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LAMONT GEOLOGICAL OBSERVATORY

PALISADES, NEW YORK

ON A POSSIBILITY OF SIMPLIFIED MEASUREMENT OF TURBULENT DIFFUSION NEAR THE OCEAN BOTTOM

Report prepared by: Takashi Ichiye

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ABSTRACT

Vertical distributions of suspended matter which has been observed by Ewing and Thorndike at three deep stations on the continental slope southeast off New York with a new nephelometer are discussed from a point of view of turbulent diffusion of sediment particles. It is shown that theoretical curves obtained with eddy diffusivity which vanishes at the bottom and the top of a nepheloid layer, varying as a quadratic function of depth, do not agree with the observed distributions, though the model of eddy diffusivity has been proved to be suitable for suspended particles in an open channel. It is found that eddy diffusivity proposed by Rossby and Montgomery for the lower layer of atmosphere yields vertical distributions of concentration similar to those observed. The eddy diffusivity determined from curve fitting techniques consist of three parts: a lowest part linearly increasing with distance from the bottom, a middle part quadratically decreasing with distance and an upper part of a constant value. When settling velocity of suspended matter is taken as 3×10^{-5} (cm/sec), the ranges of eddy diffusivity determined at three stations are 0.1 to 1 (cm^2/sec) for the upper part and 0.6 to 36 (cm^2/sec) for the maximum value at the top of the lower part. Speculation on causes of transparent zones is presented. Origins of turbulence which produces boundary layers in adjacent to the bottom and residual eddy diffusivity in the upper part of the nepheloid layer are discussed.

1. INTRODUCTION

Recent efforts to measure currents in the great depths of the ocean have shown this to be one of the most difficult tasks in oceanography. A more formidable task still is the measurement of turbulence structure near the deep sea bottom. In fact, it is more difficult to set up current measurement devices in the deep water than in the shallow water in order to record fluctuations of currents for calculating the energy spectrum of turbulence. Also, there is no reliable instrument for recording fluctuations of temperature or salinity in the deep sea owing both to engineering difficulties and to a condition that gradients and fluctuations of these quantities are so small in the great depth. Lack of gradients of temperature, salinity and concentration of other chemical components in the deep sea reduces the validity of indirect methods of determination of eddy diffusivity from distributions of these quantities, because in such methods not only the gradients, but also the second order derivatives of the concentration with space coordinates are necessary. (Ichiye, 1962) It is almost out of the question in the present state of the art to use dye diffusion techniques which have been utilized successfully in the shallow water and in the upper layer of the deep sea, considering the immense difficulties in release and detection of small dye patches in the great depth.

On the other hand, importance of information of turbulence in the deep ocean has been realized in oceanography in recent years. Particularly, knowledge of eddy diffusion is essential to disposal of waste including radioactive materials in the deep sea and dispersion of fallout from nuclear testing. So far, only three authors have estimated eddy diffusivity in the

deep part of the Philippine Trench and South Atlantic Ocean by use of indirect methods whose validity is doubtful as stated before (Defant, 1960).

Ewing and Thorndike (1964) have recently determined vertical distributions of suspended matter near the bottom of the continental slope off the Atlantic Coast of the United States by use of a new nephelometer. These distributions together with other data yield rather accurate estimations of eddy diffusivity. Since the instrument can be operated to any depth of the ocean, it will give a simple method to determine eddy diffusivity in the great depth.

2. CALCULATION OF EDDY DIFFUSIVITY

In rivers and laboratory flumes, vertical distributions of suspended sediment have been utilized to determine turbulence structure of the flow. O'Brien (1935) and Hurst (1929) considered the equation for a concentration C for a constant eddy diffusivity A.

$$w C + A \frac{dC}{dz} = 0 \qquad (1)$$

where w is settling velocity of the sediment. Equation (1) can be integrated at once to give the concentration

$$\frac{C}{C_a} = \exp \left\{ - w A^{-1} (z - a) \right\} \qquad (2)$$

in which C_a is the concentration at some arbitrary level $z = a$.

