights of following subsets need to be estimated in advance: {ø}, {A}, {B}, {C}, {A,B}, {A,C}, {B,C}, and {A,B,C}.

This estimation process, undoubtedly, can grow into a very complex undertaking, particularly when n is too large [4]. Many forms of fuzzy measures were introduced in the past to deal with this complexity, and λ^0-measure is one of them. Based on the literature, there exist a few techniques that can be used to estimate λ^0-measure values. Yet, all these techniques need some initial data from the decision-makers. In fact, to this date, none of the existing studies has attempted to develop an unsupervised technique, which is free from the need of initial data. This paper, therefore, aimed at developing one such technique, which able to estimate the complete set of λ^0-measure values by merely utilizing the available decision matrix.

The rest of this paper is organized as follows. Section II reviews the existing λ^0 -measure estimation techniques, mainly in the context of initial data requirement. Section III provides details on the steps involved in the proposed technique. Section IV demonstrates the usage of the proposed technique based on an undergraduate course evaluation problem. Section V concludes the contributions of the paper.

II. LITERATURE REVIEW

Assume $C = c_j = \{c_1, c_2, \dots, c_n\}$ is a finite set of evaluation criteria considered in a decision problem, then according to Larbani *et al.* [5], the λ^0-measure value, g of a subset consisting of two criteria, j and j' can be computed using (1), whereas the value of a subset comprising more than two criteria can be determined using (2).

$$g\{c_j, c_{j'}\} = g_j + g_{j'} + \lambda_{jj'} \quad \text{where } j \neq j',\ g_j = g\{c_j\}, \ \&\ g_{j'} = g\{c_j\} \tag{1}$$

$$g\{A\} = \sum_{c_j \in C} g_j + \bigvee_{c_j, c_{j'} \in C, j \neq j'} \lambda_{jj'} \tag{2}$$

where A denotes any subsets of C consisting more than two criteria

Note that equation (1) and (2) assure λ^0-measure to fulfill the two basic properties required by any fuzzy measure, namely the boundary and monotonicity property, where:

- boundary property means that the value of null subset is zero and the value of the subset with the presence of all criteria is one, i.e. $g\{\emptyset\} = 0$ and $g\{C\} = 1$, and
- monotonic property means that adding any new criterion into a subset will not decrease the value of the subset, i.e. $\forall A, B \in P\{C\}$, if $A \subseteq B$, then implies $g\{A\} \leq g\{B\}$.

The overall procedure of estimating λ^0-measure values, as proposed in its original work, can be summarised as follows. In the 1st stage, the decision-makers, who are expected to be familiar with the decision problem at hand, are required to subjectively estimate the interdependency degree, $\lambda_{jj'}$ for each pair of different criteria, i and j. The estimation is done by adhering to a predetermined scale that ranges from 0 to 1 where 0 implies "completely independent" and 1 denotes "completely interrelated".

In the 2nd stage, the fuzzy density of every criterion g_j, $j = 1, \dots, n$ is determined by finding the solution for the following system of inequalities (3).

In the 3rd stage, the identified $\lambda_{jj'}$ and g_j values are precisely replaced into (1) and (2) in order to estimate the whole set of λ^0-measure values. The identified λ^0-measure values and the available performance scores of each alternative can then be substituted into Choquet integral (4) to compute their final aggregated scores. With regards to (4), A_j refers to any the subsets of C for $j = 1,2, \dots, n$ and x_j represents the performance score of the alternative with respect to criterion j. Also, note that the permutation of criteria in A_n parallel to the descending order of the performance scores. For instance, if $x_{(1)} \leq x_{2)} \leq \cdots \leq x_{(n)}$, then $A_n = c_{(1)}, c_{(2)}, \dots, c_{(n)}$.

