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Taper and Volume Systems Based on Ratio Equations for *Pinus pseudostrobus* Lindl. in Mexico

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Abstract: Studies on functional relationships between relative stem diameter and height to estimate timber yield are useful in the management of commercial forest plantations. With taper analysis data of 42 *Pinus pseudostrobus* Lindl. trees in the indigenous community of Nuevo San Juan Parangaricutiro , Michoacan, Mexico, six compatible systems for predicting taper (d), merchantable volume (Vm), stem volume (Vs), total tree volume (Vt) and branch volume (Vb) were fitted and evaluated. The compatible taper and merchantable volume equations were based on volume ratio equations. Three Vs equations were tested in each system. In general, the compatible systems presented statistical accuracy in the d, Vm, Vs and Vt components but were less accurate in Vb. Three compatible systems were selected, according to their more efficient goodness-of-fit statistics, and a different total volume equation was incorporated into each system. The compatible systems based on volume ratio equations are simple, reliable tools for estimation of stand timber stocks and product classification of *P. pseudostrobus* in commercial forest plantations in Mexico.

Keywords: taper volume; compatible equation; stem volume; branch volume; total tree volume

1. Introduction

With the growing demand for wood, accurate estimation and prediction of volume is an essential activity of forest managers to determine economic yield and, therefore, select species and silvicultural treatments to be applied in a specific condition for a given period of forest rotation [1,2].

Total stem volume equations are reliable tools in quantifying timber stocks. However, merchantable volume should also be included to consider the products that have an influence in economic yield [2]. Measurements of tree stem diameter and height can be used to estimate total and merchantable volume, usually using taper and volume ratio equations [3,4]. Taper equations are useful, flexible tools for estimating total and merchantable volumes in both growth models and forest inventories [5]. When they are integrated, they can provide estimations of volume at the upper height limit as well as at the total tree height.

The concept of volume ratio functions was introduced by Honer [6] and developed by Burkhart [7]. It is a system of equations integrated by total volume and volume ratio functions [8]. The volume ratio equations predict merchantable volume as a proportion of total volume using a relative diameter or height function [1].

In previous studies, taper equations compatible with volume ratio equations were derived to a given diameter [9]. Recently, an approach was developed to derive volume equations from volume cumulative relative profiles. This methodology enables the description of changes in the ratio of cumulative volume to total stem volume over relative height [10]. Hence, compatible taper equations were developed; they were derived from volume ratio functions based on relative tree height [11]. The compatible taper equations derived from ratio functions based on relative diameters have been used for decades, but those based on relative height have been infrequently used [10,11].

Volume estimation is the main objective in forest inventories, but currently interest in quantifying volume by tree components has increased. This would allow better understanding of the relationships and exchanges among different types of forest ecosystem services, such as carbon sequestration against timber production [12,13]. The equations of d, Vm, Vs, Vt and Vb can be estimated simultaneously with compatible systems based on volume ratio equations.

In the indigenous community of Nuevo San Juan Parangaricutiro (CINSP), Michoacan, Mexico, commercial forest plantations of *Pinus pseudostrobus* Lindl. are a viable choice for managing production of short rotation forests and this permits productive diversification by reconverting agricultural or livestock land into forest use [14]. However, for the particular conditions of the study area, information on growth and yield of the plantations does not consider use of compatible taper and merchantable volume systems. Therefore, the objective of this study was to develop compatible systems based on volume ratio equations to estimate d, Vm, Vs, Vt and Vb for *P. pseudostrobus* in commercial forest plantations from Michoacan, Mexico.

2. Materials and Methods

2.1. Study Area

The study was conducted in commercial forest plantations established in the CINSP in the west-central region of the state of Michoacan, Mexico (19°25′–19°34′ N and 102°00′–102°17′ W) over an area of 18,138 ha. The type of climate is temperate humid, and the mean annual temperature is around 18 °C with mean annual precipitation of 1600 mm [15]. Soils are volcanic in origin and the units are Andosol, Regosol and Feozems. The natural vegetation is in keeping with the temperate climate. The dominant vegetation is pine-oak forest, which is managed through the Mexican method of forest regulation (MMFR) and silvicultural development method (SDM), and annual wood harvest is about 65,000 m^3 [16]. The outstanding tree components are *Pinus devoniana* Lindl., *P. montezumae* Lamb., *P. douglasiana* Martínez, *P. leiophylla* Schl. & Cham., *P. pseudostrobus* Lindl., *Quercus laurina* Humb et Bonpl., *Q. castanea* Muhl., *Q. rugosa* Neé, *Abies religiosa* Kunth Schltdl. et Cham., *Arbutus xalapensis* Kunth, *Cornus disciflora* Sessé & Moc., *Tilia mexicana* Schltdl., *Alnus acuminata* H.B.K. and *A. jorullensis* Humboldt, Bonpland & Kunth [17]. In the study area, *P. pseudostrobus* is a species of great economic importance due to the relatively wide distribution, rapid growth in well site quality, stem straightness and quality of wood [18]. The CINSP has a physiographic delimitation unique, thus, is one of the most important areas to establish forests plantations in Michoacan state [14].

2.2. Data

The following notation will be use throughout the rest of this article. Other notation specific will be listed with the equation.

D: over-bark diameter at breast height (cm); d: over-bark diameter at height h (cm); H: total height (m); h: commercial height (m); Vm: merchantable volume (m^3); Vs: stem volume (m^3); Vt: total tree volume (m^3); Vb: branch volume (m^3); $R(p) = Vm/Vs$: ratio volume; $p = \frac{h}{H}$: stem relative height.

