Approximate Reasoning in the Knowledge-Based Dynamic Fuzzy Sets

Rolly Intan*, Siana Halim, Lily Puspa Dewi
Petra Christian University, Surabaya, Indonesia.

* Corresponding author. Tel.: +62-31-8439040; email: rintan@petra.ac.id
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Abstract: Intan and Mukaidono discussed that knowledge plays an important role in determining the membership function of a given fuzzy set by introducing a concept, called *Knowledge-based Fuzzy Sets* (KFS) in 2002. Here, the membership degree of an element given a fuzzy set is subjectively determined by the knowledge. Every knowledge may have each different membership degree of the element given the fuzzy set. In 1988, Wang *et al.* extended the concept of fuzzy set, called *Dynamic Fuzzy Sets* (DFS) by considering that the membership degree of an element given a fuzzy set might be dynamically changeable over the time. Both generalized concepts, KFS and DFS, were hybridized by Intan *et al.* to be a *Knowledge-based Dynamic Fuzzy Set* (KDFS). As usually happened in the real-world application, the KDFS showed that a membership function of a given fuzzy set subjectively determined by a certain knowledge may be dynamically changeable over time. Moreover, the concept of fuzzy granularity was discussed dealing with the KDFS. Related to the concept of fuzzy granularity in KDFS, this paper discusses the concept of approximate reasoning of KDFS in representing fuzzy production rules as generally applied in the fuzzy expert system.

Key words: Knowledge-based dynamic fuzzy sets, fuzzy granularity, fuzzy expert system, fuzzy rules.

1. Introduction

The concept of fuzzy set was proposed by L.A. Zadeh in 1965 [1], [2]. The concept of fuzzy set is considered as a generalization of crisp sets dealing with the gradual membership degree of elements in a real number started from 0 (non-member) to 1 (member).

In the concept of fuzzy sets, a membership function of a given fuzzy set is consistently unchangeable during the time variable. However, in the real-world application, the membership degree of an element given a fuzzy set may be changeable dealing with time. Therefore, Wang *et al*. (1988) [3], [4] proposed an extended concept of fuzzy sets, called *Dynamic Fuzzy Sets* (DFS) in which every membership degree of an element in DFS might be dynamically changeable dealing with time's variable. In this case, the DFS may also be regarded as an example of multi-fuzzy sets by means that a given fuzzy label might be represented by many fuzzy sets dealing with time variable.

In 2002, Intan and Mukaidono [5]-[7] discussed differences between probability and fuzziness. Probability is considered as a concept to present the situation of objective uncertainty. On the other hands, fuzziness is for the situation of subjective uncertainty. Through fuzziness, a certain knowledge may subjectively determine a membership function of a given fuzzy set. To express this reality, Intan and Mukaidono (2002) [5]-[7] proposed the concept of *Knowledge-based Fuzzy Sets* (KFS) as a generalization of fuzzy sets. Similar to the DFS, the KFS is also an example of the multi-fuzzy sets by means that a given fuzzy label might be represented by many fuzzy sets dealing with the variable knowledges. In this case, fuzziness may be considered as a deterministic uncertainty by considering that a person through his/ her knowledge may be subjectively able to determine a given object even in an uncertain (unclear) situation or definition of the object. Therefore, in the KFS, a given fuzzy label may have n different membership functions (fuzzy sets) related to n different knowledge.

Both DFS and KFS are two generalized concepts of fuzzy sets dealing with different interpretations or variables, time and knowledge, respectively. Therefore, both concepts are possibly combined to construct a more comprehensive concept of fuzzy sets. By considering that a membership function of a given fuzzy set given by a certain knowledge may possibly change over time, Intan *et al* [8] proposed a hybrid concept, called *Knowledge-based Dynamic Fuzzy Set* (KDFS) as a more generalized concept of fuzzy sets compared than both KFS and DFS. In the concept of KDFS, the membership function of a given fuzzy set determined by a certain knowledge may be dynamically changeable over time as usually shown in the real-world application. Moreover, the KDFS may be considered as an example concept of two-dimensional multi-fuzzy sets dealing with both time and knowledge. Related to KDFS, three summary fuzzy sets were discussed and constructed using the functions of aggregation. Some basic operations and properties such as equality, contentment, union, intersection and complement were also defined and examined. Also, the fuzzy granularity dealing with the crisp and fuzzy coverings of knowledge was discussed in terms of KDFS [9].

Related to the *Approximate Reasoning*, this paper continually extends the concept of KDFS by discussing how to generate fuzzy production rules in KDFS. The concept of approximate reasoning dealing with KDFS as proposed in this paper plays important role in constructing rule base in Fuzzy Expert System, since the fuzzy production rule store in the rule base of fuzzy expert system is generally given by the knowledge of experts. Four categories of the fuzzy production rules, namely *Strong Implication*, *Weak Implication*, *Strong Bi-implication* and *Weak Bi-implication*, are proposed and discussed together with their properties. Several equations are proposed to measure validation's degree of the fuzzy production rules. Finally, four rules of conditions are given to justify the construction of *Strong Implication*, *Weak Implication*, *Strong Bi-implication* and *Weak Bi-implication*.

2. Knowledge-Based Dynamic Fuzzy Sets

2.1. Definition

The *Knowledge-based Fuzzy Sets* (KDFS) is a hybrid concept of both dynamic fuzzy sets and knowledge-based fuzzy sets. It can be verified that in the real-world application, even a certain knowledge k has already determined a membership function of fuzzy set A, next time the same knowledge, he/she may provide a different membership function to the fuzzy set A. It can be said that any membership function of KFS may be dynamically changeable over the time variable. Formally, the definition of knowledge-based dynamic fuzzy sets is given as follows:

Definition 1 Let U be a universal set of elements, and $K = \{k_1, k_2, \cdots, k_m\}$ be a set of knowledges, and T be a discrete set of time, where $T = \{t_1, t_2, \cdots, t_n\}$. Then a *knowledge-based dynamic fuzzy set* of A on U denoted by $\mathcal{D}(A) = \{A_{k_i}(t_j) | \forall k_i \in K, \forall t_j \in T\}$ is defined as a set of fuzzy sets dealing with the set of knowledges K and the set of time T. $A_k(t) \in \mathcal{D}(A)$ is a knowledge-based dynamic fuzzy set dealing with knowledge k at time t, and it is characterized by the following membership function.

