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Technical Report on Seismology No. 34

Seismic Model Study of Refractions from a Layer of Finite Thickness

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Seismic Model Study of Refractions from a Layer of Finite Thickness

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ABSTRACT

Two dimensional model experiments on refractions from layers of finite thickness are described. Refractions can be unreliable for velocity and depth determinations when they occur with wavelengths which are large compared to the layer thickness. Discrepancies reported between refraction velocities and bore hole velocities can be partially accounted for in this manner. Even simple two and three layer models can show such effects as misleading second arrivals, echeloning of travel time curves, masked layers, selective absorption in the overburden.

INTRODUCTION

It seems obvious that refraction shooting methods will give a proper velocity for a layer only if the wavelengths of arrivals are small compared to the layer thickness. Yet there has apparently been no discussion of this point in the literature despite the fact that refraction arrivals often occur with wavelengths so long as to cast doubt on the validity of usual methods of interpretation. Indeed, experimental investigations usually reveal discrepancies between layer velocities determined by refraction shooting and by measurements on cores or in boreholes. An excessive value of the ratio of wavelength to layer thickness may well contribute to this discrepancy. This paper is a preliminary report on model experiments where the thickness of the refracting layer is varied, while other parameters are held approximately constant. It is planned to discuss some points of the theory of refraction arrivals from a layer of finite thickness in a following paper.

Earlier work on the theory of refraction arrivals (Jeffreys 1926, Muskat 1933) was concerned primarily with proving that energy could propagate along the ray paths required by travel time data. Consequently the problem was simplified by considering an infinitely thick refractive layer.

More recently Sato (1952) and Officer (1953) studied the refracted wave at large propagation distances in a two layer medium, the bottom layer being infinitely thick and having a higher velocity. They showed independently that the predominant refraction arrivals occur with certain discrete frequencies determined by the condition for constructive interference of waves multiple reflected at the critical angle within the surface layer. Perhaps the most complete paper on the general subject of refracted and reflected waves in a system consisting of a solid layer overlying a semi-infinite solid bottom is that of Newlands (1952). All of these investigations, however, deal with an infinitely thick refracting layer.

An elementary example may be used to show one mechanism through which wavelength becomes a significant factor when the refracting layer has finite thickness. Considering for simplicity a liquid refracting layer of thickness H, sound velocity α , we find that the pulse reflected at almost grazing incidence from the bottom of the layer follows the refracted pulse after an approximate time $2H^2/x\alpha$ where x is the horizontal distance through the layer. For $x \sim 20H$ this corresponds to a time interval of about 1/100 sec when H is 1000 ft and α is 10,000 ft/sec. Now if the spectrum of the source and absorption in the overburden are such that the refractions have significant components with periods greater than 1/100 sec, interference effects occur and the resultant disturbance may travel with velocity different from α . At large distances compared to the layer thickness the refraction pulse may in this manner be inseparable from pulses multiple reflected near grazing incidence within the layer and the resulting interference pattern will have quite different characteristics from the simple refraction pulse of an infinitely thick layer.

EXPERIMENTAL PROCEDURE

The two dimensional model seismology techniques and equipment described by Oliver, Press, and Ewing (1954) were utilized in this work. Models were easily fabricated from sheets of plexiglass, brass and aluminum 1/16 inch thick bonded with Duco cement. Wavelengths long compared to this thickness were used so that the only elastic parameters involved were the plate velocity V_p, the shear velocity β and the density ρ. Except for the substitution of V_p for compressional velocity α, the results are analagous to two dimensional propagation in three dimensional media. The usual method of multiple exposure photography of a cathode ray oscillograph screen was used to simulate a refraction spread.

EXPERIMENTAL RESULTS

Propagation in a single layer. Although not of primary concern in this paper, the single layer is an important preliminary. It is necessary to understand the seismograms from this simple case before proceeding to multilayered media.

A single sheet of plexiglass 6x50 inches serves as a model for a single layer. The source was located 4 inches from a corner and a spread running from 4 - 42 inches with a detector spacing of 2 inches was utilized. The several wave types observed are shown in Figure 1. Seismograms are presented in Figure 2 for the spreads 4 - 20 inches, 22 - 38 inches, 28 - 36 inches, and 36 - 42 inches. A travel time curve appears in Figure 3.

For waves associated with the upper edge, the single sheet models the problem of propagation in a half space first considered by Lamb (1904). For waves arising from reflection or refraction at the bottom interface the single sheet is somewhat analagous to the problem considered by Lapwood (1949), that of propagation from an internal line source in a semi-infinite elastic medium.

The seismograms and travel time curves show the diffracted P wave propagating along the top edge to be the first arrival for the entire spread. A velocity of 7550 ft/sec is indicated for the plate wave in plexiglass. The first wave to emerge from the surface Rayleigh wave is PP. It is seen clearly as a second arrival in the seismogram traces at 12, 14 and 16 inches with the expected reversal in phase. For distances greater than 16 inches PP begins to interfere with P. This merging of P and PP illustrates the elementary case discussed in the introduction. However, serious alteration of P does not occur in this experiment because of poor excitation of grazing PP by the source.

A striking feature of the seismograms is the large amplitudes of the phases PS and SPS. The latter phase had been theoretically established by Nakano (1925) and this may be its first experimental verification. Since SPS follows a least time refraction path it can be used to model the refraction arrival in water covered areas discussed by Officer (1953). Here the single sheet would actually represent two liquid layers, an upper layer of finite thickness with sound velocity β and a semi-infinite lower layer having sound velocity α

Rayleigh waves associated with the upper edge were by far the largest disturbance. As predicted by Lamb they propagate without change in character. The Rayleigh wave associated with the lower edge was not observed as would be expected in view of the large distance (measured in wavelengths) of the source from the edge.

Propagation in two layers. In this model the surface layer was a plexiglass sheet 50 x 4 inches. The refracting layer consisted of an aluminum sheet 50 x 1/4 inches in one case, and 50 x 4 inches in another. Refraction spreads were run from 4 - 36 inches for each case, first on the plexiglass edge, then on the aluminum edge, with detector spacing of 2 inches.

The waves identified for both cases are shown diagrammatically in Figure 4. The direct wave P_1, the Rayleigh wave R_1, and the refraction arrival $P_1P_2P_1$ were observed for the spread on the plexiglass edge. The direct wave P_2, the Rayleigh wave R_2 were observed for the spread on the aluminum edge, as well as the phase $P_2P_1P_1P_2$. This phase has the same travel time as $P_1P_2P_1$ and differs from the latter in that it is initiated and detected in the refracting layer. Seismograms for the cases of thick and thin refracting layer are shown in Figures 5 and 6 respectively. In each figure the seismogram for the spread on the plexiglass edge is on the left and that for the spread on the aluminum edge is on the right. A combined travel time curve appears in Figure 7.

Perhaps the most significant feature of the travel time curve is the lower velocity of P_2 and $P_1P_2P_1$ for the 1/4 inch aluminum layer. That this was not due to inherent differences in the elastic parameters of the aluminum was verified by cutting the 1/4 inch strip from the 4 inch aluminum layer along the same edge used to measure P_2. The reduction in velocity from 17,750 ft/sec to 16,950 ft/sec is of the proper magnitude for the difference between V_P and the one dimensional bar wave with $\bar{V}_P \approx 2\beta\sqrt{1-\beta^2/V_P^2}$
It is not surprising that the velocity of P_2 and $P_1P_2P_1$ measured for the 1/4 inch aluminum layer is $\bar{V}_P$ in view of the large value $\lambda_2/H_2 \sim 8$ for the ratio wavelength λ_2 to thickness H_2. It may be argued that a plate wave cannot exist in a layer loaded on one side by plexiglass. Apparently such a wave can exist under these circumstances of large contrast between the layers (Press and Ewing 1951). To insure that the lower velocity was indeed due to propagation of P_2 as a one dimensional bar wave an additional test was made by freeing the 1/4 inch aluminum strip from the plexiglass and finding that the velocity of P_2 was unchanged.

This experiment contrasts the refraction arrival and direct wave through a thick and thin layer. The results show clearly that the velocities determined differ significantly for the two cases $\lambda \gtrless H$. The results suggest that the velocity of the refraction arrival is identical to that of the direct wave through the layer. Some additional conclusions may be reached by inspection of the seismograms in Figures 5 and 6.