The dataset was obtained from a taper analysis of 42 *P. pseudostrobus* trees, 26- and 28-years-old, distributed in four locations of commercial forest plantations over 12 ha; the planting density was

2500 trees per hectare. The commercial forest plantations were managed through pruning, and final wood harvest is planned at 30 years. The information on growth and yield of the plantations in the study area is limited; however, the average increment of *P. pseudostrobus* at 30 years in natural forests is around 8.18 $m^3 \cdot ha^{-1} \cdot year^{-1}$ [19]. The sample represented the variation in diameters and heights. D and H of each tree were measured. Also, d of each section h, as well as the diameters of merchantable branches (larger than 3 cm at the base), were measured. Once the trees were felled, samples were obtained at 0.3 m, 0.6 m, 1.3 m length log, and sections between 2.5 m and 3.3 m up to the total height of each tree were obtained. To obtain the volume of each section (V_l), the Smalian formula (Equation (1)) was used and the cone formula (Equation (2)) was used for the top section (V_{ts}):

$$V_l = \left(\frac{ab_0 + ab_1}{2}\right) \times ls \quad (1)$$

$$V_{ts} = \frac{ab_n}{3} \times lp \quad (2)$$

where ab_0 = the basal area of the large end of the section (m^2), ab_1 = the basal area of the small end of the section (m^2), ab_n = the basal area of the top section (m^2), ls = length of the section (m) and lp = length of the top section (m).

The descriptive statistics that include mean, minimum and maximum values and standard deviation for the variables are shown in Table 1.

Table 1. Descriptive statistics of the database.

Variable	Mean	Minimum	Maximum	SD
D	33.346	20.25	51.5	7.463
d	22.887	0	58.4	12.967
H	27.192	21.94	33	2.603
h	11.543	0.30	33	9.523
Vm	0.661	0	2.523	0.591
Vs	1.121	0.334	2.523	0.538
Vt	1.171	0.34	2.64	0.566
Vb	0.044	0.001	0.116	0.033

SD = standard deviation of the mean; D = diameter at breast height (cm); d = diameter at height h (cm); h = commercial height (m); H = total height (m); Vm = merchantable volume (m^3); Vs = stem volume (m^3); Vt = total tree volume (m^3); Vb = branch volume (m^3).

The Vs was the sum of the log volumes and top section, and Vt was the sum of Vs and Vb for each tree. Figure 1 shows the tendency of relative volume and relative height from the dataset used to fit the compatible taper and volume systems.

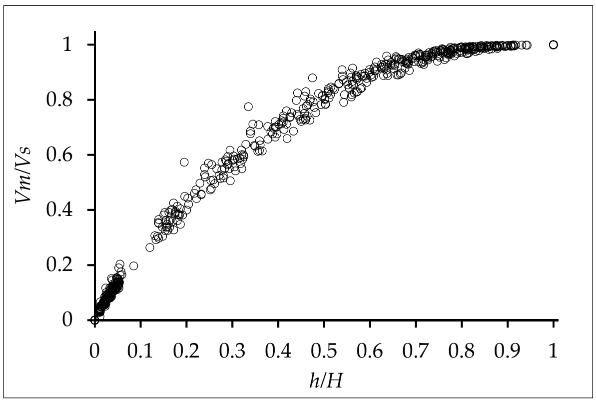

Figure 1. Dispersion diagram of volume ratio (Vm/Vs) and relative height (h/H).

2.3. Compatible Taper and Volume Systems Fitting

Volume ratio equations are based on height where the ratio function is $R(p) = Vm/Vs$. Vm (m^3) is the cumulative volume up to height h (m) above the ground, Vs (m^3) is total stem volume, and p is the ratio between commercial height and total height ($p = \frac{h}{H}$) [10,11].

Compatible taper and volume systems included five equations or components: Vm, d, Vs, Vt and Vb.

The Vm equations were developed from volume ratio functions, which describe changes in the ratio of cumulative volume to total stem volume over relative stem height [10], whereas the d equations correspond to the derivation of the expression of cumulative volume [11]. The systems were named S1, S2, S3, S4, S5 and S6 (Table 2). The S6 equation was derived from the taper function compatible with volume ratio developed by Cao, et al. [20]. The equation of commercial height (h) is given by solving the d equation. For more details, review the study conducted on these types of equations [10,11].

Table 2. Compatible taper and merchantable volume equations based on volume ratio equations.

