

UNSTEADY FREE CONVECTION NANOFLUID FLOW NEAR STAGNATION
POINT OF A THREE-DIMENSIONAL BODY

NORSHAZA ATIKA BINTI SAIDIN

A dissertation submitted in partial fulfilment of the
requirement for the award of the degree of
Master of Science

Faculty of Science
Universiti Teknologi Malaysia

APRIL 2018

Special dedication to my beloved parents and to my husband

Saidin Bin Hassan

Zaroni Binti Senapi

and

Ahmad Ghadaffi Bin Muhamad,

also to all of my brothers and sisters.

To my great supervisor and co-supervisor

Dr. Mohd Ariff Bin Admon

and

Assoc. Prof Dr. Sharidan Shafie ,

Thank you for everything.

ACKNOWLEDGEMENT

First of all, Alhamdulillah all praise be to Allah S.W.T., the Most Graceful and the Most Merciful for giving me the ability to undertake and finally complete this dissertation.

I wish to express my sincere appreciation and the deepest gratitude to my supervisor, Dr. Mohd Ariff bin Admon for his guidance, criticism, advices, suggestions, patience and encouragements. His positive attitude, support and understanding gave me the strength to do the research. Without him, this thesis would not have been completed. I would like to thank Assoc. Prof Dr. Sharidan Shafie who always shared his knowledge and suggestions to solve problems in this thesis.

I am grateful to my family especially to my parents, my husband, my parent in-laws, my brothers and my sisters for their understanding, support and always being by my side. Their love will always give me strength to face any problem, and continually drives me to complete this thesis. Last but not least, I would like to give my appreciation to my colleagues and friends especially to Zu'ain Zaizura Sofian and Azura who always help me in completing this research.

Thank you very much and may Allah S.W.T bless all of you.

ABSTRACT

Industrial systems gain a lot of benefit from unsteady free convection flow near the stagnation point of three-dimensional body such as the cooling of an infinite metallic plate in cooling baths and the boundary layer along material handling conveyers. In this study, a mathematical model of an unsteady free convection flow near the stagnation point of a three-dimensional body is developed. The problem considered involves the flow in nanofluid. The governing equations consist of continuity, momentum, energy and nanoparticle volume fraction are solved numerically through the Keller-box method. The effect of the physical parameters such as Brownian motion, thermophoresis and buoyancy parameters on the velocity, temperature and concentration profiles are investigated and discussed. Furthermore, various values of the physical parameter are examined by the skin friction coefficient in x - and y - directions, the local Nusselt number and Sherwood number. The results of the skin friction, velocity, temperature and concentration profile are presented and computed using FORTRAN and MATLAB software. The results have shown that Brownian motion, buoyancy, thermophoresis parameters and Lewis number give rises to the concentration profile. In addition, the skin friction is increased when the curvature parameter is increased.

ABSTRAK

Sistem perindustrian mendapat banyak faedah daripada aliran titik genangan tiga matra simetri sepaksi seperti penyejukan plat logam tak terhingga di dalam penyejuk mandian dan lapisan sempadan di sepanjang pengelolaan bahan-bahan penghantaran. Dalam kajian ini, model matematik dibina untuk mengkaji aliran cecair nano bagi jasad tiga matra tidak mantap berhampiran dengan titik genangan. Masalah aliran yang dipertimbangkan adalah aliran dalam nano. Persamaan menakluk yang terdiri daripada persamaan keselantaran, persamaan tenaga, persamaan momentum dan pecahan jumlah partikel nano diselesaikan secara berangka menggunakan kaedah kotak-keller. Kesan parameter fizikal seperti parameter gerakan Brownian, parameter thermophoresis, parameter keapungan pada halaju, suhu dan kepekatan dikaji dan dibincangkan. Tambahan pula, pelbagai nilai parameter fizikal diperiksa oleh pekali geseran kulit di arah x dan arah y , nombor Nusselt dan nombor Sherwood. Hasil yang ditunjukkan termasuk geseran kulit, halaju, suhu dan kepekatan dikira menggunakan perisian FORTRAN dan MATLAB. Keputusan menunjukkan bahawa parameter gerakan Brownian, parameter keapungan, parameter thermophoresis dan nombor lewis memberikan peningkatan kepada profil kepekatan. Di samping itu, geseran kulit meningkat apabila parameter kelengkungan meningkat.

