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Risk-Neutral Pricing for Arbitrage Pricing Theory

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Abstract

We consider infinite-dimensional optimization problems motivated by the financial model called Arbitrage Pricing Theory. Using probabilistic and functional analytic tools, we provide a dual characterization of the superreplication cost. Then, we show the existence of optimal strategies for investors maximizing their expected utility and the convergence of their reservation prices to the super-replication cost as their risk-aversion tends to infinity.

Keywords Infinite-dimensional optimization · Arbitrage Pricing Theory · Superreplication · Expected utility · Reservation price · Large markets

Mathematics Subject Classification 91G10 · 93E20 · 91B16

1 Introduction

We study infinite-dimensional optimization problems motivated by a celebrated financial theory called Arbitrage Pricing Theory (APT). We first expose the economic and financial background related to APT and show how important it is for both the financial mathematics and the mathematical economics communities. Then, we explain our contributions to this widely studied field together with their mathematical aspects.

Arbitrage Pricing Theory was originally introduced by Ross (see [1,2]), and later extended by [3,4], and numerous other authors. The APT assumes an approximate

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factor model and states that the risky asset returns in a "large" financial market are linearly dependent on a finite set of random variables, termed factors, in a way that the residuals are uncorrelated with the factors and with each other. One of the desirable aspects of the APT is that it can be empirically tested as argued, for example, in [5]. These conclusions had a huge bearing on empirical work: see for instance [6]. Papers on the theoretical aspects of APT mainly focused on showing that the model is a good approximation in a sequence of economies when there are "sufficiently many" assets (see for example, [1,3,4]).

Ross derives the APT pricing formula under the assumption of absence of asymptotic arbitrage in the sense that a sequence of asymptotically costless and riskless finite portfolios cannot yield a positive return in the limit. Mathematical finance subsequently took up the idea of a market involving a sequence of markets with an increasing number of assets in the so-called theory of large financial markets (see, among other papers, [7–10]). Authors mainly studied the characterization of asymptotic notions of absence of arbitrage, using sequences of portfolios involving finitely many assets, where the classical notion of no-arbitrage holds true, i.e., non-negative portfolios with zero cost should have zero return. For the sake of generality, continuous trading was assumed in the overwhelming majority of related papers. But these generalizations somehow overshadowed the highly original ideas suggested in [1], where a one-step model was considered. They did not answer the following natural question either: in the APT is there a way to consider strategies involving possibly all the infinitely many assets and to exclude exact arbitrage for them rather than considering only asymptotic notions of arbitrage? A first answer was given in [11] in a measure-theoretical setup. Then, [12,13] proposed a straightforward concept of portfolios using infinitely many assets, which we will use in the present paper, too: see Sect. 2. This notion leads to the existence of equivalent risk-neutral (or martingale) probability measures.

While questions of arbitrage for APT have been extensively studied by the economics and financial mathematics communities, other crucial topics—such as utility maximization or pricing—received little attention though these are important questions in today's markets, where there is a vast array of available assets. This is particularly conspicuous in the credit market, where bonds of various maturities and issuers indeed constitute an entity that may be best viewed as a large financial market (see [14]). Questions of pricing inevitably arise and current literature on APT does not provide satisfactory answers. A standard problem is calculating the superreplication cost of a claim G. It is the minimal amount needed for an agent selling G in order to superreplicate G by trading in the market. This is the hedging price with no risk and, to the best of our knowledge, it was first introduced in [15] in the context of transaction costs. In complete markets with finitely many assets, the superreplication cost is just the cash flow's expectation computed under the unique martingale measure. When such markets are incomplete, there exists a so-called dual representation in terms of supremum of those expectations computed under each risk-neutral probability measure, see [16] and the references in [17]. Our first contribution is such a representation theorem for APT under mild conditions (see Theorem 4.1). The proof is based on functional analytic techniques such as the Marcinkiewicz–Zygmund inequality or the Banach–Saks property. The uniform integrability property proved in Lemma 3.3 together with dual methods (using risk-neutral probabilities) allow to prove, for the first time in the context of APT, the closure in probability of the set of attainable terminal payoffs, after possibly throwing away money (see Proposition 3.1 and Corollary 3.1). We also prove a characterization of the no-arbitrage condition in a so-called quantitative form (see Proposition 3.2), which will be crucial in the rest of the paper. We mention [18], where the superhedging of contingent claims has already been considered in the general context of continuous-time large financial markets. That paper, however, relies on the notion of generalized portfolios, which fail to have a natural interpretation unlike the straightforward portfolio concept we use here.

Next, we consider economic agents whose preferences are of von Neumann-Morgenstern type (see [19]), i.e., they are represented by concave increasing utility functions. In our APT framework, we are able to prove the existence of optimizers for such utility functions on the positive real axis (see Theorem 5.1). Such results are standard for finitely many assets (see the references in [17]), but in the present context we face infinite-dimensional portfolios. In the setting of APT, we mention [20], which relies on the notion of generalized portfolios. Utility functions defined on the real line (i.e., admitting losses) have been considered in [12,13] (we expose the differences between these two papers and ours in Remark 3.2). Our quantitative no-arbitrage characterization allows to prove a key boundedness condition on the set of admissible strategies (see Lemma 3.4) and the existence of an optimal solution. Finally, we establish that, when risk aversion tends to infinity, the utility indifference (or reservation) prices (see [21]) tend to the superreplication price. This links in a nice way investors' price calculations to the preference-free cost of superhedging (see Theorem 6.1). It also justifies the use of a cheaper, preference-based price instead of the super-replication price, which may be too onerous.

The model is presented in Sect. 2. Concepts of no-arbitrage are discussed in Sect. 3. The dual characterization of superreplication prices is given in Sect. 4, the utility maximization problem is treated in Sect. 5. The asymptotics of reservation prices in the high risk-aversion regime is investigated in Sect. 6, and Sect. 7 concludes.