$$\mu_{A_k(t)}: U \rightarrow [0,1] \tag{1}$$

Related to (1), $\mu_{A_k(t)}(u) \in [0,1]$ is a the membership degree of element $u \in U$ on fuzzy set A dealing with the knowledge $k \in K$ at the time $t \in T$. Similarly, $\mu_{A_k(t)}(u) = 1$ means u has full membership in A according to k at the time t. On the other hand, $\mu_{A_k(t)}(u) = 0$ means u is not a member of $A_k(t)$. Thus, the membership degree of u in A could be changeable depending on both k and t. Here, $A_k(t) \in \mathcal{F}(U)$ is considered as a *knowledge-based dynamic fuzzy set* of A dealing with knowledge k at the time t. In this case, $A_k(t)$ is a fuzzy set that has a similar concept with the concept of fuzzy set proposed by Zadeh in 1965 [1], [2], where $\mathcal{F}(U)$ is a fuzzy power set of U. Every $A_k(t) \in \mathcal{D}(A)$ has its membership function given by $\mu_{A_k(t)}$. Therefore, a knowledge-based fuzzy set of A, $\mathcal{D}(A) = \{A_{k_i}(t_j) | \forall k_i \in K, \forall t_j \in T\}$ has $m \times n$ membership functions as given by $\{\mu_{A_{k_i}(t_j)} | \forall k_i \in K, \ \forall t_j \in T\}$.

2.2. Summary Fuzzy Sets

The relationship among DFS, KFS and KDFS as shown in Table 1. Let A be a fuzzy set on U, K be a set of knowledges and T be a set of times, where $K = \{k_1, k_2, \ldots, k_m\}$ and $T = \{t_1, t_2, \ldots, t_n\}$.

Table 1. Relation among KFS, DFS and KDFS

	$A(t_1)$	$\cdots$	$A(t_n)$
A_{k_1}	$A_{k_1}(t_1)$	$\cdots$	$A_{k_1}(t_n)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_{k_m}	$A_{k_m}(t_1)$	$\cdots$	$A_{k_m}(t_n)$

Furthermore, both A_{k_i} and $A(t_j)$ could be interpreted as results of aggregating $A_{k_i}(t_j)$ by taking two different aggregate functions, Υ and Θ, respectively over their membership degrees as follows.

$$\forall u \in U, \mu_{A_{k_i}}(u) = \Upsilon(\mu_{A_{k_i}(t_1)}(u), \cdots, \mu_{A_{k_i}(t_n)}(u)) \tag{2}$$

where $\Upsilon: [0,1]^n \rightarrow [0,1]$

$$\forall u \in U, \mu_{A(t_j)}(u) = \Theta(\mu_{A_{k_1}(t_j)}(u), \cdots, \mu_{A_{k_m}(t_j)}(u)) \tag{3}$$

where $\Theta: [0,1]^m \rightarrow [0,1]$

According to the need and the context of applications, Υ and Θ may utilize any existed functions of aggregation such as *maximum*, *minimum*, *average*, etc. in order to summarize from KDFS to KFS and DFS, respectively. Here, (3) is the same as *the knowledge-based summary fuzzy set* which discussed by Intan and Mukaidono in 2002 [5]-[7]. In practical application, the knowledge-based summary fuzzy set of A as defined in (3) could be understood as an agreement among a group of persons represented by a set of knowledge to describe fuzzy set A at the time t_j. Similarly, (2) might be considered to provide a *time-based summary fuzzy set*. For it is usually happened in the real-world application, subjective opinion of someone toward a given fuzzy set A may be changeable according to the changing of times. Thus, the objective of the time-based summary fuzzy set as given in (2) is to summarize the multiple opinions of a certain knowledge k_i to the fuzzy set A because of time variable. Depending on the reason behind calculating, to be more flexible and accurate, both summary fuzzy sets as given in (2) and (3) may use the weighted average as their aggregate function as shown in the following equations.

$$\Upsilon(\mu_{A_{k_i}(t_1)}(u),\cdots,\mu_{A_{k_i}(t_n)}(u)) = \frac{\sum_{j=1}^{m} w_j \cdot \mu_{A_{k_i}(t_j)}(u)}{\sum_{j=1}^{m} w_j} \tag{4}$$

where $w_j \in R^+,\ R^+ = [0,\infty)$

$$\Theta(\mu_{A_{k_1}(t_j)}(u),\cdots,\mu_{A_{k_m}(t_j)}(u)) = \frac{\sum_{i=1}^{n} w_i \cdot \mu_{A_{k_i}(t_j)}(u)}{\sum_{i=1}^{n} w_i} \tag{5}$$

where $w_i \in R^+,\ R^+ = [0,\infty)$

The usage of weighted average in calculating the summary fuzzy sets may have some benefits in which w_i and w_j as are able to express the importance of an opinion. For instance, in the case to calculate the knowledge-based summary fuzzy sets, more prominent knowledge k_i is considered to determine the summary fuzzy set, a larger w_i is given to k_i. In the case to calculate the time-based summary fuzzy sets, a larger weight may usually be given to the more recent opinion, since a more recent opinion may represent a more real-time situation. Therefore, in the case of calculating the time-based summary fuzzy sets, the relationship between time and weight may satisfy $t_j > t_p \Rightarrow w_j \geq w_p,\ \forall t_j, t_p \in T$, where t_j is considered more recent than t_p.

It is also necessary to propose a *general summary fuzzy set* in order to summarize all interpretation/ opinion based on the knowledge as well as the times into only one summary fuzzy set. In this case, the general summary fuzzy set may be interpreted as an agreement made to sum up all opinions given by multiple knowledge over several times. Formally, given A be a fuzzy set on U. Let $K = \{k_1,\cdots,k_m\}$ and $T = \{t_1,\cdots,t_n\}$. Similar to the concept of weighted average as defined in (4) and (5), three different equations of general summary fuzzy set are introduced as follows.