TABLE OF CONTENTS

CHAPTER	TITLE	PAGE
	DECLARATION	
	DEDICATION	iii
	ACKNOWLEDGEMENT	iv
	ABSTRACT	v
	ABSTRAK	vi
	TABLE OF CONTENTS	vii
	LIST OF TABLES	x
	LIST OF FIGURES	xi
	LIST OF SYMBOLS	xiv
	LIST OF APPENDICES	xvii
1	INTRODUCTION	
	1.1 Introduction	1
	1.2 Research Background	2
	1.3 Problem Statement & Research Questions	4
	1.4 Objectives of the Study	4
	1.5 Scope of the Study	4
	1.6 Significance of the Study	5
	1.7 Research Organization	5
2	LITERATURE REVIEW	
	2.1 Introduction	7
	2.2 Viscous and Incompressible Fluid near the Stagnation	7

	Point	
2.3	Fluid Flow on Nanofluid	11
	2.3.1 Model of Buongiorno	13
	2.3.2 Tiwari and Das Model	15
2.4	Terminology	17
3	DERIVATION OF THE EQUATION OF MOTION	
3.1	Introduction	21
3.2	Equation of Motion	21
	3.2.1 Continuity Equation	22
	3.2.2 Momentum Equation	25
	3.2.3 Energy Equation	34
3.3	Equation of Nanofluid	46
	3.3.1 Conservation of Mass	46
	3.3.2 Conservation of Momentum	51
	3.3.3 Conservation of Energy	55
4	MATHEMATICAL FORMULATION	
4.1	Introduction	58
4.2	Basic Equations	59
4.3	Solution Procedure	67
	4.3.1 Finite Difference Method	68
	4.3.2 Newton's Method	74
	4.3.3 Block-elimination Method	84
	4.3.4 Starting Conditions	96
5	RESULTS AND DISCUSSION	
5.1	Introduction	98
5.2	Result and Discussion	98
5.3	Conclusion	115
6	CONCLUSION	
6.1	Introduction	117

6.2	Summary of Research	117
6.3	Suggestion for Future Research	119
REFERENCES		121
Appendix A		127

LIST OF TABLES

TABLE	TITLE	PAGE
5.1	Comparison of reduced skin frictions $f''(0), h''(0)$ and heat flux rate $-\theta'(0)$ for $\xi = 1$ (final steady-state), $Pr = 0.72$ and various values of c .	99
6.1	Conclusion for effect of nanofluid on the skin friction.	118
6.2	Conclusion for effect of nanofluid on velocity profile.	118
6.3	Conclusion for effect of nanofluid on temperature profile.	119
6.4	Conclusion for effect of nanofluid on concentration profile.	119

LIST OF FIGURES

FIGURE	TITLE	PAGE
3.1	Mass flow through infinitesimal fixed element.	22
3.2	Illustration of (a) shear stress (related to the time rate of the shearing deformation) and (b) normal stress (related to the time of change of volume).	27
3.3	The forces acting on the z -direction.	27
3.4	The energy fluxes acting on the z -direction.	36
3.5	Nanofluid control volume for conservation mass.	47
3.6	Nanofluid control volume for conservation energy.	56
4.1	Physical model and coordinate system.	60
4.2	Net rectangle for difference approximation.	69
5.1	Graph of skin friction coefficient, Nusselt and Sherwood number. $N_b = N_t = N_r = 0.1$ and $Le = 1.0$.	101
5.2	Graph of skin friction coefficient for $N_b = 0.2$, $Le = Pr = 1.0$ and $c = 0.5$ with varying N_t and Gr .	102
5.3	Graph of skin friction coefficient for $N_b = 0.2$, $Le = Pr = 1.0$ and $c = 0.5$ with varying N_r and Gr .	102
5.4	Graph of skin friction coefficient for $N_t = 0.2$, $Le = Pr = 1.0$ and $c = 0.5$ varying N_r and Gr .	103
5.5	(a) Velocity profile in x - and y -direction for various ξ with $N_r = N_t = 0$, $N_b = 0.001$, $Gr = Pr = 1.0$, $c = 0.5$ and $Le = 1$. (b) Velocity profile in x - and y -direction for various ξ with $N_r = N_t = 0$, $N_b = 0.001$, $Gr = Pr = 1.0$, $c = 0.5$ and $Le = 5$. (c) Velocity profile in x - and y -direction for various ξ with	104

$N_r = N_t = 0$, $N_b = 0.001$, $Gr = Pr = 1.0$, $c = 0.5$ and $Le = 10$.

