Columbia University
in the City of New York

LAMONT GEOLOGICAL OBSERVATORY
PALISADES, NEW YORK

The Use of Topographic Highlights for the Measurement of Ship's Ground Speed

by

B. Luskin, J. E. Nafe and M. Ewing

Technical Report No. 15

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CONTENTS

	Page
INTRODUCTION	1
THEORY OF THE METHOD	
A. General	2
B. Two-Point Method	7
C. Three-Point Method	9
D. Hyperbolic Approximation	11
ERRORS	
A. General	13
B. Timing Errors	14
C. Sound Propagation Errors	16
D. Basic Assumptions	19
E. Hyperbolic Assumption	25
F. Summary of Errors	27
HIGHLIGHT POPULATION	30
APPLICATIONS	
A. Current Measurement	38
B. Inertial Navigation	41
CONCLUSIONS	46
ACKNOWLEDGMENTS	48
REFERENCES	49

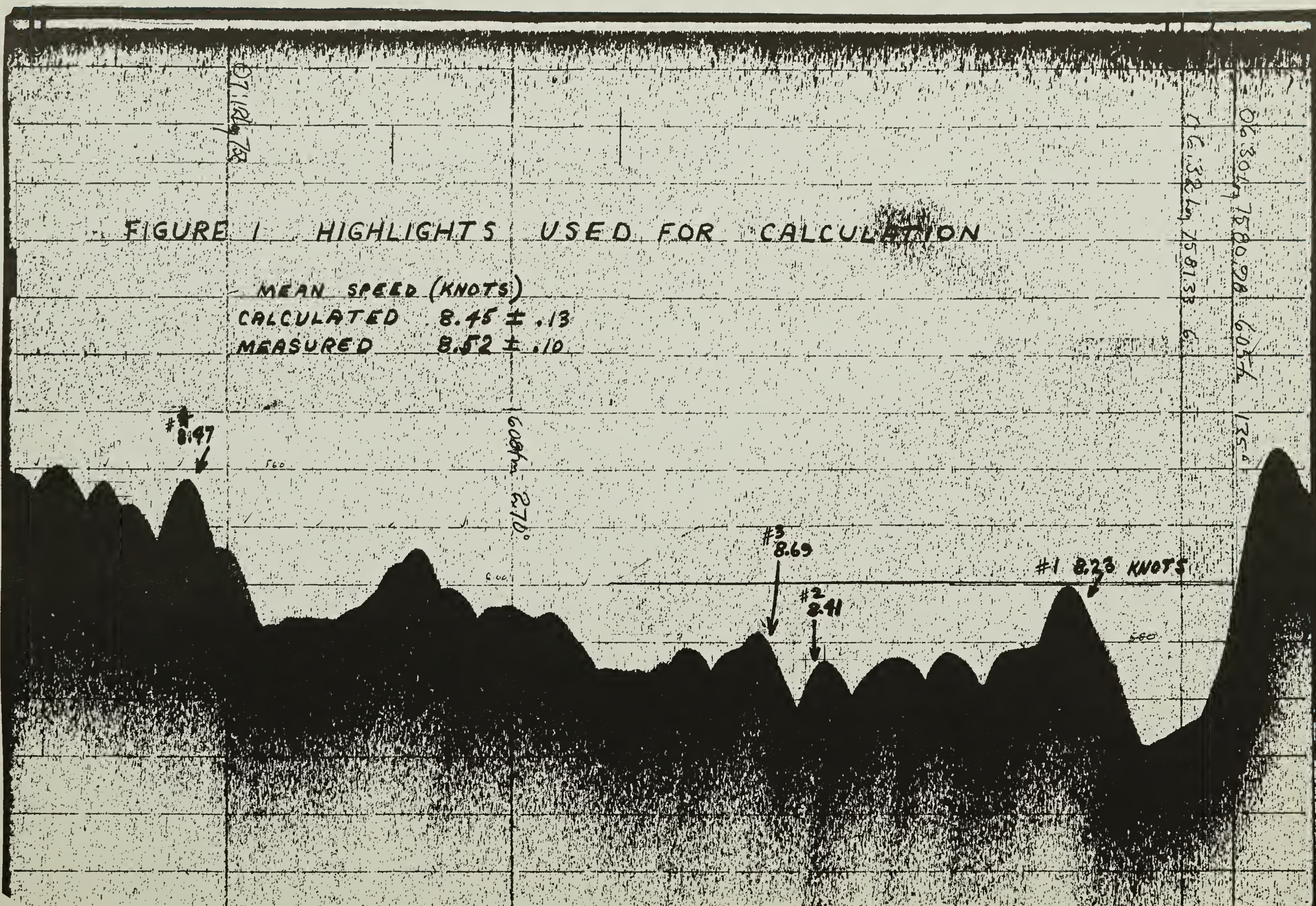


FIGURE 1 HIGHLIGHTS USED FOR CALCULATION

INTRODUCTION

A feature frequently observed on the records of expanded scale echo sounders is the characteristic near-hyperbolic trace of a reflecting area of dimensions small compared to the depth of water. Following the initial detection of such a small reflecting spot or highlight, the recorded distance to the highlight decreases to a minimum value and then increases again until the echo is lost. The shape of the trace so obtained depends upon the