In Figure 5 the direct wave P_2 through the thick aluminum layer may be characterized by its content of both low and high frequencies. Apparently a thick layer will support propagation of P_2 over a large (though not necessarily continuous) range of frequencies. In the same figure the refracted wave $P_1P_2P_1$ is characterized by the presence of only the low frequency components of P_2. This is interpreted as an effect of absorption of the high frequency components ($>$ 100 kc) in the plexiglass along the incident and emergent portions of the $P_1P_2P_1$ path.

In Figure 6 the P_2 arrival for the thin aluminum layer contains mostly high frequency ($\sim$ 100 kc) components. The corresponding wavelength of about 2 inches is large compared to the 1/4 inch layer thickness, hence P_2 for this case is a one dimensional bar wave as discussed earlier. The refraction arrival $P_1P_2P_1$, especially at large distances, contrasts markedly with the corresponding arrival for the thick layer. The former is weak and low frequency components are almost absent. These results are interpreted as an indication of poor excitation of low frequency P_2 energy in a thin layer and absorption by the plexiglass of the predominantly high frequency energy that can be transmitted along the P_2 portion of the refraction path.

The similarity in character of the events $P_2P_1P_1P_2$ and $P_1P_2P_1$ is not surprising since the paths traversed by these phases are identical. However this similarity despite the difference in source and detector location for these phases again suggests that much of the character of the $P_1P_2P_1$ phase is determined by the effects of propagation in the competent P_2 layer and selective absorption of high frequencies in the plexiglass layer.

Propagation in three layers. The three layer model was fabricated from sheets of plexiglass, brass and aluminum 72 inches long. The widths of the layers were as follows:

Case I	Case II
H_1 = 6 inches plexiglass	H_1 = 6 inches plexiglass
H_2 = 1 1/4 inches brass	H_2 = 6 inches brass
H_3 = 8 inches aluminum	H_3 = 8 inches aluminum

Refraction spreads were run along the plexiglass edge from 4 - 60 inches for Case I and 2 - 64 inches for Case II, with detector spacing of 2 inches. In Figure 8 the various types of waves observed are depicted. Seismograms for the two cases are shown in Figures 9, 10, 11, 12, and a travel time curve appears in Figure 13.

As shown in the travel time curve and indicated on the seismograms refractions $P_1P_2P_1$ and $P_1P_3P_1$ were obtained for both cases. For Case I, however, the thin brass layer was masked and $P_1P_2P_1$ could be read only as a second arrival at distances from 12 - 20 inches. This same phase appears clearly in Case II as a second arrival from 8 - 24 inches and as a first arrival from 24 - 38 inches. Examination of the seismograms in Figure 9 shows that $P_1P_2P_1$ has essentially the same pulse-like character for both cases at these small shot-detector distances. With increasing distance, however (Figure 10), the character of these arrivals changes profoundly. For Case I at distances greater than 24 inches $P_1P_2P_1$ and P_1 are overtaken by the weak high frequency aluminum refraction $P_1P_3P_1$. The only event following $P_1P_3P_1$ on the seismogram for these distances is a large amplitude low frequency wave $P_1P_{32}P_1$ which plots in Figure 13 with a velocity of 15, 250 ft/sec, intermediate to that of brass and aluminim. This event is a prominent feature of the seismogram at all distances greater than 24 inches. It is interpreted as a "composite refraction" which because of its long wavelength has a velocity determined by the elastic properties of both the second and third layers. For the thick refracting layer of Case II, $P_1P_2P_1$ maintains its impulsive beginning after it emerges as a first arrival but increases in complexity with distances. However it continues to plot with a velocity appropriate for brass. At 40 inches the aluminum refraction $P_1P_3P_1$ emerges as the first arrival. An attempt was made to pick $P_1P_2P_1$ as a second arrival in the range 56 - 64 inches for Cast II. Although a sharp event $P_1P_{23}P_1$ is present on the seismogram its velocity of 13,900 ft/sec is again too high for brass. The phase $P_1P_{23}P_1$ is also interpreted as a composite refraction, with velocity closer to that of brass than was the case with $P_1 P_{32}P_1$, indicative of the smaller value of λ_2/H_2. These results for $P_1P_2P_1$ suggest that only at relatively short shot-detector distances do refraction arrivals from a layer of finite thickness provide reliable velocity (and depth) determinations. At larger distances, especially when the refraction occurs as a second arrival, erroneous determinations of velocity may occur when the principal wavelengths of the refractions are large compared to the layer thickness.

An additional significant result is made evident by comparing the character and velocity of $P_1P_3P_1$ for cases I and II. In Figure 12 it is seen that the character of these refractions are entirely different for the two cases and the travel time curve of Figure 13 indicates a velocity difference of about 2%. The identical sheet of aluminum was used for the P_3 refracting layer in both these cases so that differences in elastic constants or thickness of this layer are ruled out. One must conclude that even refractions from a very thick layer are effected by the layering in the overburden. This is not surprising in view of the somewhat analagous results of Officer (1953) for the refraction arrival in water covered areas.

<u>Discussion.</u> With great simplification we may ascribe the character of a refraction arrival to three factors: (1) the spectrum of the source; (2) absorption and scattering in the layers above the refracting horizon; (3)(the effects of transmission through the refracting horizon. This picture would be particularly applicable when the refracting layer is an excellent transmitter of elastic waves in contrast to an absorbing and scattering overburden. Under these conditions multiple reflection and constructive interference in the overburden need not be considered. Factors 1 and 2 determine the nature of the pulse delivered to the refracting layer. The third factor effects both the velocity and character of the refraction arrival. If predominantly low frequency energy is available for transmission through the refracting layer, our results suggest that the velocity determination can be unreliable if the ratio of wavelength to layer thickness is too large. This is especially true when refractions are picked as second arrivals. In addition an excitation function for horizontal transmission through the refraction layer also effects the relative amplitudes of the component frequencies that make up the refraction arrival. Comparison of P_2 and $P_1P_2P_1$ in the two layer model suggests that this excitation function is related to the corresponding function for source and receiver in the transmitting layer. Excitation functions for special cases have been discussed by Pekeris (1948), Press and Ewing (1950).

A common difficulty encountered in refraction shooting may be termed* "shingling" or "echeloning" of the travel time curve. Instead of plotting as continuous straight line segments the travel time curves appear as discontinuous, offset segments, the velocities indicated by the segments often being erratic. Although it is hazardous to extrapolate from our as yet too simple models to the more complicated conditions known to exist in the field, similar features are observable in the model. For example, had we decreased our initial pulse amplitude, or had we used a more absorbent medium for high frequencies than plexiglass the $P_1P_3P_1$ refraction for Case I in Figure 10, 11 and 12 would not have been observed. Similarly the $P_1P_3P_1$ refraction for Case II in Figure 12 could not have been picked. In both cases a second arrival would have been plotted with a resultant echeloning of the travel time. The velocities indicated by these second arrival travel time segments $P_1P_{32}P_1$ and $P_1P_{23}P_1$ in Figure 13 are misleading and would certainly not agree with well shooting determinations. The models show how echeloning can occur even under relatively simple conditions.

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* Terms used by Karl Dyk, personal communication, 1953.

