

# STOCHASTIC AND MODIFIED SEQUENT PEAK ALGORITHM FOR RESERVOIR PLANNING ANALYSIS CONSIDERING PERFORMANCE INDICES

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## STOCHASTIC AND MODIFIED SEQUENT PEAK ALGORITHM FOR RESERVOIR PLANNING ANALYSIS CONSIDERING PERFORMANCE INDICES

by

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### **Dedicated to My Dear Parents**

Esmaeil Saket Oskoui and Nahid Azari

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#### LIST OF ABBREVIATIONS

AR	Auto Regressive
ARMA	Auto Regressive Moving Average
BL	Broken Line
CDF	Cumulative Distribution Function
СР	Critical Period
DF	Degrees of Freedom
DID	Department of Irrigation and Drainage
DP	Dynamic Programming
DRI	Drought Risk Index
F	Failure
FGN	Fractional Gaussian Noise
FM	Figure of Merit
ISM	Indexed Sequential Method
LP	Linear Programming
MA	Moving Average
MAF	Mean Annual Flow
MLE	Maximum Likelihood Estimates
NLP	Non-Linear Programming
PDF	Probability Density Function
PPCC	Probability Plot Correlation Coefficient
RMSE	Root Mean Square Error
RNG	Random Number Generator
S	Success

- SC Storage Capacity
- SD Standard Deviation
- SE Standard Error
- SPA Sequent Peak Algorithm
- SPSS Statistical Package for Social Sciences
- SROC Spearman Rank Order Correlation
- WMO World Meteorological Organization

#### LIST OF SYMBOLS

А	Coefficient matrix of (MN×N) in the V-S model
AR(1)	auto regressive model of 1 order
AR(p)	autoregressive model of p order
ARMA(p, q)	autoregressive moving average models
Ac	regression coefficient for modeling the critical period
As	regression coefficient for modeling the storage capacity
A <sub>k</sub>	Kirby parameter for modifying Pearson type III reduced variate
At	reservoir water surface area at the beginning period t
$A_{t+1}$	reservoir water surface area at the end of period t
a	parameter, linearized slope of the area-storage curve for the reservoir
В	Coefficient matrix of (MN×MN) in the V-S model
Bc	regression coefficient for modeling the critical period
Bs	regression coefficient for modeling the storage capacity
$\mathbf{B}_k$	Kirby parameter for modifying Pearson type III reduced variate
B0, B1, B2	specific coefficients for autoregressive model
b	parameter, reservoir water surface area at the dead storage level
Cov	Covariance
CPE	critical period for English systems
CPI	critical period for Iranian systems
CP (Johor)	critical period for Johor system
CP (Melaka)	critical period for Melaka system
CP (Muar)	critical period for Muar system
CP (3 Systems)	critical period for the three study systems together

Cv	coefficient of variation
c	minimum fraction of target demand to be guaranteed in a failure period
D	demand expressed as a ratio of mean annual flow
Demand	annual demand from the reservoir as a ratio of mean annual flow
$D_{\rm w}$	a window width of a given duration in moving window method
E[]	Expectation function
EPt	depth of evaporation from reservoir surface in period t
EVt	volumetric storage-dependent fluxes during period t
$\mathrm{EV}_{\mathrm{t}}^{\mathrm{sys}}$	net evaporation losses of the system during period t
e <sup>()</sup>	exponential function employed for modeling the storage capacity
ei	independent zero mean and unit variance normal random variable
ē	the average of independent zero mean and unit variance normal random variables
ent	net evaporation (i.e. evaporation minus rainfall) in period t
F-stat	<i>F</i> -statistic
$\mathbf{f}_{s}$	the number of continuous sequences of failure periods
f	total number of failure periods
f	annual risk i.e. $1$ - $f$ is the annual time-based reliability
G <sub>k</sub>	Kirby parameter for modifying Pearson type III reduced variate
g	skewness of Pearson type III variable corrected for serial correlation
<b>g</b> 0	original sample skewness
H <sub>k</sub>	Kirby parameter for modifying Pearson type III reduced variate
Ho	the null hypothesis
h	Hurst coefficient
It	volumetric inflow into reservoir during period t