2 The Large Market Model

Let $(\Omega, \mathcal{F}, P)$ be a probability space. We consider a one-step economy, which contains a countable number of tradeable assets. The price of asset $i \in \mathbb{N}$ is given by $(S_t^i)_{\{t\in\{0,1\}\}}$. The returns R_i, $i \in \mathbb{N}$ represent the profit (or loss) created tomorrow from investing one dollar's worth of asset i today, i.e., $R_i = S_1^i/S_0^i - 1$. We briefly describe below our version of the Arbitrage Pricing Model, identical to that of [8,12,13,22], which is a special case of the model presented in [1,3]. Asset 0 represents a riskless investment and, for simplicity, we assume a zero rate of return, i.e., $R_0 = 0$. We assume that the other assets' returns are given by

$$R_i := \mu_i + \bar{\beta}_i \varepsilon_i, \quad 1 \le i \le m; \quad R_i := \mu_i + \sum_{j=1}^{m} \beta_i^j \varepsilon_j + \bar{\beta}_i \varepsilon_i, \quad i > m,$$

where the ε_i are random variables and μ_i, β_i^j, $\overline{\beta}_i$ are constants. The random variables ε_i, $1 \le i \le m$ serve as *factors*, which influence the return on all the assets $i \ge 1$, while ε_i, $i > m$ are random sources particular to the individual assets R_i, $i > m$.

Assumption 1 The ε_i are square-integrable, independent random variables satisfying $E(\varepsilon_i) = 0$ and $E\left(\varepsilon_i^2\right) = 1$ for all $i \ge 1$.

Assuming that $\bar{\beta}_i \neq 0$, $i \ge 1$, we reparametrize the model using

$$b_i := -\frac{\mu_i}{\bar{\beta}_i}, \quad 1 \le i \le m; \quad b_i := -\frac{\mu_i}{\bar{\beta}_i} + \sum_{j=1}^{m} \frac{\mu_j \beta_i^j}{\bar{\beta}_j \bar{\beta}_i}, \quad i > m$$

and set $b := (b_i)_{i \ge 1}$. Asset returns then take the following form:

$$R_i = \bar{\beta}_i(\varepsilon_i - b_i), \quad 1 \le i \le m; \quad R_i = \sum_{j=1}^{m} \beta_i^j (\varepsilon_j - b_j) + \bar{\beta}_i(\varepsilon_i - b_i), \quad i > m.$$

For some $n \in \mathbb{N}$, a portfolio ϕ in the assets $0, \ldots, n$ is an arbitrary sequence $(\phi_i)_{0 \le i \le n}$ of real numbers satisfying $\sum_{i=0}^{n} \phi_i S_0^i = x$, where x is a given initial wealth. As $S_1^0 = S_0^0$ such a portfolio will have value tomorrow given by

$$V_n^{x,\phi} := \sum_{i=0}^{n} \phi_i S_1^i = x + \sum_{i=1}^{n} \phi_i S_0^i R_i = x + \sum_{i=1}^{n} h_i(\varepsilon_i - b_i) =: V_n^{x,h},$$

for some $(h_1, \ldots, h_n) \in \mathbb{R}^n$, using our parametrization.

The value tomorrow that can be attained using finitely many assets is given by $J^x := \cup_{n \ge 1} V_n^{x,h} : (h_1, \ldots, h_n) \in \mathbb{R}^n$. As J^x fails to be closed in any reasonable sense, we consider strategies, which can use infinitely many assets. This is desirable from an economic point of view (see [11]). Let

$$\Phi : \ell_2 := \left\{ (h_i)_{i \ge 1} : \sum_{i=1}^{\infty} h_i^2 < \infty \right\} \to L^2(P) := \{X : \Omega \to \mathbb{R},\ E|X|^2 < \infty\}$$
$$x \mapsto \Phi(h) := \sum_{i=1}^{\infty} h_i \varepsilon_i.$$

Recall that the spaces ℓ_2 and $L^2(P)$ are Hilbert spaces with the respective norm $||h||_{\ell_2} := \sqrt{\sum_{i=1}^{\infty} h_i^2}$ and $||X||_{L^2(P)} := \sqrt{E(|X|^2)}$. The infinite sum in $\Phi(h)$ has to be understood as the limit in $L^2(P)$ of $(\sum_{i=1}^{n} h_i \varepsilon_i)_{n \ge 1}$, which are Cauchy sequences. Indeed, let $h \in \ell_2$, under Assumption 1, for $p > n$,

$$E\left(\left(\sum_{i=1}^{p} h_i \varepsilon_i - \sum_{i=1}^{n} h_i \varepsilon_i\right)^2\right) = \sum_{i=n+1}^{p} h_i^2 \le \sum_{i=n+1}^{\infty} h_i^2,$$

which can be arbitrarily small for n large enough. Actually, under Assumption 1, Φ is even an isometry, i.e., $||\Phi(h)||^2_{L^2(P)} = \sum_{i=1}^{\infty} h_i^2 = \|h\|^2_{\ell_2}$. We would like to give sense (as an $L^2(P)$ limit of a sequence of finite sums) to the portfolio value $V^{x,h} := x + \sum_{i=1}^{\infty} h_i(\varepsilon_i - b_i)$. Since

$$E\left(\left(\sum_{i=1}^{p} h_i(\varepsilon_i - b_i) - \sum_{i=1}^{n} h_i(\varepsilon_i - b_i)\right)^2\right) = \sum_{i=n+1}^{p} h_i^2 + \sum_{i=n+1}^{p} h_i^2 b_i^2, \quad (1)$$

we need the following hypothesis.

Assumption 2 We have that $b \in \ell_2$.

Then, (1) shows that $(\sum_{i=1}^{n} h_i(\varepsilon_i - b_i))_{n\geq 1}$ is a Cauchy-sequence in $L^2(P)$ and $V^{x,h}$ is well defined. Notice furthermore that

$$E\left(\left(\sum_{i=1}^{\infty} h_i(\varepsilon_i - b_i)\right)^2\right) = \sum_{i=1}^{\infty} h_i^2 + \sum_{i=1}^{\infty} h_i^2 b_i^2 \leq (1 + \|b\|^2_{\ell_2})\|h\|^2_{\ell_2} < \infty. \quad (2)$$

From now on, we will use the notation $\langle h, \varepsilon - b\rangle := \sum_{i=1}^{\infty} h_i(\varepsilon_i - b_i)$. Under Assumptions 1 and 2, the portfolio values tomorrow that can be attained using infinitely many assets with a strategy in ℓ_2 is thus given by

$$K^x := \{V^{x,h} : h \in \ell_2\} = \{x + \langle h, \varepsilon - b\rangle : h \in \ell_2\}.$$

3 No-Arbitrage in Large Markets

In Arbitrage Pricing Theory, the classical notion of arbitrage is the asymptotic arbitrage in the sense of [1] and [3].

