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ON HERMITE-HADAMARD TYPE INEQUALITIES FOR MULTIPLICATIVE FRACTIONAL INTEGRALS

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Abstract. In this study, we first establish two Hermite-Hadamard type inequality for multiplicative (geometric) Riemann-Liouville fractional integrals. Then, by using some properties of multiplicative convex function, we give some new inequalities involving multiplicative fractional integrals.

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1. INTRODUCTION

The inequalities discovered by C. Hermite and J. Hadamard for convex functions are considerable significant in the literature (see, e.g.,[8], [11], [18, p.137]). These inequalities state that if $f : I \to \mathbb{R}$ is a convex function on the interval I of real numbers and $a, b \in I$ with $a < b$, then

$$f\left(\frac{a+b}{2}\right) \le \frac{1}{b-a}\int_a^b f(x)dx \le \frac{f(a)+f(b)}{2}. \tag{1.1}$$

Both inequalities hold in the reversed direction if f is concave. We note that Hadamard's inequality may be regarded as a refinement of the concept of convexity and it follows easily from Jensen's inequality.

Over the last twenty years, the numerous studies have focused on to obtain new bound for left hand side and right and side of the inequality (1.1). For some examples, please refer to ([3], [5], [6], [7], [8], [9], [10], [12], [13], [14], [15], [16], [17], [19], [20], [21], [22], [23], [24], [25]).

1.1. *Multiplicative Calculus*

Recall that the concept of the multiplicative integral called * integral is denoted by $\int_a^b (f(x))^{dx}$ which introduced by Bashirov et al. in [4]. While the sum of the terms of product is used in the definition of a classical Riemann integral of f on $[a,b]$, the product os terms raised to power is used in the definition multiplicative integral of f on $[a,b]$.

There is the following relation between Rimann integral and multiplicative integral [4]:

Proposition 1. *If f is Riemann integrable on $[a,b]$, then f is multiplicative integrable on $[a,b]$ and*

$$\int_a^b (f(x))^{dx} = e^{\int_a^b \ln(f(x))dx}.$$

Moreover, Bashirov et al [4] show that multiplicative integrable has the following results and properties:

Proposition 2. *If f is positive and Riemann integrable on $[a,b]$, then f is *integrable on $[a,b]$ and*

(i) $\int_a^b ((f(x))^p)^{dx} = \int_a^b ((f(x))^{dx})^p,$

(ii) $\int_a^b (f(x)g(x))^{dx} = \int_a^b (f(x))^{dx} . \int_a^b (g(x))^{dx},$

(iii) $\int_a^b \left(\frac{f(x)}{g(x)}\right)^{dx} = \frac{\int_a^b (f(x))^{dx}}{\int_a^b (f(x))^{dx}},$

(iv) $\int_a^b (f(x))^{dx} = \int_a^c (f(x))^{dx} . \int_c^b (f(x))^{dx},\ a \leq c \leq b.$

(v) $\int_a^a (f(x))^{dx} = 1$ *and* $\int_a^b (f(x))^{dx} = \left(\int_b^a (f(x))^{dx}\right)^{-1}.$

On the other hand, Abdeljawed and Grossman [1] intoduce the following Multiplicative Riemann-Liouville fractional integrals.

Definition 1. The multiplicative left Riemann-Liouville fractional integral $({}_aI_*^{\alpha} f)(x)$ of order $\alpha \in \mathbb{C}$, $Re(\alpha) > 0$ starting from α is defined by

$$({}_aI_*^{\alpha} f)(x) = e^{(J_{a^+}^{\alpha}(\ln \circ f))(x)}$$

and the multiplicative right one is defined by

$$({}_*I_b^{\alpha} f)(x) = e^{(J_{b^-}^{\alpha}(\ln \circ f))(x)},$$

Here $J_{a^+}^{\alpha} f(x)$ and $J_{b^-}^{\alpha} f(x)$ denote the left and right Riemann-Liouville fractional integral, defined by

$$J_{a^+}^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-t)^{\alpha-1} f(t)dt, x > a$$

and

$$J_{b-}^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_{x}^{b} (t-x)^{\alpha-1} f(t)dt, x < b$$

respectively.

1.2. *Hermite-Hadamard inequality and convexity*

For the our main results we need to following definition.

