An Improvement of Computing Newton's Direction for Finding Unconstrained Minimizer for Large-Scale Problems with an Arrowhead Hessian Matrix

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Abstract. In large-scale problems, classical Newton's method requires solving a large linear system of equations resulting from determining the Newton direction. This process often related as a very complicated process, and it requires a lot of computation (either in time calculation or memory requirement per iteration). Thus to avoid this problem, we proposed an improved way to calculate the Newton direction using an Accelerated Overrelaxation (AOR) point iterative method with two different parameters. To check the performance of our proposed Newton's direction, we used the Newton method with AOR iteration for solving unconstrained optimization problems with its Hessian is in arrowhead form and compared it with a combination of the Newton method with Gauss-Seidel (GS) iteration and the Newton method with Successive Over Relaxation (SOR) iteration. Finally, comparison results show that our proposed technique is significantly more efficient and more reliable than reference methods.

Keywords: Newton method, AOR iteration, Unconstrained optimization problems, Large-scale optimization, Arrowhead matrix.

1 Introduction

We consider large-scale unconstrained minimization problem

$$\min_{\underline{x}\in\mathbb{R}^n} f\left(\underline{x}\right) \tag{1}$$

where the objective function $f:\mathbb{R}^n \rightarrow \mathbb{R}$ is smooth with at least twice continuously differentiable and n is large. These problems appear in a wide range of applications, including electric power systems for the nonlinear large mesh-interconnected system [1], discrete-time optimal control [2], the DNA reproduction process [3] or even in the sign recurrent neural network [4]. Therefore, developing an efficient method to solve problems (1) is a significant and vigorous scientifically research area.

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There are various methods listed in [5] to solve unconstrained minimization, and Newton's method is listed as one of the best-known methods that possess the fast quadratic rate of convergence with excellent performance when the initial point is chosen correctly. However, if it involves a large-scale problem, computation and storage of the Hessian for the classical Newton method are too costly to be practical. For that reason, many researchers have modified this classical Newton's method to overcome its disadvantages, such as in [6-10].

Shen et al. [6] presented a new regularized Newton method for solving unconstrained minimization problems by using the modified Cholesky factorization algorithm to replace the objective function's Hessian. On top of that, they used a first-order method to find the first-order stationary point and only mention that the algorithms can be extended to second-order algorithms by the help of negative curvature. Grapsa [7] proposed modification on Newton's direction for solving unconstrained optimization using the gradient vector modification at each iteration, while Tahera et al. [8] combined two types of descent direction to propose a new algorithm for solving unconstrained optimization problems. Recently, Abiodun and Adelabu [9] investigated the performance of the Newton method and compared it with two iterative modification of Newton's method for solving unconstrained minimization problems. However, they concluded that the computational numerical result obtained was depend on the nature of the objective function.

Thus, we modified the classical Newton method via its Newton's direction, and this modification is different from the existing combination of Newton's method with other methods, as stated in the previous paragraphs. This Newton's direction is obtained by solving a linear system that involves the Hessian matrix and the gradient. Therefore, solving it using the direct method for large-scale problems is not a smart choice. On the other hand, we used the AOR point iteration. This AOR iteration has been introduced by Hadjidimos in [11] using a two-parameter generalization of the SOR method and classified as one of the most straightforward and powerful techniques for solving any linear systems.

As a result, in this paper, motivated by the advantages of the classical Newton method, we proposed the AOR point iteration to find the Newton direction and then solve problem (1) using Newton's method. We called this method as Newton-AOR method. This improvement is a new contribution to the large-scale unconstrained optimization problem where the Hessian is in the arrowhead matrix form. To verify the performance of the improved Newton direction, we used a combination of Newton's method with Gauss-Seidel iteration and a combination of Newton's method with SOR iteration as reference methods, and they are called as Newton-GS method and Newton-SOR method respectively.

The rest of this paper is organized as follows. In the next section, we described the Newton scheme with an arrowhead Hessian matrix, while in section 3, we formulated the AOR iteration for computing Newton's direction. In section 4, we presented our numerical experiment and result. Lastly, the conclusion is stated in section 5.