- 5.6** (a) Temperature and concentration profile $\theta(\eta)$ and $\phi(\eta)$ for various ξ with $N_r = N_t = 0$, $N_b = 0.001$, $Gr = Pr = 1.0$, $c = 0.5$ and $Le = 1$. (b) Temperature and concentration profile $\theta(\eta)$ and $\phi(\eta)$ for various ξ with $N_r = N_t = 0$, $N_b = 0.001$, $Gr = Pr = 1.0$, $c = 0.5$ and $Le = 5$. (c) Temperature and concentration profile $\theta(\eta)$ and $\phi(\eta)$ for various ξ with $N_r = N_t = 0$, $N_b = 0.001$, $Gr = Pr = 1.0$, $c = 0.5$ and $Le = 10$. 106
- 5.7** (a) Velocity profile in x - and y -direction for various ξ with $N_t = N_b = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_r = 0$. (b) Velocity profile in x - and y -direction for various ξ with $N_t = N_b = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_r = 0.2$. (c) Velocity profile in x - and y -direction for various ξ with $N_t = N_b = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_r = 0.4$. 107
- 5.8** (a) Temperature and concentration profile $\theta(\eta)$ and $\phi(\eta)$ for various ξ with $N_t = N_b = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_r = 0$. (b) Temperature and concentration profile $\theta(\eta)$ and $\phi(\eta)$ for various ξ with $N_t = N_b = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_r = 0.2$. (c) Temperature and concentration profile $\theta(\eta)$ and $\phi(\eta)$ for various ξ with $N_t = N_b = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_r = 0.4$. 109
- 5.9** (a) Velocity profile in x - and y -direction for various ξ with $N_r = N_b = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_t = 0$. (b) Velocity profile in x - and y -direction for various ξ with $N_r = N_b = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_t = 0.2$. (c) Velocity profile in x - and y -direction for various ξ with $N_r = N_b = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_t = 0.4$. 110

- 5.10** (a) Temperature and concentration profile $\theta(\eta)$ and $\phi(\eta)$ for various ξ with $N_r = N_b = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_t = 0$. (b) Temperature and concentration profile $\theta(\eta)$ and $\phi(\eta)$ for various ξ with $N_r = N_b = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_t = 0.2$. (c) Temperature and concentration profile $\theta(\eta)$ and $\phi(\eta)$ for various ξ with $N_r = N_b = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_t = 0.4$. 112
- 5.11** (a) Velocity profile in x - and y -direction for various ξ with $N_r = N_t = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_b = 0.1$. (b) Velocity profile in x - and y -direction for various ξ with $N_r = N_t = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_b = 0.3$. (c) Velocity profile in x - and y -direction for various ξ with $N_r = N_t = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_b = 0.5$. 113
- 5.12** (a) Temperature and concentration profile $\theta(\eta)$ and $\phi(\eta)$ for various ξ with $N_r = N_t = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_b = 0.1$. (b) Temperature and concentration profile $\theta(\eta)$ and $\phi(\eta)$ for various ξ with $N_r = N_t = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_b = 0.3$. (c) Temperature and concentration profile $\theta(\eta)$ and $\phi(\eta)$ for various ξ with $N_r = N_t = 0.1$, $Le = Gr = Pr = 1.0$, $c = 0.5$ and $N_b = 0.5$. 115

