Introduction to the papers of TWG19: Mathematics Teaching and Teacher Practice(s)

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Since its birth in the aftermath of CERME10, discussions in TWG19 concentrated on identifying its research territory and on creating opportunities for collaboration among the participants beyond the conference sessions. These discussions led to shifting the focus of the group on mathematics teaching and teacher practice (s) and to an initiative of working on shared data that fueled fruitful explorations related to conceptualizations and theorization of mathematics teaching. The participants' contributions and the work carried out during CERME11 sessions challenged further the core ideas of how mathematics teaching can be defined and studied while an emerging distinction between teaching as an activity and teaching as work stimulated further the group's discussions.

Introduction

Thematic Working Group 19 (TWG19) emerged as a result of the splitting of a TWG called "From a study of teaching practices to issues in teacher education" into three sub-groups after CERME8. One of the new groups targeted teacher education and professional development (TWG18), another focused on teacher knowledge, beliefs and identity (TWG20), and TWG19 targeted "Mathematics teacher and classroom practices". Since its conception, TWG19 has been in a process of developing its identity. In discussions, core concepts have been taken for granted, without explicit or shared definitions. This became apparent at CERME10, where participants called for a development of common grounds — or at least for efforts to increase awareness about differences in conceptualizations, theories and methods, and of the implications of these differences. After CERME10, two changes have been made to move forward TWG19's work: 1) the name of the working group has been adjusted, and 2) an initiative of offering sets of shared data for analysis and use in the working group has been initiated.

This introductory paper discusses the ongoing development of TWG19's identity and where the continued development appears to be heading. Due to a large number of submitted papers, the group split into two (TWG19a and TWG19b) for CERME11, but we tried to maintain a sense of community by having three common sessions, and by discussing the same overarching questions in split sessions. In the final session, team leaders shared reports to the whole group and facilitated a

concerted discussion around these questions. This joint introduction to TWG19a and TWG19b testifies to these efforts. There are three main sections in this introduction. The first section discusses emerging issues and critical considerations as reflected in the section name. The second section reports from the joint activity around the initiative of analyzing and using shared data, and the third section discusses patterns and trends in the individual papers that were presented. The introductory paper concludes with a discussion of the collective efforts made to deal with the emerging issues and some reflections on the way ahead.

Emerging issues to pursue: some critical considerations

After CERME10, the name of TWG19 changed from "Mathematics teacher and classroom practices" to "Mathematics teaching and teacher practice(s)". The change of wording is intentional and deserves elaboration. First, the change from teacher to teaching signals that the group targets teaching rather than teachers and their characteristics; teacher knowledge, beliefs and identity are the focus of TWG20. This shift reflects a development that Skott, Mosvold and Sakonidis (2018) described and reflected upon in their recollection of research on classroom practice, knowledge, beliefs and identity over ten biannual CERME conferences and corresponds with a general shift of focus in research on teaching. Although the shift from teachers to teaching has emerged over several decades, conceptualizations of teaching appear to be underdeveloped in our field. While discussing the problem of theorizing teaching, Jaworski (2006, p. 188) suggests that "the big theories do not seem to offer clear insights to teaching and ways in which teaching addresses the promotion of mathematics learning". Furthermore, Jaworski suggests that one of the problems lies in how learning and teaching are connected with practice, and this relates to a second change in the name of TWG19.

The shift from classroom practices to teacher practice(s) signals that the primary focus of the research in TWG19 is on practices of teachers and teaching rather than on any practices in classrooms. Use of the term practice has changed over the years. Jaworski (2006) discussed "teaching as learning in practice", whereas Schön and DeSanctis (2011) consider teaching to be a reflective practice. The former suggests conceiving teaching as a social practice in which teachers are practitioners, whereas the latter offers a view of how teachers "act in the classroom as informed, concerned professionals and about how they continue to learn" (Lerman, 1998, p. 33). The discussions by these authors imply some of the different meanings that might be attached to the term practice. Lampert (2010) distinguishes between four different meanings of the word practice, and three of those are relevant for TWG19. First, practice might refer to what teachers do in contrast to theory. Second, the word is used to describe a collection of practices, some of which might be more core or high-leverage than others. A third use is to consider teaching as a professional practice, like the practice of law or medicine. The word practice in the new name of TWG19 might encompass all three meanings.