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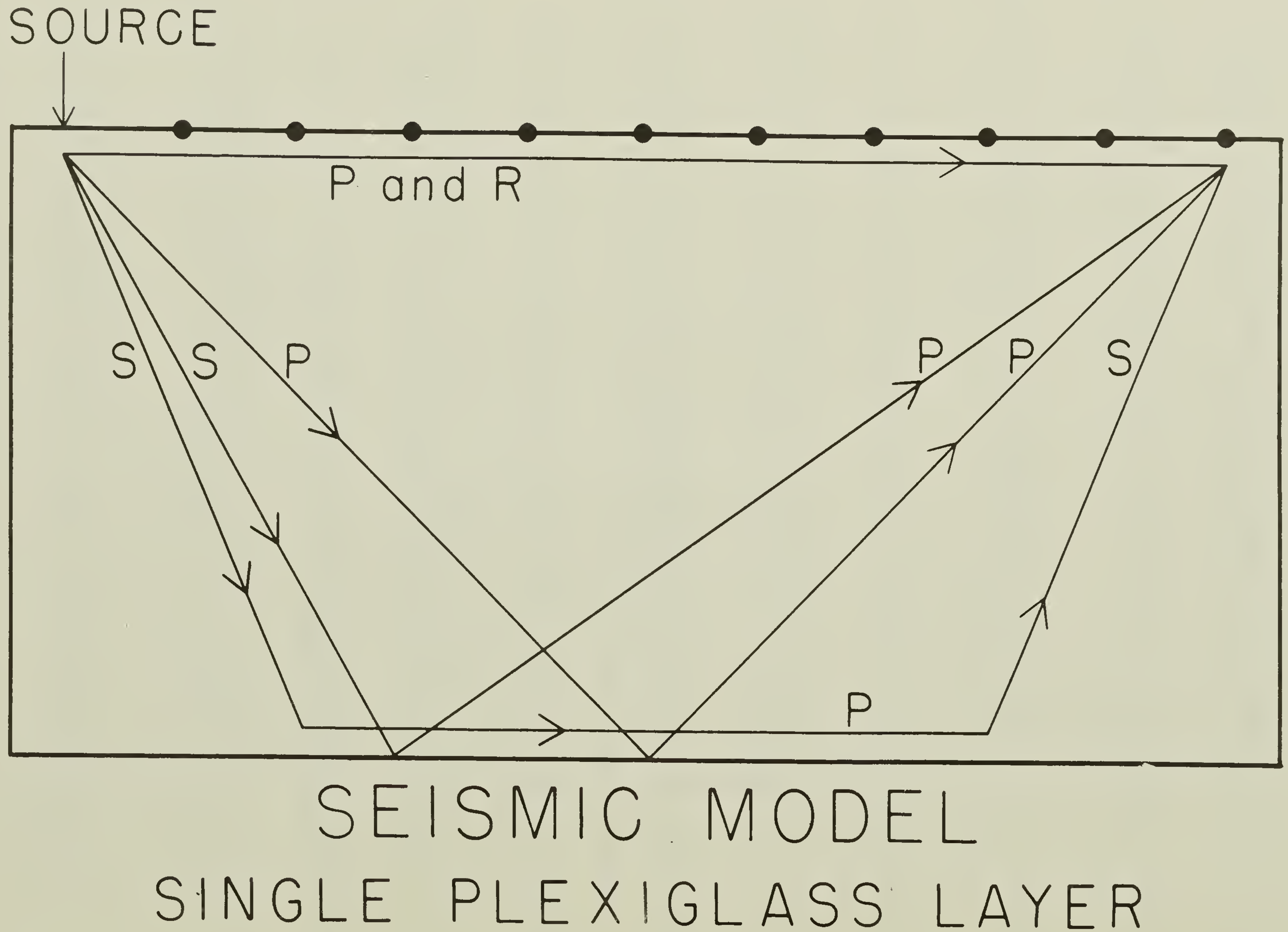

Figure 1. Types of waves observed in single layer model.

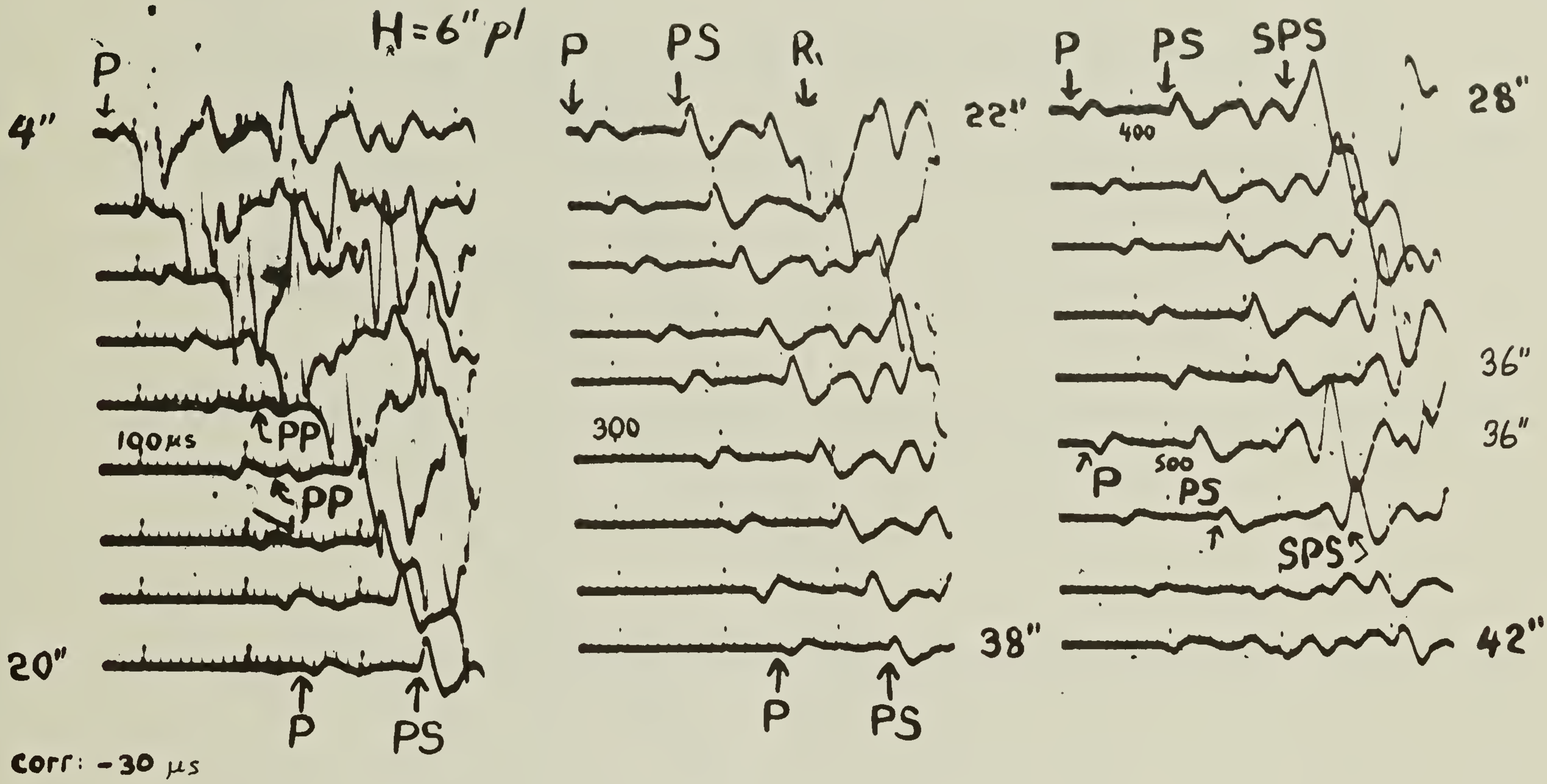

Figure 2. Seismograms for spreads 4 - 20 inches, 22 - 38 inches, 28 - 36 inches on a single layer of plexiglass, H = 6 inches. Time marks shown are at 100 and 10 microsecond intervals.