LIST OF SYMBOLS

a	-	acceleration
a₁, a₂	-	unit vectors
<i>a, b</i>	-	principles curvature in the <i>x</i> -and <i>y</i> -planes
<i>c</i>	-	curvature parameter
<i>c_p</i>	-	heat at constant pressure
<i>D_B</i>	-	Brownian diffusion coefficient
<i>D_T</i>	-	thermophoretic diffusion coefficient
F	-	force
<i>g</i>	-	gravity acceleration
<i>J_σ</i>	-	surface curvature
<i>k</i>	-	fluid conductivity
	-	thermal conductivity
<i>m</i>	-	mass
n	-	unit normal
<i>n</i>	-	index point on ξ plane
<i>O</i>	-	nodal stagnation point
<i>P</i>	-	pressure
<i>P_D</i>	-	dynamic pressure
R	-	vector position
r	-	surface of the body <i>S</i>
<i>S</i>	-	body surface
<i>t</i>	-	time
<i>T</i>	-	fluid temperature
<i>T_w</i>	-	wall temperature
<i>T_∞</i>	-	ambient temperature

u, v, w	-	velocity component along x -, y -, z -axes
\mathbf{V}	-	velocity vector
x, y, z	-	Cartesian coordinates

Greek Symbols

α	-	thermal diffusivity
β	-	thermal expansion
∇	-	gradient operator
∇_s	-	surface gradient operator
η	-	plane along y -axis
h	-	volumetric heat addition
μ	-	dynamic viscosity
ν	-	kinematic viscosity
ϕ	-	viscous dissipation
ρ	-	density
τ	-	dimensionless parameter
	-	viscous stress
θ	-	dimensionless parameter
ξ	-	plane along x -axis

Superscripts

,	-	differentiation with respect on η
---	---	--

Subscripts

s	-	steady-state flow
w	-	wall condition
∞	-	far field condition

Nondimensional number

$C_{f\,x}$	-	skin friction coefficient in x -direction
$C_{f\,y}$	-	skin friction coefficient in y -direction
Gr	-	Grashof number

Le	-	Lewis number
Nu	-	Nusselt number
N_b	-	Brownian motion parameter
N_r	-	buoyancy parameter
N_t	-	thermophoresis parameter
Pr	-	Prandtl number
Ra	-	Rayleigh number
Re	-	Reynold number
Sh	-	Sherwood number
	-	mass transfer Nusselt number

LIST OF APPENDICES

APPENDIX	TITLE	PAGE
A	This Fortran Program to Find the Solution of Unsteady Free Convection Nanofluid Flow Near Stagnation Point of a Three-Dimensional body	127

CHAPTER 1

INTRODUCTION

1.1 Introduction

The study of convective heat transfer in nanofluids is gaining a lot of attention. As we know, a nanofluid is a fluid that contains nanometer-sized particles called nanoparticles. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid, which is the term proposed by Choi (1995) to describe the new class of nanotechnology-based heat transfer fluids that exhibit thermal properties superior to those of their base fluids or conventional particle fluid suspensions. The nanoparticles used in nanofluids are typically made of metals, oxides, carbides, or carbon nanotubes. Common base fluids include water, ethylene glycol and oil. The nanofluids have novel properties that make them potentially useful in many engineering applications in the industry including microelectronics, fuel cells, pharmaceutical processes and hybrid-powered engines.

These fluids can be assumed to be single phase or two-phase models. Classical theory of single phase models can be applied when nanoparticles and base fluids are considered as a single homogeneous fluid while two-phase models can be useful when equations of motion for handling nanoparticles and base fluids separately. Since the inception of the nanofluid concept about two decades ago, the potentials of nanofluid in heat transfer applications have attracted more attention from researchers. Accordingly, some of the applications of nanofluid are electronic

applications, transportations and industrial cooling applications. Therefore, the present work has been undertaken in order to study the unsteady free convection nanofluid flow near stagnation point of a three-dimensional body.

1.2 Research Background

Free convection on boundary layer flow near a stagnation point has gained a great deal of attention from several researchers in this field. Free convection or natural convection may occur in many applications, such as cooling molten metals, fluid flows around shrouded heat dissipation fins, and solar ponds. This convection occurs naturally without any forces and is considered as an important finding in solving many applications in real life. Due to this reason, many researchers have studied theoretically and performed experiments that are related to this problem. For example, the steady three-dimensional boundary layer equations were derived by Poots (1964). The investigation was focused on the isothermal curved surface. The researcher maintained the temperature above the ambient temperature of the fluid. Thus, the stagnation point is defined at the lowest point of the surface and it shows that the stagnation point was solved numerically depending on Prandtl number and Grashof number.