- *General Summary Fuzzy Set* (A^{G_1}) is constructed from the Knowledge-based Summary Fuzzy Sets:

$$\mu_{A^{G_1}}(u) = \Psi(\mu_{A_{k_1}}(u),\ \cdots,\mu_{A_{k_m}}(u)) = \frac{\sum_{i=1}^{m} w_i \cdot \mu_{A_{k_i}}(u)}{\sum_{i=1}^{m} w_i} \tag{6}$$

where $w_i \in R^+,\ R^+ = [0,\infty)$

- *General Summary Fuzzy Set* (A^{G_2}) is constructed from the Time-based Summary Fuzzy Sets:

$$\mu_{A^{G_2}}(u) = \Gamma\left(\mu_{A(t_1)}(u),\cdots,\mu_{A(t_n)}(u)\right) = \frac{\sum_{j=1}^{n} w_j \cdot \mu_{A(t_j)}(u)}{\sum_{j=1}^{n} w_j} \tag{7}$$

where $w_i \in R^+,\ R^+ = [0,\infty)$

- *General Summary Fuzzy Set* (A^{G_3}) is constructed from the Knowledge-based Dynamic Fuzzy Sets:

$$\mu_{A^{G_3}}(u) = \Omega\begin{pmatrix} \mu_{A_{k_1}(t_1)}(u) & \cdots & \mu_{A_{k_m}(t_1)}(u) \\ \vdots & \ddots & \vdots \\ \mu_{A_{k_1}(t_n)}(u) & \cdots & \mu_{A_{k_m}(t_n)}(u) \end{pmatrix} = \frac{\sum_{j=1}^{n}\sum_{i=1}^{m} w_{ij} \cdot \mu_{A_{k_i}(t_j)}(u)}{\sum_{j=1}^{n}\sum_{i=1}^{m} w_{ij}} \tag{8}$$

where $w_{ij} \in R^+$, $R^+ = [0, \infty)$.

The calculation of these three different equations of General Summary Fuzzy Set may provide different results in which it depends on the need and context of application to choose which one is better to use.

2.3. Basic Operations and Properties

Related to the concept of KDFS, this paper proposes some basic operations of the KDFS, and verifies their properties. The basic operations of the KDFS are defined as the following definition.

Definition 2 Let U be a universe of elements and $K = \{k_1, k_2, \cdots, k_m\}$ be a set of knowledges, and T be a discrete set of time, where $T = \{t_1, t_2, \cdots, t_n\}$. $\mathcal{D}(A)$ and $\mathcal{D}(B)$ are two KDFS on U dealing with K. Some basic operations and properties of *Equality*, *Containment*, *Complementation, Intersection* and *Union* are given by the following equations.

Equality

a) $A_k(t) = B_k(t) \Leftrightarrow \mu_{A_k(t)}(u) = \mu_{B_k(t)}(u), \forall u \in U,$

b) $A_k = B_k \Leftrightarrow \mu_{A_k(t)}(u) = \mu_{B_k(t)}(u), \forall u \in U, \forall t \in T,$

c) $A_k \equiv B_k \Leftrightarrow \mu_{A_k(t_i)}(u) = \mu_{B_k(t_j)}(u), \forall u \in U, \forall t_i, t_j \in T,$

d) $A(t) = B(t) \Leftrightarrow \mu_{A_k(t)}(u) = \mu_{B_k(t)}(u), \forall u \in U, \forall k \in K,$

e) $A(t) \equiv B(t) \Leftrightarrow \mu_{A_{k_i}(t)}(u) = \mu_{B_{k_j}(t)}(u), \forall u \in U,\ \forall k_i, k_j \in K,$

f) $A = B \Leftrightarrow \mu_{A_k(t)}(u) = \mu_{B_k(t)}(u), \forall u \in U, \forall k \in K, \forall t \in T,$

g) $A \cong B \Leftrightarrow \mu_{A_k(t_i)}(u) = \mu_{B_k(t_j)}(u), \forall u \in U, \forall k \in K,\ \forall t_i, t_j \in T,$

h) $A \triangleq B \Leftrightarrow \mu_{A_{k_i}(t)}(u) = \mu_{B_{k_j}(t)}(u), \forall u \in U, \forall k_i, k_j \in K,\ \forall t \in T,$

i) $A \equiv B \Leftrightarrow \mu_{A_{k_i}(t_i)}(u) = \mu_{B_{k_j}(t_j)}(u), \forall u \in U,\ \forall k_i, k_j \in K,\ \forall t_i, t_j \in T,$

j) $k_i = k_j \Leftrightarrow \mu_{A_{k_i}(t)}(u) = \mu_{A_{k_j}(t)}(u), \forall u \in U, \forall t \in T,\ \forall A \in \mathcal{F}(U),$ where $\mathcal{F}(U)$ is fuzzy power set on U.

k) $t_i = t_j \Leftrightarrow \mu_{A_k(t_i)}(u) = \mu_{A_k(t_j)}(u), \forall u \in U, \forall k \in K,\ \forall A \in \mathcal{F}(U),$ where $\mathcal{F}(U)$ is fuzzy power set on U.

Containment

a) $A_k(t) \subseteq B_k(t) \Leftrightarrow \mu_{A_k(t)}(u) \le \mu_{B_k(t)}(u), \forall u \in U,$

b) $A_k \subseteq B_k \Leftrightarrow \mu_{A_k(t)}(u) \le \mu_{B_k(t)}(u), \forall u \in U, \forall t \in T,$

c) $A_k \Subset B_k \Leftrightarrow \mu_{A_k(t_i)}(u) \le \mu_{B_k(t_j)}(u), \forall u \in U, \forall t_i, t_j \in T,$

d) $A(t) \subseteq B(t) \Leftrightarrow \mu_{A_k(t)}(u) \le \mu_{B_k(t)}(u), \forall u \in U, \forall k \in K,$

e) $A(t) \Subset B(t) \Leftrightarrow \mu_{A_{k_i}(t)}(u) \le \mu_{B_{k_j}(t)}(u), \forall u \in U,\ \forall k_i, k_j \in K,$

f) $A \subseteq B \Leftrightarrow \mu_{A_k(t)}(u) \le \mu_{B_k(t)}(u), \forall u \in U, \forall k \in K, \forall t \in T,$

g) $A \sqsubseteq B \Leftrightarrow \mu_{A_k(t_i)}(u) \le \mu_{B_k(t_j)}(u), \forall u \in U, \forall k \in K,\ \forall t_i, t_j \in T,$

h) $A \preccurlyeq B \Leftrightarrow \mu_{A_{k_i}(t)}(u) \le \mu_{B_{k_j}(t)}(u), \forall u \in U, \forall k_i, k_j \in K,\ \forall t \in T,$

i) $A \Subset B \Leftrightarrow \mu_{A_{k_i}(t_i)}(u) \le \mu_{B_{k_j}(t_j)}(u), \forall u \in U,\ \forall k_i, k_j \in K,\ \forall t_i, t_j \in T,$

j) $k_i \trianglelefteq k_j \Leftrightarrow \mu_{A_{k_i}(t)}(u) \le \mu_{A_{k_j}(t)}(u), \forall u \in U, \forall t \in T,\ \forall A \in \mathcal{F}(U),$ where $\mathcal{F}(U)$ is fuzzy power set on U.

k) $t_i \trianglelefteq t_j \Leftrightarrow \mu_{A_k(t_i)}(u) \le \mu_{A_k(t_j)}(u), \forall u \in U, \forall k \in K,\ \forall A \in \mathcal{F}(U),$ where $\mathcal{F}(U)$ is fuzzy power set on U.