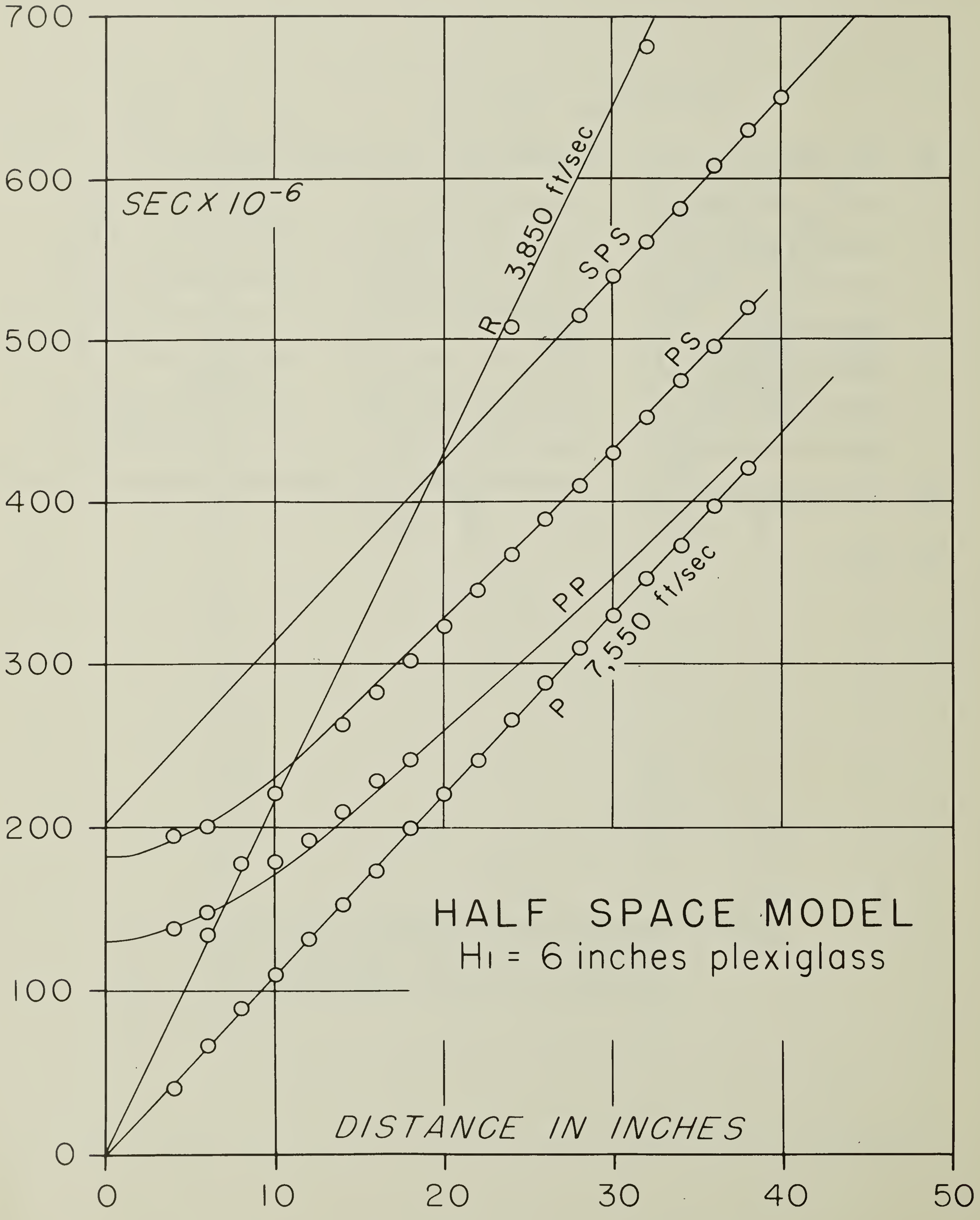

Figure 3. Travel time curve for a single layer model.

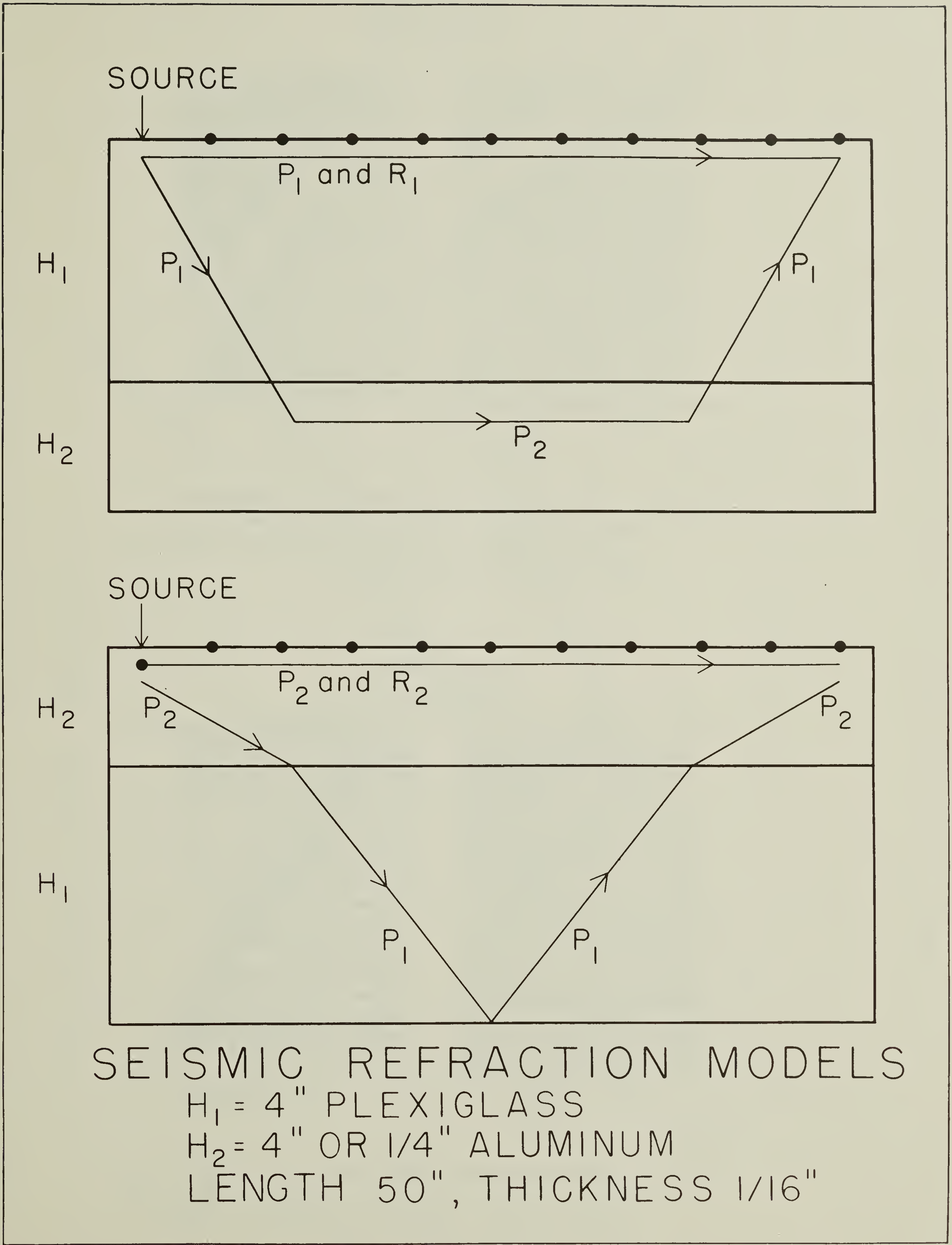

Figure 4. Types of waves observed in double layer model.