Next, this paper was continued by Xu *et al.* (2008) that studied the series solution of unsteady free convection flow in the stagnation-point region of three-dimensional body in ambient fluid. Xu *et al.* (2008) focused to continue the Poots (1964) work but in an unsteady case. The original momentum and energy equation have been reformulated and applied to the homotopy analysis method to obtain the accurate series solutions of resulting equations.

Most importantly, this research focused to expand the following works by Admon *et al.* (2011) who studied the unsteady free convection flow near the stagnation point of three-dimensional body and to investigate the behavior of heat and mass transfer on viscous and incompressible fluid. This problem has similar problems with Poots (1964) but was solved numerically using different methods.

The governing boundary layer equation was transformed first into non-similar boundary layer and solved numerically using finite-difference scheme known as Keller-box method. This research investigated the effect of curvature parameter, c and Prandtl number, Pr to the flow and heat transfer characteristics.

Nanoparticles are defined to be an object with at least one dimension smaller than 100 nanometer-sized. When a particle is that small and the area of the particle which got the surface is much higher than its weight, this causes the particle to develop new and exciting properties. The addition of nanoparticles in a base fluid like water and a solution where the suspension of nanoparticles can be stabilized will result in the creation of something extraordinary. This set-up is defined as nanofluids.

Choi (1995) is the first among many to use the term nanofluids. The purpose of the nanofluids is to intensify the heat transfer properties and the development of the nanofluids is potentially useful for several applications such as cooling of electronics and cooling of data center where a computer ejects a lot of heat, coolants, and so forth. The in-depth study on nanofluids can be found in the book, *Nanofluids: Science and Technology*, by Das *et al.* (2007), in the review paper by Wang and Mujumdar (2008), Yang *et al.* (2013), and Jahani *et al.* (2013). Kuzetsov and Nield (2010) used Buongiorno (2006) model to investigate a natural convection flow of a nanofluid over a vertical plate.

Since the heat and mass transfer are very extensive in the industry, the unsteady three-dimensional body near stagnation point can give a significant impact on the heat transfer process. Thus, it can improve the previously developed result. Therefore, on the present study, this research will be focusing on the numerical study of the unsteady free convection near the stagnation point of the three-dimensional body in nanofluid.

1.3 Problem Statement and Research Questions

This present study will concentrate on these problems and explore the following questions:

- (1) What is the mathematical model for the unsteady free convection nanofluid flow near stagnation point problem?
- (2) How to apply Keller-box method for this problem?
- (3) What happens to the skin friction, velocity, temperature and concentration profile of unsteady free convection near stagnation point flow in the present nanofluid particle?

1.4 Objective of the Study

The main objective of this study is to investigate the unsteady boundary layer flow near the three-dimensional stagnation point in nanofluid theoretically. It is required to construct the correct mathematical model and solve the equation numerically. Following are the objectives for this study:

- (1) To develop a mathematical model for unsteady free convection nanofluid flow near stagnation point of a three-dimensional body.
- (2) To simulate the governing equation using the Keller-box method.
- (3) To investigate the effects of nanofluid on skin friction, velocity, temperature and concentration profile of unsteady free convection near the stagnation point.

1.5 Scope of the Study

This study focused on the unsteady free convection flow near the stagnation point of a three-dimensional body in nanofluid. Three-dimensional mathematical

modeling will be formulated. Numerical solution is obtained using implicit finite-difference scheme of Keller-box method. Mathematical software of FORTRAN and MATLAB were used to compute and plot all interested profiles of the skin friction, velocity, temperature and concentration profile. In this research, no experimentation is needed.

1.6 Significance of the Study

Boundary layer and stagnation point flow have important impacts on the industry and technology application. Many applications such as in the food processing industry can be demonstrated using boundary layer flow near stagnation point theory. Boundary layer theory calculates the skin friction drag which acts on the body movement through all types of fluid. Furthermore, this theory is suitable to identify the shape that is right for the body to avoid bad results. Besides that, it is also suitable to be used in turbine blade, aeroplane wing and drag of the ship.