Definition 2. A non-empty set K is said to be convex, if for every $a, b \in K$ we have

$$a + \mu(b-a) \in K, \ \forall \mu \in [0,1].$$

Definition 3. A function f is said to be convex function on set K, if

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y), \ \forall t \in [0,1].$$

Definition 4. A function f is said to be *log* or multiplicatively convex function on set K, if

$$f(tx + (1-t)y) \leq [f(x)]^{t} [f(y)]^{1-t}, \forall t \in [0,1].$$

Proposition 3. *If f and g are log (multiplicatively) convex functions, then the functions fg and $\frac{f}{g}$ are log (multiplicatively) convex functions.*

The classical Hermite-Hadamard inequality for convex function given by the inequality (1.1).

Hermite-Hadamard inequality for multiplicatively convex function is proved by Ali et al. in [2] as follows:

Theorem 1. *Let f be a positive and multiplicatively convex function on interval $[a,b]$,then the following inequalities hold*

$$f\left(\frac{a+b}{2}\right) \leq \left(\int_{a}^{b} (f(x))^{dx}\right)^{\frac{1}{b-a}} \leq G(f(a), f(b)), \tag{1.2}$$

where $G(.,.)$ is a geometric mean.

The inequality (1.2) is the same result proved by Dragomir in [8, Page 197, inequality (5.3)]

On the other hand, Sarikaya et al.[22] proved following important inequality which is the Hermite-Hadamard inequality for Riemann-Liouville fractional integrals.

Theorem 2. *Let $f : [a,b] \to \mathbb{R}$ be a positive function with $0 \leq a < b$ and $f \in L_1[a,b]$. If f is a convex function on $[a,b]$, then the following inequalities for fractional integrals hold:*

$$f\left(\frac{a+b}{2}\right) \leq \frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}} \left[J_{a+}^{\alpha} f(b) + J_{b-}^{\alpha} f(a)\right] \leq \frac{f(a)+f(b)}{2} \tag{1.3}$$

with $\alpha > 0$.

In this paper, we establish Hermite-Haddamard inequality for multiplicative Riemann-Liouville fractional integrals.

2. Main results

In this section we obtain some Hermite-Hadamard type inequalities for multiplicatively convex function via multiplicative Riemann-Liouville fractional integrals.

Theorem 3. *Let f be a positive and multiplicatively convex function on interval $[a,b]$,Then we have the following Hermite-Haddamard inequality for multiplicative Riemann-Liouville fractional integrals*

$$f\left(\frac{a+b}{2}\right) \leq \left[{}_{a}I_{*}^{\alpha} f(b) \cdot {}_{*}I_{b}^{\alpha} f(a)\right]^{\frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}} \leq G(f(a), f(b)) \tag{2.1}$$

where $G(.,.)$ is a geometric mean.

Proof. Since f is multiplicatively convex function on interval $[a,b]$, then we have

$$\begin{aligned} f\left(\frac{a+b}{2}\right) &= f\left(\frac{at+(1-t)b+(1-t)a+tb}{2}\right) \\ &\leq [f(at+(1-t)b)]^{\frac{1}{2}} [f((1-t)a+tb)]^{\frac{1}{2}}, \end{aligned}$$

i.e.

$$\ln f\left(\frac{a+b}{2}\right) = \frac{1}{2}\ln f(at+(1-t)b) + \frac{1}{2}\ln f((1-t)a+tb) \tag{2.2}$$

Multiplying both sides of (2.2) by $t^{\alpha-1}$then integrating the resulting inequality with respect to t over $[0,1]$, we obtain

$$\int_0^1 t^{\alpha-1} \ln f\left(\frac{a+b}{2}\right) dt \leq \frac{1}{2}\int_0^1 t^{\alpha-1} \ln f(at+(1-t)b)dt + \frac{1}{2}\int_0^1 t^{\alpha-1} \ln f((1-t)a+tb)dt.$$

By using the change variable, we have

$$\frac{1}{\alpha}\ln f\left(\frac{a+b}{2}\right) dt \leq \frac{1}{(b-a)^{\alpha}}\int_a^b (b-x)^{\alpha-1} \ln f(x)dx + \frac{1}{2}\frac{1}{(b-a)^{\alpha}}\int_a^b (x-a)^{\alpha-1} \ln f(x)dx.$$

That is,

$$\ln f\left(\frac{a+b}{2}\right) \leq \frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}\left[J_{b-}^{\alpha} \ln f(a) + J_{a+}^{\alpha} \ln f(b)\right].$$