$$\begin{aligned} &0 \leq g_j + g_{j'} + \lambda_{jj'} \leq 1, \text{ for all } c_j \ \&\ c_{j'} \text{ in } C \text{ where } j \neq j' \\ &g(C) = \sum_{c_j \in C} g_j + \bigvee_{c_j, c_{j'} \in C, j \neq j'} \lambda_{jj'} = 1 \\ &g_j \geq 0, j = 1, \dots, n \end{aligned} \tag{3}$$

$$Choquet_g(x_1, x_2, \dots, x_n) = \sum_{j=1}^{n} (x_j - x_{j-1})\, g(A_n) \tag{4}$$

Note that the original λ^0-measure estimation technique fails to clearly reveal the complete relationship that presence across the criteria, and therefore the decision-makers may not be able to develop the best possible strategies to improve the performance of the targeted alternatives. Owing to this limitation, two revised techniques, namely the DEMATEL [6] and interpretive structural modeling (ISM) [7] based technique, were then introduced. However, all these available techniques still require some amount of initial data from the decision-makers. The type and amount of initial data required by each of these techniques are summarised in Table I. Meanwhile, Fig. 1 shows how the initial data requirement for each technique grows with an increasing number of decision criteria. Based on Table 1 and Fig. 1, it can be concluded that the initial data requirement for all the three techniques are still at a manageable level when n is small, but it increases exponentially as n becomes larger. This means the involved decision-makers may have a complication in providing consistent or precise initial data when the decision problem entails large n. It gets even more complex if the decision-makers are not familiar or ill-informed about the problem that they are dealing with. Therefore, in such a situation, it will really be helpful if an unsupervised technique, which can derive the λ^0-measure values solely based on the available decision matrix is developed.

TABLE I. COMPARISON OF THE AVAILABLE ESTIMATION TECHNIQUES

Technique	Type of initial data required	Amount of initial data required
Original technique	Interdependency degree of each pair of criteria	$n(n-1)/2$
DEMATEL based technique	• Direct influence between every two criteria • Interdependency degree of each pair of criteria	$3n(n-1)/2$
ISM based technique	• Contextual relationship between every two criteria • Interdependency degree of each pair of criteria	$2n(n-1)/2$

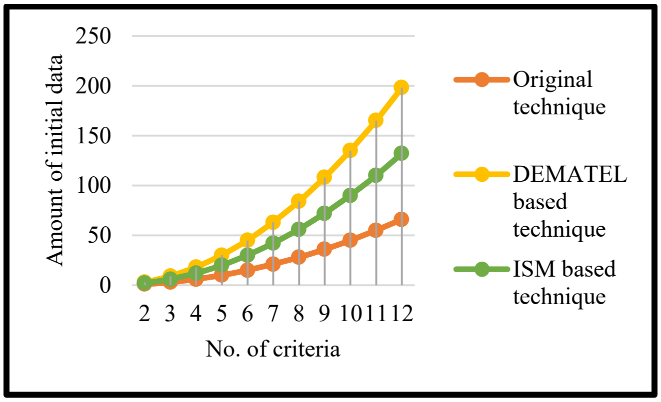

Figure 1. Amount of initial data required vs. no. of criteria

III. THE PROPOSED TECHNIQUE

The proposed technique is developed by integrating the CRITIC (Criteria Importance through Intercriteria Correlation) method with the original λ^0-measure estimation technique. All in all, the execution of the proposed technique entails four important phases: (1) Applying CRITIC method, (2) identifying the inputs required for estimation, (3) estimating λ^0 -measure values, and (4) applying Choquet integral.

In Phase 1, the CRITIC method is used mainly for the purpose of determining the objective weights of the evaluation criteria. The CRITIC method is suggested for this purpose as it has the ability to determine the weights by taking into consideration the contrast intensity and the conflicting character of the criteria. Say $[x_{ij}]$ is a decision matrix where $Alt_i = \{a_1, a_2, \dots, a_m\}$ is a set of alternative evaluated based on a predetermined set of criteria, $C_j = C = \{c_1, c_2, \dots, c_n\}$. The steps involved in applying the CRITIC method can then be summarized as follows:

- Step 1- Translate the scores of each criterion within the range of 0 to 1 using the following normalization technique (5).