ship's speed over the bottom and may be used to measure that speed. Furthermore, the calculation required to obtain speed is not modified in any way when the highlight is displaced laterally from the ship's track. The method is extremely simple, can be utilized with existing equipment, and promises to attain an accuracy of speed determination of the order of 1% with little development.

THEORY OF THE METHOD

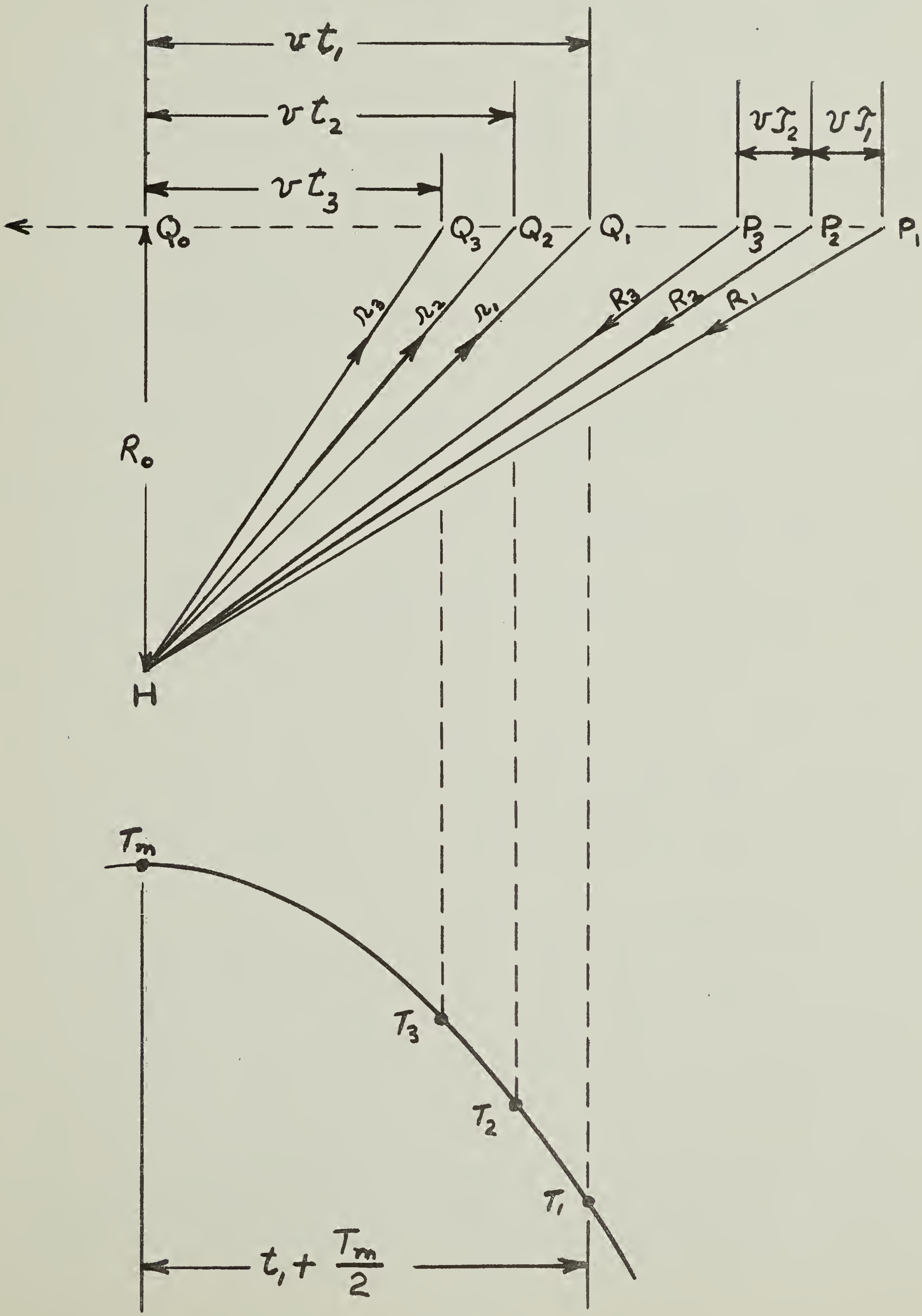
A. General

Several recorded traces of the highlight echoes are shown in Figure 1. The record was made on a Precision Depth Recorder Mark V operating with the Transmitter-Transducer-Receiver equipment of a SONAR Sounding Set AN/UQN-1B (Luskin and Israel, 1956),(NAVSHIPS 91420, 1951). The recorder is an expanded scale type scanning the recording surface once per second so that the depth scale is 400 fathoms equals 18.85 inches at the standard sounding velocity of 800 fathoms per second. The recording chart is fed at 24 inches per hour. The depth scale is subdivided into 20-fathom (50 millisecond) intervals by internally generated pulses which are automatically printed as the record is made. The 20-fathom mark lines are interrupted every 3 minutes to provide calibration of the horizontal time scale.