Union

a) $\mu_{(A\cup B)_k(t)}(u) = \max(\mu_{A_k(t)}(u), \mu_{B_k(t)}(u))$,

b) $\mu_{A_{k_i}(t_i)\cup B_{k_j}(t_j)}(u) = \max\left(\mu_{A_{k_i}(t_i)}(u), \mu_{B_{k_j}(t_j)}(u)\right)$,

c) $\mu_{A_{k_i\vee k_j}(t)}(u) = \max\left(\mu_{A_{k_i}(t)}(u), \mu_{A_{k_j}(t)}(u)\right)$,

d) $\mu_{A_k(t_i\vee t_j)}(u) = \max\left(\mu_{A_k(t_i)}(u), \mu_{A_k(t_j)}(u)\right)$,

e) $\mu_{A_{k_i\vee k_j}(t_i\vee t_j)}(u) = \max\left(\mu_{A_{k_i}(t_i)}(u), \mu_{A_{k_i}(t_j)}(u), \mu_{A_{k_j}(t_i)}(u), \mu_{A_{k_j}(t_j)}(u)\right)$,

Intersection

a) $\mu_{(A\cap B)_k(t)}(u) = \min(\mu_{A_k(t)}(u), \mu_{B_k(t)}(u))$,

b) $\mu_{A_{k_i}(t_i)\cap B_{k_j}(t_j)}(u) = \min\left(\mu_{A_{k_i}(t_i)}(u), \mu_{B_{k_j}(t_j)}(u)\right)$,

c) $\mu_{A_{k_i\wedge k_j}(t)}(u) = \min\left(\mu_{A_{k_i}(t)}(u), \mu_{A_{k_j}(t)}(u)\right)$,

d) $\mu_{A_k(t_i\wedge t_j)}(u) = \min\left(\mu_{A_k(t_i)}(u), \mu_{A_k(t_j)}(u)\right)$,

e) $\mu_{A_{k_i\wedge k_j}(t_i\wedge t_j)}(u) = \min\left(\mu_{A_{k_i}(t_i)}(u), \mu_{A_{k_i}(t_j)}(u), \mu_{A_{k_j}(t_i)}(u), \mu_{A_{k_j}(t_j)}(u)\right)$,

Complementation

a) $\mu_{\neg A_k(t)}(u) = 1 - \mu_{A_k(t)}(u)$,

b) $\mu_{A_{\neg k_i}(t)}(u) = \begin{cases} \mu_{A_{k_j}(t)}(u), j \neq i, |K| = 2, \\ \Phi(\alpha_{k_1}, \cdots, \alpha_{k_{i-1}}, \alpha_{k_{i+1}}, \cdots, \alpha_{k_m}), |K| > 2, \text{ where } \Phi \text{ is an aggregate function.} \\ \alpha_{k_p} = \mu_{A_{k_p}(t)}(u), \end{cases}$

c) $\mu_{A_k(\neg t_i)}(u) = \begin{cases} \mu_{A_k(t_j)}(u), j \neq i, |T| = 2, \\ \Phi(\alpha_{t_1}, \cdots, \alpha_{t_{i-1}}, \alpha_{t_{i+1}}, \cdots, \alpha_{t_n}), |T| > 2, \text{ where } \Phi \text{ is an aggregate function.} \\ \alpha_{t_p} = \mu_{A_k(t_p)}(u), \end{cases}$

The basic operations as defined in Definition 2 provide some properties as follows.

- From *Equality*:
  $(A \equiv B) \Rightarrow \{(A \cong B), (A \triangleq B)\} \Rightarrow (A = B)$.
- From *Containment*:
  $(A \Subset B) \Rightarrow \{(A \sqsubseteq B), (A \preccurlyeq B)\} \Rightarrow (A \subseteq B)$.

3. Granularity of Knowledge

As discussed by Intan and Mukaidono [5]-[7] in proposing the concept of knowledge-based fuzzy sets, the granularity of knowledge was constructed to obtain the similarity classes of knowledge. All knowledge in a specific similarity class will consider having a similar perception subjectively toward a given fuzzy set. Through the similarity classes of knowledge, Intan *et al.* (8) discussed and introduced three necessary measures, namely Objectivity Measures, Individuality Measures and Consistency Measure in the knowledge-based dynamic fuzzy sets. Here, the similarity classes of knowledge are provided by a fuzzy conditional probability relation [5]-[7] which is an asymmetric relation as defined in Definition 3.

Definition 3 A *fuzzy conditional probability relation* is a mapping, $R: \mathcal{F}(U) \times \mathcal{F}(U) \to [0,1]$ such that for $X, Y \in \mathcal{F}(U)$,

$$R(X,Y) = \frac{\sum_{u \in U} \min(\mu_X(u), \mu_Y(u))}{\sum_{u \in U} \mu_Y(u)} \tag{9}$$

where $R(X,Y)$ means the degree Y supports X or the degree Y is similar to X or similarity degree of X given Y.

An interesting mathematical relation characterizes the concept of fuzzy conditional probability relation. This relation is called weak fuzzy similarity relationship and defined as follows.