Rouse (1937) assumed the similarity between the eddy diffusivity A and eddy viscosity ν , the latter of which is given by

$$\nu = k_c u_* z (1 - z / h) \qquad (3)$$

when it is assumed that the vertical profile of the current velocity obeys Prandtl's logarithmic law. In Equation (3) ko is Karman's constant, h is the depth of the flume and u_* is the shearing velocity defined by

$$u_* = k_o z (du / dz) \tag{4}$$

where u is the velocity. With the assumption that

$$A = \beta \nu \tag{5}$$

integration of Equation (1) yields

$$C/C_a = \left(\frac{h - z}{h - a} \quad \frac{a}{z} \right)^{\gamma} \tag{6}$$

where

$$\gamma = w (\beta k_o u_*)^{-1} \tag{7}$$

In order to determine the eddy diffusivity from the vertical distribution C by use of (6), the shearing velocity u_* must be obtained from (4) by measuring the shear and the coefficient γ must be determined by curve fitting method. However, it is obvious that the distribution of suspended matter in the deep sea does not correspond to the model which yields Equation (6) as indicated in Fig. 1, in which typical readings of the Thorndike nephelometer as the concentrations are plotted against the distance from the bottom besides the theoretical curves of (6) for different values of γ. The data plotted in Fig. 1 were obtained at Station RC 8-5 which is located at about 37.5° N and 70° W and 4023 meters deep, as shown in Fig. 3 (Ewing and Thorndike, 1964) The theoretical curves are calculated by taking h = 431 m and a = 1 m, considering the fact that the reading of the nephelometer showed the minimum value of -0.2 throughout above 431 m from the bottom.

Since it is unbelievable that the logarithmic law for the velocity profiles is valid throughout this entire layer, the conspicuous disagreement between the observed curves is natural.

The observed distribution of Fig. 1 may be obtained from the theoretical relationships (2) by taking two different values of A which can be determined by fitting two straight lines to the observed curve plotted in semi-logarithmic scale near the bottom and near the upper limit of the nepheloid zone. However, this process yields the discontinuous distribution of vertical eddy diffusivity. On the other hand, Rossby and Montgomery (1935) developed a theory on turbulent shear flow in the lower atmospheric layer based on Prandtl's boundary layer theory. According to them, the eddy viscosity increases linearly with height from the surface to the upper limit of the frictional boundary layer, yielding the logarithmic profile of wind speed and then it decreases to zero at the upper limit of the planetary boundary layer, in which the wind vector changes with height similar to Ekman's spiral. It turned out that this modal of the eddy diffusivity also does not yield the vertical distribution of the concentration as observed in the nepheloid zone. However, if the existence of a layer of constant eddy diffusivity is assumed in the upper part of the turbidity layer as susggested by points scattered on a straight line in Fig. 1, the theoretical curve can well be fitted to the observed one. This method will be shown next.

The eddy diffusivity A is considered to consist of three parts:

$$A = A_0 = \text{const} \qquad (z > D) \tag{8a}$$

$$A = k_1 (h - z)^2 \qquad (D > z > H) \tag{8b}$$

$$A = k_2 z \qquad (H > z > 0) \tag{8c}$$

The distributions expressed by (8b) and (8c) are the same as proposed by Rossby and Montgomery (1935). The relation (8a) is introduced by considering the observed curve as stated above. The coefficients k_1 and k_2 can be determined by continuity of A at $z = H$ and $z = D$. The distribution of concentration C is obtained by integrating Equation (1) with z for values of A given by (8a), (8b) and (8c). When the distance from the bottom is expressed by a non-dimensional quantity $s = z / D$, the distribution of C is given by

$$C = C_d \exp\left\{ - m(s - 1) \right\} \qquad (\text{for } s > 1) \qquad (9a)$$