Those highlights which truly satisfy the criterion that they be small in all dimensions relative to the depth give traces which show no deviation from a smooth near-hyperbolic shape. The shape may be calculated with the aid of Figure 2.

Consider a ship at point P_1 proceeding toward Q_0 at constant speed v, and on a steady course. Sonic pulses are transmitted at P_1, P_2, P_3. The sound is reflected from the highlight at H, and the pulses received when the ship is respectively at Q_1, Q_2, Q_3. Let Q_0 be the point of closest approach along the ship's track to the highlight at H,

FIGURE 2



vt₁
vt₂
vt₃
vτ₂
vτ₁
Q₀
Q₃
Q₂
Q₁
P₃
P₂
P₁
r₃
r₂
r₁
R₃
R₂
R₁
R₀
H
Tm
T₃
T₂
T₁
t₁ + Tm/2

and R_0 be the distance $\overline{Q_0 H}$. vt_1, vt_2, and vt_3 are the distances $\overline{Q_1 Q_0}$, $\overline{Q_2 Q_0}$ respectively.

The speed of sound propagation is assumed to be known, constant, and equal to V. The transmitted pulse at P_1 travels a distance R_1 to H, is reflected, and travels a distance r_1 to Q_1, where it is received. The corresponding travel time is T_1, a point on the sounding trace. At a time $\mathcal{T}_1$, after passing P_1, a second pulse is transmitted at P_2, travels the distances R_2 and r_2 to and from H, and is received at Q_2 after a travel time T_2. Similarly, a travel time T_3 is recorded for a pulse transmitted at P_3, at a time $\mathcal{T}_2$ after passing P_2. The soundings are recorded at short time intervals so that the curve T_1-T_2-T_3 is essentially continuous. Let T_m be the minimum recorded sounding.

For each pulse, we may write the following set of equations:

$$\left.\begin{aligned} R + r &= VT \\ R^2 &= R_0^2 + v^2(t+T)^2 \\ r^2 &= R_0^2 + v^2 t^2 \end{aligned}\right\} \qquad (1)$$

Eliminating R and r, we have

$$T^2 - \alpha^2 tT - \alpha^2 t^2 = \beta^2 T_0^2 \qquad (2)$$

where

$$\alpha = \frac{2v}{\left[V^2 - v^2\right]^{\frac{1}{2}}} = \frac{2v}{V}\left\{1 + \frac{1}{2}\left(\frac{v}{V}\right)^2 + \cdots\right\} \qquad (3a)$$

$$\beta = \frac{V^2}{V^2 - v^2} = 1 + \left(\frac{v}{V}\right)^2 + \left(\frac{v}{V}\right)^4 + \cdots \qquad (3b)$$

$$T_o = \frac{2\,R_o}{V} \qquad (3c)$$

Differentiating (2) with respect to t, and setting the derivative equal to zero, we have

$$\frac{dT}{dt} = \frac{\alpha^2 (2t + T)}{2T - \alpha^2 t} = 0$$

so that the surface time range for the minimum sounding is

$$t_m = -\frac{T_m}{2}$$

and

$$T_m = T_o\left[1 - \left(\frac{v}{V}\right)^2\right]^{\frac{1}{2}} = T_o\left\{1 + \frac{1}{2}\left(\frac{v}{V}\right)^2 + \cdots\right\} \qquad (4)$$

From (2), the normalized speed, α, is given by

$$\alpha = \left[\frac{T^2 - \beta^2\, T_o^2}{t\,(t + T)}\right]^{\frac{1}{2}} \qquad (5)$$

We note now that the ratio of the ship's speed and the speed of sound is of the order of 10^{-2}. If V is 800 fms/sec and the ship's speed is 20 knots, v/V is equal to .007 and $(v/V)^2$ is .000049. Neglecting terms of the order of $(v/V)^2$ compared to unity, from (3) and (4)

$$\left.\begin{aligned} \alpha &= \frac{2v}{V} \\ \beta &= 1 \\ T_m &= T_0 \end{aligned}\right\} \qquad (6)$$

Therefore, with negligible error (less than .1%), we may rewrite (5) as

$$v = \frac{V}{2}\left[\frac{T^2 - T_m^2}{t(t+T)}\right]^{\frac{1}{2}} \qquad (7)$$