ICRIT	the time period corresponding to the end of the critical period
ICY	current critical period
IOVF	the time period corresponding to the beginning of the critical period
i	time / rank of observation values
K	reduced variate or frequency factor
к <sup>*</sup> а	minimum active storage capacity obtained by the modified SPA
K <sub>t-1</sub>	volumetric sequential storage deficits at the beginning of period t
Kt	volumetric sequential storage deficits at the end of period t
Ka, K'a	capacity estimates during any consecutive iterations for modified SPA
Ke	reduced variate of Gumbel distribution
Kg	reduced variate of Pearson type III distribution
$K_g^m$	modified Pearson type III reduced variate developed by Kirby (1972)
Ki	i <sup>th</sup> order reduced variate or frequency factor
Kn	reduced variate of the standard normal distribution
Ko	over-year storage capacity expressed as a ratio of Mean Annual Flow
K <sub>T</sub>	total (i.e. over-year plus within-year) storage capacity
k	continuous failure period
Le	the length of streamflow data
LN2	the two-parameter log-normal distribution
LN3	the three-parameter log-normal distribution
LP3	the log-Pearson type III distribution
Lt	other losses during period t in the SPA
М	the total number of seasons per year in the V-S model
MA(1)	moving average model of order 1

т	standard demand parameter / standardized demand parameter
max(sh <sub>k</sub> )	maximum shortfall during the k <sup>th</sup> continuous failure period
Ν	the length of the observed data record
Ν	the total number of sites in the V-S model
Ns	the number of independent samples or replicates
n	the number of the data
nu	the total number of data / the number of years of data
<b>n</b> 1	the number of successes in the randomness test
n2	the number of failures in the randomness test
Р	cumulative probability
Pi	the cumulative probability of i <sup>th</sup> ordered data observations
Pt	depth of rainfall on reservoir surface in period t
Р3	Pearson type III distribution
p	annual reliability
$\overline{q}$	historical mean annual flow
R	the total number of the runs in randomness test
R	the range of cumulative departures of annual flows from the mean
Re	the time-based reliability
Rt	release of the reservoir system during period t
$R_t^{sys}$	release of the system during period t
$R^*_t$	target demand of the reservoir system during period t
$R_t^{*sys}$	target demand of the system during period t
$R^2$	Coefficient of determination for the regression analysis
r ( )	lag-zero cross-correlation function for the V-S model

round { }	is a function that rounds to the closest integer number
rui	the rank of time series value during i period
S	standard deviation of annual flows
Skew	Skewness
SN	normalized storage capacity
Storage	active storage capacity
S-Y-P	storage-yield-performance
S-Y-R	storage yield relationships
SN (Johor)	normalized storage capacity for Johor system
SN (Melaka)	normalized storage capacity for Melaka system
SN (Muar)	normalized storage capacity for Melaka system
SN (3 Systems)	normalized storage capacity for the three systems together
St	active storage for the reservoir at the beginning of period t
$S_{t+1}$	active storage for the reservoir at the end of period t
Sĸ	the coefficient of skewness for annual flows
Sx	standard deviation of the observed data
S <sub>xx</sub>	The matrix of (MN×MN) in the V-S model
S <sub>xz</sub>	The matrix of (MN×N) in the V-S model
Szz	the covariance matrix of $(N \times N)$ in the V-S model
<b>S</b> 0	the covariance matrix of $(N \times N)$ in the V-S model
sh <sub>k</sub>	shortfall during the k-th continuous failure period
Т	total number of time periods in the record or inflow sequence
Т	one divided by cumulative probability
T-F	Thomas-Fiering

t	time period
tr	the coefficient of the trend for trend test
tu	the value of the test statistic for the trend test
tui	time of occurrence of time series data
$t\alpha_t/2$	value of trend test limit at $\alpha_t$ significance level
u	observed data record
u*	a bootstrap sample
ui	time series data during i period
Var	variance
V-S	Valencia-Schaake
Vi	the vector of (MN $\times$ 1) independent standard normal random variables
$V_u$	Vulnerability
$\overline{\mathbf{X}}$	mean of the observed data
Xi	the vector of (MN×1) transformed normally distributed seasonal flows at i year
Xi, Xi+1	annual flows for the $i^{th}$ and $(i+1)^{th}$ years, respectively
$X_{mi}^{n}$	streamflow for season m, year i and site n in the V-S model
$X_j^n$	flows of season j at the site n in the V-S model
$X_l^k \\$	flows of season l at the site k in the V-S model
Xi	i-th ordered data observations
X <sub>max</sub>	the largest observation
Xmed	sample medium
X <sub>min</sub>	the smallest observation
х	streamflow data