The first joint session of TWG19a and TWG19b at CERME11 was devoted to discussing some of these foundational issues, and attempts were made to contribute to the discussion of the underdeveloped conceptualizations and theorization of mathematics teaching. To introduce the discussion, two of the presented papers drew on different theoretical foundations. In the first

presentation, Watson drew on developments in cognitive psychology, neuroscience and ontology to provide a theoretical account of teacher decision making and how this influences practice. He argued that the momentary decisions influence the character of the lesson and proceeds to focusing on algorithmic reasoning, a conscious process making use of heuristics and pre-established routines and processes to reduce the demand on working memory. In mathematics teaching, in-the-moment decisions have to be immediate and thus well-practiced. The algorithmic mind quickly accesses routines and procedures that are learnt through participation and provide well-rehearsed and culturally-embedded approaches to respond to situations in the classroom. Watson concluded that it is important to further study teachers' decision making, to better understand the affective dimensions, the culturally embedded routines and strategies acquired through participation in the professional community. In the second paper, Sakonidis argued that mathematics teaching tends to be seen today as a professional practice shaped by the expectations and norms of the learning settings where is exercised. Sociocultural theories provide useful lenses for the relevant research and three of them appear to readily lend themselves to this direction: the theory of learning participating in a community of practice, Cultural-historical activity theory, and Skott's (2013) patterns of participation theory. Sakonidis concluded by highlighting the need for dynamic perspectives to disentangle and understand the ever-evolving outcome of individual and communal acts of meaning-making (by both teachers and students) characterizing the practice of mathematics teaching.

Following the presentation of these two papers, Hoover and Mosvold presented their reflections on the developments in the field of research on mathematics teaching. They started by reflecting on the different meanings of teaching, and how the field has proposed dichotomies like pedagogic versus didactic, art versus science, ambitious versus conventional, and profession versus occupation. Based on these initial reflections, they proposed a distinction between teaching as *activity* and teaching as *work*. The first interprets teaching as activities done by teachers, whereas the second considers teaching to be work to be done. Each of these understandings has different implications for studying teaching. Studies of teaching as activity tend to seek empirical descriptions of what teachers are doing, and they study methods of expert teachers in order to improve teaching. On the other hand, studies of teaching as work tend to investigate the demands of instruction by applying conceptual analysis, and they conceptualize professional practice for teacher education as a means to improve teaching. This distinction was taken up by participants in the group and stimulated productive discussions.

In search of answers to the emerging issues in a joint research activity

As another effort to move the group forward, TWG19 decided to prepare sets of shared data for CERME11. These shared data sets were made available for analysis and use in participants' papers, and they were also available for use in presentations and as a common reference point for discussions in the group. Four sets of data were prepared and uploaded to a secure server that all participants in TWG19 were given access to. Three of the data sets contained videos of mathematics teaching, whereas one data set contained transcripts only. Each data set was accompanied by a document that laid out policies for sharing and reuse, and a document with information about the person or institution that shared the data as well as with permissions and limitations for usage.

Videos are extensively used in research on teaching. In a presentation to the group about sharing data, Mosvold, Hoover and Suzuka suggested that study of a complex phenomenon like teaching requires good data, and that video data provides access to nuances in communication and interaction. Videos also provide more detail about context. While adding value, video data also introduce some challenges — including practical as well as ethical concerns. Sharing and reusing video data provide benefits like better utilization of data, access to high-quality data, saved costs, as well as increased quality and scope of analysis. However, successful sharing and reuse of video data demand careful considerations around issues of usability and reuse of data. Many find the challenges entailed in sharing and reusing data so severe that they are reluctant to get involved. Engaging in this initiative has provided us with useful experiences that we hope might help us as a field move forward towards overcoming the anticipated difficulties.