When the ship has passed Q_0, Equation (1) becomes

$$\left.\begin{aligned} R + r &= VT \\ R^2 &= R_0^2 + v^2(t-T)^2 \\ r^2 &= R_0^2 + v^2 t^2 \end{aligned}\right\} \qquad (8)$$

Eliminating R and r as before, we have

$$T^2 + \alpha^2 tT - \alpha^2 t^2 = \beta^2 T_0^2 \qquad (9)$$

and with the same approximations as before

$$v = \frac{V}{2}\left[\frac{T^2 - T_m^2}{t(t-T)}\right]^{\frac{1}{2}} \qquad (10)$$

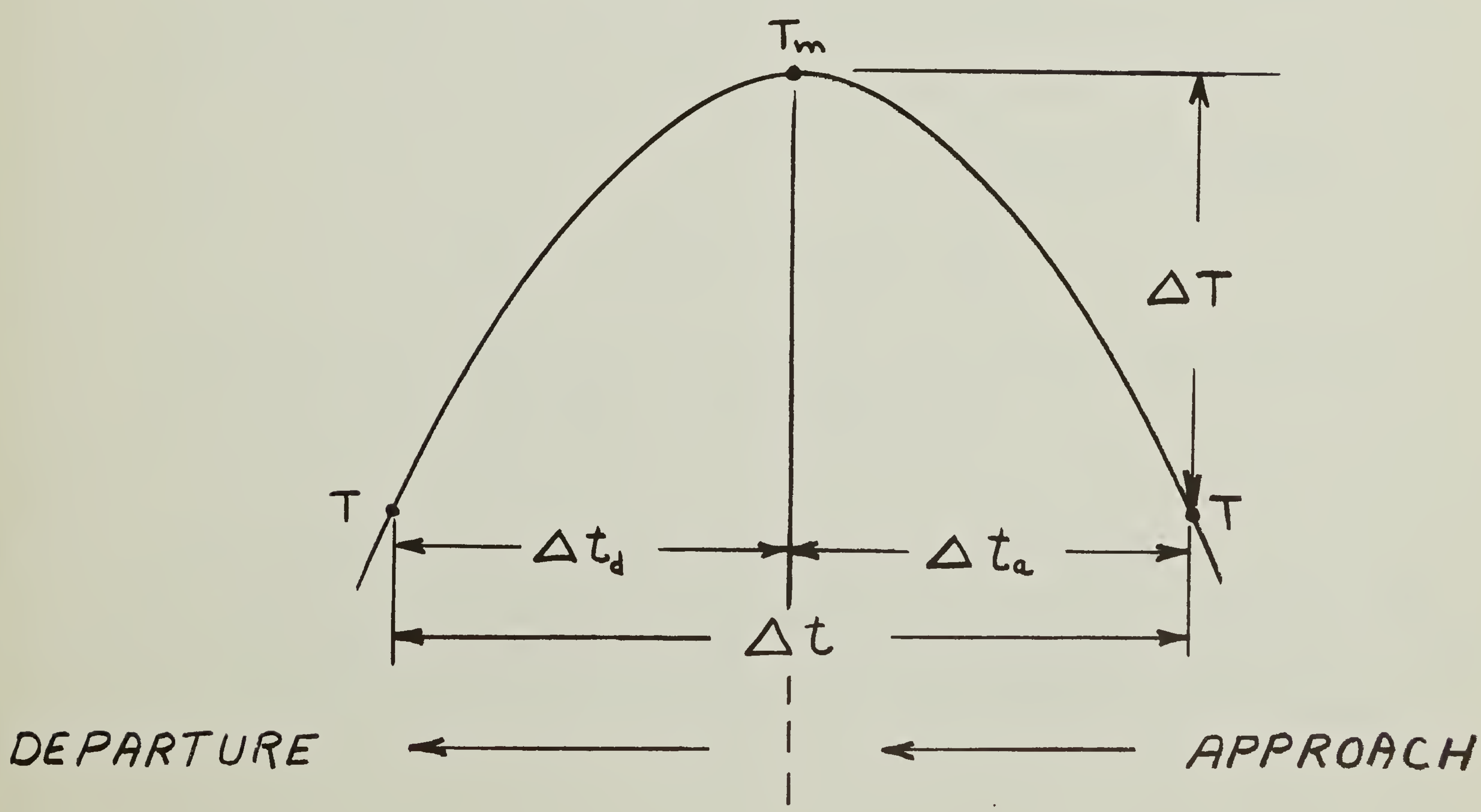
Let the subscripts a and d refer the quantities to the approach and departure portions of the ship's track with respect to Q_0. If $T_a = T_d = T$, from (2) and (9)

$$t_d = t_a + T \qquad (11)$$

B. <u>Two-Point Method</u>

We can now state the simple formulas for calculating the ship's speed from the sounding trace. Refer to Figure 3.