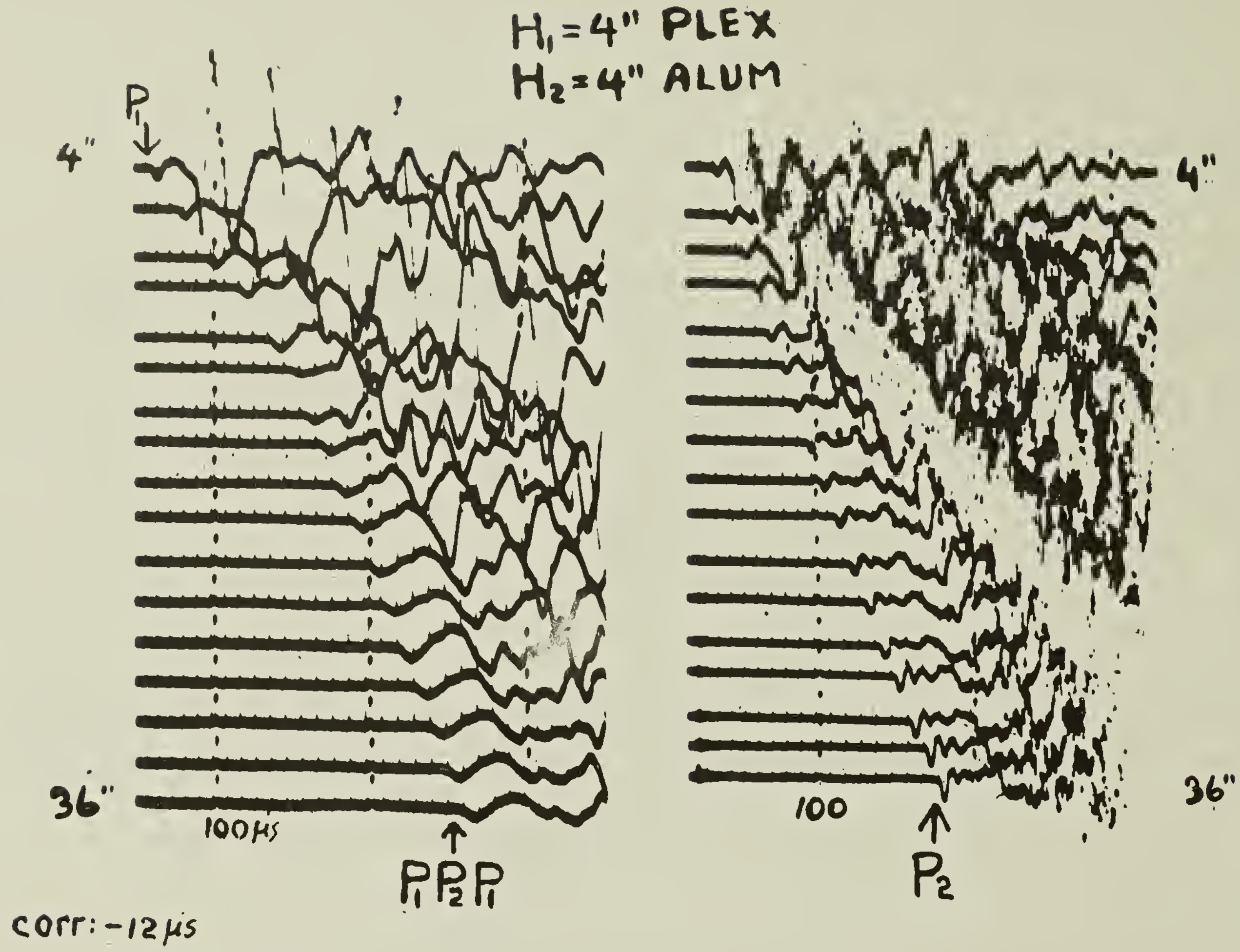

Figure 5. Seismograms for spreads 4 - 36 inches on a double layer, H_1 = 4 inches plexiglass, H_2 = 4 inches aluminum. Spread on plexiglass edge on left, aluminum edge on right.

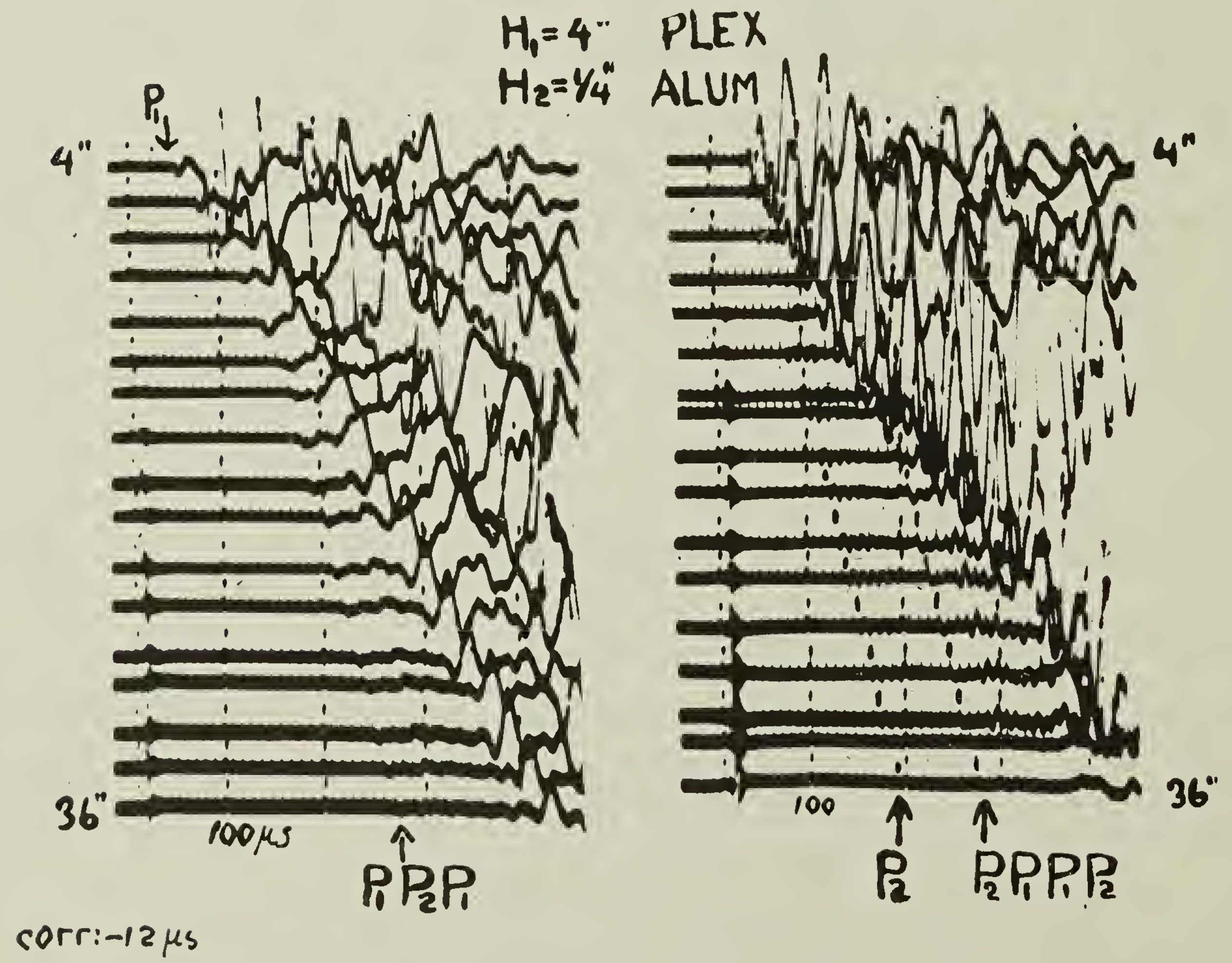

Figure 6. Seismograms for spreads 4 - 36 inches on a double layer H_1 = 4 inches plexiglass, H_2 = 1/4 inch aluminum. Spread on plexiglass edge on left, aluminum edge on right.