Equation	System
$R(h) = \left(1-\left(1-\frac{h}{H}\right)^{\beta_0}\right)^{\beta_1}\ (0 < \beta_1 \leq 1, \beta_0 > 1, 0 \leq p \leq 1)$	(S1)
$d(h) = \sqrt{\frac{Vs\beta_0\beta_1}{kH}\left(1-\left(\frac{h}{H}\right)^{\beta_0}\right)^{\beta_1-1}\left(1-\frac{h}{H}\right)^{\beta_0-1}}$	
$R(h) = 1-\left(1-\frac{h}{H}\right)^{\beta_0}\exp\left(-\beta_1\frac{h}{H}\right)\ (\beta_0 \geq 1, \beta_1 > 0, 0 \leq p \leq 1)$	(S2)
$d(h) = \sqrt{\frac{Vs}{Hk}\exp\left(-\beta_1\frac{h}{H}\right)\left(1-\frac{h}{H}\right)^{\beta_0-1}\left(\beta_0+\beta_1\left(1-\frac{h}{H}\right)\right)}$	
$R(h) = 1-\left(1-\frac{h}{H}\right)\frac{\beta_0}{\beta_0+\frac{h}{H}}\ (\beta_0 > 0, 0 \leq p \leq 1)$	(S3)
$d(h) = \sqrt{\frac{Vs\left(\beta_0{}^2-\beta_0\right)}{Hk\left(\beta_0+\frac{h}{H}\right)^2}}$	
$R(h) = 1-\beta_1(1-p)^{\beta_0}\ (\beta_0 > 1, \beta_1 \geq 0, 0 \leq p \leq 1)$	(S4)
$d(h) = \sqrt{Vs\frac{\beta_0\beta_1}{kH}\left(1-\frac{h}{H}\right)^{\beta_0-1}}$	
$R(h) = a_0D^{a_1}H^{a_2}\left(1-\left(1-\frac{h}{H}\right)^{\beta_0}\right)^{1-\beta_1\exp(-\exp(\beta_2D^{a_1}H^{a_2}))}$	(S5)
$d(h) = \sqrt{\frac{\beta_0}{kH}a_0D^{a_1}H^{a_2}\left(1-\frac{h}{H}\right)^{\beta_0-1}(1-\beta_1\exp(-\exp(\beta_2D^{a_1}H^{a_2})))\left(1-\left(1-\frac{h}{H}\right)^{\beta_0}\right)^{(-\beta_1D^{a_1}H^{a_2})}}$	
$R(h) = 1+\beta_0\left(\frac{(H-h)^{\beta_1}}{H^{\beta_2}}\right)$	(S6)
$d(h) = \sqrt{-\frac{Vs}{k}\beta_0\beta_1\left(\frac{(H-h)^{\beta_1-1}}{H^{\beta_2}}\right)}$	

h: commercial height (m); H: total height; p: relative height (h/H); Vs: stem volume; α_i, β_i: parameters to be estimated.

Total Volume Models

In compatible taper and merchantable volume systems (Table 2), three equations of total volume were combined. These equations were applied as a multiplication on both sides of the ratio function ($R(h)$) to obtain the variable merchantable volume [21]. The equations of total volume used were those of Schumacher and Hall [22]—the non-linear combined variable [23] and coefficient of constant form [24]—which have been used in previous studies [25–27] and were named Equations (3)–(5), respectively.

$$V = a_0 D^{a_1} H^{a_2} \tag{3}$$

$$V = a_0 \left(D^2\right) H^{a_1} \tag{4}$$

$$V = a_0 D^2 H \tag{5}$$

where V = volume (Vs or Vt); D = diameter at breast height; H = total height; a_0, a_1 and a_2 = equation parameters.

The parameters of the Vs models correspond to a_i (i = 0, 1, 2) and are totally compatible with the d and Vm equations and partially compatible with Vb. Vt was modeled with the same equations as Vs, but the parameters were rewritten as δ_i (i = 0, 1, 2) and these were compatible only with Vb. The component of Vb was modeled as the subtraction between Vt and Vs. For example, for Equation (3), the equation was given as $Vb = \delta_0 D^{\delta_1} H^{\delta_2} - a_0 D^{a_1} H^{a_2}$.

2.4. Compatible Systems Fitting

Components of the compatible system were fitted simultaneously with the iterative of seemingly unrelated regression technique (ITSUR) with the MODEL procedure of the statistical software SAS/ETS® [28]. The ITSUR technique generates estimation of more efficient parameters when the error components are correlated in systems of equations. Compatibility among equations allowed the estimated parameters to be the same for the equations and this guaranteed the optimization of the fitting process [29,30]. To assure compatibility by degrees of freedom between taper and merchantable volume equations and the equations of volume components, we used the ratio of one over the number of sections of each tree (n_i), which generated the weight variable ($w = 1/n_i$). This procedure was programed into the SAS code as $resid.(Vs, Vt, Vb) = resid.(Vs, Vt, Vb)\sqrt{w}$ for Vs, Vt and Vb, respectively.

2.5. Autocorrelation and Heteroscedasticity

The problems associated with taper and volume that do not satisfy the fundamental assumptions of regression are autocorrelation and heteroscedasticity. The presence of these can affect predictions and the standard error of parameters estimates, and the statistical tests can be inconsistent. Therefore, when modeling with longitudinal series, statistical procedures should be used to correct the dependence and variability of the errors [1,31,32]. The data used considered measurement of diameters and heights of each tree and thus autocorrelation of residuals can be expected. We used a second-order continuous autoregressive error structure (CAR2) in the taper functions, given in Equation (6) [33,34]:

$$e_{ij} = d_1 {\rho_1}^{h_{ij} - h_{ij-1}} e_{ij-1} + d_2 {\rho_2}^{h_{ij} - h_{ij-2}} e_{ij-2} + \varepsilon_{ij} \tag{6}$$

where e_{ij} = is the jth ordinary residual of the ith tree; d_1 = 1 for j > 1; d_2 = 1 for j > 2; d_1 = 0 for j = 1; d_2 = 0 for $j \leq 2$; $h_{ij} - h_{ij-1}$ and $h_{ij} - h_{ij-2}$ are the distances between j to $j - 1$ and between j and $j - 2$ observations within each tree, $h_{ij} > h_{ij-1}$ and $h_{ij} > h_{ij-2}$; ρ_1 and ρ_2, are the autoregressive parameters of first and second order, respectively.