FIGURE 3



If a complete sounding trace is available, we measure the quantities T_m, T and Δt. Then v is given by

$$v = .7071\, V \left[\frac{\Delta T \quad \bar{T}}{t_a \, t_d} \right]^{\frac{1}{2}} \tag{12}$$

where

$$\left.\begin{aligned} \Delta T &= T - T_m \\ \bar{T} &= \tfrac{1}{2}\left(T + T_m\right) \\ t_a &= \tfrac{1}{2}\left(\Delta t - T\right) \\ t_d &= \tfrac{1}{2}\left(\Delta t + T\right) \end{aligned}\right\} \tag{13}$$

If only enough of the approach portion of the sounding trace is available so that T_m may be determined, we measure T_m, T, and Δt_a. In (12) we now have

$$\left.\begin{aligned} t_a &= \Delta t_a - \frac{T_m}{2} \\ t_d &= \Delta t_a + T - \frac{T_m}{2} \end{aligned}\right\} \tag{14}$$

If only enough of the departure portion of the sounding trace is available so that T_m may be determined, we measure T_m, T and Δt_d.

In (12) we now have

$$\left.\begin{aligned} t_a &= \Delta t_d + \frac{T_m}{2} - T \\ t_d &= \Delta t_d + \frac{T_m}{2} \end{aligned}\right\} \qquad (15)$$

If t and T are measured in the same units (e.g. seconds), v has the units of V.

C. Three-Point Method

If only a portion of the sounding trace is available with T_m indeterminate, we require a formula involving three sounding measurements. In Figure 2, suppose that we know T_1, T_2, T_3, $\mathcal{T}_1$ and $\mathcal{T}_2$. Rewriting (2), we have

$$T_i^2 - \alpha^2 t_i T_i - \alpha^2 t_i^2 = \beta^2 T_0^2 \; ; \quad i = 1, 2, 3 \qquad (16)$$

Also

$$t_1 = t_2 + \mathcal{T}_1 + T_2 - T_1$$

and

$$t_3 = t_2 - \mathcal{T}_2 + T_2 - T_3 \qquad (17)$$

We have 5 unknown quantities: α, T_0, t_1, t_2, t_3; and 5 equations. Eliminating all the unknowns but α, we have

$$\alpha = \left[\frac{(T_1^2 - T_2^2)(T_2 - T_3 - 2\mathcal{T}_2) - (T_3^2 - T_2^2)(T_2 - T_1 + 2\mathcal{T}_1)}{(T_2 + \mathcal{T}_1)(T_2 - T_1 + \mathcal{T}_1)(T_2 - T_3 - 2\mathcal{T}_2) - (T_2 - \mathcal{T}_2)(T_2 - T_3 - \mathcal{T}_2)(T_2 - T_1 + 2\mathcal{T}_1)} \right]^{\frac{1}{2}} \qquad (18)$$

In usual sounding procedure, pulses are transmitted at regular intervals corresponding to the scanning periods of the recorder. For this case, $\tau_1 = \tau_2 = \tau$ and

$$\alpha = \left[\frac{(T_1^2 - T_2^2)(T_2 - T_3 - 2\tau) - (T_3^2 - T_2^2)(T_2 - T_1 + 2\tau)}{\tau \left\{ (T_1 - \tau)(T_3 - T_2 + 2\tau) + (T_3 + \tau)(T_1 - T_2 - 2\tau) \right\}} \right]^{\frac{1}{2}} \qquad (19)$$

A considerable simplification of (18) may be obtained by operating the sounder as a transponder. Suppose that a pulse is transmitted at P_1 (Figure 2) as before. Let no pulses be transmitted until time T_1, when the pulse is received. Assume an arrangement where the received pulse instantaneously triggers the transmitted. For this case, $\tau_1 = T_1$ and $\tau_2 = T_2$ and

$$v = \frac{V}{2} \left[\frac{T_1 - 2T_2 + T_3}{T_2} \right]^{\frac{1}{2}} \qquad (20)$$

Furthermore, knowing v, the range to Q_0 may be predicted from

$$t_m = \frac{T_m - T_{m+1}}{\left(\frac{2v}{V}\right)^2} \quad ; \quad m = 1, 2, 3, \cdots \qquad (21)$$

and the minimum sounding predicted from

$$T_m = \left[T^2 - \left(\frac{2v}{V}\right)^2 t\,(t + T) \right]^{\frac{1}{2}} \qquad (22)$$