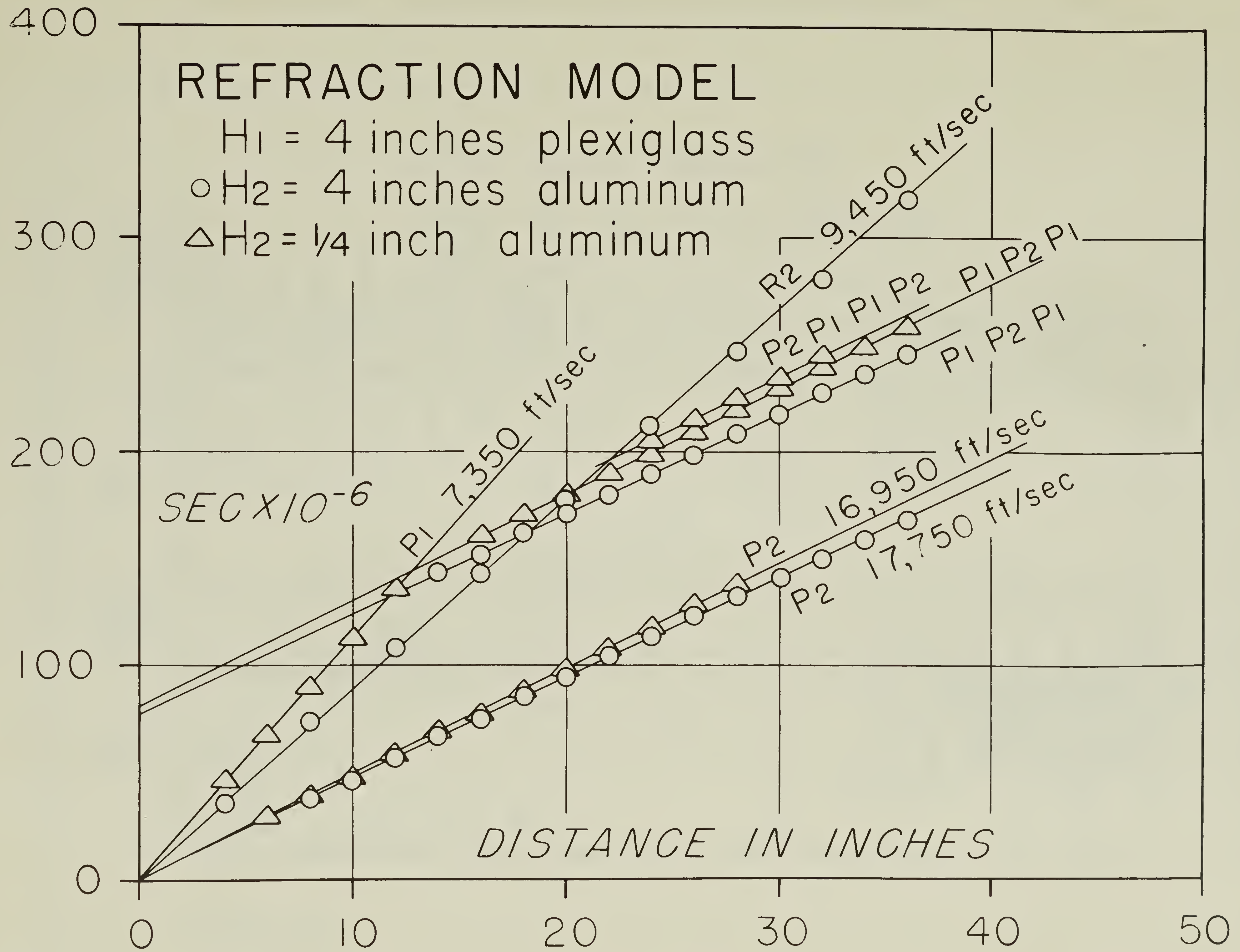

Figure 7. Travel time curves for double layer model.

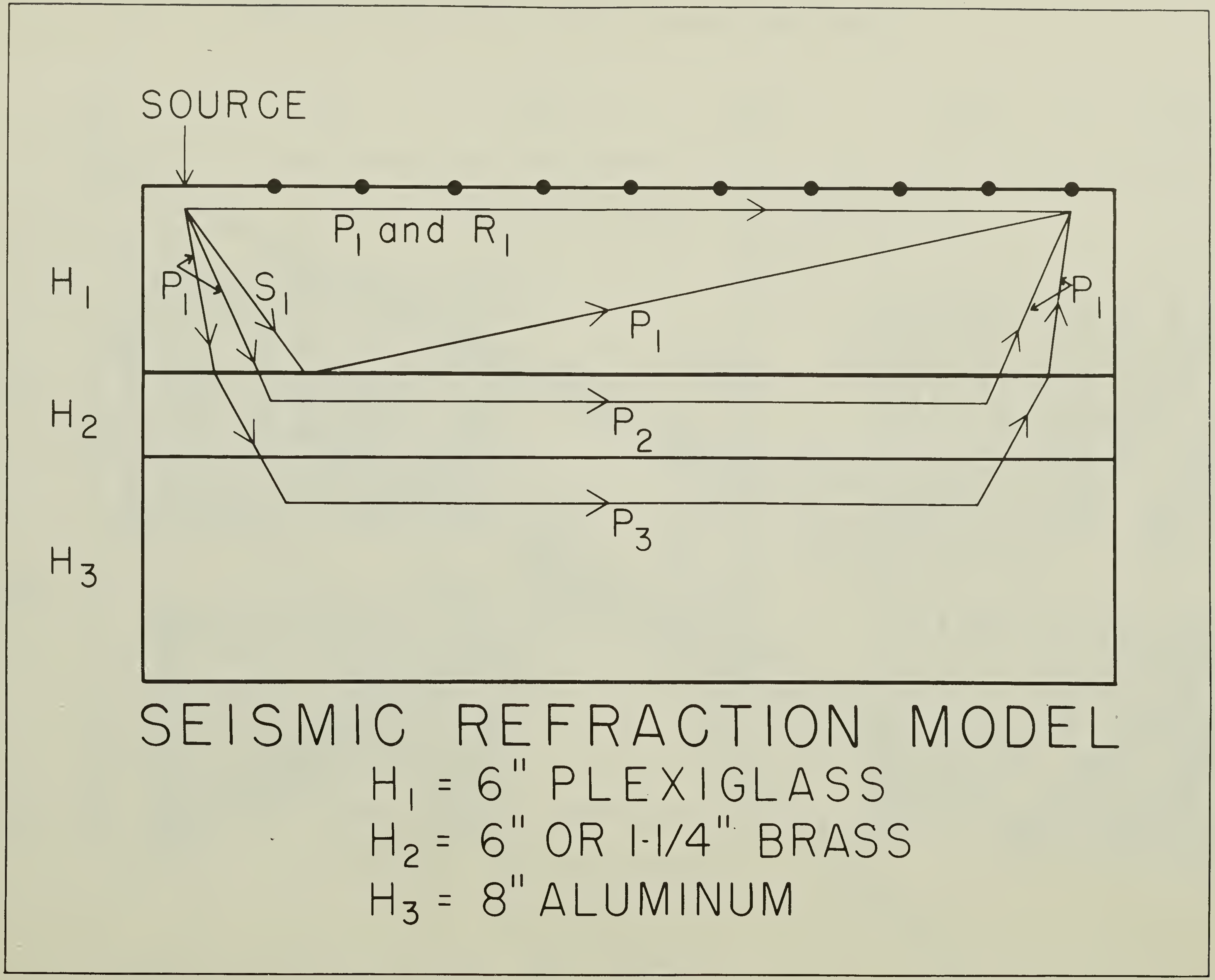

Figure 8. Types of waves observed in triple layer model.