$$C = C_d \exp\left[- m(s_h - 1) - (s_h - 1)^2 (s_h - s)^{-1} \right\}\Big] \qquad \text{for } 1 > s > s_h \quad (9b)$$

$$C = C_a (s_a / s)^{\alpha} \quad \text{for} (s_h > s > s_a) \qquad (9c)$$

where

$$m = w D / A_o, \quad s_h = h / D, \quad s_H = H/D, \quad s_a = a/D, \quad \alpha = s_H (s_h - 1)^2$$

$$\times \ (s_h - s_H)^{-2}$$

and C_d and C_a are the values of C at $z = D$ and $z = a$, respectively. The eddy diffusivity of Equation (8b) and (8c) can be expressed by

$$A = A_o (s_h - s)^2 \ (s_h - 1)^{-2} \qquad (\text{for } 1 > s > s_H) \qquad (8b')$$

$$A = A_o \ s_H \ s (s_h - s_H) \qquad (8c')$$

The numerical constant w / A_o can be determined from the slope C represented in semi-logarithmic scale as in Fig. 1 against the depth for the upper linear part of the curve. The value of D is determined as the lower depth of the linear part of the curve which is represented by Equation (9a). The reference distance a is arbitrary, and is taken as 1 meter, which is not very far from a truth, considering some observations indicating existence

of slime with a few feet depth. Two unknown depths h and H can be determined by fitting the curves computed from (9b) and (9c) with the observed one. The numerical values of these constants thus determined are as follows:

$D = 350$ (m), $w / A_o = 2.54 \times 10^{-4}$ (cm^{-1})

$C_d = 0.36$ (NU or Nephelometer Unit), $C_a = 1.17$ (NU)

$s_h = h / D = 1.34$, $s_H = H / D = 0.57$

The value of s_h corresponds to 200 m of the thickness of the frictional boundary layer. The theoretical curve of C computed from (9a) (9b) (9c) for these numerical constants is plotted in Fig. 1. It is seen that the computed curve is in good agreement with the observed data. It may be argued that the agreement is obvious because several numerical constants are determined by curve fitting. However, it should be remembered that there are a number of degrees of freedom in the curve fitting and particularly the agreement in both curves in the regions expressed by (9b) and (9c) suggests that the mechanism of suspension might actually correspond to the proposed theory. The vertical distribution of eddy diffusivity A corresponding to the theoretical curve shown in Fig. 1 is plotted as a ratios to A_o in Fig. 2.

In order to determine eddy diffusivity from the estimated value of w / A_o, we have to know the settling velocity w, which can be computed by Stokes' law, if the mean diameter of the suspended matter is known. On the other hand, according to Arrhenius (1963), the settling velocity of the suspended matter in the ocean is several orders of magnitude larger than the value obtained from Stokes' law for particles whose diameters are of the order of 1μ or less. The settling velocity of the finer fraction of the suspenoid (0.01 to 0.5μ) whose average diameter is assumed to be 0.05μ is estimated by him as 3×10^{-5}

(cm/sec). Using this value for w, we obtain $A_o = 0.12$ (cm^2/sec) with the value of w / A_o determined from the observed data in Fig. 1. This value seems to be small compared with values of eddy diffusivity estimated by others in the deep sea using temperature or salinity data (Defant, 1961). However, it is still several orders of magnitude larger than the molecular diffusivity of 2×10^{-5} (cm^2/sec). Since in this calculation settling velocity of the finer fraction of particles is assumed, the actual settling velocity may be larger than the assumed one, if the suspendoid consists of mineral particles. All this estimation of the eddy diffusivity depends on accurate value of w. Also, it is more important to note that the eddy diffusivity determined from the nepheloid distribution shows maximum at about 200 m from the bottom than to determine numerical values of the eddy diffusivity.