D. Hyperbolic Approximation

At great ranges from the highlight, T becomes insignificant compared to t and the sounding trace approaches an exact hyperbolic shape. For this case, the ship's speed is approximately

$$v_h = \frac{V}{2t}\left[T^2 - T_m^2\right]^{\frac{1}{2}} \qquad (23)$$

At even greater ranges, T_m^2 becomes insignificant compared to T^2 and the sounding trace approaches a straight line which is the asymptote of the hyperbola. For this case,

$$v_a = \frac{VT}{2t} \qquad (24)$$

ERRORS

A. General

We consider now the errors in the measurement of ship's ground speed by the methods described in the preceding section. The errors of measurement may be estimated by differentiating the basic equations:

(a) Two-point method

$$v = .7071\ V \left[\frac{\Delta T\ \overline{T}}{t_a\ t_d} \right]^{\frac{1}{2}} \tag{12}$$

$$\frac{dv}{v} = \left| \frac{dV}{V} \right| + \frac{1}{2} \left\{ \left| \frac{d\overline{T}}{\overline{T}} \right| + \left| \frac{d\Delta T}{\Delta T} \right| + \left| \frac{dt_a}{t_a} \right| + \left| \frac{dt_d}{t_d} \right| \right\} \tag{12a}$$

(b) Three-point method

$$v = \frac{V}{2} \left[\frac{T_1 - 2T_2 + T_3}{T_2} \right]^{\frac{1}{2}} \tag{20}$$

$$\frac{dv}{v} = \left| \frac{dV}{V} \right| + \frac{1}{2} \left\{ \left| \frac{d\delta}{\delta} \right| + \left| \frac{dT_2}{T_2} \right| \right\} \tag{20a}$$

where

$$\delta = T_1 - 2T_2 + T_3$$

(c) Hyperbolic Assumption

The errors in using the hyperbolic assumption may be estimated by considering:

$$\epsilon_h = 1 - \left[1 - \left(\frac{T}{\Delta t} \right)^2 \right]^{\frac{1}{2}} \tag{25}$$

where

$$\epsilon_h = 1 - \frac{v_h}{v}$$

and v_h is given by (23).

The error in the use of the asymptote formula is

$$\epsilon_a = 1 - \left[1 + \frac{T}{t}\right]^{\frac{1}{2}} \left[1 - \left(\frac{T_m}{T}\right)^2\right]^{\frac{1}{2}} \qquad (26)$$

where

$$\epsilon_a = 1 - \frac{v_a}{v}$$

and

v_a is given by (24).

For convenience, the sources of error are considered in three parts. First are the timing errors inherent in the recording instrument. Second are the errors due to the lack of knowledge of the sound propagation process. Third are errors due to deviations of the operating conditions from the assumptions upon which the speed measuring equations are based.

The object of making error estimates is to determine the minimum required capabilities of the several methods and of the instrumentation. For this purpose, we choose 1% as a limiting error in v and we assume that the values of all the contributing errors are additive. Furthermore, the maximum allowable errors will be distributed equally (.33% each) among the three sources of error considered.

In order to introduce practical calculations into the discussion of errors, certain assumptions must be made. First, we assume a minimum sounding of 2 seconds, about 800 fathoms. Second, we note that the maximum range at which a highlight echo is recorded is limited by the effective half-angle beamwidth of the transducer, and assume this to be 30°. Thus the maximum value of T is $3200/V\ \sqrt{3}$ seconds and the corresponding maximum value of t is $800/v\ \sqrt{3}$ seconds. The speed of sound in water is assumed to be 800 fms/sec. The ship speeds of interest are approximately 3 knots, 9 knots and 20 knots. The first speed is that of submerged submarines on a gravity station. The second speed is that of many oceanographic research vessels. The third speed is in the range of operation of naval vessels. T_{max} is 2.3 seconds for all speeds. t_{max} is approximately 554 seconds at 3 knots, 185 seconds at 9 knots, and 80 seconds at 20 knots.

B. <u>Timing Errors</u>

(a) Two-point method

The timing errors of (12a) are given by one-half the sum of

$$\left|\frac{d\bar{T}}{\bar{T}}\right| + \left|\frac{d\,\Delta T}{\Delta T}\right| + \left|\frac{d t_a}{t_a}\right| + \left|\frac{d t_d}{t_d}\right| \qquad (27)$$

The total error of the terms in (27) is restricted, by the assumptions above, to .66%. Let us assume, then, a maximum error for ΔT of .15%, and a maximum error for t_a of .15%. It is apparent that the error in $\bar{T}$ is less than the error in ΔT and the error in t_d is less than that of t_a. With these assumptions, then, the terms in (27) will not exceed .66%.

For the given example, ΔT is 0.3 seconds, so that the required precision of the PDR sounding scale is 0.45 milliseconds. The required precision of the traverse scale is 0.12 seconds at 20 knots, and approximately 0.25 seconds at 9 knots and 0.84 seconds at 3 knots.

Assuming 1/64 inch to be the minimum readability of the record, the precision of the PDR MK V is about 1 millisecond for T and 2.4 seconds for t. To achieve the required precision at 20 knots, then, the T scale must be expanded by 2 and the traverse scale by 20.