To guarantee homoscedasticity in the volume components, we used a power function of the residual variance $\sigma_i^2 = \left(D^2 H\right)^{\phi}$, the value ϕ was estimated by the method proposed by Harvey [35], using the errors of the fitted model as dependent variable. The parameters were programmed with the specification $resid.(Vm, Vs, Vt\,y\,Vb) = resid.(Vm, Vs, Vt\,y\,Vb)/\left(\left(D^2 H\right)^{\phi}\right)^{0.5}$, for Vs, Vt and Vb, respectively. This has been used by Simental-Cano, López-Sánchez, Wehenkel, Vargas-Larreta, Álvarez-González and Corral-Rivas [27] in volume equations for twelve forest species in the state of Durango, Mexico, and by Tamarit-Urias, et al. [36] in a cubing system for individual *Quercus* spp. trees in Puebla, Mexico.

2.6. Evaluation of Compatible Systems

The goodness-of-fit of the compatible systems was determined with statistical criteria used in forest modeling [26,36–38], which includes the adjusted coefficient of determination (R^2_{adj}), root mean square error (*RMSE*), absolute average bias ($\bar{E}$) and Akaike information criterion (*AIC*). In addition, the graphic analysis of the residuals was used.

To compare the systems, we used a rating criterion [39] which consisted of ranking the fit statistics for each component. Values from 1 to 6 were assigned consecutively in order of precision: 1 is the most efficient statistic and 6 the least efficient. The sum of the scores of the components represented the total of each system. The rating system was used to compare within six systems integrated by a specific volume equation; that is, Equation (3), Equation (4) or Equation (5).

Finally, the *Vs* and *Vt* estimations of top three best systems were compared with a previously fitted compatible system for *P. pseudostrobus* in natural forests reported by Vargas-Larreta, et al. [40], which was based on segmented system developed by Fang et al. [41]. The compatible system used as control is part of a forest biometric system (SiBiFor) for forest management in Mexico. The predictions were tested with average bias and percent bias statistics.

3. Results

Table 3 shown the estimated parameters and standard errors for the fitted compatible systems and the combinations of the total volume equations. All parameters were different from zero ($p < 0.0001$). Also, the second-order continuous autoregressive structure parameters, which enabled independence of the residuals in each tree, are presented. In general, the *d*, *Vm*, *Vs* and *Vt* equations had statistical accuracy with R^2_{adj} values greater or equal than 0.92, but the *Vb* component was between 0.68 and 0.69. The RMSE values were 1.37–2.40 cm, 0.028–0.08 m^3, 0.115–0.145 m^3, 0.120–0.147 m^3 and 0.018–0.019 m^3 for *d*, *Vm*, *Vs*, *Vt* and *Vb*, respectively (Table 4).

The total score of each system was evaluated as a group of equations, depending on the rating system used. According to the rating system, S2E3 and S5E3 presented the lowest scores; that is, the best fit statistics of the systems conformed by Equation (3). Although the S2E3 and S5E3 systems had the same score, S2E3 was better in the components of *d* and *Vm*, with R^2_{adj} values of 0.982 and 0.998, respectively, whereas S5E3 obtained 0.973 and 0.986. Values of *RMSE* and $\bar{E}$ were lower in S2E3.

S1E4 was better within the group of systems integrated by Equation (4), with the highest R^2_{adj} (0.982) and the lowest *RMSE* (1.736 cm) and $\bar{E}$ (0.028 cm) for the *d* component, but statistical values were close to other systems in *Vm*, *Vs* and *Vt*.

The S1E5 and S2E5 systems got the lowest scores of the systems which used Equation (5) to estimate stem and total volume. Their statistical values were similar, but S2E5 had less bias in the *d*, *Vs* and *Vt* components (Table 4).

Figure 2 shows the distribution of residuals in box and whisker of the *d* and *Vm* components, by relative height of the compatible systems S2E3, S1E4 and S2E5, which were selected because they had lower total scores and correspond to each total volume equation.

In the *d* and *Vm* graphs by category of relative height, we observe a trend of the residuals associated to the accuracy of the selected systems (Table 2). In systems S2E3 and S2E5 the trends were similar in the *d* component, while S2E3 had a distribution closer to 0. In *Vm*, the S1E4 residuals were near 0.

Of the selected systems, S2E3 had a better total score, with higher R^2_{adj} values in the *d* and *Vm* components, and lower values of *RMSE* and $\bar{E}$ for all components. S2E3 was better in *Vm* and S2E5 had lower *AIC* values in all components, except in *d*, which in this case was better in S1E5 (Table 4).

Table 3. Estimated parameters and standard error of the simultaneous fitting of the compatible systems.