Definition 3.1 There is an asymptotic arbitrage, if there exists a sequence of strategies $(h(n))_{n\geq 1}$, with $h(n) = (h(n)_i)_{1\leq i\leq n}$, such that

$$E\left(V_n^{x,h(n)}\right) \underset{n\to+\infty}{\longrightarrow} \infty \text{ and } \mathrm{Var}\left(V_n^{x,h(n)}\right) \underset{n\to+\infty}{\longrightarrow} 0.$$

If there exists no such sequence, then we say that there is absence of asymptotic arbitrage (AAA).

We would like to understand the link between AAA and the classical definition of no-arbitrage, as formulated in the next definition.

Definition 3.2 The no-arbitrage condition on a "small market" with N random sources for some $N \geq 1$ holds true, if $P(\sum_{i=1}^{N} h_i(\varepsilon_i - b_i) \geq 0) = 1$ for $(h_1, \ldots, h_N) \in \mathbb{R}^N$ implies that $h_1 = \ldots = h_N = 0$. This is called AOA(N).

We prove that under the following assumption there is absence of arbitrage in any of the small markets containing N assets (see Lemma 3.1) and also in the large market (see Lemma 3.2).

Assumption 3 For all $i \geq 1$, $P(\varepsilon_i > b_i) > 0$ and $P(\varepsilon_i < b_i) > 0$.

Lemma 3.1 *Under Assumption 1, Assumption 3 implies AOA(N) for any $N \geq 1$. Moreover, AOA(N) implies the so-called quantitative no-arbitrage condition: there exists some $\alpha_N \in]0, 1[$, such that for every $(h_1, \ldots, h_N) \in \mathbb{R}^N$ satisfying $\sum_{i=1}^N h_i^2 = 1$, $P(\sum_{i=1}^N h_i(\varepsilon_i - b_i) < -\alpha_N) > \alpha_N$.*

Proof Fix some $N \geq 1$ and let $(h_1, \ldots, h_N) \in \mathbb{R}^N$ such that $\sum_{i=1}^N h_i(\varepsilon_i - b_i) \geq 0$ a.s. We proceed by contradiction.
Assume that $I_N := \{i \in \{1, \ldots, N\},\ h_i \neq 0\} \neq \emptyset$. Let $B_i := \{h_i(\varepsilon_i - b_i) < 0\}$. Then, $\bigcap_{i \in I_N} B_i \subset \{\sum_{i=1}^N h_i(\varepsilon_i - b_i) < 0\}$. As the $(\varepsilon_i)_{i \geq 1}$ are independent and for $i \in I_N$, $P(B_i) \geq \min\{P(\{\varepsilon_i - b_i < 0\}), P(\{\varepsilon_i - b_i > 0\})\} > 0$, we get that $P(\bigcap_{i \in I_N} B_i) = \prod_{i \in I_N} P(B_i) > 0$, a contradiction. The proof of the last result is standard (see for example [23]) and thus omitted. □

It is well known that absence of arbitrage in markets with finitely many assets is equivalent to the existence of an equivalent martingale measure, see [24] and the references in [17]. In the present setting with infinitely many assets, we need to consider equivalent martingale measures having a finite second moment. Let

$$\mathcal{M}_2 := \left\{Q \sim P : dQ/dP \in L^2(P),\ E_Q(\varepsilon_i) = b_i,\ i \geq 1\right\}.$$

Remark 3.1 If $Q \in \mathcal{M}_2$ and if Assumptions 1 and 2 hold true, then for all $h \in \ell_2$, $E_Q\left(V^{0,h}\right) = 0$. This is Cauchy–Schwarz inequality, see also Lemma 3.4 of [13].

Unfortunately Assumptions 1–3 are not known to be sufficient to ensure that $\mathcal{M}_2 \neq \emptyset$ (see Proposition 4 of [22]). So we also postulate the following.

Assumption 4 We have that $\sup_{i \geq 1} E\left(|\varepsilon_i|^3\right) < \infty$.

Remark 3.2 We comment on the main differences with [12,13,22]. First, we use [22] to show $\mathcal{M}_2 \neq \emptyset$. This justifies Assumption 4. In [13], both conditions

$$\inf_{i \geq 1} P(\varepsilon_i > x) > 0 \text{ and } \inf_{i \geq 1} P(\varepsilon_i < -x) > 0 \text{ for all } x \geq 0, \tag{3}$$

$$\sup_{i \in \mathbb{N}} E\left(\varepsilon_i^2 1_{\{|\varepsilon_i| \geq N\}}\right) \to 0,\ N \to \infty, \tag{4}$$

were postulated. It was proved that the set K^x is closed in probability and that for concave, non-decreasing utility functions $U : \mathbb{R} \to \mathbb{R}$ there exist optimizers. In [12], the rather restrictive assumption (3), which excludes, e.g., the case where all the ε_i are bounded random variables, was relaxed at the price of requiring more integrability on the ε_i than (4). Assumption 3 was postulated together with $\sup_{i \geq 1} E(e^{\gamma|\varepsilon_i|}) < \infty$, for some $\gamma > 0$. This strong moment condition was not justified in the APT problem, and in this paper, we manage to use instead the weaker Assumption 4. Moreover, we will be able to prove that $\mathcal{C}^x := K^x - L^2_+(P)$ is closed in probability.

In Corollary 1 of [22] it is shown that, under Assumptions 1, 3 and 4,

$$\text{AAA} \iff \text{Assumption 2} \iff \mathcal{M}_2 \neq \emptyset. \tag{5}$$

Based on (5), one can show that AAA implies the classical no-arbitrage condition stated with infinitely many assets.