For decades, the research on heat and mass transfer has been carried out by many researchers and there are many methods that have been used and put forward. For this recent year, there are many researchers studying the flow of heat transfer in nanofluid. This fluid can extremely enhance the heat transfer characteristics of the base fluid. Thus, there are many applications of nanofluid in the industry such as heat exchangers, coolants, micro-channel heat sinks and lubricants. Finally, this study is a medium to give better understanding on the behaviors of fluid motion, the effects of nanofluid on the boundary layer flow near the stagnation point and phenomena of fluid mechanics and heat transfer.

1.7 Research Organization

This study consists of six chapters. Chapter 1 includes introduction, research background, problem statement, objectives of study, scope of the study, significance

of study. The literature review is a summary of research on previous works related to unsteady free convection boundary layer near stagnation points and will be discussed in Chapter 2. Chapter 3 discusses the derivation of the equation of motion. The governing equations of the unsteady free convection nanofluid flow near stagnation point of a three-dimensional body are discussed in Chapter 4. Chapter 5 will explain the result and discussion. Lastly, Chapter 6 describes the conclusion and followed by some suggestions for future research.

REFERENCES

- Abbas, Z. *Numerical and Series Solutions for Flows of Nonlinear Fluids*. Ph.D. Thesis. Quaid-I-Azam University; 2009.
- Abu Bakar, N. A., Bachok, N., and MD. Ariffin, N. (2016). Nanofluid Flow Using Buongiorno Model over a Stretching Sheet and Thermophysical Properties of Nanoliquids. *Indian Journal of Science and Technology*, 9(31), 0974-6846.
- Admon, M.A., Shafie, S., and Pop, I. (2011). Unsteady Free Convection Flow near the Stagnation Point of a Three-Dimensional Body. *Journal of Applied Sciences*. 11 (8), 1441-1444.
- Ahmad, K., Nazar, R., Ishak, A., and Pop, I. (2011). Unsteady three-dimensional boundary layer flow due to a stretching surface in a micropolar fluid. *International Journal of Numerical Method in Fluids*. 68, 1561-1573. Wiley Online Library.
- Aldoss, T. K., Ali, Y. D., and Al-Nimr, M. A. (1996). MHD Mixed Convection from a Horizontal Circular Cylinder. *Numerical Heat Transfer, Part A*. 30, 379-396.
- Aleng, N. L., Bachok, N., and MD.Ariffin, N. (2014). Studied About Boundary Layer Flow of a Nanofluid and Heat Transfer Over an Exponentially Shrinking Sheet: Copper-Water.

- Ali, F. M., Nazar, R. and Arifin, N., M. (2010). MHD Viscous Flow and Heat Transfer Induced by a Permeable Shrinking Sheet with Prescribed Surface Heat Flux *WSEAS Transactions on Mathematics*. 9(5), 365-375.
- Alizadeh, R., Rahimi, A.B., Arjmandzadeh, R., Najafi, M., and Alizadeh, A. (2016). Unaxisymmetric stagnation point flow and heat of a viscous fluid with variable viscosity on cylinder in constant heat flux. *Alexandria Engineering Journal*. 55, 1271-1283.
- Amin, N., and Riley, N. (1995). Mixed Convection at a Stagnation Point. *Quarterly Journal of Mechanics and Applied Mathematics*. 48(1), 111-121.
- Anderson, J. D. (1995). *Computational Fluid Dynamics: The Basic with Applications*. Singapore: McGrawHill.
- Attia, H.A. (2008). Stagnation Point Flow and Heat Transfer of a Micropolar Fluid with Uniform Suction or Blowing: *Journal of the Braz. Soc. of Mech. Sci. & Eng.* Vol xxx, No.1.
- Bejan, A. (2004). *Convection Heat Transfer*. 3rd. Edition. U.S.A.: John Wiley & Sons.
- Buongiorno, J. (2006). Convective transport in nanofluids. *Journal of Heat Transfer*. 128 (3), 2006.
- Cebeci, T., and Bradshaw, P. (1977). *Momentum Transfer in Boundary Layers*. Washington: Hemisphere.
- Choi, S.U. (1995). Enhancing thermal conductivity of fluids with Nanoparticles. *Developments and applications of non-Newtonian flows*, AJME, FED 231/MD 66, 99105.
- Das, S.K., Choi, S.U., Yu, W. and Pardeep, T. (2007). *Nanofluids: Science and Technology*, JohnWiley & Sons, New York, NY, USA.