(b) Three-point method

The timing errors of (20a) are given by one-half the sum of

$$\left|\frac{d\delta}{\delta}\right| + \left|\frac{dT_2}{T_2}\right| \qquad (28)$$

The total error of the terms in (28) is restricted to .66% as before. Since $\delta = \left(\frac{2v}{V}\right)^2 T_2$, δ is of the order of $10^{-4}T_2$ and the error in T_2 is negligible compared to the error in δ. Let the error in δ = .66%. Assume T_2 is measured at the maximum range (T_2 = 2.3 seconds). Then δ equals 5 microseconds at 3 knots, 50 μs at 9 knots and 200 μs at 20 knots, approximately. At 20 knots, the least severe requirement on $d\delta$ is 1.4 microseconds. Timing precision of this order is more suitable to electronic counters than to mechanical recorders.

C. Sound Propagation Errors

In all the methods of measurement considered, the error in the speed of sound contributes directly to the error in the ship's speed. We may estimate this error by considering the comprehensive sound velocity data compiled by the British Admiralty (Matthews, 1939).

Matthews divides the oceanic areas into 52 different types. For each type of area, a table of sound velocity as a function of depth is given. The velocities are computed from direct measurements of temperature and salinity, and indirect measurements from which the pressure is calculated. The precision of the given velocities is about 0.1%. The accuracy of the velocities has been checked by, among others, the U. S. Coast and Geodetic Survey (Swainson, 1936). In a depth of 1000 fathoms, the agreement between the tests and the tables was found to be 0.2% or better.

In very shallow water, less than 200 fathoms, the variations in the physical properties of the water may be so large as to cause a disagreement with the tables of 2%. Along the East Coast of North America, from about Cape Hatteras to the Grand Banks, the water is also highly variable although the depth is of the order of 2000 fathoms. On a practical basis, the error in this region may be considered to be due to the uncertainty in deciding which of the velocity tables applies.

For example, in the area about 500 miles east of Nova Scotia, the tables for areas 5 through 12 may apply. For a depth of 2000 fathoms, the velocities in the tables range from 816 to 823 fathoms/second, and the average velocity is 819.2 fathoms/sec. If the average is assumed to be correct, the maximum error for this case is 0.46%.

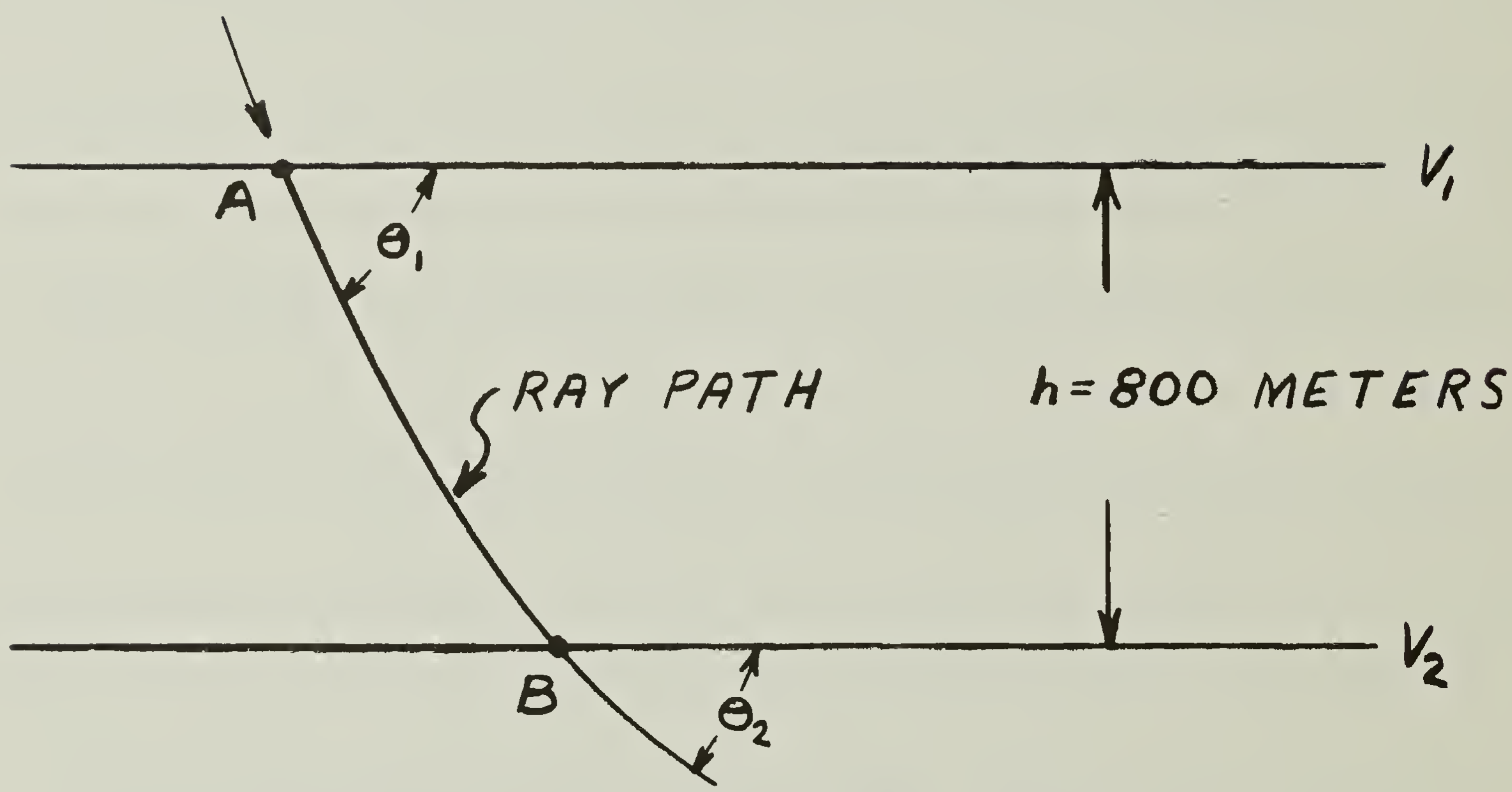
In a less variable area, off the West Coast of the British Isles, there is a region of uncertainty where the tables of areas 9 or 10 may apply. For this case, the maximum errors are about 0.1% for 500 and 1000 fathoms, and 0% for deeper water.