CS		a_0	a_1	a_2	β_0	β_1	β_2	ρ_1	ρ_2	δ_0	δ_1	δ_2
S1E3	θ	9.1×10^{-5}	1.669	1.062	2.384	0.958		0.764	0.306	8.6×10^{-5}	1.693	1.066
	ε	1.1×10^{-5}	0.024	0.052	0.025	0.002		0.029	0.043	1.0×10^{-5}	0.023	0.051
S2E3	θ	9.7×10^{-5}	1.682	1.031	1.675	1.364		0.805	0.251	9.1×10^{-5}	1.705	1.036
	ε	1.2×10^{-5}	0.025	0.053	0.109	0.130		0.025	0.035	1.1×10^{-5}	0.024	0.052
S3E3	θ	9.8×10^{-5}	1.683	1.035	0.421			0.922	0.843	9.2×10^{-5}	1.705	1.041
	ε	9.7×10^{-6}	0.019	0.042	0.006			0.022	0.018	9.0×10^{-6}	0.019	0.041
S4E3	θ	9.3×10^{-5}	1.687	1.035	2.497	1.155		0.814	0.245	8.8×10^{-5}	1.710	1.041
	ε	1.2×10^{-5}	0.025	0.054	0.026	0.013		0.026	0.032	1.1×10^{-5}	0.025	0.053
S5E3	θ	9.7×10^{-5}	1.725	0.977	2.642	−0.33	3.8×10^{-5}	0.772	0.226	9.3×10^{-5}	1.747	0.985
	ε	8.6×10^{-6}	0.017	0.036	0.021	0.073	1.1×10^{-5}	0.031	0.031	8.1×10^{-6}	0.017	0.036
S6E3	θ	7.8×10^{-5}	1.735	1.039	−0.97	2.566	2.558	0.671	0.186	7.4×10^{-5}	1.756	1.044
	ε	7.2×10^{-6}	0.017	0.039	0.093	0.020	0.035	0.045	0.028	6.8×10^{-6}	0.017	0.038
S1E4	θ	1.3×10^{-4}	0.872		2.380	0.958		0.763	0.296	1.2×10^{-4}	0.882	
	ε	9.5×10^{-6}	0.007		0.025	0.002		0.029	0.042	8.7×10^{-6}	0.007	
S2E4	θ	1.2×10^{-4}	0.875		1.680	1.358		0.804	0.248	1.0×10^{-4}	0.886	
	ε	9.3×10^{-6}	0.007		0.110	0.130		0.025	0.035	8.5×10^{-6}	0.007	
S3E4	θ	1.3×10^{-4}	0.872		0.423			0.922	0.842	1.2×10^{-4}	0.882	
	ε	7.4×10^{-6}	0.005		0.006			0.022	0.019	6.9×10^{-6}	0.005	
S4E4	θ	1.2×10^{-4}	0.875		2.47	1.159		0.814	0.243	1.2×10^{-4}	0.885	
	ε	9.6×10^{-6}	0.007		0.025	0.013		0.026	0.032	8.6×10^{-6}	0.007	
S5E4	θ	1.1×10^{-4}	0.883		3.080	−1.499	1.7×10^{-4}	0.920	0.796	1.0×10^{-4}	0.893	
	ε	9.1×10^{-6}	0.007		0.0366	0.533	3.3×10^{-5}	0.022	0.023	8.3×10^{-6}	0.007	
S6E4	θ	4.4×10^{-5}	0.974		−1.193	2.581	2.637	0.830	0.241	4.2×10^{-6}	0.982	
	ε	7.8×10^{-7}	0.001		0.119	0.020	0.036	0.023	0.033	8.8×10^{-7}	0.001	
S1E5	θ	3.3×10^{-5}			2.467	0.958		0.766	0.233	3.5×10^{-5}		
	ε	1.3×10^{-7}			0.028	0.002		0.029	0.037	1.3×10^{-7}		
S2E5	θ	3.3×10^{-5}			1.758	1.374		0.809	0.212	3.5×10^{-5}		
	ε	1.4×10^{-7}			0.120	0.142		0.025	0.031	1.4×10^{-7}		
S3E5	θ	3.5×10^{-5}			0.410			0.921	0.828	3.6×10^{-5}		
	ε	1.2×10^{-7}			0.006			0.023	0.020	1.2×10^{-7}		
S4E5	θ	3.3×10^{-5}			2.533	1.180		0.817	0.223	3.4×10^{-5}		
	ε	1.4×10^{-7}			0.027	0.013		0.025	0.030	1.4×10^{-7}		
S5E5	θ	3.3×10^{-5}			2.682	−0.376	7.0×10^{-6}	0.772	0.207	3.4×10^{-5}		
	ε	9.9×10^{-8}			0.022	0.062	2.6×10^{-6}	0.030	0.029	1.1×10^{-7}		
S6E5	θ	3.4×10^{-5}			−1.252	2.637	2.701	0.697	0.184	3.5×10^{-5}		
	ε	1.1×10^{-7}			0.131	0.021	0.038	0.041	0.028	1.1×10^{-7}		

CS: compatible system; E3: Equation (3); E4: Equation (4); E5: Equation (5); θ: estimated parameters; ε: standard error of the estimated parameters. The system is represented by the combination of S1, S2, S3, S4 and S5 with E3, E4 and E5.

Table 4. Goodness-of-fit statistics of the compatible systems.

CS	Component	$R^2{}_{adj}$	*RMSE*	$\bar{E}$	*AIC*	Rating
S1E3	*d*	0.982	1.722	0.033	732.1	54
	Vm	0.998	0.028	0.001	−4731.6	
	Vs	0.953	0.116	0.002	−175.9	
	Vt	0.953	0.122	0.006	−171.9	
	Vb	0.699	0.018	0.001	−333.2	
S2E3	*d*	0.976	2.012	0.033	938.7	52
	Vm	0.998	0.028	0.001	−4732.8	
	Vs	0.953	0.116	0.002	−176.067	
	Vt	0.953	0.122	0.006	−172.053	
	Vb	0.699	0.018	0.001	−333.228	