- Dessie, H., and Naikoti, K. (2015). Scaling Group Analysis on MHD Effects on Heat Transfer Near a Stagnation Point on a Linearly Stretching Sheet With Variable Viscosity and Thermal Conductivity, Viscous Dissipation and Heat Source/Sink. *theoretical and applied mechanics*. 42, 111-133.
- Edalati, Z., Heris, S.Z., and Noie, S.H. (2012). The Study of Laminar Convective Heat Transfer of CuO/Water Nanofluid Through an Equilateral Triangular Duct at Constant Wall Heat Flux. *Heat Transfer—Asian Research*. 41(5), 2012.
- Farooq, U., and Xu, H. (2014). Free Convection Nanofluid Flow in the Stagnation-Point Region of a Three-Dimensional Body. *The Scientific World Journal*. Vol 2014.
- Hajipour, M., Dehkordi, A.M., and Jamshidi, M. (2013). Numerical Investigation of Nanofluid Mixed-Convection Flow in the Entrance Region of a Vertical Channel Partially Filled with Porous Medium. *Heat Transfer-Asian Research*. 43 (7), 2014.
- Hasani, M. and M. Mohammad Tabar. (2011). An Analytical Solution for Boundary Layer Flow of Nanofluid Past a Stretching Sheet. *International Journal of Thermal Sciences*. 50(11), 2256-2263.
- Hayat, T., Rashid, M., Imtiaz, M., and Alsaedi, A. (2015). Magnetohydrodynamic (MHD) Stretched Flow of Nanofluid with Power-Law Velocity and Chemical Reaction. 5, 117121. American of Physics.
- Jahani, K., Mohammadi, M., Shafii, M.B., and Shiee, Z. (2013). Promising technology for electronic cooling: nanofluidic micro pulsating heat pipes. *Journal of Electronic Packaging*. 135(2), 2013.
- Kandasamy, R., Loganathan, P., and Puvi Arasu, P. (2011). Scaling group transformation for MHD boundary-layer flow of a nanofluid past a vertical

stretching surface in the presence of suction/injection, *Nuclear Engineering and Design* 241(6): 2053-2059.

- Keller, H. B. (1971). *A New Difference Scheme for Parabolic Problems*, in *Numerical Solutions of Partial Differential Equations*. B. Hubbard Edition. New York: Academic Press. 2, 327-350.
- Khalid, A., Jian, L.Y., Khan, I., and Shafie, S. (2017). Exact Solutions for Unsteady Free Convection Flow of Carbon Nanotubes over an Oscillating Vertical. *The 4th International Conference on Mathematical Sciences*.
- Khan, J.A., Mustafa, M., Hayat, T., Sheikholeslami, M., and Alsaedi, A. (2015). Three-Dimensional Flow of Nanofluid Induced by an Exponentially Stretching Sheet: An Application to Solar Energy. *Journal. Pone.* 10(3), 1371.
- Kuznetsov, A.V., and Nield, D.A. (2010). Natural convective boundarylayer flow of a nanofluid past a vertical plate. *International Journal of Thermal Sciences*. 49(2), 2010.
- Maboob, F., Pochai, N., and Shateyi, S. (2016). Stagnation Point Flow of Nanofluid Over a Moving Plate with Convective Boundary Condition and Magnetohydrodynamic. *Journal of Engineering*. <http://dx.doi.org/10.1155/2016/5874864>
- Mansur, S., and Ishak, A. (2013). The Flow and Heat Transfer of a Nanofluid Past a Stretching/Shrinking Sheet with a Convective Boundary Condition. *Abstract and Applied Analysis*. Vol 2013.
- Nazar, R., Amin, N., and Pop, I. (2003). Unsteady Mixed Convection near the Forward Stagnation Point of a Two-Dimensional Symmetric Body. *International Communication in Heat and Mass Transfer*. 30(5), 673-682.
- Ozisik, M. N. (1977). *Basic Heat Transfer*. New York: McGrawHill.