Lemma 3.2 *Assume that Assumptions 1, 3, 4 together with AAA hold true. Then,* $\langle h, \varepsilon - b\rangle \geq 0$ *a.s. for some* $h \in \ell_2$ *implies that* $\langle h, \varepsilon - b\rangle = 0$ *a.s.*

Proof Let $h \in \ell_2$ and assume that $\langle h, \varepsilon - b\rangle \geq 0$. Fix some $Q \in \mathcal{M}_2$ given by (5), then $E_Q(\langle h, \varepsilon - b\rangle) = 0$ (see Remark 3.1). Thus $\langle h, \varepsilon - b\rangle = 0$ Q-a.s. and also P-a.s. since P and Q are equivalent. □

The following lemma is crucial to prove the closure property of $\mathcal{C}^x$ (see Corollary 3.1).

Lemma 3.3 *Let Assumptions 1 and 2 hold true and assume, for some* $\gamma \geq 2$, *that* $\sup_{i\geq 1} E|\varepsilon_i|^\gamma < \infty$. *Then, there is a constant* C_γ *such that, for all* $h \in \ell_2$

$$E|\langle h, \varepsilon - b\rangle|^\gamma \leq C_\gamma \|h\|_{\ell_2}^\gamma \left(1 + \|b\|_{\ell_2}^\gamma\right).$$

Moreover, if $\gamma = 3$, *for any* $c > 0$, $\{|V^{x,h}|^2 : h \in \ell_2, \|h\|_{\ell_2} \leq c\}$ *and also* $\{|V^{x,h}| : h \in \ell_2, \|h\|_{\ell_2} \leq c\}$ *are uniformly integrable.*

Proof Let $h(n) := (h_1, \ldots, h_n, 0, 0, \ldots)$ and $b(n) := (b_1, \ldots, b_n, 0, 0, \ldots)$, for $n \geq 1$.

$$E|\langle h(n), \varepsilon - b\rangle|^\gamma = E\left|\sum_{i=1}^n h_i(\varepsilon_i - b_i)\right|^\gamma \leq 2^{\gamma-1} E\left|\sum_{i=1}^n h_i \varepsilon_i\right|^\gamma + 2^{\gamma-1} E\left|\sum_{i=1}^n h_i b_i\right|^\gamma.$$

The Marcinkiewicz–Zygmund and triangle inequalities imply for some $\bar{C} > 0$

$$\begin{aligned} E\left|\sum_{i=1}^n h_i \varepsilon_i\right|^\gamma &\leq \bar{C} E\left(\left(\sum_{i=1}^n h_i^2 \varepsilon_i^2\right)^{\gamma/2}\right) = \bar{C}\left\|\sum_{i=1}^n h_i^2 \varepsilon_i^2\right\|_{L^{\gamma/2}(P)}^{\gamma/2} \\ &\leq \bar{C}\left(\sum_{i=1}^n |h_i|^2 \|\varepsilon_i\|_{L^\gamma(P)}^2\right)^{\gamma/2} \leq \bar{C}\left(\sup_{i\geq 1} \|\varepsilon_i\|_{L^\gamma(P)}^2 \sum_{i=1}^n |h_i|^2\right)^{\gamma/2} \\ &\leq \bar{C} \sup_{i\geq 1} E|\varepsilon_i|^\gamma \|h(n)\|_{\ell_2}^\gamma. \end{aligned}$$

Thus, $E|\langle h(n), \varepsilon - b\rangle|^\gamma \leq C_\gamma \|h(n)\|_{\ell_2}^\gamma (1 + \|b(n)\|_{\ell_2}^\gamma)$ and Fatou's lemma finishes the proof. □

For all $x \geq 0$, the set of attainable wealth at time 1, allowing the possibility of throwing away money, is $C^x := K^x - L_+^2(P)$.

Proposition 3.1 *Let Assumptions 1–4 hold true. Fix some $z \in \mathbb{R}$ and let $B \in L^2(P)$ such that $B \notin \mathcal{C}^z$. Then, there exists some $\eta > 0$ such that*

$$\inf_{h \in \ell_2} P(z + \langle h, \varepsilon - b \rangle < B - \eta) > \eta. \tag{6}$$

Proof Assume that (6) is not true. Then, for all $n \geq 1$, there exists some $h(n) \in \ell_2$ such that $P(V_n < B - \frac{1}{n}) \leq \frac{1}{n}$, where we have introduced the following notation: $V_n := z + \langle h(n), \varepsilon - b \rangle$. Let $G_n := \{V_n \geq B - \frac{1}{n}\}$ and set $\kappa_n := (V_n - (B - \frac{1}{n}))1_{G_n}$. Then, $P(|V_n - \kappa_n - B| > \frac{1}{n}) = P(\Omega \setminus G_n) \leq \frac{1}{n}$ and thus, $(V_n - \kappa_n)_{n \geq 1}$ converges to B in probability.

First, we claim that $\sup_n ||h(n)||_{\ell_2} < \infty$. Else, $\sup_n ||h(n)||_{\ell_2} = \infty$. So extracting a subsequence (which we continue to denote by n), we may and will assume that $||h(n)||_{\ell_2} \to \infty, n \to \infty$. Let $\tilde{h}_i(n) := h_i(n)/||h(n)||_{\ell_2}$ for all n, i. Clearly, $\tilde{h}(n) \in \ell_2$ with $||\tilde{h}(n)||_{\ell_2} = 1$. Then,

$$W_n := V^{0,\tilde{h}(n)} - \frac{\kappa_n}{||h(n)||_{\ell_2}} \to 0 \text{ a.s.}, n \to \infty.$$

Let $Q \in \mathcal{M}_2$ (which is not empty: see (5)). We claim that $E_Q(W_n) \to 0$. By the Cauchy–Schwarz inequality, $\left|E_Q(W_n)\right| \leq \sqrt{E\,(dQ/dP)^2}\sqrt{E\left(W_n^2\right)}$ and it remains to show the uniform integrability of W_n^2, $n \in \mathbb{N}$ under P.

$$\begin{aligned} |W_n|^2 &= \frac{|B - z - n^{-1}|^2}{||h(n)||_{\ell_2}^2} 1_{G_n} + |V^{0,\tilde{h}(n)}|^2 1_{\Omega \setminus G_n} \\ &\leq \frac{|B|^2 + |z|^2 + n^{-2}}{||h(n)||_{\ell_2}^2} + |V^{0,\tilde{h}(n)}|^2 \leq c|B|^2 + |V^{0,\tilde{h}(n)}|^2, \end{aligned}$$

for n big enough, with some constant c. Using Assumption 4 and Lemma 3.3, $|V^{0,\tilde{h}(n)}|^2$, $n \in \mathbb{N}$ for $\|\tilde{h}(n)\|_{\ell_2} \leq 1$ is uniform integrable under P. As B^2 is also integrable, we get that $E_Q(W_n)$ goes to 0.