$$\overline{x_{ij}} = \frac{x_{ij} - x_j^{worst}}{x_j^{best} - x_j^{worst}} \tag{5}$$

- Step 2- Calculate the standard deviation of each criterion, s_j based on the normalized matrix.
- Step 3- Find the correlation coefficient of every pair of criteria, $r_{jj'}$ based on the normalized matrix to form the correlation matrix, $[r_{jj'}]$.
- Step 4- Calculate the amount of information contained in criterion j using equation (6).

$$I_j = s_j \sum_{j'=1}^{n} (1 - r_{jj'}) \tag{6}$$

- Step 5- Determine the objective weight of criterion j using equation (7).

$$w_j = \frac{I_j}{\sum_{j=1}^{n} I_j} \tag{7}$$

In phase 2, the inputs needed to estimate λ^0 -measure values are determined. It is important to recall that the estimation actually requires two types of inputs. The first and perhaps the most crucial input is the degree of interdependency, $\lambda_{jj'}$ of every pair of criteria. The second input is the fuzzy density of each criterion, where, according to Larbani *et al.* [5], these fuzzy densities can simply be determined by solving equation (3). However, Krishnan *et al.* [6] claimed that a more reliable set of fuzzy densities could be determined if the decision-makers could provide some additional information about the densities such as the probable relative differences among them. As such, in this proposed technique, the correlation coefficients and the objective weights derived via the usage of the CRITIC method, are utilized to determine the said two inputs, respectively.

- Input 1- Degree of interdependency

A correlation coefficient lies between -1 and 1. The sign and absolute value of a coefficient indicate the direction and intensity of the relationship, respectively. The closer the value to -1 and 1, the more interrelated are the two variables or criteria. In fact, Carbunaru-Bacescu and Condruz-Bacescu [8] claimed that a correlation coefficient can be considered as the degree of interdependency between two criteria. Given this, in this technique, the following simple rules (8) are used to transform $r_{jj'}$ to the needed $\lambda_{jj'}$ values.

$$\begin{aligned} &\text{If } r_{jj'} < 0, \text{ then } \lambda_{jj'} = |r_{jj'}| \text{ and} \\ &\text{If } r_{jj'} \geq 0, \text{ then } \lambda_{jj'} = r_{jj'} \end{aligned} \tag{8}$$

- Input 2- Fuzzy densities

The objective weights, w_j and $\lambda_{jj'}$ values determined earlier are then used to develop and solve the following system of inequalities (9) to identify the fuzzy density of each criterion, g_j.

$$\begin{aligned} &0 \leq g_j + g_{j'} + \lambda_{jj'} \leq 1, \text{ for all } c_j \ \& \ c_{j'} \text{ in } C \\ &g(C) = \sum_{c_j \in C} g_j + \bigvee_{c_j, c_{j'} \in C, j \neq j'} \lambda_{jj'} = 1 \\ &g_j \geq 0, j = 1, \dots, n \\ &{}^*g_1 : g_2 : \dots : g_n = w_1 : w_2 : \dots : w_j \end{aligned} \tag{9}$$

It can be noticed that the proposed system of inequalities (9) somewhat varies from the original one (3) with the presence of (*), where it ensures that the ratio of the densities, g_j complies with the ratio of the objective weights, w_j determined via CRITIC method. For instance, if $w_1 = 0.1$, $w_2 = 0.7$, and $w_3 = 0.2$, then (*) can be expressed as follows: $g_2 = 7g_1$ and $g_2 = 3.5g_3$.

In phase 3, the identified inputs (i.e. λ_{ij} and g_1) are substituted accordingly into (1) and (2) to estimate the whole set of λ^0 -measure values, before applying them to the Choquet integral operator in phase 4.