Table 4. *Cont.*

CS	Component	R^2_{adj}	*RMSE*	$\bar{E}$	*AIC*	Rating
S3E3	*d*	0.971	2.221	0.586	1069.1	100
	Vm	0.998	0.029	0.002	−4687.8	
	Vs	0.949	0.120	0.030	−172.947	
	Vt	0.951	0.125	0.026	−169.9	
	Vb	0.699	0.018	0.001	−333.3	
S4E3	*d*	0.972	2.187	0.602	1049.6	74
	Vm	0.998	0.028	0.001	−4730.0	
	Vs	0.953	0.116	0.006	−175.9	
	Vt	0.953	0.122	0.010	−171.8	
	Vb	0.699	0.018	0.001	−333.2	
S5E3	*d*	0.973	2.129	0.108	1014.9	52
	Vm	0.986	0.071	0.002	−3514.9	
	Vs	0.953	0.116	0.001	−176.0	
	Vt	0.953	0.122	0.005	−172.1	
	Vb	0.699	0.018	0.004	−333.3	
S6E3	*d*	0.969	2.277	0.311	1104.6	80
	Vm	0.998	0.028	0.001	−4723.4	
	Vs	0.952	0.117	0.005	−175.5	
	Vt	0.952	0.123	0.009	−171.4	
	Vb	0.699	0.018	0.001	−333.3	
S1E4	*d*	0.982	1.736	0.028	741.5	36
	Vm	0.998	0.028	0.001	−4729.8	
	Vs	0.954	0.115	0.002	−177.7	
	Vt	0.955	0.120	0.006	−174.2	
	Vb	0.699	0.018	0.001	−333.3	
S2E4	*d*	0.976	2.020	0.040	942.9	46
	Vm	0.998	0.028	0.000	−4732.1	
	Vs	0.954	0.115	0.002	−177.7	
	Vt	0.954	0.120	0.006	−174.2	
	Vb	0.699	0.018	0.001	−333.2	
S3E4	*d*	0.970	2.232	0.590	1075.0	88
	Vm	0.998	0.029	0.002	−4683.6	
	Vs	0.950	0.119	0.031	−174.6	
	Vt	0.952	0.123	0.027	−171.9	
	Vb	0.699	0.018	0.001	−5335.8	
S4E4	*d*	0.971	2.209	0.633	1062.0	60
	Vm	0.998	0.028	0.001	−4727.7	
	Vs	0.954	0.115	0.005	−177.6	
	Vt	0.954	0.120	0.010	−173.9	
	Vb	0.699	0.018	0.001	−333.3	
S5E4	*d*	0.971	2.217	0.176	1067.7	79
	Vm	0.998	0.028	0.000	−4727.4	
	Vs	0.954	0.115	0.005	−177.6	
	Vt	0.954	0.120	0.009	−173.9	
	Vb	0.699	0.018	0.001	−333.3	
S6E4	*d*	0.970	2.240	0.566	1081.5	103
	Vm	0.997	0.032	0.001	−4566.4	
	Vs	0.941	0.130	0.021	−167.4	
	Vt	0.942	0.136	0.025	−163.7	
	Vb	0.699	0.018	0.000	−333.2	

Table 4. *Cont.*

CS	Component	R^2_{adj}	*RMSE*	$\bar{E}$	*AIC*	Rating
S1E5	*d*	0.980	1.845	0.363	821.7	56
	Vm	0.997	0.034	0.001	−4491.8	
	Vs	0.935	0.136	0.038	−164.5	
	Vt	0.938	0.140	0.040	−162.2	
	Vb	0.686	0.019	0.003	−331.4	
S2E5	*d*	0.972	2.158	0.352	1029.8	51
	Vm	0.997	0.034	0.001	−4492.3	
	Vs	0.935	0.136	0.037	−164.6	
	Vt	0.938	0.140	0.039	−162.2	
	Vb	0.686	0.019	0.003	−331.4	
S3E5	*d*	0.968	2.326	0.455	1128.5	84
	Vm	0.996	0.036	0.001	−4411.9	
	Vs	0.927	0.145	0.013	−159.3	
	Vt	0.931	0.147	0.010	−157.9	
	Vb	0.687	0.019	0.002	−331.5	
S4E5	*d*	0.970	2.238	0.416	1078.2	87
	Vm	0.997	0.034	0.002	−4488.6	
	Vs	0.935	0.136	0.042	−164.3	
	Vt	0.938	0.140	0.044	−161.9	
	Vb	0.686	0.019	0.002	−331.5	
S5E5	*d*	0.972	2.175	0.024	1041.3	67
	Vm	0.979	0.086	0.020	−3255.4	
	Vs	0.935	0.136	0.039	−164.5	
	Vt	0.938	0.140	0.042	−162.1	
	Vb	0.687	0.019	0.002	−331.5	
S6E5	*d*	0.968	2.324	0.464	1129.8	67
	Vm	0.997	0.034	0.001	−4489.9	
	Vs	0.935	0.136	0.022	−164.4	
	Vt	0.938	0.140	0.025	−162.3	
	Vb	0.687	0.019	0.002	−331.5	

CS: compatible system; R^2_{adj}: adjusted coefficient of determination; *RMSE*: root mean square error; $\bar{E}$: average absolute bias; *AIC*: Akaike information criterion; Rating: total score.

The best three systems showed lower values of Bias than SiBiFor system used as a control for *P. pseudostrobus* in natural forests of the study area [40] (Table 5). S2E3 had the lowest values of Bias in (−0.17% and 0.64%) for *Vs* and *Vt*, respectively.