As $E_Q V^{0,\tilde{h}(n)} = 0$ (see Remark 3.1), we deduce that $\kappa_n/||h(n)||_{\ell_2}$ goes to zero in $L^1(Q)$ and also Q-a.s. (along a subsequence) and, as Q is equivalent to P, P-a.s. This implies that $V^{0,\tilde{h}(n)}$ goes to 0 P-a.s. and in $L^2(P)$ as well (recall that the family $|V^{0,\tilde{h}(n)}|^2$, $n \geq 1$ for $\|\tilde{h}(n)\|_{\ell_2} \leq 1$ is uniformly integrable). But this is absurd since using the isometry property [see (2)], we get that

$$\|V^{0,\tilde{h}(n)}\|_{L^2}^2 = \|\tilde{h}(n)\|_{\ell_2}^2 + \sum_{i=1}^{\infty} \tilde{h}^2(n)_i b_i^2 \geq 1 \text{ for all } n \geq 1.$$

This contradiction shows that necessarily $\sup_n ||h(n)||_{\ell_2} < \infty$.

We have concluded that $\sup_n ||h(n)||_{\ell_2} < \infty$. Since ℓ_2 has the Banach–Saks property, there exists a subsequence $(n_k)_{k\geq 1}$ and, some $h^* \in \ell_2$, such that

$$\widehat{h}(N) := \frac{1}{N}\sum_{k=1}^{N} h(n_k), \quad \left\|\widehat{h}(N) - h^*\right\|_{\ell_2}^2 \to 0, \; N \to \infty.$$

Hence, using (2), $E((V^{z,\widehat{h}(N)} - V^{z,h^*})^2) \leq (1 + \|b\|_{\ell_2}^2)\|\widehat{h}(N) - h^*\|_{\ell_2}^2$, which tends to zero as $N \to \infty$. So $V^{z,\widehat{h}(N)} \to V^{z,h^*}$ a.s. as well. Then,

$$V^{z,\widehat{h}(N)} - \frac{1}{N}\sum_{k=1}^{N}\kappa_{n_k} = \frac{1}{N}\sum_{k=1}^{N}\left(V^{z,h(n_k)} - \kappa_{n_k}\right) \to B, \; N \to \infty,$$

in probability, and also a.s., for a subsequence for which we keep the same notation. Thus, $\frac{1}{N}\sum_{k=1}^{N}\kappa_{n_k}$ converges a.s. and $B \in \mathcal{C}^z$, a contradiction. □

Corollary 3.1 *Let Assumptions 1–4 hold true and fix some $z \in \mathbb{R}$. Then, $\mathcal{C}^z$ is closed in probability.*

Proof Assume that $\mathcal{C}^z$ is not closed in probability. Then, one can find some $h(n) \in \ell_2$ and $\kappa_n \in L_+^2(P)$ such that $\theta_n := z + \langle h(n), \varepsilon - b\rangle - \kappa_n \in \mathcal{C}^z$ converges in probability to some $\theta^* \notin \mathcal{C}^z$. Then, for any $\eta > 0$,

$$\inf_{h\in\ell_2} P(z + \langle h, \varepsilon - b\rangle < \theta^* - \eta) \leq P(z + \langle h(n), \varepsilon - b\rangle - \kappa_n < \theta^* - \eta) \to 0,$$

when η goes to zero. This contradicts (6), showing closedness of C^z. □

We now provide a quantitative version of the no-arbitrage condition (see Assumption 3).

Proposition 3.2 *Let Assumptions 1–4 hold true. Then, there exists $\alpha > 0$, such that for all $h \in \ell_2$ with $\|h\|_{\ell_2} = 1$, $P(\langle h, \varepsilon - b\rangle < -\alpha) > \alpha$ holds.*

Proof We argue by contradiction. Assume that for all $n \geq 1$, there exist $h(n)$ with $\|h(n)\|_{\ell_2} = 1$ and $P\left(\langle h(n), \varepsilon - b\rangle < -1/n\right) \leq 1/n$.
Clearly, $\langle h(n), \varepsilon - b\rangle_- \to 0$ in probability as $n \to \infty$. Let $Q \in \mathcal{M}_2$ [see (5)]. We claim that $E_Q(\langle h(n), \varepsilon - b\rangle_-) \to 0$. Using Cauchy-Schwarz inequality

$$E_Q\left(\langle h(n), \varepsilon - b\rangle_-\right) \leq \|dQ/dP\|_{L^2(P)}\left(E\left(\langle h(n), \varepsilon - b\rangle_-^2\right)\right)^{1/2},$$

and it remains to show uniform integrability of $\langle h(n), \varepsilon - b\rangle_-^2$, $n \in \mathbb{N}$ under P. This follows from $\langle h(n), \varepsilon - b\rangle_-^2 \leq |V^{0,h(n)}|^2$, Assumption 4 and Lemma 3.3. So $E_Q(\langle h(n), \varepsilon - b\rangle_-) \to 0$ but, since $E_Q(\langle h(n), \varepsilon - b\rangle) = 0$ by Remark 3.1, we also get that $E(\langle h(n), \varepsilon - b\rangle_+) \to 0$. It follows that $E_Q(|\langle h(n), \varepsilon - b\rangle|) \to 0$, hence $\langle h(n), \varepsilon - b\rangle$ goes to zero Q-a.s. (along a subsequence) and, as Q is equivalent to

P, P-a.s. Using again that $|\langle h(n), \varepsilon - b\rangle|^2$, $n \in \mathbb{N}$ is uniformly P-integrable, we get $E(|\langle h(n), \varepsilon - b\rangle|^2) \to 0$. But this contradicts the fact that $E(|\langle h(n), \varepsilon - b\rangle|^2) = \|h(n)\|_{\ell_2}^2 + \sum_{i=1}^{\infty} h_i^2(n) b_i^2 \geq 1$ [see (2)]. □

The following lemma proves that, under the no-arbitrage condition (see Assumption 3), any strategy with a non-negative final wealth is bounded.