Having multiple eyes look at the same data opens up for richer and more nuanced interpretations. For instance, five papers analyzed a Norwegian video where a teacher facilitates a group of students' presentations of their solutions to a problem of figuring out when the king of Norway was born, given that he celebrated his 80^{th} birthday on that particular day. Some participants focused on how the teacher was moderating the dialogue, how he elicited students' mathematical thinking (Drageset), and how he positioned students as mathematical authorities whose thinking were put on public display and collectively analyzed (Bass & Mosvold). Others emphasized how the teacher's use of revoicing might interrupt and even prevent further development of the discourse (Kleve, Solem, & Aanestad), and Nic Mhuiri followed up by observing that student interactions were always funneled through the teacher and never developed into discussions in whole class. Whereas most papers applied particular frameworks or theories in their analysis of this video, Hoover and Goffney used conceptual analysis to identify aspects of mathematics teaching that might disrupt patterns of inequity.

Another frequently analyzed video displays Deborah Ball teaching fractions on the number line. From his analysis of turns in the conversation, Drageset noticed how the teacher often requested students to ask questions instead of entering a traditional pattern of initiation-response-evaluation. He also discussed how the teacher moderated the discourse while simultaneously guiding students' participation and attending to norms. Mosvold and Bjuland also mentioned attending to norms of participation in their use of the data set to unpack the role of positioning in the work of teaching mathematics. A couple of papers also observed how assigning students with agency and authority becomes visible in this instance of mathematics teaching (Bass & Mosvold; Nic Mhuiri). Again, the nuances provided by the sum of the analyses exceeds the contributions of each individual paper.

Engaging in analysis of shared data prompts some new questions: Why do we see different things when analyzing the same data? Can we reach agreement of some kind in our analyses? Is agreement even a goal? After all, our efforts to analyze and discuss these shared data sets did not lead to a shared understanding of mathematics teaching, but they stimulated productive discussions about different conceptualizations of teaching.

In search of answers to the emerging issues in individual participants' research

The papers reporting on studies of mathematics teaching and teacher practices that did not use the shared data sets can be placed in three groups: 1) teachers' practices, actions and resources before or during daily instruction; 2) teachers' implementation of teaching practices and actions developed in the context or in the aftermath of a particular purposeful professional development (PD) activity; 3) developing tools or practices of monitoring classroom instructional activity. Emerging issues from these groups of papers are discussed below.

Papers reporting on issues related to daily teaching practices, actions and resources apply established theoretical or conceptual frameworks to examine their functionality and effectiveness. For instance, Pericleous used cultural-historical activity theory to investigate how the activity of proving was constituted in a primary classroom. She analyzed tensions that arose for the teacher and identified points of contradiction between the classroom micro-system and elements of the macro-system, such as curriculum context. In a series of focus group interviews, Grundén used critical discourse analysis to interrogate teachers' planning practice(s) in mathematics. The results described how teachers construct their own discourse of mathematics teaching in resistance to the official discourse of the National Agency of Education. Grundén contended that critical discourse analysis is useful for attending to the complexity of mathematics teaching. Rudsberg, Sundhäll and Nilsson inquired into teachers' monitoring of students' meaning making. To this purpose, they adopted a framework of epistemological move analysis and a pragmatic perspective on learning to analyze lesson transcripts. The results suggested that epistemological move analysis might enhance our understanding of the relationship between students' meaning-making and the related actions of teachers.

Stouraitis and Potari studied a prospective secondary school teacher's first field experiences. Using activity theory, the authors focused on emerging contradictions considering prospective teachers' planning and enactment of a lesson plan, as well as what led to such contradictions and how contradictions could potentially influence prospective teacher learning. Calor et al. examined teachers' use of a model based on small-group work, drawing on socio-cultural notions such as collaborative learning and scaffolding as well as on cognitive ideas like Janvier's approach to representation systems. The analysis of a pre- and post-test on mathematical level raising shows that more mathematical level raising occurred in the small-group than in the control condition. Olsson and Teledahl described an investigation into how principles of feedback might be used by a mathematics teacher to encourage students' creative reasoning. Their analysis illustrates the challenges that this presents for teachers and highlights associated issues such as the existing classroom norms for interaction, as well as teachers' beliefs about the object of teaching.

Andrews explored what happens mathematically when teachers and learners engage in between-desk-teaching across different mathematics topics. From a multiple case study, he focused on the case of one teacher who used between-desk-teaching practice with a strategic purpose that could potentially support student understanding. Taylan and Esmer used the instructional actions framework to examine a novice teacher's instructional actions in response to unexpected classroom moments. Analyzing lesson planning meetings and observations as well as teacher interviews, they

found out that responses to unexpected moments include both supporting and extending actions. Kayali and Biza studied a teacher's use of the available resources in relation to their teaching aim. Based on the 'Knowledge Quartet' (Rowland, 2010), their analysis highlights different aspects of a teacher's classroom work and leads to suggesting replacing code 'instructional materials' to 'resources' and expanding the Knowledge Quartet by adding a code named 'connection among resources'.