The vertical distributions of A/A_o for Stations RC 8-4 and RC 8-6 (Fig. 3) are also determined from the nephelometer data in the same way as for Station RC 8-5. The vertical distributions of the nephelometer readings are plotted in Fig. 4 with the theoretical distributions obtained through curve-fitting processes. For Station RC 8-6, two readings taken while lowering and raising the instrument are shown. At these stations, existence of transparent zones makes the data points scattered more wildly than at RC 8-6. However, if we discard the transparent zones, agreement between the theoretical curves and observed values is good. In Fig. 5, the vertical profiles of A/A_o is plotted. The characteristic quantities for theoretical distributions of concentration and eddy diffusivity are as follows:

	D (m)	H (m)	w / A_o (cm^{-1})	A_H / A_o
RC 8-4	350	125	4.4×10^{-5}	53.5
RC 8-6	100	50	4.8×10^{-5}	18.1

A_H is the maximum of A which is reached at $z = H$. Comparison of the numbers of the last two columns in the above table with those of RC 8-5 indicates that the eddy diffusivity A_o is one order of magnitude larger and A_H is one to two orders of magnitude larger at these two stations than at RC 8-5. This suggests that the transparent zones which are abundant at these two stations are related to intense turbulence in the nepheloid layer.

3. Various Factors Pertinent to Estimation of Eddy Diffusivity

There are many arguments against the validity of the Fickian equation in describing the diffusion phenomenon in the ocean (Ichiye, 1962). However, the equation may be valid when it is applied to the process which occurs in a limited area for a steady state or in a limited period of time. Also, the idea of eddy diffusivity on which the equation is based is not inadequate to indicate the state of turbulence if we interpret the estimated value with care. Even so, there remain many problems to be solved in order to determine the eddy diffusivity by use of distributions of suspended matter.

Firstly, the relationships between readings of the nephelometer and concentration of suspended material must be determined. There might be a saturation concentration beyond which the nephelometer gives the constant maximum reading. Secondly, the particle sizes of suspended matter must be measured at several heights from the bottom. As discussed, in the end of the previous section, the absolute values of eddy viscosity cannot be determined from the distribution of concentration without knowledge of the settling velocity of particles. Also, there is a possibility that the distribution of particle sizes may change with the depth and thus, the concentration merely reflects the

vertical distribution of the settling velocity with a constant diffusivity.

Even if the validity of the Fickian equation for the suspended matter problem is established, there remains the doubt about applicability of Equation (1) which is so much simplified from the complete Fickian equation. Firstly, the assumption of steady state must be questioned. Although the nephelometer readings in most stations show the same values within an accuracy of the instrument while lowering and hauling up, there is no information on the change of concentration with time longer than a few hours.

Secondly, neglect of the convection terms due to water movements must be checked. It can be shown that particularly the effect of vertical velocity of water is important. The Fickian equation in a steady state becomes

$$u\,\partial G/\partial x + v\partial C/\partial y + (w_s + w)\,\partial C/\partial z = \text{Diffusion terms} \qquad (10)$$

where u and v are components of horizontal velocity and w_s and w are settling velocity of particles and vertical velocity of the water, respectively. So far, there is no data which gives exact values of currents and horizontal gradient of the concentration and thus, we have to be satisfied with estimation of orders of magnitude from the existing data. The order of magnitude of horizontal gradients of C can be estimated from the difference of vertically averaged concentration at three stations, RC 8-6, RC 8-5 and RC 8-4 (Fig. 3) and is given by

$$\text{Ord (Hor. Grad } C\text{)} = 0.1\ \text{NU}/400\ (\text{km}) = 2.5 \times 10^{-9}\ (\text{NU/cm})$$

where NU means a unit of the nephelometer. The averaged vertical gradient $\partial C / \partial z$ is estimated from the vertical distribution of C at one station, for instance, at RC 8-5 and is equal to 1 (NU) / 500 (m) = 2×10^{-5} (NU / cm).