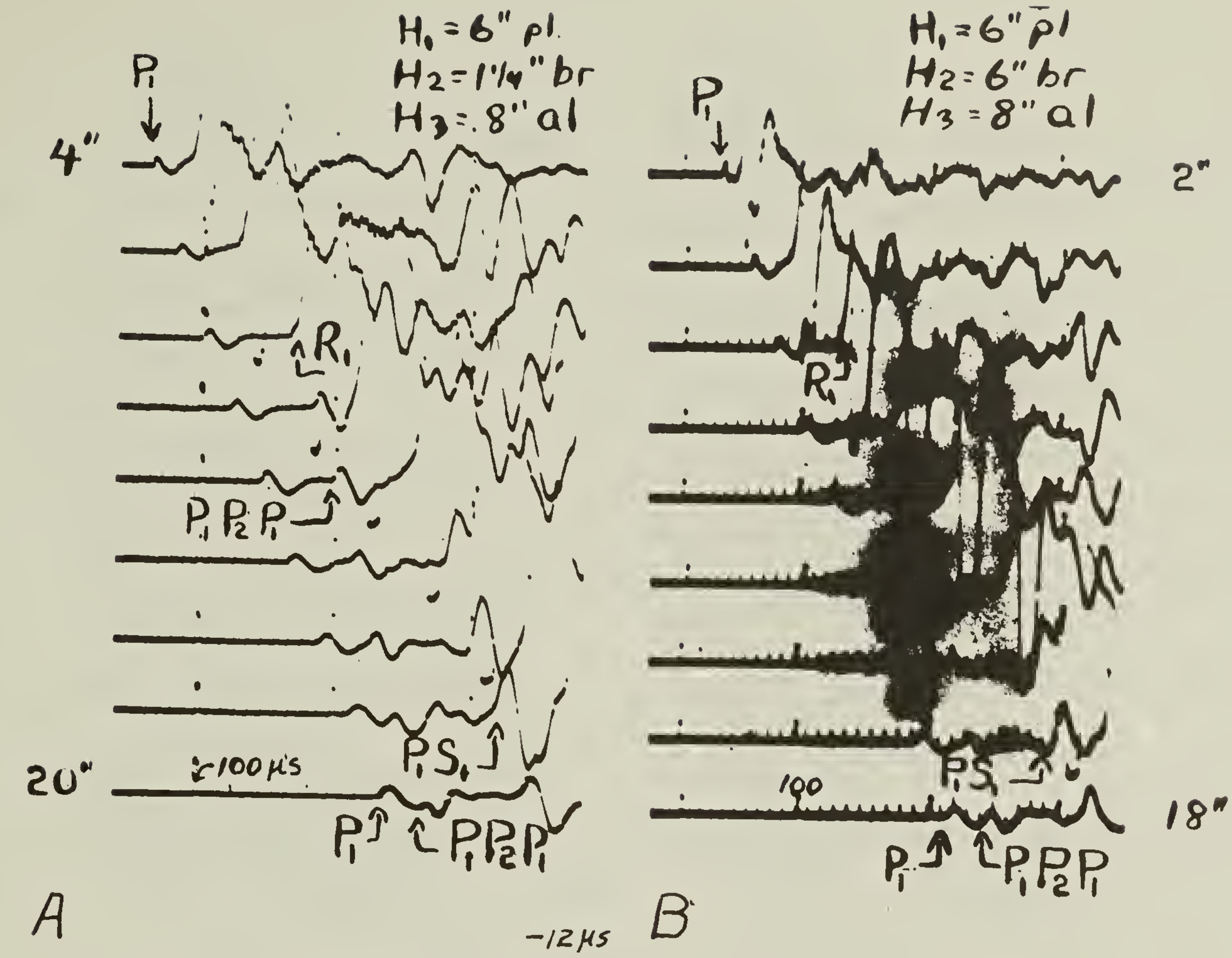

Figure 9. A. Seismogram for spread 4 - 20 inches on triple layer model Case I. B. Spread 2 - 18 inches Case II.

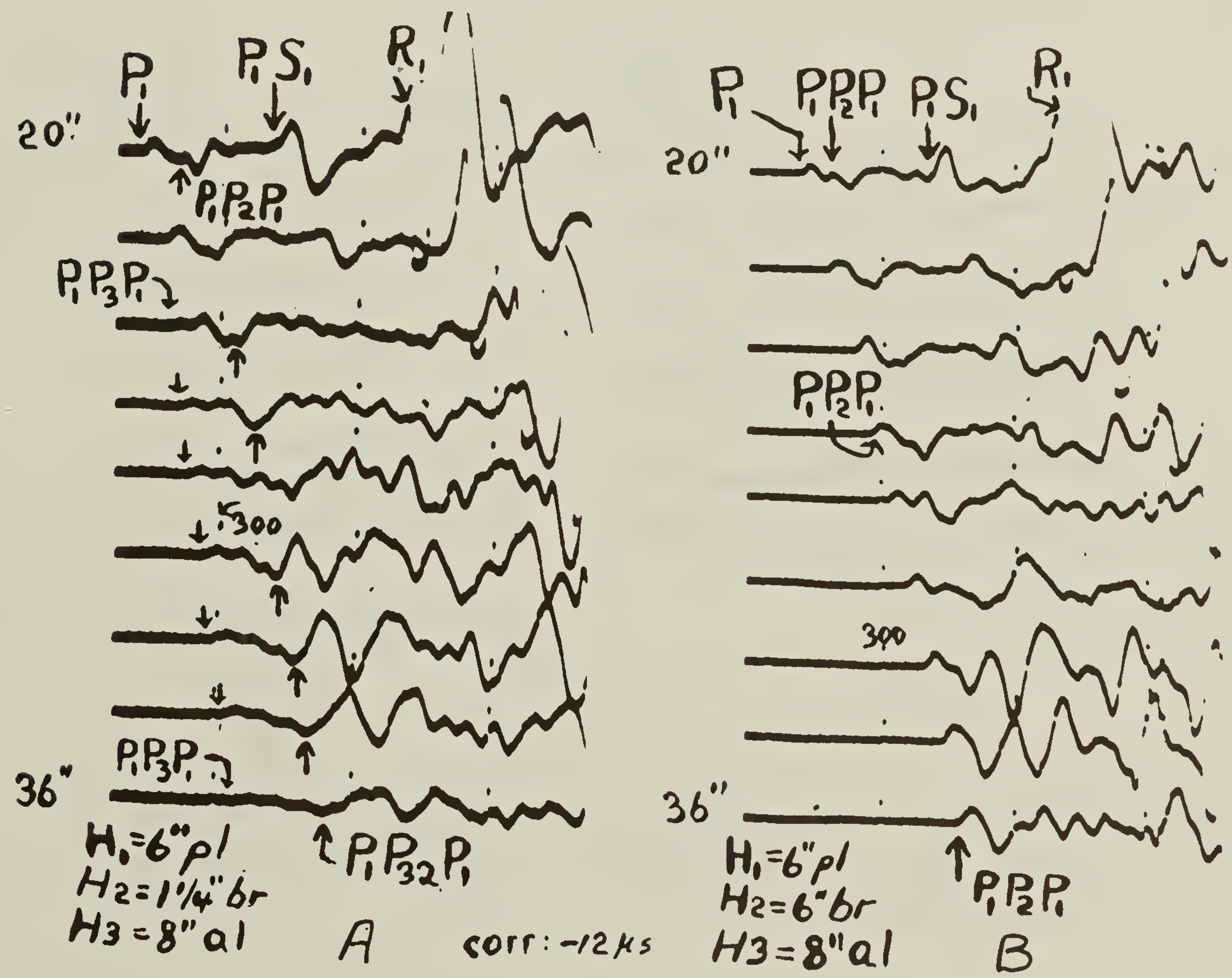

Figure 10. A. Seismogram for spreads 20 - 36 inches on triple layer model Case I. B. Case II.

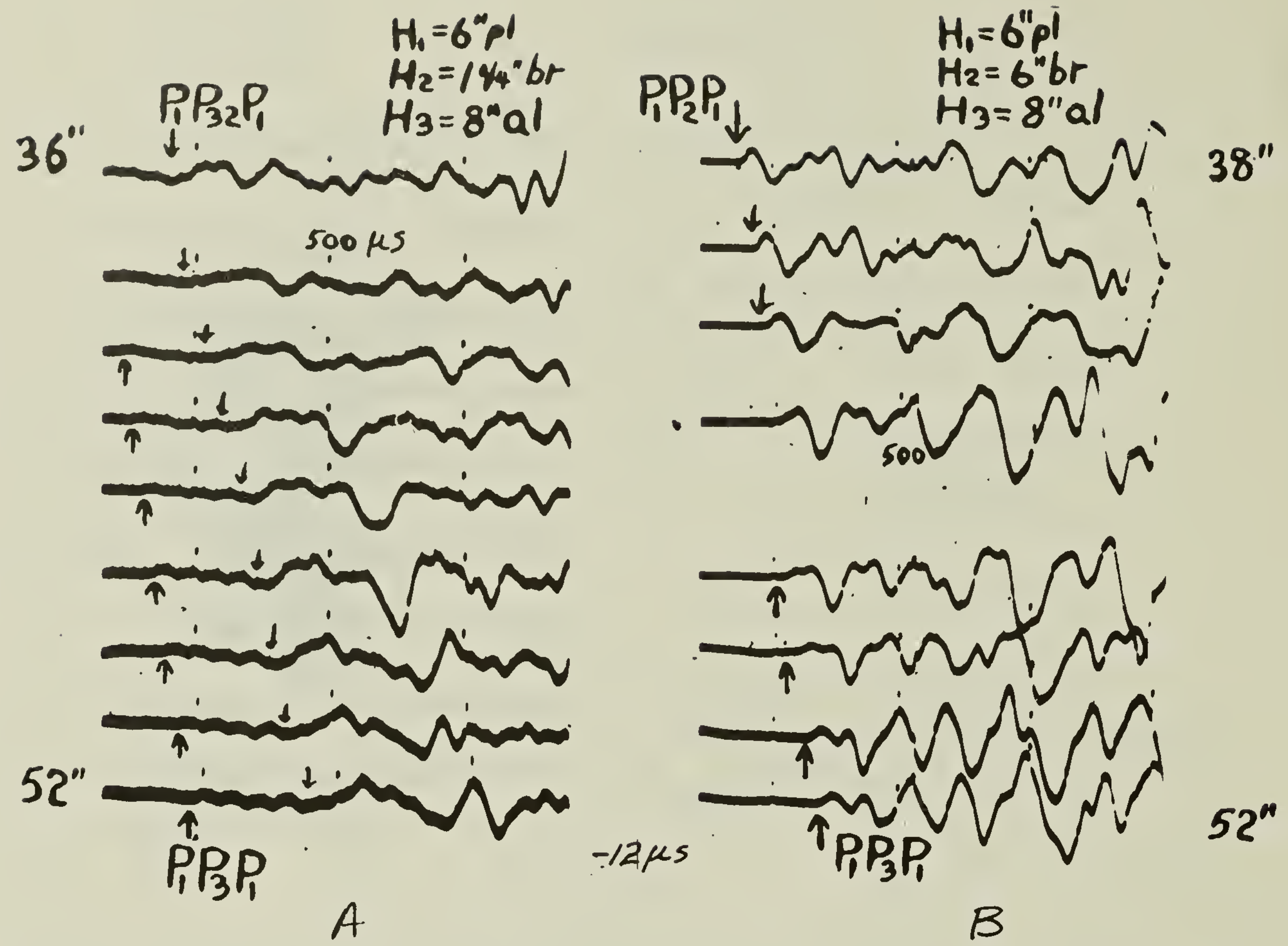

Figure 11. A. Seismogram for spreads 36 - 52 inches on triple layer model Case I. B. Spread 38 - 52 inches Case II.