The submissions related to teachers' implementation of instructional practices and tools developed in professional development contexts aim at providing rich mathematical classroom activity for all students. In particular, Büscher focused on teachers' designed tasks during a professional development course to reconstruct their categories of differentiation for percentage problems. The results show that teachers tended to differentiate in ways that exclude low-achieving students from conceptually rich learning opportunities due to their partitioning of their envisioned ideal-typical solution paths. Psycharis, Potari, Triantafillou and Zachariades investigated how secondary school teachers attempted to balance mathematical challenge and differentiation in whole class settings in the context of a professional development program which aimed to support teachers in engaging all learners while teaching challenging tasks.

Klothou, Sakonidis and Arsenidou used a multiple case study approach to investigate the use of learning trajectories in teaching fractions by three primary teachers participating in a PD project. The analysis shows that all teachers tended to view learning trajectories primarily as a way of organizing mathematical content more than as an elaboration of the possible development of student thinking. The study by Medová, Bulková, and Čeretková was conducted in the context of a professional development program focusing on inquiry-based learning in upper secondary mathematics. Results indicate that students' independent learning was observable more often in the inquiry-based lessons than in regular lessons, whereas teachers played a more dominant role in the regular lessons. Maugesten explored Norwegian lower primary teachers' views about good mathematics teaching by using a focus group interview with teachers after a two-year school-based professional development. The results reveal that teachers focused on communication in classroom and use of representations, student thinking in and about the subject of mathematics, and resources, textbooks, and tasks that are related to everyday mathematics.

Finally, the papers related to developing tools or practices of monitoring classroom instructional activity concentrated on cognitive as well as social determinants of this activity. Thus, Nowińska reported on the development of a rating system for analyzing metacognitive activities: Metacognitive-discourse instructional quality. This is illustrated with examples of how one can identify different parts of metacognition related to planning, monitoring, and reflection. She emphasized that such a rating system needs to identify patterns that are stable across lessons and practice. Arnesen and Grimeland also presented an investigation of teachers' planning. They asked teachers to identify the mathematical ideas they would focus on and use Bloom's Taxonomy to classify the learning goals teachers had identified for lessons. Findings indicate that higher-order cognitive process categories were poorly represented in the learning goals and teachers' descriptions of mathematical ideas were vague or absent.

The study by Mellroth and Boesen provides a proposal for teachers to help them notice mathematically highly able students by illuminating aspects of students' problem-solving processes. The paper of Ableitinger, Anger and Dorner is unique in its efforts to attend to student voice. Student feedback was collected in post-lesson interviews and quantitative and qualitative methods were used to compare students' and researchers' choice of significant events in mathematics lessons. The analysis indicates differences between students' and researchers' choices of significant events. Even when both groups identified the same event as significant, different reasons were given for this choice.

Summarizing, all papers presented above highlight challenges of accessing interesting data in consistent ways within or across instructional settings. The adopted theoretical frameworks view teachers as professionals coordinating the instructional activity aiming at rich learning environments and maximum participation. These frameworks offer analytical tools and sometimes provide — combined or coordinated — new theoretical lenses allowing useful insights into various aspects of mathematics teaching and teacher practice(s). The accompanying methodologies are mostly qualitative. Although these methodologies vary, they tend to concentrate on examining instructional routines, actions, tools and practice(s) by analyzing teachers' designs, classroom implementations or classroom excerpts and so on.