Therefore, in order to yield the same effect on the distribution of the concentration, the horizontal component of the velocity must be 10^4 times the vertical component. It is instructive to compare this figure with the average bottom slope of the strip of 400 km width, at the center of which the station RC 8-5 is located. The slope is 6×10^{-3} and thus for the current flowing parallel to the bottom slope, the term $w \partial G / \partial z$ is one to two orders of magnitude larger than the first two terms on the r. h. s. of Equation (10).

4. Discussions

Although it is highly speculative, it is worthwhile to consider causes of transparent zones in the nepheloid layer at some stations (RC 8-6, for example) reported by Ewing and Thorndike (1964). Since the effect of vertical component of the water movement seems to be predominant on the distribution of suspended matter, there must be some kind of vertical motion in order to generate the transparent zones. One speculation for such causes is internal waves of the higher modes which are generated due to the stratification caused by the suspended material. Another speculation is stationary cellular motion of convective origin. The vertical streakiness due to local bottom topography of the current flowing parallel to the bottom cannot be eliminated as a cause, since such current has effective vertical components as explained before. If data of the nepheloid concentration are obtained repeatedly at the same station or at several stations located closely to each other, we shall be able to determine causes of the transparent zones as well as the details of bottom current from the change of these zones with time or space.

The vertical distributions of eddy diffusivity determined at three stations

indicate the presence of two different layers ($z > D$ and $D > s > 0$) in the deep water with regard to turbulence characteristics. In the layer between the bottom and $z = D$, eddy diffusivity varies with height in a similar way to the atmospheric boundary layer near the ground. This similarity seems to be superficial, because in the atmosphere there are two dominant effects which generate turbulence: shear in the lower layer of geostrophic winds and free convection due to differential heating on the ground (Priestly, 1959) while in the ocean both effects seem to be negligibly small. However, a quasi-geostrophic nepheloid current which will be discussed elsewhere (Ichiye, 1964) may generate turbulence in the boundary layer.

Finite eddy viscosity in the layer above D was, in case of atmosphere, attributed to 'residual turbulence' whose energy source is found partly in convective motion and partly in shearing forces due to vertical variations in geostrophic winds (Rossby and Montgomery, 1935). However, the convective motion seems to be very small in the great depth of the ocean. Some preliminary experiments on effects of winds and gravity waves on diffusion of dye in water were made in a racetrack-type flume of 23 cm wide and 50 cm deep of an outward periphery of a major axis of 3.0 m and a minor axis of 1.5 m. (Ichiye, 1963) . The results indicated that the dye released near the bottom was diffused much more rapidly when winds were blown on the water surface, than when gravity waves were generated, although there was no noticeable motion near the bottom in the former case compared with occurrence of oscillatory motions in the latter case. This suggests that wind stresses applied at the surface may become an effective energy source of turbulence near the bottom. In an oceanic scale, energy input at the sea surface due to

meteorological disturbances is mostly consumed in the upper layer above a thermocline and only a small portion of it reaches the bottom. However, simplified analyses for a two-layered ocean model indicate that meteorological disturbances with scales and duration of ordinary fronts and cyclones can effectively generate barotropic currents which may reach the bottom (Veronis and Stommel, 1956) and that such perturbations in the deep ocean are amplified near the western coast of the ocean due to reflection of the Rossby waves there (Ichiye, 1958). It is necessary to measure turbulence in the deep water on a worldwide scale in order to determine transport of turbulence energy from meteorological disturbances to the deep water.

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Figure 1 — Distribution of nepheloid concentration (Nephelometer Unit) versus distance from the bottom (meter)

Closed circles: Data at RC 8-5

Broken curves: Theoretical curves computed from Equation (6)

Solid curve: Theoretical curve computed from Equations (9a) (9b) and (9c). A, B and C represent portions of the curve corresponding to (9c), (9b) and (9a), respectively.