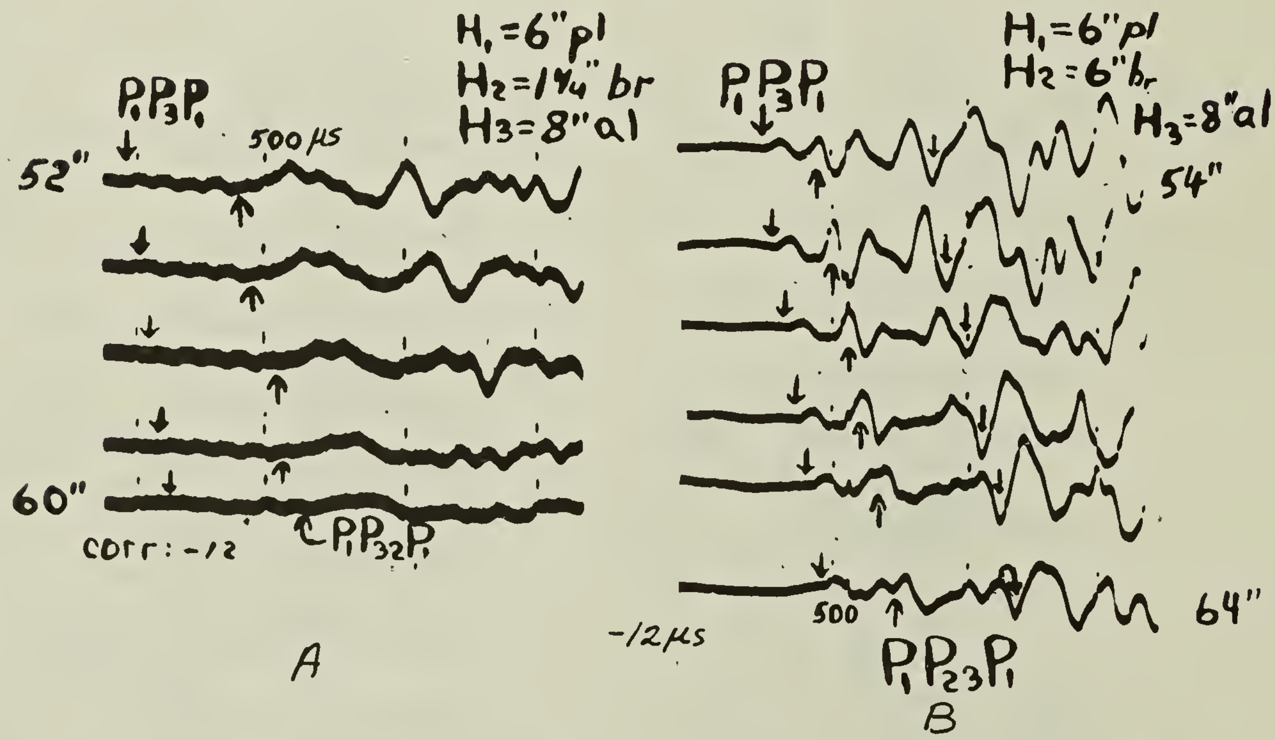

Figure 12. A. Seismogram for spread 52 - 60 inches on triple layer model case I. B. Spread 54 - 64 inches Case II.

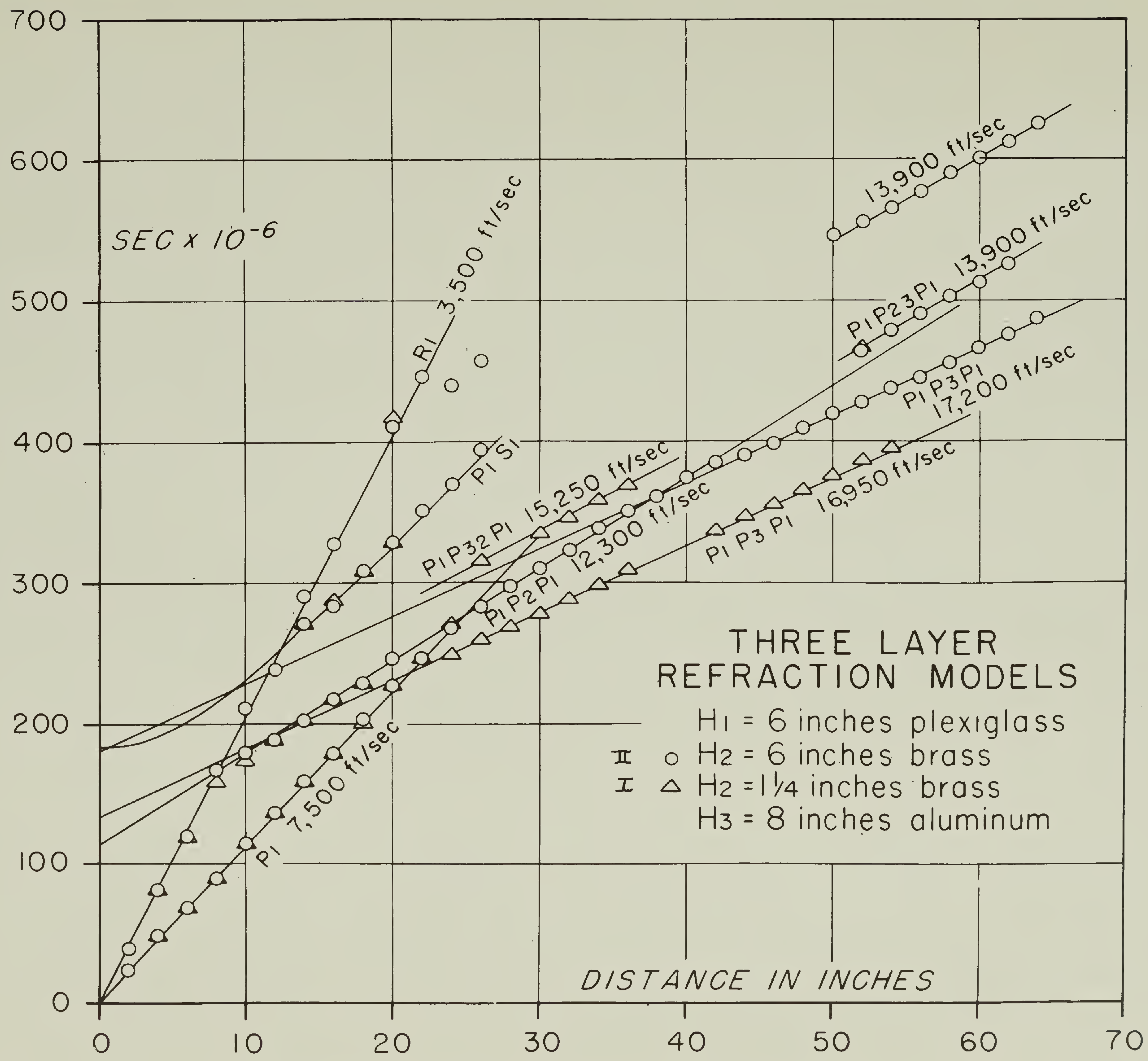

Figure 13. Combined travel time curves for Cases I and II of three layer model.