Towards a collective effort to deal with the emerging issues

Discussions in TWG19 at CERME11 often revolved around the core questions of what we mean by teaching and how we study it, and the distinction between teaching as activity and teaching as work was discussed frequently. The majority of participants in TWG19 tend to consider teaching as what teachers do, but there is considerable variation in this view of teaching as activity and how it is studied. Some consider teaching as what teachers do, but they also include teachers' thinking and emotions (e.g., Watson). Others focus more squarely on teacher actions and study patterns of such actions (e.g., Sakonidis). Yet others emphasize that teaching is a communicational activity (e.g., Nachlieli et al.). Among the papers viewing teaching as activity, many were concerned with the connection between teaching and learning (e.g., Ayan; Grundén, Serrazina et al.). Some consider the teacher as an agent who shapes new classroom practices (Nic Mhuiri). Others consider the teacher as a facilitator or an orchestrator (Baldry), who provides students with opportunities to explore (e.g., Pericleous). Yet others consider the teacher as a transmitter of knowledge (Ableitinger et al.). Teachers' use of feedback is supposed to influence student learning, and Olsson and Teledahl investigated development of productive feedback practices.

Other papers lean towards a conceptualization of teaching as work to be done. A couple of papers are explicitly directed towards such a conceptualization of mathematics teaching (e.g., Hoover & Goffney; Mosvold & Bjuland), whereas others point in this direction implicitly. For instance, Nachlieli et al. studied tasks that teachers are faced with, and Santos et al. investigated demands of teaching mathematics. It is also worth noticing that some papers appear to study what teachers do, but they still make claims about the work to be done (e.g., Baldry).

Having the distinction between teaching as activity or work out there seemed to fuel the discussions. People started engaging with questions about the purpose of distinguishing between

teaching as activity and teaching as work, and how the conceptualization of one might be independent of the other. The discussion of papers often targeted foundational questions about implications of our views on teaching, of applying different theoretical frameworks to analyze teaching, and about how different methods and types of data might or might not inform us about teaching and the demands of teaching. The effort of introducing and using shared data was another initiative that seemed to stimulate productive discussions in the group. Some reflected on how seeing different things in the same data must prompt careful reflection about use of different theoretical frameworks, whereas others suggested that seeing different things might be fruitful and provide a broader and more complete image of teaching.

Conclusion

TWG19 is still developing its identity, but the group is taking some important steps forward. The name change stimulated some important foundational discussions about conceptualizations of teaching and methods of studying teaching. The distinction between teaching as activity and teaching as work was useful, but further conceptual work is needed. The participants' contributions, despite the polyphony identified in their attempts to conceptualize mathematics teaching as well as in the theoretical lenses and the methodologies employed to study its classroom manifestations, somehow highlight the issues to consider and the questions to pursue in the way ahead.

The initiative of sharing data was also interesting, and several participants were interested in contributing to and using such data sets in upcoming conferences. Sharing and reusing data provide opportunities for richer analysis, but it also appears to be a productive space for developing shared understandings and possibly also better consensus about the object of study. However, further efforts are needed to investigate what kinds of (video) data are most suitable for joint analysis of mathematics teaching, how to organize an even more productive space for such joint analysis, and how such analysis might be scaled up. It is also imperative to investigate ways of organizing analysis of shared data in order for it to be more than an interesting exercise in data analysis.

References

Jaworski, B. (2006). Theory and practice in mathematics teaching development: critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education, 9*, 187–211.

Lampert, M. (2010). Learning teaching in, from, and for practice: What do we mean? *Journal of Teacher Education, 61*(1-2), 21–34.

Lerman, S. (1998). The intension/intention of teaching mathematics. In C. Kanes (Ed.), *Proceedings of MERGA* (Vol. 1, pp. 29-44). Griffith University: MERGA Inc.

Rowland, T. (2010). Foundation knowledge for teaching: Contrasting elementary and secondary mathematics. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of the VI Congress of the European Society for Research in Mathematics Education* (CERME 6) (pp. 1841–1850). Lyon: Institut National de Recherche Pedagogique.

Schön, D. & DeSanctis, V. (2011). The Reflective Practitioner: How Professionals Think in Action. *The Journal of Continuing Higher Education, 34*(3), 29–30.

Skott, J. (2013). Understanding the role of the teacher in emerging classroom practices: searching for patterns of participation. *ZDM - the International Journal on Mathematics Education, 45*(4), 547–559.

Skott, J., Mosvold, R., & Sakonidis, C. (2018). Classroom practice and teachers' knowledge, beliefs, and identity. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger & K. Ruthven (Eds.), *Developing research in mathematics education: Twenty years of communication, cooperation and collaboration in Europe* (pp. 162–180). Oxon, UK: Routledge.