Lemma 3.4 *Let Assumptions 1–4 hold true. Let $y \in \mathbb{R}$ and $h \in \ell_2$ such that $y + \langle h, \varepsilon - b\rangle \geq 0$. Then, $\|h\|_{\ell_2} \leq |y|/\alpha$, see Proposition 3.2 for α.*

Proof On $\{\langle h, \varepsilon - b\rangle < -\alpha||h||_{\ell_2}\}$, which is of positive measure by Proposition 3.2, $|y| - \alpha||h||_{\ell_2} > y + \langle h, \varepsilon - b\rangle \geq 0$ and $\|h\|_{\ell_2} \leq |y|/\alpha$ follows. □

4 Superreplication Price

Let $G \in L^0$ be a random variable, which will be interpreted as the payoff of some derivative security at time T. The superreplication price $\pi(G)$ is the minimal initial wealth needed for hedging G without risk. For all $x \in \mathbb{R}$, let

$$\mathcal{A}(G, x) := \left\{h \in \ell_2 : \; V^{x,h} \geq G \text{ a.s.}\right\} \text{ and } \pi(G) := \inf\{z \in \mathbb{R} : \; \mathcal{A}(G, z) \neq \emptyset\},$$

where $\pi(G) = +\infty$ if $\mathcal{A}(G, z) = \emptyset$ for every z. The so-called dual representation of the superreplication price (see Theorem 4.1) in terms of supremum over the different risk-neutral probability measures has a long history: see [16] and also the textbook [17] for more details about this preference-free price.

Lemma 4.1 *Let Assumptions 1–4 hold true. Then, $\pi(G) > -\infty$ and $\mathcal{A}(G, \pi(G)) \neq \emptyset$.*

Proof Assume that $\pi(G) = -\infty$. Then, for all $n \geq 1$, there exists $h_n \in \ell_2$ such that $-n + \langle h_n, \varepsilon - b\rangle \geq G$ a.s. Thus, $\langle h_n, \varepsilon - b\rangle \geq G + n \geq (G + n) \wedge 1$ a.s. It follows that $(G + n) \wedge 1 \in C^0$, which is closed in probability (see Corollary 3.1). Thus, $1 \in C^0$, i.e., $\langle h, \varepsilon - b\rangle \geq 1$ a.s. for some $h \in \ell_2$, which contradicts AAA (or Assumption 2, see (5)), see Lemma 3.2. So $\pi(G) > -\infty$.

If $\pi(G) = +\infty$, the second claim is trivial. So, assume that $\pi(G) < \infty$. Then, for all $n \geq 1$, there exists $h_n \in \ell_2$ such that $\pi(G) + 1/n + \langle h_n, \varepsilon - b\rangle \geq G$ a.s. It follows that $G - \pi(G) - 1/n \in C^0$. Thus, as C^0 is closed, $G - \pi(G) \in C^0$. □

We are now in position to prove our duality result.

Theorem 4.1 *Let Assumptions 1–4 hold true and let $G \in L^2(P)$. Then, $\pi(G) = \sup_{Q \in \mathcal{M}_2} E_Q(G)$.*

Proof Let $s := \sup_{Q \in \mathcal{M}_2} E_Q(G)$. Let x be such that there exists $h \in \ell_2$ verifying $x + \langle h, \varepsilon - b\rangle \geq G$ a.s. Fix $Q \in \mathcal{M}_2$ [see (5)]. As $G \in L^2(P)$, $E_Q(G)$ is well defined by the Cauchy–Schwarz inequality. Using Remark 3.1, we get that $E_Q(x + \langle h, \varepsilon - b\rangle) = x$. Thus, $x \geq E_Q(G)$ and $\pi(G) \geq s$ follows. For the other inequality, it is enough to prove that $G - s \in C^0$. Indeed, this will imply that there exists $h \in \ell_2$ such that $s + \langle h, \varepsilon - b\rangle \geq G$ a.s., which shows, by definition of $\pi(G)$, that $s \geq \pi(G)$. Assume this is not true. Then, $\{G - s\} \notin C^0 \cap L^2(P)$. As C^0 is closed in probability (see Corollary 3.1), we can apply classical Hahn–Banach argument (see, e.g., [17]) to find some $Q \in \mathcal{M}_2$ such that $E_Q(G) > s$. □

Remark 4.1 One may wonder whether $\pi_n(G)$, the superreplication price of G in the small market with n random sources $(\varepsilon_i)_{1\leq i\leq n}$, converges to $\pi(G)$, the superreplication price of G in the large market. The answer is *no* in general.

Let ε_i, $i \in \mathbb{N}$ be standard Gaussian random variables, let $b_i = 0$ for all $i \in \mathbb{N}$ and define $G := \sum_{i=1}^{\infty} i^{-1}\varepsilon_i$. There exists no $x, h_1, \ldots, h_n$ with $x + \sum_{j=1}^{n} h_j \varepsilon_j \geq G$, since this would mean that $\sum_{j=1}^{n}(h_j - j^{-1})\varepsilon_j - \sum_{j\geq n+1} j^{-1}\varepsilon_j \geq -x$, where the left-hand side is a Gaussian random variable with nonzero variance. It follows that $\pi_n(G) = \infty$ while $\pi(G) = 0$, trivially.