Figure 2 — Vertical profile of eddy diffusivity corresponding to the theoretical distribution of nepheloid expressed with the solid curve in Fig. 2. A, B and C correspond to each portion of the solid curve in Fig. 1.

Figure 3 — Locations of three nephelometer stations (A) and the bottom profile of the transection (B) . (A broken line in (B) indicates the upper limit of the nepheloid zone).

Figure 4 — Vertical distributions of nepheloid concentration. (Closed circles and open circles indicate the readings of the nephelometer while lowering and raising it, respectively, at Station RC 8-6. Triangles indicate the readings at Station RC 8-4 while lowering the meter. Solid and broken curves indicate theoretical curves obtained from the distribution of eddy diffusivity plotted in Fig. 5).

Figure 5 — Vertical profiles of eddy diffusivity (A/A_0) at Station RC 8-6 (solid curve) and at Station RC 8-4 (broken curve).

FIG. 1

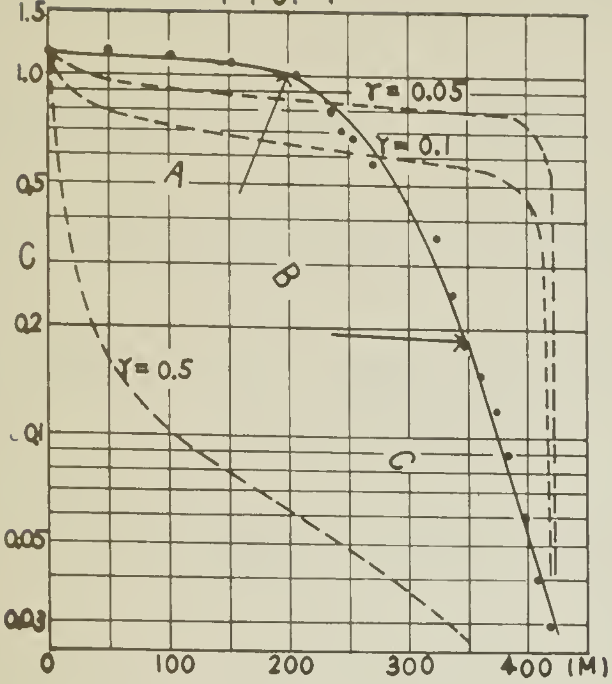


FIG. 4

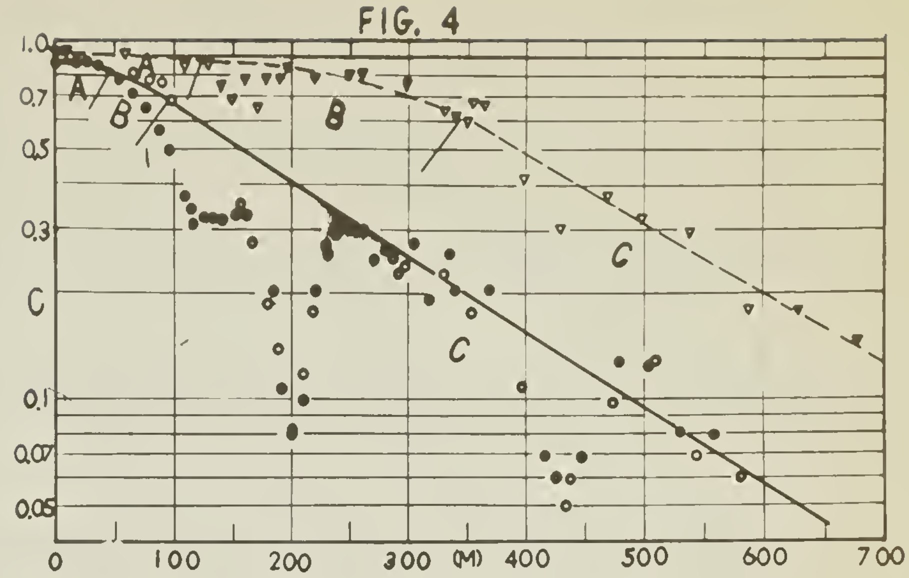


FIG. 2

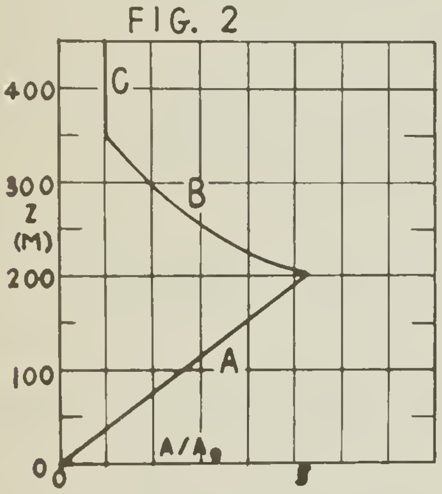


FIG. 5

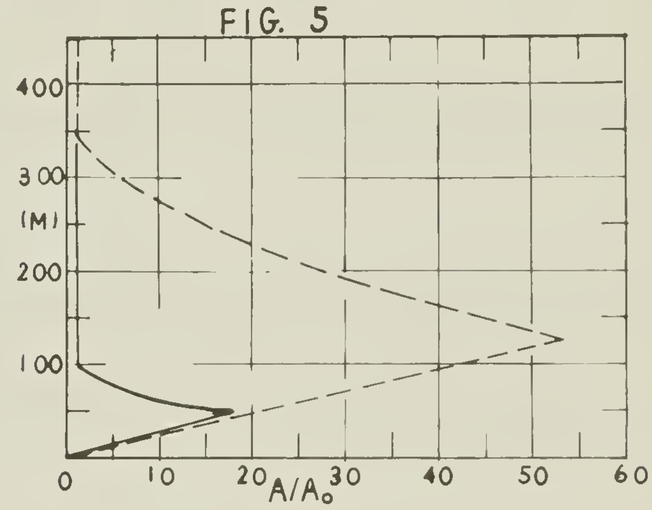
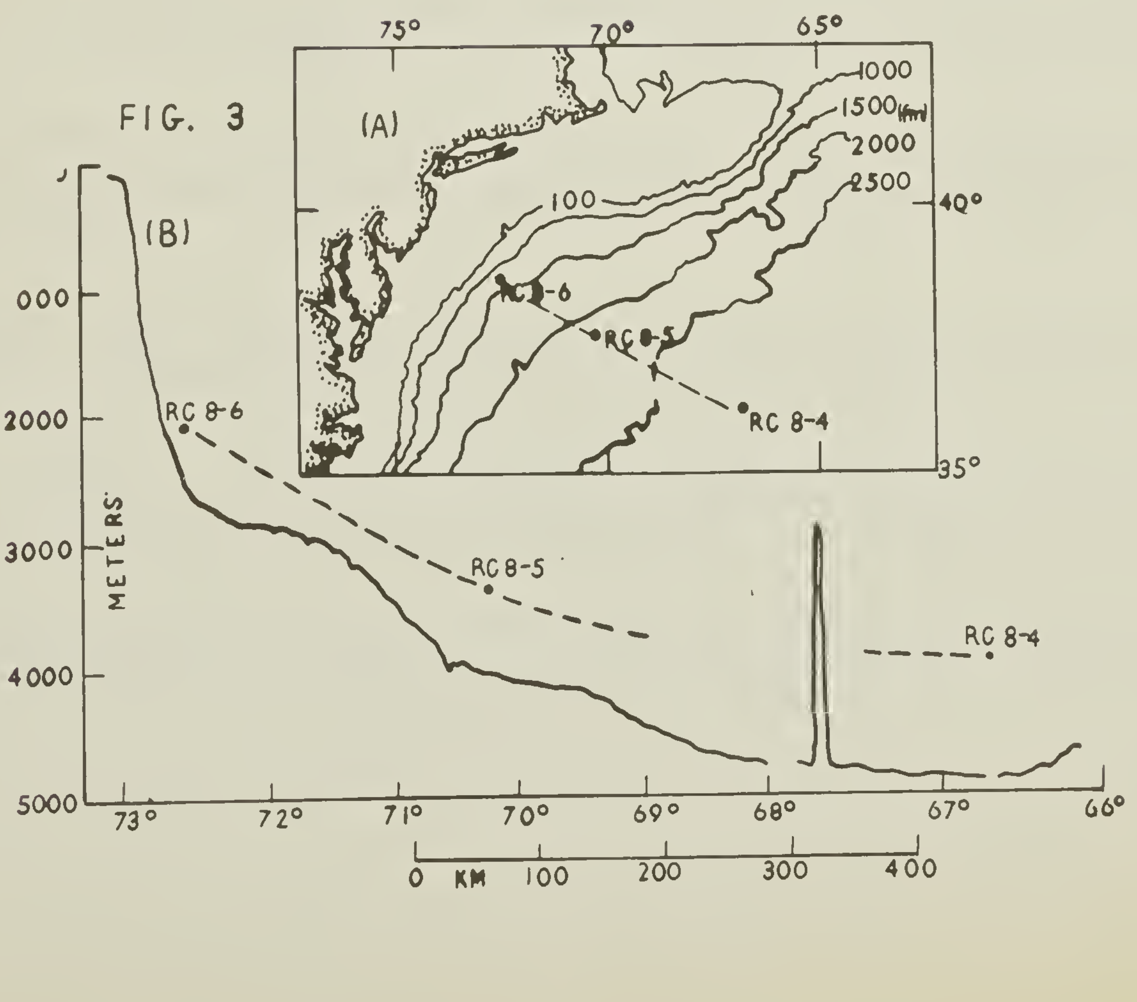