Definition 4 A *weak fuzzy similarity relation* is a mapping, $S: \mathcal{F}(U) \times \mathcal{F}(U) \to [0,1]$, such that for $X, Y, Z \in \mathcal{F}(U)$,

1. Reflexivity: $S(X,X) = 1$
2. Conditional symmetry: if $S(X,Y) > 0$ then $S(Y,X) > 0$
3. Conditional transitivity:
   If $S(X,Y) \geq S(Y,X) > 0$ and $S(Y,Z) \geq S(Z,Y) > 0$ then
   $S(X,Z) \geq S(Z,X) > 0$

where U is an ordinary set of elements and $\mathcal{F}(U)$ is fuzzy power sets of U.
Furthermore, in the relation to (9), similarity degree of k_i given k_j concerning fuzzy set A in time t is given by the following equation.

$$R(A_{k_i}(t), A_{k_j}(t)) = \frac{\sum_{u \in U} \min(\mu_{A_{k_i(t)}}(u), \mu_{A_{k_j(t)}}(u))}{\sum_{u \in U} \mu_{A_{k_j(t)}}(u)} \tag{10}$$

It can be followed clearly that the degree of similarity between two knowledge satisfy the following properties.

r1. $\left[R(A_{k_i}(t), A_{k_j}(t)) = R(A_{k_j}(t), A_{k_i}(t)) = 1, \forall A \in \mathcal{F}(U), \forall t \in T\right] \Leftrightarrow k_i = k_j$

r2. $\left[R(A_{k_j}(t), A_{k_i}(t)) = 1, R(A_{k_i}(t), A_{k_j}(t)) < 1, \forall A \in \mathcal{F}(U), \forall t \in T\right] \Leftrightarrow k_i \trianglelefteq k_j$

r3. $\left[R(A_{k_j}(t), A_{k_i}(t)) = R(A_{k_i}(t), A_{k_j}(t)) > 0, \forall A \in \mathcal{F}(U), \forall t \in T\right] \Leftrightarrow k_i \sim k_j$

r4. $\left[R\left(A_{k_i}(t), A_{k_j}(t)\right) < R\left(A_{k_j}(t), A_{k_i}(t)\right), \forall A \in \mathcal{F}(U), \forall t \in T\right] \Leftrightarrow k_i \preccurlyeq k_j$

r5. $R\left(A_k(t), A_k(t)\right) = 1, \forall t \in T, \forall k \in K, \forall A \in \mathcal{F}(U)$

r6. $\left[R\left(A_{k_i}(t), A_{k_j}(t)\right) > 0, \forall A \in \mathcal{F}(U), \forall t \in T\right] \Leftrightarrow \left[R\left(A_{k_j}(t), A_{k_i}(t)\right) > 0\right]$

r7. $\left[R\left(A_{k_i}(t), A_{k_j}(t)\right) \geq R\left(A_{k_j}(t), A_{k_i}(t)\right),\ R\left(A_{k_j}(t), A_{k_m}(t)\right) \geq R\left(A_{k_m}(t), A_{k_j}(t)\right), \forall A \in \mathcal{F}(U),\ \forall t \in T\right]$
$\Rightarrow \left[R(A_{k_i}(t), A_{k_m}(t)) \geq R(A_{k_m}(t), A_{k_i}(t))\right]$

Property (r1) proves that both k_i and k_j are the same, and it is similar to *Equality* (j). (r2) shows that k_j covers k_i, or k_i contains in k_j. It means that in all the time, k_j gives a higher degree of membership for all element of all fuzzy sets than k_i, and it is the same as *Containment* (j). Property (r3) points to similar cardinality between k_i and k_j for all fuzzy sets in all the time. On the other hand, (r4) means the cardinality of all fuzzy sets and all the time is given by k_i is always less or equal to k_j. As related to the weak fuzzy similarity relation, (r5) is the property of reflexivity. (r6) is a conditional similarity, and (r7) is a conditional transitivity.

Using degree of similarity between two pieces of knowledge as calculated by (10), two asymmetric similarity classes of a given element of knowledge k.

Definition 5 Let K be a non-empty universal set of knowledge, and A be a fuzzy set on U**.** For any $k_i \in K$, $S_\alpha^A(k_i,t)$ and $P_\alpha^A(k_i,t)$ are defined as the set of knowledge that supports k_i and the set supported by k_i at time $t \in T$, respectively by:

$$S_\alpha^A(k_i,t) = \{k \in K | R(A_{k_i}(t), A_k(t)) > \alpha\} \tag{11}$$

$$P_\alpha^A(k_i,t) = \{k \in K | R(A_k(t), A_{k_i}(t)) > \alpha\} \tag{12}$$

where $\alpha \in [0,1]$.

$S_\alpha^A(k_i,t)$ can also be interpreted as the set of knowledge that is similar to k_i at time t with respect to fuzzy set A. On the other hand, $P_\alpha^A(k_i,t)$ can be considered as the set of knowledge to which k_i is similar at time t. In this case, $S_\alpha^A(k_i,t)$ and $P_\alpha^A(k_i,t)$ are regarded as two different semantic interpretations of similarity classes in providing the crisp granularity of knowledge.

For two asymmetric similarity classes of knowledge, $S_\alpha^A(k_i,t)$ and $S_\alpha^A(k_j,t)$, the complement, intersection and union are defined by:

$$\neg S_\alpha^A(k_i,t) = \{k \in K | k \notin S_\alpha^A(k_i,t)\} \tag{13}$$

$$S_\alpha^A(k_i,t) \cap S_\alpha^A(k_j,t) = \{k \in K | k \in S_\alpha^A(k_i,t) \; and \; k \in S_\alpha^A(k_j,t)\} \tag{14}$$

$$S_\alpha^A(k_i,t) \cup S_\alpha^A(k_j,t) = \{k \in K | k \in S_\alpha^A(k_i,t) \; or \; k \in S_\alpha^A(k_j,t)\} \tag{15}$$

Similarly, the complement, intersection and union might be defined on $P_\alpha^A(k_i,t)$ and $P_\alpha^A(k_j,t)$. Since the similarity classes of knowledge are crisp sets, they satisfy the Boolean Lattice. Based on these two asymmetric similarity classes, we then construct two dynamic crisps covering of the universal knowledge regarding fuzzy set A, $\Upsilon_A^\alpha(t) = \{P_\alpha^A(k,t) | k \in K\}$ and $\Psi_A^\alpha(t) = \{S_\alpha^A(k,t) | k \in K\}$, where $\alpha \in [0,1]$. Here the crisp, dynamic covering means that the crisp covering will be dynamically changed depending on time t.