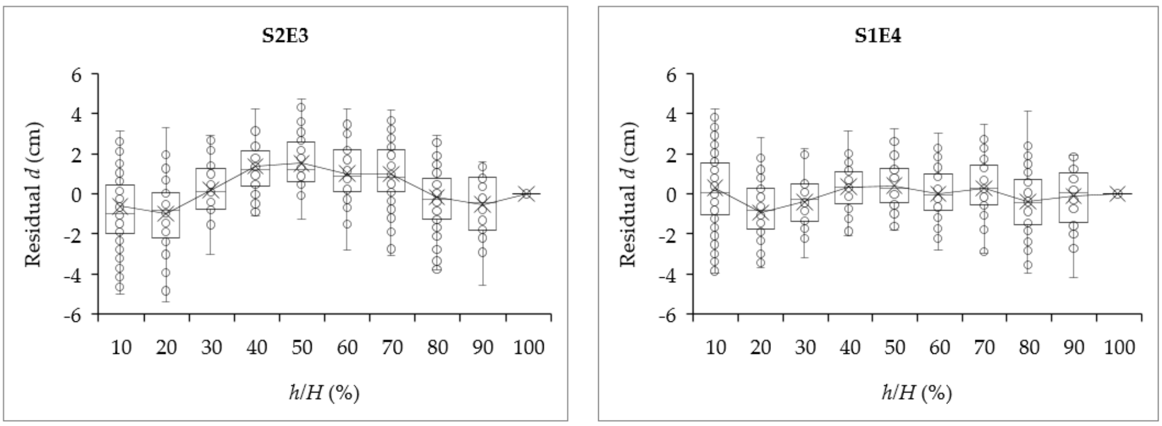

Figure 2. *Cont.*

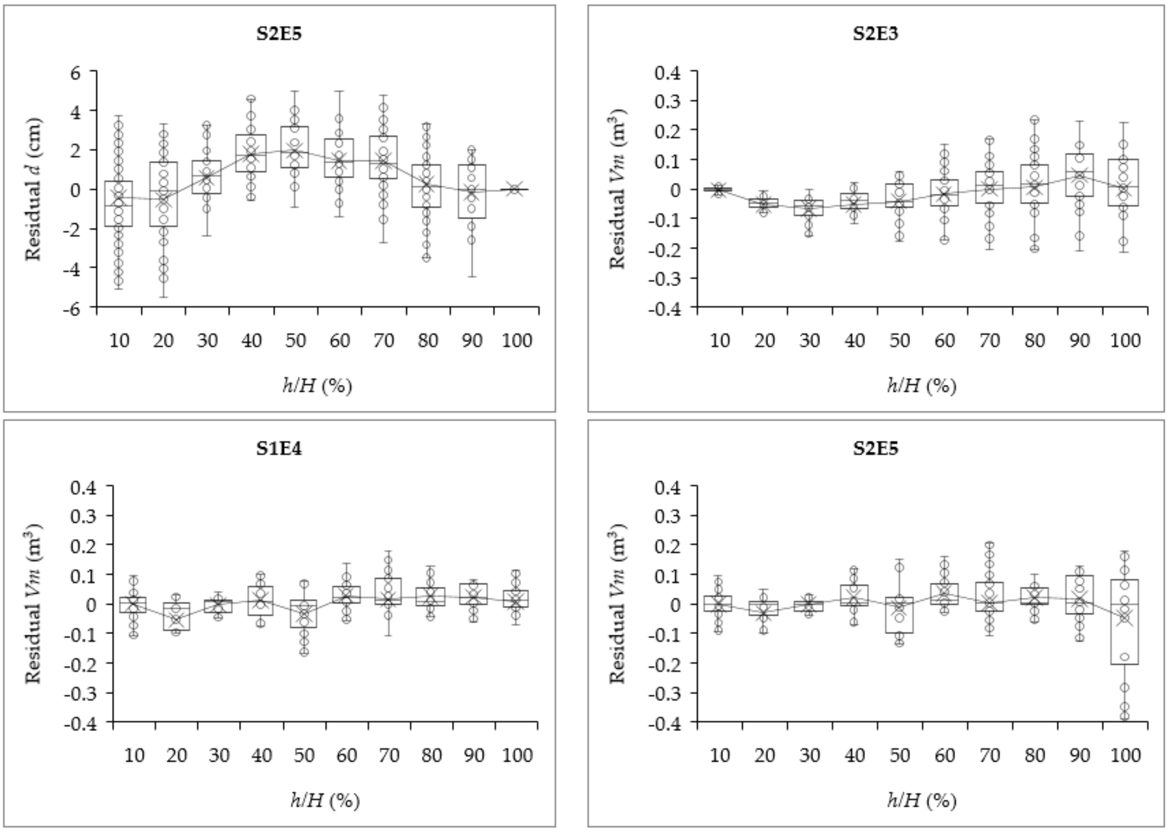

Figure 2. Distribution of residuals in box and whisker for *d* and *Vm* of the compatible systems S2E3, S1E4 and S2E5.

Table 5. Bias and % bias for *Vs* and *Vt* equations.

CS	*Vs*		*Vt*	
	Bias (m^3)	**% Bias**	**Bias (m^3)**	**% Bias**
S2E3	−0.001	−0.17%	0.007	0.64%
S1E4	0.014	1.36%	0.036	3.16%
S2E5	0.046	4.22%	0.094	8.25%
SiBiFor	−0.100	−9.22%	−0.111	−9.81%

CS: Compatible system; % bias is based on the average observed *Vs* and *Vt* values.

4. Discussion

The procedure for fitting systems based on volume ratio equations implies that the taper equation is derived from the ratio equation, thus guaranteeing compatibility of the components [11]. This methodology allowed simultaneous fitting of the components of *Vs*, *Vt* and *Vb*. The compatible systems derived from volume ratio equations have demonstrated compatibility in *d* and *Vm* [10,42].