IV. APPLICATION

Similar to any other public universities in Malaysia, Universiti Malaysia Sabah has its own course evaluation system, popularly known as PK07. The system requires the students to rate their agreement or satisfaction over the course, based on a prefixed set of criteria. The rating is done using a 5-point Likert scale, where 1 and 5 implies "very unsatisfied" and "very satisfied", respectively. The scorings resulted from this evaluation will be used as the indicators for improving the course content and delivery, apart from taking them into consideration for the lecturer's performance appraisal. The university has therefore made it mandatory for all university undergraduate students to fill out and submit the online PK07 form before sitting for their final examination. However, in the current PK07 system, the ratings across all the criteria are aggregated by simply assuming that every criterion has an equal weightage, and also without capturing the interrelationships held by those criteria; thus, the final aggregated scores may not exactly reflect the lecturers' actual performance. On that note, this section aimed at demonstrating the possibility of extending the proposed technique to the existing PK07 system, apart from illustrating the usage of the technique.

Table 2 is an example of a decision matrix showing the average satisfaction ratings of five undergraduate courses with respect to four PK07 evaluation criteria, namely lecturer's preparation (c_1) delivery (c_2), assessment (c_3) and course learning outcomes (c_4). Note that for the sake of simplicity, only four PK07 criteria were retained in this example. The aggregated score of each course and its respective ranking was then computed based on the data in Table II using the proposed fuzzy measure estimation technique along with the Choquet integral operator. The overall calculation process involved and the results obtained are summarized in Table III-VIII.

Table 3 shows the normalized decision matrix derived using equation (5), together with the standard deviation of each criterion. Meanwhile, Table IV presents the correlation among the involved criteria. The information content and objective weight of each criterion identified using (6) and (7) are summarized in Table V. The objective weights are hinting that students care more on c_2 and c_3 as compared to c_1 and c_2. This means that the lecturers will have to pay extra attention to refine their delivery technique and the design of assessments if they wish to significantly improve their student satisfaction in the future.

TABLE II. DECISION MATRIX (COURSES VS CRITERIA)

	c_1	c_2	c_3	c_4
Course A	3.90	4.60	3.75	4.64
Course B	3.80	3.90	4.20	4.52
Course C	3.75	4.52	4.48	4.00
Course D	3.88	3.86	4.63	4.61
Course E	3.67	4.60	3.98	4.64

TABLE III. NORMALIZED DECISION MATRIX AND STANDARD DEVIATION OF EACH COURSE

	c_1	c_2	c_3	c_4
Course A	1.00	1.00	0.00	1.00
Course B	0.57	0.05	0.51	0.81
Course C	0.35	0.89	0.83	0.00
Course D	0.91	0.00	1.00	0.95
Course E	0.00	1.00	0.26	1.00
s_j	0.411	0.515	0.407	0.428

TABLE IV. CORRELATION MATRIX OF CRITERIA

Criteria	c_1	c_2	c_3	c_4
c_1	1	-0.382	0.029	0.286
c_2	-0.382	1	-0.601	-0.198
c_3	0.029	-0.601	1	-0.473
c_4	0.286	-0.198	-0.473	1

TABLE V. INFORMATION CONTENT AND OBJECTIVE WEIGHT OF EACH CRITERION

Criteria	I_j	w_j
c_1	1.262	0.194
c_2	2.155	0.331
c_3	1.647	0.253
c_4	1.449	0.222

On the other hand, based on the values in Table 4, and also by adhering to the rules stated in (8), the following the interdependency degrees were determined: $\lambda_{12} = 0.382$, $\lambda_{13} = 0.029$, $\lambda_{14} = 0.286$, $\lambda_{23} = 0.601$, $\lambda_{24} = 0.198$, and $\lambda_{34} = 0.473$. The following system of inequalities was then constructed based on the available λ_{ij} values and objective weights, w_j, and it was solved with the help of EXCEL Solver to determine the fuzzy densities (in this case, the fuzzy densities are $g_1 = 0.077, g_2 = 0.132, g_3 = 0.101$, and $g_4 = 0.089$.