Thus we get,

$$f\left(\frac{a+b}{2}\right) \leq e^{\frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}\left[J_{a+}^{\alpha} \ln f(b) + J_{b-}^{\alpha} \ln f(a)\right]}$$

$$= \left[e^{J^{\alpha}_{a+}\ln f(b)}.e^{J^{\alpha}_{b-}\ln f(a)}\right]^{\frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}}$$

$$= \left[{}_aI^{\alpha}_{*}f(b)._{*}I^{\alpha}_{b}f(a)\right]^{\frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}}.$$

which completes the proof of the first inequality in (2.1).
As f is multiplicatively convex function on interval $[a,b]$, then we have

$$f(at+(1-t)b) \leq [f(a)]^{t}[f(b)]^{1-t}$$

$$f((1-t)a+tb) \leq [f(a)]^{1-t}[f(b)]^{t}$$

i.e.

$$\ln f(at+(1-t)b)+\ln f((1-t)a+tb) \tag{2.3}$$

$$\leq t\ln f(a)+(1-t)\ln f(b)+(1-t)\ln f(a)+t\ln f(b)$$

$$= \ln f(a)+\ln f(b)$$

Multiplying both sides of (2.3) by $t^{\alpha-1}$then integrating the resulting inequality with respect to t over $[0,1]$,we establish

$$\int_0^1 t^{\alpha-1}\ln f(at+(1-t)b)\,dt+\int_0^1 t^{\alpha-1}\ln f((1-t)a+tb)dt \leq \frac{\ln f(a)+\ln f(b)}{\alpha}.$$

Hence,

$$\frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}[J^{\alpha}_{b-}f(a)+J^{\alpha}_{a+}f(b)] \leq \frac{1}{2}\ln[f(a).f(b)].$$

Thus, we have the inequality

$$e^{[J^{\alpha}_{b-}\ln f(a)+J^{\alpha}_{a+}\ln f(b)]^{\frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}}} \leq \sqrt{f(a).f(b)}$$

and

$$\left[I^{\alpha}_{*}f(b)._{*}I^{\alpha}_{b}f(a)\right]^{\frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}} \leq \sqrt{f(a).f(b)}.$$

The proof is completed □

Remark 1. If we choose $\alpha = 1$ in Theorem 3, then Theorem 3 reduces to the Theorem 1.

Corollary 1. *If f and g are two positive and multiplicative convex functions, then we have the following inequality*

$$f\left(\frac{a+b}{2}\right)g\left(\frac{a+b}{2}\right) \leq \left[{}_{a}I_{*}^{\alpha} fg(b) \cdot {}_{*}I_{b}^{\alpha} fg(a)\right]^{\frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}} \leq G(f(a),f(b)).G(g(a),g(b)). \tag{2.4}$$

Proof. Since f and g are positive and multiplicative convex functions, then fg is positive and multiplicative convex function. Thus if we apply Theorem 3 to the function fg, then we obtain the desired inequality (2.4). □

Remark 2. If we take $\alpha = 1$ in Corollary 1, then we have the following inequality

$$f\left(\frac{a+b}{2}\right)g\left(\frac{a+b}{2}\right) \leq \left(\int_{a}^{b}(f(x))^{dx} . \int_{a}^{b}(g(x))^{dx}\right)^{\frac{1}{b-a}} \leq G(f(a),f(b)).G(g(a),g(b)) \tag{2.5}$$

which is given by Ali et al. in [2].

Theorem 4. *Let f be a positive and multiplicatively convex function on interval $[a,b]$,Then we have the following Hermite-Hadamard inequality for multiplicative Riemann-Liouville fractional integrals*

$$f\left(\frac{a+b}{2}\right) \leq \left[{}_{\frac{a+b}{2}}I_{*}^{\alpha} f(b) \cdot {}_{*}I_{\frac{a+b}{2}}^{\alpha} f(a)\right]^{\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}} \leq G(f(a),f(b)) \tag{2.6}$$

where $G(.,.)$ is a geometric mean.

Proof. By using the multiplicatively convexity of f, we have

$$f\left(\frac{a+b}{2}\right) = f\left[\frac{1}{2}\left(\frac{t}{2}a+\frac{2-t}{2}b\right)+\frac{1}{2}(\frac{2-t}{2}a+\frac{t}{2}b)\right]$$

$$\leq \left(f\left(\frac{t}{2}a+\frac{2-t}{2}b\right)\right)^{\frac{1}{2}} . \left(f\left(\frac{2-t}{2}a+\frac{t}{2}b\right)\right)^{\frac{1}{2}},$$

i.e.

$$\ln f\left(\frac{a+b}{2}\right) \leq \frac{1}{2}\left[\ln f\left(\frac{t}{2}a+\frac{2-t}{2}b\right)+\ln f\left(\frac{2-t}{2}a+\frac{t}{2}b\right)\right]. \tag{2.7}$$