5 Utility Maximization

We follow the traditional viewpoint of [19] and model economic agents' preferences by some concave strictly increasing differentiable utility function denoted by $U :]0, \infty[\to \mathbb{R}$. Note that we extend U to $[0, \infty[$ by (right)-continuity ($U(0)$ may be $-\infty$). We also set $U(x) = -\infty$ for $x \in]-\infty, 0[$. For a contingent claim $G \in L^0$ and $x \in \mathbb{R}$, we define $\Phi(U, G, x) := \left\{h \in \ell_2,\ EU^+(V^{x,h} - G) < +\infty\right\}$, the set of strategies, where the expectation is well defined. Then, we set $\mathcal{A}(U, G, x) := \Phi(U, G, x) \cap \mathcal{A}(G, x)$. Note that even for $x \geq \pi(G)$, $\mathcal{A}(U, G, x)$ might be empty. Indeed, from Lemma 4.1, we know that there exists some $h \in \mathcal{A}(G, x)$, but h might not belong to $\Phi(U, G, x)$. But this holds true under appropriate assumptions, as proved in the lemma below.

Lemma 5.1 *Let Assumptions 1–4 hold true. Assume that* $G \geq 0$ *a.s. and* $U(x_0) = 0$, $U'(x_0) = 1$, *for some* $x_0 \geq 0$. *Then,* $\mathcal{A}(G, x) = \mathcal{A}(U, G, x)$ *for all* $x \in \mathbb{R}$.

Proof As U is concave, increasing and differentiable with $U(x_0) = 0$, $U'(x_0) = 1$, we can bound it from above by its first order Taylor approximation, for all $x \in]0, \infty[$, as follows:

$$U(x) \leq U(\max(x_0, x)) \leq U(x_0) + \max(x - x_0, 0)U'(x_0) \leq |x - x_0| \leq |x|,$$

since $x_0 \geq 0$. If $x < \pi(G)$ then $\mathcal{A}(G, x) = \emptyset$ and $\mathcal{A}(G, x) = \mathcal{A}(U, G, x) = \emptyset$. Let $x \geq \pi(G)$. Then, by Lemma 4.1, $\mathcal{A}(G, x) \neq \emptyset$. Let $h \in \mathcal{A}(G, x)$. Then, $V^{x,h} \geq G \geq 0$ a.s. and $h \in \mathcal{A}(0, x)$. Let $A := \{x + \langle h, \varepsilon - b\rangle \geq x_0\}$.

$$\begin{aligned} U^+(x + \langle h, \varepsilon - b\rangle - G) &\leq U^+(x + \langle h, \varepsilon - b\rangle)1_A + U^+(x_0)1_{\Omega\setminus A} \\ &= U(x + \langle h, \varepsilon - b\rangle)1_A \leq |x + < h, \varepsilon - b > |. \end{aligned} \tag{7}$$

Using (2), the Cauchy–Schwarz inequality and Lemma 3.4, we get that

$$\begin{aligned} EU^{+}(x+\langle h,\varepsilon-b\rangle-G) &\leq |x|+\sqrt{E\left(\langle h,\varepsilon-b\rangle^{2}\right)} \leq |x|+\|h\|_{\ell_2}\sqrt{1+\|b\|_{\ell_2}^{2}} \\ &\leq |x|+\frac{|x|}{\alpha}\sqrt{1+\|b\|_{\ell_2}^{2}}<+\infty. \end{aligned} \tag{8}$$

□

We now define the supremum of the expected utility at the terminal date when delivering the claim G, starting from initial wealth $x \in \mathbb{R}$:

$$u(G,x) := \sup_{h\in\mathcal{A}(U,G,x)} EU\{V^{x,h}-G\}, \tag{9}$$

where $u(G,x)=-\infty$, if $\mathcal{A}(U,G,x)=\emptyset$. The following result establishes that there exists an optimal investment for the investor we are considering.

Theorem 5.1 *Let Assumptions 1–4 hold true. Let $G \geq 0$ and $x \in \mathbb{R}$ such that $x \geq \pi(G)$. Then, there exists $h^* \in \mathcal{A}(U,G,x)$ such that*

$$u(G,x)=EU(V^{x,h^*}-G).$$

Proof If U is constant, there is nothing to prove. Else, there exists $x_0>0$ such that $U'(x_0)>0$. Replacing U by $(U-U(x_0))/U'(x_0)$, we may and will suppose that $U(x_0)=0$ and $U'(x_0)=1$. Note that $\pi(G)\geq 0$, as $G\geq 0$ a.s. (see Theorem 4.1). Let $h_n \in \mathcal{A}(G,x)=\mathcal{A}(U,G,x)$ (see Lemmata 4.1 and 5.1) be a sequence such that

$$EU(V^{x,h_n}-G)\uparrow u(G,x),\ n\to\infty.$$

By Lemma 3.4, $\sup_{n\in\mathbb{N}}\|h_n\|_{\ell_2}\leq x/\alpha<\infty$. Hence, as ℓ_2 has the Banach–Saks Property, there exists a subsequence $(n_k)_{k\geq 1}$ and some $h^*\in\ell_2$ such that for

$$\tilde{h}_n := \frac{1}{n}\sum_{k=1}^{n} h_{n_k},\ \|\tilde{h}_n-h^*\|_{\ell_2}\to 0,\ n\to\infty.$$

Note that $\tilde{h}_n\in\mathcal{A}(G,x)$ and $\sup_{n\in\mathbb{N}}\|\tilde{h}_n\|_{\ell_2}\leq x/\alpha<\infty$. Using (2), we get that

$$E\langle\tilde{h}_n-h^*,\varepsilon-b\rangle^2\leq\|\tilde{h}_n-h^*\|_{\ell_2}^2(1+\|b\|_{\ell_2}^2)\to 0,\ n\to\infty.$$

In particular, $\langle\tilde{h}_n-h^*,\varepsilon-b\rangle\to 0$, $n\to\infty$ in probability. Hence, we also get that $U(V^{x,\tilde{h}_n}-G)\to U(V^{x,h^*}-G)$ in probability, by continuity (right-continuity in 0) of U on $[0,\infty[$. We also have (up to a subsequence) that $V^{x,\tilde{h}_n}-G\to V^{x,h^*}-G$ a.s. and thus, $h^*\in\mathcal{A}(G,x)$. Now, using (7), we have that $U^{+}(V^{x,\tilde{h}_n}-G)\leq|V^{x,\tilde{h}_n}|$. So