$$g_1 + g_2 + 0.382 \leq 1$$
$$g_1 + g_3 + 0.029 \leq 1$$
$$g_1 + g_4 + 0.286 \leq 1$$
$$g_2 + g_3 + 0.601 \leq 1$$
$$g_2 + g_4 + 0.198 \leq 1$$
$$g_3 + g_4 + 0.473 \leq 1$$
$$g_1 + g_2 + g_3 + g_4 + 0.601 = 1$$
$$g_1, g_2, g_3, g_4 \geq 0$$
$$g_2 = 1.706 * g_1$$
$$g_2 = 1.308 * g_3$$
$$g_2 = 1.491 * g_4$$

Table 6 depicts the λ^0-measure values for every possible coalition of the criteria, which were estimated based on (2) and (3). It can be seen that the estimated values comply with the boundary and monotonicity condition.

TABLE VI. COMPLETE SET OF λ^0-MEASURE VALUES

Subset	Value, g
$\{\emptyset\}$	0
$\{c_1\}$	0.077
$\{c_2\}$	0.132
$*\{c_1, c_2\}$	0.591
$\{c_3\}$	0.101
$\{c_1, c_3\}$	0.208
$\{c_2, c_3\}$	0.834
$**\{c_1, c_2, c_3\}$	0.911
$\{c_4\}$	0.089
$\{c_1, c_4\}$	0.452
$\{c_2, c_4\}$	0.419
$\{c_1, c_2, c_4\}$	0.680
$\{c_3, c_4\}$	0.663
$\{c_1, c_3, c_4\}$	0.740
$\{c_2, c_3, c_4\}$	0.922
$\{c_1, c_2, c_3, c_4\}$	1

Calculation for selected subsets of criteria:
$*g_1 + g_2 + \lambda_{12} = 0.077 + 0.132 + 0.382 = 0.591$
$**g_1 + g_2 + g_3 + \lambda_{12} = 0.077 + 0.132 + 0.101 + 0.601 += 0.911$

The estimated λ^0 -measure values together with the ratings in Table II were then replaced into the Choquet integral formula (4) to compute the aggregated score of each course. Based on the results in Table VII, C can be considered as the 'most satisfying' course with the aggregated score of 4.386, followed by D, E, A, and B. The same evaluation was also performed based on the objective weights derived through CRITIC method and a common additive operator, namely simple weighted average (SWA). A different set of ranking was then obtained, where Course E tops the list followed by A, C, D, and B. The key reason that has led to this difference is that the latter operator has failed to capture the interdependencies between criteria while aggregating the ratings, unlike the former one which was able to model the said interdependencies with the help of the estimated fuzzy measure values.

TABLE VII. AGGREGATED SCORE AND RANKING OF EACH COURSE

Course	Aggregated score (ranking) using the proposed technique & Choquet integral operator	Aggregated score (ranking) using SWA operator
A	4.149 (4)	4.258 (2)
B	4.120 (5)	4.094 (5)
C	4.386 (1)	4.245 (3)
D	4.361 (2)	4.225 (4)
E	4.219 (3)	4.272 (1)

V. CONCLUSION

This paper has made two important contributions. Firstly, from a theoretical viewpoint, we have presented a completely unsupervised technique to estimate λ^0-measure values. The technique enables the ill-informed decision makers to directly estimate the whole set of the fuzzy measure values based on the available decision matrix, without the need for any additional initial data from them.

Meanwhile, from an application viewpoint, we have demonstrated the possibility of integrating the usage of the proposed technique into the PK07 system, a course evaluation system owned by UMS. Such integration will allow a more realistic aggregated score for each course is computed without disregarding the existing interdependencies among the evaluation criteria. It will also help to furnish information about the weights of criteria, and thus enable the lecturers to plan the right strategies to improve their scores in the future.

However, the proposed technique has a drawback. It fails to characterize the exact causal relationships held by the criteria under consideration, unlike the existing DEMATEL and ISM based technique which clearly visualize such relationships through diagraphs. Future research may address this limitation by developing an unsupervised λ^0-measure estimation technique that can also deliver clear-cut information about the causal relationships as such information could lead to better decision-making.

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