By removing α, crisp similarity classes, $S_\alpha^A(k_i,t)$ and $P_A^\alpha(k_i,t)$ will be generalized to the fuzzy similarity classes, $S_{k_i}^A(t)$ and $P_{k_i}^A(t)$, respectively. Naturally, the fuzzy similarity classes of a specific knowledge k_i with respect to fuzzy set A at time t is given by the following equations.

$$S_{k_i}^A(t,k) = R(A_{k_i}(t), A_k(t)), \forall k \in K \tag{16}$$

$$P_{k_i}^A(t,k) = R(A_k(t), A_{k_i}(t)), \forall k \in K \tag{17}$$

Basic operations, such as the complement, intersection and union of the fuzzy similarity classes are defined by:

$$\neg S_{k_i}^A(t,k) = 1 - S_{k_i}^A(t,k), \forall k \in K \tag{18}$$

$$S_{k_i}^A(t,k) \wedge S_{k_j}^A(t,k) = \min(S_{k_i}^A(t,k), S_{k_j}^A(t,k)), \forall k \in K \tag{19}$$

$$S_{k_i}^A(t,k) \vee S_{k_j}^A(t,k) = \max(S_{k_i}^A(t,k), S_{k_j}^A(t,k)), \forall k \in K \quad (20)$$

Furthermore, two dynamic fuzzy coverings of the universal set of knowledge are constructed dealing with a fuzzy set A as defined by $\Theta^A(t) = \{P_k^A(t)|k \in K\}$ and $\Omega^A(t) = \{S_k^A(t)|k \in K\}$. Here, the fuzzy coverings of the universal set of knowledge are also dynamically changed based on the time t.

4. Approximate Reasoning

Related to the granularity of knowledge as discussed in Section 3, this paper introduces the concept of approximate reasoning dealing with KDFS. Approximate reasoning provides approximate solution using fuzzy production rules. Let fuzzy label of A be a given premise and fuzzy label of B be the conclusion. Through fuzzy production rules [9], relation between A and B will connect problem with solution, antecedent with consequence, or premise with conclusion, as usually applied in representing knowledge in fuzzy expert system. In general, fuzzy production rules have the form of *if-then* rule as follows:

$$\text{If } A, \text{then } B,$$

where A and B are fuzzy sets.

In constructing a fuzzy production rule, assume two persons represented by two knowledge, k_i and k_j have different conclusions at the time t given a certain premise. Related to the concept of knowledge-based dynamic fuzzy sets, conclusions of k_i and k_j are denoted by $B_{k_i}(t)$ and $B_{k_j}(t)$, respectively, in which $B_{k_i}(t) \neq B_{k_j}(t)$. The problem is how to determine which one has the right conclusion, k_i or k_j. Possibly, different views or understanding of premise perceived by k_i and k_j is the cause of different conclusions. Comparing perception of k_i and k_j regarding premise and conclusion may be summarized into four possibilities of relations:

1. Premise: $A_{k_i}(t) = A_{k_j}(t)$, Conclusion: $B_{k_i}(t) = B_{k_j}(t)$: There is no problem because both k_i and k_j have exactly the same perception of premise and conclusion.
2. Premise: $A_{k_i}(t) = A_{k_j}(t)$, Conclusion: $B_{k_i}(t) \neq B_{k_j}(t)$: Both k_i and k_j have the same perception of premise, but different perception of conclusions; That is the problem.
3. Premise: $A_{k_i}(t) \neq A_{k_j}(t)$, Conclusion: $B_{k_i}(t) = B_{k_j}(t)$: Since both k_i and k_j have different perception of the premise, even though they have the same conclusion, their conclusions should be treated independently.
4. Premise: $A_{k_i}(t) \neq A_{k_j}(t)$, Conclusion: $B_{k_i}(t) \neq B_{k_j}(t)$: Similar to point 3, their conclusions are independent so that their different conclusions can be understood and tolerated.

From all four possibilities, the problem is only in Point 2. Suppose there are only two knowledge, k_i and k_j, the situation as happened in Point 2 gives the same validation's degree to k_i and k_j. In probability measure, their validation's degree will be 0.5 each. For there are more than two knowledge, intuitively, validation's degree will depend on support of other knowledges. More supports should cause higher validation's degree. Therefore, validation's degree of a fuzzy production rule given by a certain knowledge can be approximately calculated using granularity of knowledge as proposed in the previous section, as follows.

Definition 6 Let K be a non-empty universe of knowledge, and $S_\alpha^A(k,t)$, $S_\alpha^B(k,t)$ be crisp granularity of knowledge of $k \in K$ at the time t dealing with fuzzy label A and fuzzy label B, respectively. $\delta_\alpha^t(A \overset{k}{\rightarrow} B)$ is defined as the validation's degree of a fuzzy production rule (if A then B) given by k at the time t as follows.

$$\delta_\alpha^t(A \overset{k}{\rightarrow} B) = \frac{|S_\alpha^A(k,t) \cap S_\alpha^B(k,t)|}{|S_\alpha^A(k,t)|} \tag{21}$$

where $\alpha \in [0,1]$ and $|\cdot|$ be a cardinality of set.

The set of knowledge, K, provides a family of values $\{\delta_\alpha^t(A \overset{k}{\rightarrow} B)\ |k \in K\}$. To summarize all degrees of correctness, three aggregate formulas will be defined as follows.

a) Minimum: $\delta_\alpha^t(A \overset{K}{\rightarrow} B)^m = \min\{\delta_\alpha^t(A \overset{k}{\rightarrow} B)\ |k \in K\}$ (22)

b) Maximum: $\delta_\alpha^t(A \overset{K}{\rightarrow} B)^M = \max\{\delta_\alpha^t(A \overset{k}{\rightarrow} B)\ |k \in K\}$ (23)

c) Average: $\delta_\alpha^t(A \overset{K}{\rightarrow} B)^* = \text{avg}\{\delta_\alpha^t(A \overset{k}{\rightarrow} B)\ |k \in K\}$ (24)

Some properties and summaries can be verified from (21) to (24) such as:

- if relation between premise A and conclusion B is totally valid at time t then $S_\alpha^A(k,t) \subseteq S_\alpha^B(k,t)$ for all $k \in K$; if relation between premise A and conclusion B is totally valid all the time then $S_\alpha^A(k,t) \subseteq S_\alpha^B(k,t), \forall k \in K, \forall t \in T$. Here, the similarity classes of knowledge dealing with fuzzy label A is finer than the similarity classes of knowledge dealing with fuzzy label B.
- $\forall t \in T,\ \delta_\alpha^t(A \overset{K}{\rightarrow} B)^m = 1 \Leftrightarrow \delta_\alpha^t(A \overset{K}{\rightarrow} B)^* = 1$;
- Similarly, $\forall t \in T,\ \delta_\alpha^t(A \overset{K}{\rightarrow} B)^m < 1 \Leftrightarrow \delta_\alpha^t(A \overset{K}{\rightarrow} B)^* < 1$;
- If $\forall t \in T,\ \delta_\alpha^t(A \overset{K}{\rightarrow} B)^* = 1$ then B is regarded as *permanent absolute conclusion* given premise A.
- If $\forall t \in W, W \subset T,\ \delta_\alpha^t(A \overset{K}{\rightarrow} B)^* = 1$ then B is regarded as *temporary absolute conclusion* given premise A during W.
- If $P \subset K, \forall t \in T, \delta_\alpha^t(A \overset{P}{\rightarrow} B)^* = 1$ then B is regarded as *permanent relative conclusion* given premise A according to some knowledge in P.
- If $P \subset K, \forall t \in W, W \subset T, \delta_\alpha^t(A \overset{P}{\rightarrow} B)^* = 1$ then B is regarded as *temporary relative conclusion* given premise A during W according to some knowledge in P.
- If $\forall t \in T,\ \delta_\alpha^t(A \overset{K}{\rightarrow} B)^* < 1$ then B is regarded as *permanent partial conclusion* with the validation's degree equals to $\delta_\alpha^t(A \overset{K}{\rightarrow} B)^*$ given premise A.

Validation's degree as define in Definition 6 may also be reformulated and generalized dealing with fuzzy granularity of knowledge as follows.

$$\delta^t(A \overset{k_i}{\rightarrow} B) = \frac{\sum_{k \in K} \min\left(S_{k_i}^A(t,k), S_{k_i}^B(t,k)\right)}{\sum_{k \in K} S_{k_i}^A(t,k)} \tag{25}$$

where intersection is defined as minimum and cardinality is given by sum of membership degree.

Similarly, the set of knowledge, K, provides a family of values $\{\delta^t(A \overset{k}{\rightarrow} B) \mid k \in K\}$. Three aggregate formulas to summarize all validation's degrees will be defined as follows.

a) Minimum: $\delta^t(A \overset{K}{\rightarrow} B)^m = \min\{\delta^t(A \overset{k}{\rightarrow} B) \mid k \in K\}$

b) Maximum: $\delta^t(A \overset{K}{\rightarrow} B)^M = \max\{\delta^t(A \overset{k}{\rightarrow} B) \mid k \in K\}$

c) Average: $\delta^t(A \overset{K}{\rightarrow} B)^* = \text{avg}\{\delta^t(A \overset{k}{\rightarrow} B) \mid k \in K\}$

In the real-world application, it is well known that A and B have a causal relationship. However, it is still unclear to determine which one is the premise, and which one is the conclusion. For example, let $K = \{k_1, k_2, k_3\}$ be set of knowledge. Interpretation or perception of fuzzy labels A and B according to K at the time t is arbitrarily given in several fuzzy sets as shown in Fig. 1.

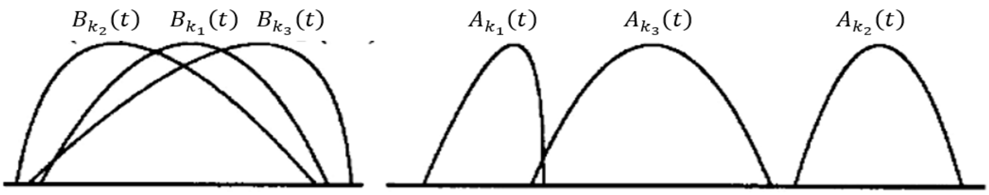

Fig. 1. Fuzzy sets A and B given by let $K = \{k_1, k_2, k_3\}$.

It is clearly shown in Fig. 1 that all elements of K have very similar interpretation of B, but they have enough different interpretation of A. If B is considered as premise and A as conclusion, problem in Point 2 will happen. The very similar interpretations of B as premise should also have similar interpretations of A as conclusion. However, the interpretations of A are very different. The problem is which interpretation of A should be used as conclusion. On the other hand, if A is considered as premise, and B is used as conclusion, no matter in the beginning A as premise has different interpretations, finally it will have the same conclusion (in α level set) of B. Therefore, related to the example in Figure 1, A should be used as premise and, B should be used as conclusion. Related to the concept of fuzzy granularity that have been discussed before, in the causal relationship between A and B, we can determine which one should be a premise, and which one should be the conclusion. Here, similarity classes of knowledge dealing with premise should be finer than similarity classes of knowledge dealing with conclusion. Several categories of fuzzy production rules, $if\ A\ then\ B$, representing A as premise and B as conclusion in element of knowledge $k \in K$ at the time t may be defined by:

a) $A \overset{k(t)}{\longrightarrow} B \Leftrightarrow \delta_\alpha^t(B \overset{k}{\rightarrow} A) < \delta_\alpha^t(A \overset{k}{\rightarrow} B) = 1$, (strong implication)

b) $A \overset{k(t)}{\sim\!\!\longrightarrow} B \Leftrightarrow \delta_\alpha^t(B \overset{k}{\rightarrow} A) < \delta_\alpha^t(A \overset{k}{\rightarrow} B) < 1$, (weak implication)

c) $A \overset{k(t)}{\longleftrightarrow} B \Leftrightarrow \delta_\alpha^t(B \overset{k}{\rightarrow} A) = \delta_\alpha^t(A \overset{k}{\rightarrow} B) = 1$, (strong bi-implication)

d) $A \overset{k(t)}{\sim\!\!\longleftrightarrow} B \Leftrightarrow \delta_\alpha^t(B \overset{k}{\rightarrow} A) = \delta_\alpha^t(A \overset{k}{\rightarrow} B) < 1$, (weak bi-implication)

Here, $\delta_\alpha^t(B \overset{k}{\rightarrow} A)$ and $\delta_\alpha^t(A \overset{k}{\rightarrow} B)$ can be generalized and changed to $\delta^t(B \overset{k}{\rightarrow} A)$ and $\delta^t(A \overset{k}{\rightarrow} B)$, respectively. it is also necessary to consider some subsets of K that is related to the categories of fuzzy production rules as follows.