FIG. 3



REFERENCES

Arrhenius, G. (1963) Pelagic Sediments, "The Sea", Vol. III Edited by M. N. Hill, Interscience Publishers, New York (655-727)

Defant, A. (1961) Physical Oceanography, Vol. I, New York, Pergamon Press

Ewing, M. and E.M. Thorndike (1964) Suspended matter in deep ocean water, Science (to be published)

Hurst, H. R. (1929) The suspension of sand in water, Proc. Roy. Soc. London A, 124 (196-201)

Ichiye, T. (1958) On the response of a stratified bounded ocean to variable wind stresses, Oceanog. Mag. 10, (19-63)

Ichiye, T. (1962) Oceanic turbulence (a Review) Technical Report No. 2 to ONR, pp 200 (unpublished manuscript)

Ichiye, T. (1963) A progress report to the National Science Foundation on Research Project supported by GP 1806, pp 7 (unpublished manuscript)

Ichiye, T. (1964) Some hydrodynamic problems in a nepheloid layer (in preparation)

O'Brien, M.P. (1935) Review of turbulent flow and its relation to sediment transportation. Trans. Amer. Geophys. Union, p. 487

Priestly, C.H.B. (1959) Turbulent Transfer in the Lower Atmosphere pp 130 Univ. of Chicago Press

Rossby, C.G. and R.B. Montgomery (1935) The layer of frictional influence in wind and ocean currents, Papers in Phys. Oceanog. Meteorol., 3, (3) 101 pp.

Rouse, H. (1938) Experiments in the mechanics of sediment suspension, Proc. Fifth Int. Congress for Applied Mech. (550-554)

Vanoni, V.A. (1953) A summary of sediment transportation mechanics, Proc. Third Midwestern Conf. on Fluid Mech. (129-160)

Veronis, G. and H. Stommel (1958) The action of variable wind stresses on a stratified ocean, J. Mar. Res. 15(1) (43-76)