Assumption 4 and Lemma 3.3 imply that $\{U^+(V^{x,\tilde{h}_n}-G) : h_n \in \ell_2,\ \|h_n\|_{\ell_2} \le x/\alpha\}$ is uniformly integrable and

$$\lim_{n\to\infty} E\left(U^+(V^{x,\tilde{h}_n}-G)\right) = E\left(U^+(V^{x,h^*}-G)\right).$$

Then, $E(-U^-(V^{x,h^*}-G)) \ge \limsup_{n\to\infty} E(-U^-(V^{x,\tilde{h}_n}-G))$, by Fatou's lemma. As by concavity of U,

$$U(V^{x,\tilde{h}_n}-G) = U\left(\frac{1}{n}\sum_{k=1}^{n}(V^{x,h_{n_k}}-G)\right) \ge \frac{1}{n}\sum_{k=1}^{n} U\left(V^{x,h_{n_k}}-G\right),$$

we get that

$$EU(V^{x,h^*}-G) \ge \limsup_{n\to\infty} EU(V^{x,\tilde{h}_n}-G) \ge u(G,x).$$

The proof is finished since $h^* \in \mathcal{A}(G,x) = \mathcal{A}(U,G,x)$ (see Lemma 5.1). □

6 Convergence of the Reservation Price to the Superreplication Price

We go on incorporating a sequence of agents in our model.

Assumption 5 Suppose that $U_n :]0,\infty[\to \mathbb{R},\ n \in \mathbb{N}$ is a sequence of concave strictly increasing twice continuously differentiable functions such that

$$\forall x \in]0,\infty[\quad r_n(x) := -\frac{U_n''(x)}{U_n'(x)} \to \infty,\ n \to \infty.$$

Again we extend each U_n to $[0,\infty[$ by (right)-continuity, and set $U_n(x) = -\infty$ for $x \in]-\infty, 0[$. We define the value functions $u_n(G,x)$ for our sequence of utility functions $(U_n)_{n\ge1}$ changing U by U_n in (9).

Assumption 5 says that the sequence of agents we consider have asymptotically infinite aversion towards risk. Indeed, [25] shows that an investor n has greater absolute risk-aversion than investor m (i.e., $r_n(x) > r_m(x)$ for all x) if and only if investor n is more risk averse than m (i.e., the amount of cash for which she would exchange the risk is smaller for n than for m).

The utility indifference (or reservation) price $p_n(G,x)$, introduced by [21], is

$$p_n(G,x) := \inf\{z \in \mathbb{R} : u_n(G,x+z) \ge u_n(0,x)\}.$$

Intuitively, it seems reasonable that under Assumption 5 the utility prices $p_n(G,x)$ tend to $\pi(G)$ and this was proved for finitely many assets in [26]. Now, we treat the case of APT.

Theorem 6.1 *Assume that Assumptions 1–5 hold true. Suppose that* $x > 0$ *and* $G \in L^2_+(P)$. *Then, the utility indifference prices* $p_n(G, x)$ *are well defined and converge to* $\pi(G)$ *as* $n \to \infty$.

Proof Applying affine transformations to each U_n, we may and will assume that $U_n(x) = 0$ and $U_n'(x) = 1$ for all $n \in \mathbb{N}$.

If $\pi(G) = +\infty$ then for all $z \in \mathbb{R}$, $n \geq 1$, $\emptyset = \mathcal{A}(G, z) = \mathcal{A}(U_n, G, z)$ and $u_n(G, x + z) = -\infty$. But $u_n(0, x) \geq EU_n(x) = 0$. Thus, $p_n(G, x) = +\infty$ for all $n \geq 1$ and the claim is proved.

Assume now that $\pi(G) < \infty$. Just like in the proof of Theorem 3 in [26], $p_n(G, x) \leq \pi(G)$. So, it remains to show that $\liminf_{n\to\infty} p_n(G, x) \geq \pi(G)$. If this is not the case, we can find a subsequence (still denoted by n) and some $\eta > 0$ such that $p_n(G, x) \leq \pi(G) - \eta$ for all $n \geq 1$. We may and will assume that $x \geq \eta$. By definition of $p_n(G, x)$, we have that

$$u_n(G, x + \pi(G) - \eta) \geq u_n(0, x).$$

Let $y := x + \pi(G) - \eta < x + \pi(G)$. If we prove that $\lim_{n\to+\infty} u_n(G, y) = -\infty$, $\liminf_{n\to+\infty} u_n(0, x) \geq \liminf_{n\to+\infty} U_n(x) \geq 0$ will provide a contradiction.

First, remark that $x + G \notin \mathcal{C}^y$. Applying Proposition 3.1, we get some $\gamma > 0$ such that $\inf_{h\in\ell_2} P(A_h) > \gamma$, where $A_h := \{y + \langle h, \varepsilon - b\rangle < x + G - \gamma\}$. Note that we can always assume that $x \geq \gamma$. As $y \geq \pi(G) \geq 0$, Lemmata 4.1 and 5.1 imply that $\mathcal{A}(U_n, G, y) \neq \emptyset$. Hence, for all $h \in \mathcal{A}(U_n, G, y)$, we get that

$$\begin{aligned} EU_n(y + \langle h, \varepsilon - b\rangle - G) &\leq E1_{A_h} U_n(x - \gamma) + E1_{\Omega\setminus A_h} U_n^+(y + h\langle \varepsilon - b\rangle) \\ &\leq \gamma U_n(x - \gamma) + EU_n^+(y + \langle h, \varepsilon - b\rangle). \end{aligned}$$

Using (8), $u_n(G, y) \leq \gamma U_n(x - \gamma) + y + \frac{y}{\alpha}\sqrt{1 + \|b\|^2_{\ell_2}}$ goes to $-\infty$ when n goes to infinity, by Lemma 4 of [26]. □

7 Conclusions

The current paper, just like [12,13,22], is based on techniques that are at the intersection of probability and functional analysis. These permit to state a dual representation for the superreplication cost, to prove existence in the problem of maximization of expected utility and to show the convergence of the reservation prices to the superreplication cost in markets with infinitely many assets, which form an important model class of financial mathematics, pertinent to, e.g., bond markets. In future work, our approach is hoped to be extended to other infinite market models (e.g., complete ones, where ε_i are not independent but form a complete orthonormal system) so as to gain further insight about how these complex systems operate.

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