The parameters of the systems proved to be efficient (Table 4) and the use of the autoregressive structure of second order errors (CAR2) allowed correction of residual dependency in *d*. Similar procedures have been used in compatible taper and volume systems for the same component. Hernández-Ramos, De los Santos-Posadas, Valdéz-Lazalde, Tamarit-Urias, Ángeles-Pérez, Hernández-Ramos, Méndez-Lopez and Peduzzi [26] used this errors structure to fit a cubing system from volume ratio models for *Swietenia macrophylla* King in southeastern Mexico, Vargas-Larreta, et al. [40] used it to develop a biometric system for temperate and tropical forests of Mexico, and Tang, Pérez-Cruzado, Fehrmann, Álvarez-González, Lu and Kleinn [1] used it for *Cunninghamia lanceolata* Lamb. in China.

In general, the compatible systems were highly precise in the *d*, *Vm*, *Vs*, *Vt* components but were less accurate in the *Vb* equation. This is common in this type of model [43,44] because of the diversity of sizes and shapes present in the tree crowns that can vary widely among individuals and localities [45]. Although tree structure and size should be similar in plantations, the variation may be attributed to the silvicultural treatments applied in the study area for which we have no record.

Tree branches are an important component of forest structure and contribute volume in forest timber or environmental service inventories. The *Vb* can be estimated directly with an equation in function of diameter and height or as an equation such as that of *Vs* [46]. *Vb* obtained as the subtraction between *Vt* and *Vs* allowed a lower number of parameters within the system. This equation has a simple mathematical structure and showed reliable statistics, which were similar to those reported by Corral-Rivas, Vega-Nieva, Rodríguez-Soalleiro, López-Sánchez, Wehenkel, Vargas-Larreta, Álvarez-González and Ruiz-González [12] for a compatible system of *Pinus* in Durango, Mexico, where they calculated *Vb* directly in function of diameter at breast height and crown radius.

The S2E3, S1E4 and S2E5 compatible systems were selected because they had better statistical values (Table 4), for each stem and total volume equation. In contrast, system S5 has given accurate results for taper, stem volume and green weight to any height for plantations of *Pinus taeda* L. The selected systems satisfy properties I–IV [10,11].

The distribution of residuals of the selected systems showed that for *d* there is an overestimation in height categories of 10 and 20% and an underestimation (40–60%) in the central categories for S2E3 and S2E5, this could be attributed the different lengths of the logs cut after diameter at breast height. Quiñonez-Barraza, De los Santos-Posadas, Álvarez-González and Velázquez-Martínez [4] found a similar distribution of residuals in the first height categories for *d* in a segmented taper and merchantable volume system for the principal species of *Pinus* in Durango, Mexico. The largest errors for *Vm* were found as of 60% of the relative height category in S2E3, 70% in S1E4, and 100% in S2E5.

Equations (4) and (5), which integrate the selected compatible systems, had one and two parameters fewer than Equation (3), respectively, but the results were similar. Although it is preferable to use the system integrated by Equation (3) as it has been used in studies in the region [40,47], the system that uses Equation (4) has very efficient estimations. Barrios, López and Nieto [25] in a commercial forest plantations of *Eucalyptus grandis* W in Colombia and Hernández-Ramos, De los Santos-Posadas, Valdéz-Lazalde, Tamarit-Urias, Ángeles-Pérez, Hernández-Ramos, Méndez-Lopez and Peduzzi [26] in commercial forest plantations of *Eucalyptus urophylla* S.T. Blake in Tabasco, Mexico, adjusted systems based on models of total and ratio volume dependent on relative diameter and reported statistical accuracy results when the equation corresponding to Equation (3) was incorporated. Fonweban, et al. [48] compared total volume and volume ratio equations and they found the best results with Equation (3) for plantations of *Pinus sylvestris* L. and *Picea sitchensis* (Bong.). In modeling volume equations, a simple function is recommended since the incorporation of additional parameters in some cases does not contribute significantly to accuracy of stem volume predictions [25].

In Mexico, previous studies have developed taper and volume systems for natural forests and commercial forest plantations [49–53], which were derived from taper equations. In these systems, the segmented model developed by Fang, et al. [41] has been the most frequently used, but the mathematical structure is complex.

The S2E3 is integrated by total volume equation developed by Schumacher and Hall [22], which is also implicit in the Fang et al. [41] compatible system, S2E3 provided lower values of bias in the *Vs* and *Vt* estimations. The best three compatible systems had lower values of bias than the SiBiFor equations (Table 5). Even though SiBiFor equations were developed for natural forest, estimations for *P. pseudostrobus* in forest plantation should be analyzed. The equations that integrate the systems based on volume ratio functions exhibit better parsimony, generated consistent results, and showed accurate estimations of *Vm* for heights and diameters without implementing numerical methods of integration [3,21].

5. Conclusions

The volume ratio equations allowed fit-compatible systems of taper, merchantable volume, stem volume, branch volume and total tree volume for *P. pseudostobus*. Even though the last two components are not totally compatible in the system, the results were efficient. S2 had better results when Equation (3) or Equation (5) was used in the ratio function. However, with Equation (4), S1 was the best of the rest of the fitted systems. The use of the Equation (3) enables accurate estimation of timber stock; however, Equation (4) suggests satisfactory results with two parameters in the volume equation. The compatible systems based on volume ratio equations are a simple, reliable tool that can determine timber stocks and classification of products for management and valuation of commercial forest plantations.

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