a) $\mathcal{K}\left(A \overset{k(t)}{\longrightarrow} B\right) = \{k \in K \mid A \overset{k(t)}{\longrightarrow} B\}$, (subset of K which provide strong implication)

b) $\mathcal{K}\left(A \sim\xrightarrow{k(t)} B\right) = \{k \in K | A \sim\xrightarrow{k(t)} B\}$, (subset of K which provide weak implication)

c) $\mathcal{K}\left(A \xleftrightarrow{k(t)} B\right) = \{k \in K | A \xleftrightarrow{k(t)} B\}$, (subset of K which provide strong bi-implication)

d) $\mathcal{K}\left(A \sim\xleftrightarrow{k(t)} B\right) = \{k \in K | A \sim\xleftrightarrow{k(t)} B\}$, (subset of K which provide weak bi-implication)

where they are satisfied the following equation:

$$\mathcal{K}\left(A \xrightarrow{k(t)} B\right) \cup \mathcal{K}\left(A \sim\xrightarrow{k(t)} B\right) \cup \mathcal{K}\left(A \xleftrightarrow{k(t)} B\right) \cup \mathcal{K}\left(A \sim\xleftrightarrow{k(t)} B\right) \cup \mathcal{K}\left(B \xrightarrow{k(t)} A\right) \cup \mathcal{K}\left(B \sim\xrightarrow{k(t)} A\right) = K,$$

where $\mathcal{K}\left(A \xrightarrow{k(t)} B\right), \mathcal{K}\left(A \sim\xrightarrow{k(t)} B\right), \mathcal{K}\left(A \xleftrightarrow{k(t)} B\right), \mathcal{K}\left(A \sim\xleftrightarrow{k(t)} B\right), \mathcal{K}\left(B \xrightarrow{k(t)} A\right), \mathcal{K}\left(B \sim\xrightarrow{k(t)} A\right)$ are disjoint subsets in K. In order to measure validation's degree of a fuzzy production rules, A implies B $(A \xrightarrow{t} B)$, we propose the following equations.

$$\mathcal{C}\left(A \xrightarrow{t} B\right) = \frac{\left|\mathcal{K}\left(A \xrightarrow{k(t)} B\right)\right| + 0.75 \times \left|\mathcal{K}\left(A \sim\xrightarrow{k(t)} B\right)\right| + 0.5 \times \left|\mathcal{K}\left(A \sim\xleftrightarrow{k(t)} B\right)\right| + 0.25 \times \left|\mathcal{K}\left(B \sim\xrightarrow{k(t)} A\right)\right|}{|K|} \quad (26)$$

Similarly,

$$\mathcal{C}\left(B \xrightarrow{t} A\right) = \frac{\left|\mathcal{K}\left(B \xrightarrow{k(t)} A\right)\right| + 0.75 \times \left|\mathcal{K}\left(B \sim\xrightarrow{k(t)} A\right)\right| + 0.5 \times \left|\mathcal{K}\left(B \sim\xleftrightarrow{k(t)} A\right)\right| + 0.25 \times \left|\mathcal{K}\left(A \sim\xrightarrow{k(t)} B\right)\right|}{|K|} \quad (27)$$

where $\mathcal{C}\left(A \xrightarrow{t} B\right) \in [0,1]$ and $\mathcal{C}\left(B \xrightarrow{t} A\right) \in [0,1]$ are defined as validation's degree of $A \xrightarrow{t} B$ and $B \xrightarrow{t} A$, respectively. The cardinality of $\mathcal{K}(A \xleftrightarrow{k(t)} B)$ is not included in Equation (26) and (27) in order to treat the strong bi-implication, $A \xleftrightarrow{t} B$, as a special condition. Coefficients of cardinality of sets are simply given with intervals of 0.25 because there are four sets of fuzzy production rules that involve in the calculation. In general, the fuzzy production rules might be defined at the time t if they satisfy the following rules of conditions.

(Rule 1) $A \xrightarrow{t} B \Leftrightarrow \mathcal{C}\left(A \xrightarrow{t} B\right) = 1,$

(Rule 2) $A \xleftrightarrow{t} B \Leftrightarrow \mathcal{C}\left(A \xrightarrow{t} B\right) = \mathcal{C}\left(B \xrightarrow{t} A\right) = 0,$

(Rule 3) $A \sim\xrightarrow{k(t)} B \Leftrightarrow \mathcal{C}\left(B \xrightarrow{t} A\right) < \mathcal{C}\left(A \xrightarrow{t} B\right) < 1,$

(Rule 4) $A \sim\xleftrightarrow{t} B \Leftrightarrow \mathcal{C}\left(A \xrightarrow{t} B\right) = \mathcal{C}\left(B \xrightarrow{t} A\right) > 0.$

5. Conclusion

Knowledge-based Dynamic Fuzzy Sets (KDFS) is a hybrid concept of *the Knowledge-based Fuzzy Sets* and *the Dynamic Fuzzy Sets*. The KDFS shows that a membership function of a given fuzzy set subjectively determined by a certain knowledge may be dynamically changeable over time. This paper discussed how the concept of KDFS applied in Approximate Reasoning. In this case, this paper proposed several concept and method how to generate fuzzy production rules dealing with the KDFS. The proposed concept played important role in constructing fuzzy rule base in Fuzzy Expert System, since the fuzzy production rule store in the fuzzy rule base of fuzzy expert system is generally provided by the knowledge of experts. Four categories of the fuzzy production rules, namely, *Strong Implication*, *Weak Implication*, *Strong Bi-implication* and *Weak Bi-implication*, were introduced and discussed together with their properties. Several equations were proposed to measure validation's degree of the fuzzy production rules. Finally, four rules of conditions were given to justify the construction of *Strong Implication*, *Weak Implication*, *Strong Bi-implication* and *Weak Bi-implication*. Our future work is to apply the proposed concept in the real-world application.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

This paper is a partial result of research conducted by all authors in a research group leading by the first author. The first author wrote the paper, and all authors had approved the final version.

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Rolly Intan received his B.Sc. in computer engineering from Sepuluh Nopember Institute of Technology, Surabaya Indonesia in 1991, and his M.A.Sc. in computer science from International Christian University, Tokyo, Japan in 2000. His doctor of engineering in computer science and engineering was obtained from Meiji University, Tokyo, Japan in 2003. He is currently a professor in informatics engineering at Petra Christian University, Surabaya, Indonesia. His primary research interests are fuzzy logic and systems, rough sets, granular computing and data mining.

Siana Halim got her Ph.D. in statistics from TU Kaiserslautern Germany. She is now an associate professor in Industrial Engineering Department, Petra Christian University, Surabaya, Indonesia. Her research interest areas are data mining, statistical modeling, machine learning and dynamic fuzzy time series

Lily Puspa Dewi obtained her master's degree from Sepuluh Nopember Institute of Technology, Surabaya, Indonesia. Her current field placement is a lecturer in Informatics Department, Petra Christian University, Surabaya, Indonesia. She is interested with decision support system, information system and software engineering.