Multiplying both sides of (2.7) by $t^{\alpha-1}$then integrating the resulting inequality with respect to t over $[0,1]$, we get

$$\frac{1}{\alpha}\ln f(\frac{a+b}{2}) \leq \frac{1}{2}\int_{0}^{1} t^{\alpha-1}\ln f\left(\frac{t}{2}a+\frac{2-t}{2}b\right)dt + \int_{0}^{1} t^{\alpha-1}\ln f\left(\frac{2-t}{2}a+\frac{t}{2}b\right)dt$$

$$= \frac{\Gamma(\alpha)}{2}\left[\int_{\frac{a+b}{2}}^{b}\left(\frac{2(b-x)}{b-a}\right)^{\alpha-1}\ln f(x)\frac{2}{b-a}dx\right.$$

$$\left.+\int_{a}^{\frac{a+b}{2}}\left(\frac{2(x-a)}{b-a}\right)^{\alpha-1}\ln f(x).\frac{2}{b-a}dx\right]$$

$$= \frac{2^{\alpha-1}\Gamma(\alpha)}{(b-a)^{\alpha}}\left[J^{\alpha}_{(\frac{a+b}{2})^{+}}\ln f(b)+J^{\alpha}_{(\frac{a+b}{2})^{-}}\ln f(a)\right].$$

Then it follows that,

$$f\left(\frac{a+b}{2}\right)\leq e^{\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[J^{\alpha}_{(\frac{a+b}{2})^{+}}\ln f(b)+J^{\alpha}_{(\frac{a+b}{2})^{-}}\ln f(a)\right]}$$

$$= \left[{}_{\frac{a+b}{2}}I^{\alpha}_{*}f(b)._{*}I^{\alpha}_{\frac{a+b}{2}}f(a)\right]^{\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}}.$$

This completes the proof of first inequality in the inequality (2.6).

On the other hand, since f is multiplicatively convex function, we get

$$f\left(\frac{t}{2}a+\frac{2-t}{2}b\right)+f\left(\frac{2-t}{2}a+\frac{t}{2}b\right)\leq[f(a)]^{\frac{t}{2}}+[f(b)]^{\frac{2-t}{2}}+[f(a)]^{\frac{2-t}{2}}+[f(b)]^{\frac{t}{2}}.$$

Thus, we have

$$\ln f\left(\frac{t}{2}a+\frac{2-t}{2}b\right)+\ln f\left(\frac{2-t}{2}a+\frac{t}{2}b\right)\leq\ln f(a)+\ln f(b) \tag{2.8}$$

Multiplying both sides of (2.8) by $t^{\alpha-1}$then integrating the resulting inequality with respect to t over $[0,1]$, we have

$$\frac{2^{\alpha}}{(b-\alpha)^{\alpha}}\Gamma(\alpha)\left[J^{\alpha}_{(\frac{a+b)}{2})^{+}}\ln f(b)+J^{\alpha}_{(\frac{a+b}{2})^{-}}\ln f(a)\right]\leq\frac{\ln f(a)+\ln f(b)}{\alpha}$$

i.e.

$$\frac{2^{\alpha-1}}{(b-\alpha)^{\alpha}}\Gamma(\alpha+1)\left[J^{\alpha}_{(\frac{a+b}{2})^{+}}\ln f(b)+J^{\alpha}_{(\frac{a+b}{2})^{-}}\ln f(a)\right]\leq\frac{\ln[f(a).f(b)]}{2}.$$

Hence, we get the inequality

$$\left[{}_{\frac{a+b}{2}}I^{\alpha}_{*}f(b)._{*}I^{\alpha}_{\frac{a+b}{2}}f(a)\right]^{\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}}\leq\sqrt{f(a)f(b)}.$$

The proof is completed □

Corollary 2. *If f and g are positive and multiplicative convex function, then we have the following inequality*

$$
\begin{aligned}
& f\left(\frac{a+b}{2}\right) g\left(\frac{a+b}{2}\right) \\
& \leq \left[{}_{\left(\frac{a+b}{2}\right)}I_{*}^{\alpha} f.g(b) \cdot {}_{*}I_{\left(\frac{a+b}{2}\right)}^{\alpha} f.g(a) \right]^{\frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}} \leq G(f(a),f(b))G(g(a),g(b)).
\end{aligned} \tag{2.9}
$$

Proof. The proof is similar to the proof of Corollary 1. □

Remark 3. If we take $\alpha = 1$ in Corollary 2, then the inequality (2.9) reduces to the inequality (2.5).

Corollary 3. *If f and g are positive and multiplicative convex function, then we get the following inequality*

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