- Panigrahi, S., and Reza, M. (2014). Analytic Solution of MHD Stagnation Point Flow over a Stretching Permeable Surface with Effect of Viscous Dissipation and Joule Heating. *Journal of Engineering*. 6, 827-840.
- Pfautsch, E. *Forced Convection in Nanofluid over a Flat Plate*. Faculty of the Graduate School, University of Missouri;2008.
- Poots, G. (1964). Laminar Free Convection near the Lower Stagnation Point on an Isothermal Curved Surface. *International Journal of Heat and Mass Transfer*. 7, 863-874.
- Pop, S.R., Grosam, T., and Pop, I. (2004). Radiation Effects on the Flow near the Stagnation Point of a Stretching Sheet. *Technische Mechanik*. 25(2), 100-106.
- Raj, M., Jha, A., and Sharma, A. (2016). Effect of Thermal Radiation and Variable Fluid Viscosity on Stagnation Point Flow Past a Porous Stretching Sheet Embedded in Porous Medium with Partial Slip Condition. *International Journal of Mathematics and Its Application*. 3, 139-148.
- Salleh, M.Z., Nazar, R., and Pop, I. (2010). Modelling of Free Convection Boundary Layer Flow on a Solid Sphere with Newtonian Heating. *Acta Appl Math*. 112, 263-274.
- Sayyar, R. O., and Saghafian, M. (2017). Numerical Simulation of Convective Heat Transfer of Nonhomogeneous Nanofluid Using Buongiorno Model.
- Seshadri, R., Sreeshylan, N., and Nath, G. (2002). Unsteady Mixed Convection Flow in the Stagnation Region of a Heated Vertical Plate due to Impulsive Motion. *International Journal of Heat and Mass Transfer*. 45, 1345-1352.
- Sharidan, S., Amin, N., and Pop, I. (2007). G-Jitter Free Convection Flow in the Stagnation Point Region of a Three-Dimensional Body. *Mechanics Research Communications*. 34, 115-122.

- Sharma, R., Ishak, A., and Pop, I. (2014). Dual Solution of Unsteady Separated Stagnation-Point Flow in a Nanofluid with Suction: A Finite Element Analysis. *Indian Journal of Pure & Applied Physics*. 55, 275-283.
- Shateyi, S., and Mabood, F. (2015). Mhd Mixed Convection Slip Flow Near a Stagnation Point on a Nonlinearly Vertical Stretching Sheet In The Presence Of Viscous Dissipation. South Africa: *Univesity of Venda*.
- Shen, M., Wang, F., Xiao, T., and Chen, H. (2016). Stagnation Point Flow of a Nanofluid Past a Permeable Cylinder with Chemical Reaction. *Heat Transfer-Asian Research*. 0(0), 2016.
- Tiwari, R. K., and Das, M. K. (2007). Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. *International Journal of Heat and Mass Transfer*, vol. 50.
- Wang, X. and Mujumdar, A.S. (2008). A review on nanofluids—partI: theoretical and numerical investigations. *Brazilian Journal of Chemical Engineering*. 25(4), 2008.
- Xu, H., Liao, S., and Pop, I. (2008). Series Solutions of Unsteady Free Convection Flow in the Stagnation-Point Region of a Three-Dimensional Body. *International Journal of Thermal Sciences*. 47, 600-608.
- Yang, C., Li, W., Sano, Y., Mochizuki, M., and Nakayama, A. (2013). On the anomalous convective heat transfer enhancement in nanofluids: a theoretical answer to the nanofluids controversy. *Journal of Heat Transfer*. 135(5), 2013.
- Yasin, M.H.M., Ishak, A., and Pop, I. (2015). MHD Stagnation-Point Flow and Heat Transfer of Viscous Dissipation, Joule Heating and Partial Velocity Slip.