Matthews' tables are computed by assigning an average temperature, salinity and pressure to every 200 meter layer to 1000 meters and to every 500 meter layer at greater depths. A mean velocity is then computed for each layer and the velocity given in the tables is a harmonic average of the mean layer velocities. This method ignores the fact that sound energy in the ocean follows curved ray paths due to the variable velocity structure. We may estimate the discrepancies by considering the data on p. 38 of the tables. The velocity at 0 meters depth is 1490.9 m/sec and at 800 meters depth is 1475.1 m/sec. Assume a linear velocity gradient down to 800 meters. The ray paths are then circles and the following relations hold (see Figure 4).

$$T = \frac{1}{g}\left[\coth^{-1}\csc\theta_1 - \coth^{-1}\csc\theta_2\right] \qquad (29)$$

$$\overline{AB} = h\csc\left(\frac{\theta_1+\theta_2}{2}\right) \qquad (30)$$

FIGURE 4



where T is the ray path travel time from A to B, g is the velocity gradient, $\overline{AB}$ is the straight line distance from A to B.

Assuming $\theta_1 = 30^\circ$, the travel time along the ray path differs from that along the straight line path at Matthews' velocity by about 1 millisecond in 1 second or .1%. This does not mean, necessarily, that a .1% error is inherent in the given velocities, but does serve to show the smallness of the discrepancy between the ray path and the straight line path. We note that $\theta_1 = 30^\circ$ implies an effective transducer half-angle of 60° which is much greater than that of conventional sounding equipment. With an actual transducer, the path discrepancy would be much less than calculated. In any case, with sufficient hydrographic data, velocity tables can be computed which take into account the curvature of the sound rays.

From the above analysis, then, we may reasonably estimate that the velocity of sound given in the Admiralty tables is accurate to 0.2% for ground speed calculations.

D. Basic Assumptions

It has been assumed that the ship's ground speed and course are constant and that the dimensions of the reflecting highlight feature are small compared to its depth.

(a) Constant Speed.

In the two-point calculation method, a change of speed during a highlight crossing produces an error in t. Suppose that a ship approaching a highlight at speed v_1 records a travel time T. At a time τ_1 after this, the speed changes to v_2 and the ship proceeds at speed v_2 for a time τ_2 before obtaining a minimum highlight sounding. The actual time range from the highlight to the place where T was measured is then $\tau_1 + \tau_2$.

If the speed v_2 is desired, then the range used in (12) should be t_2 where

$$t_2 = \frac{v_1 \tau_1 + v_2 \tau_2}{v_2} \qquad (31)$$

Since $t = \tau_1 + \tau_2$ is used instead, the error in t_2 and, very nearly, the error in v_2, is

$$\epsilon_2 = \frac{\tau_1 (v_1 - v_2)}{v_1 \tau_1 + v_2 \tau_2} \qquad (32)$$

If the average speed, $\bar{v}$, is desired, then the range used in (12) should be $\bar{t}$ where

$$\bar{t} = \frac{v_1 \tau_1 + v_2 \tau_2}{\bar{v}} \qquad (33)$$

The error in $\bar{v}$ for this case is approximately

$$\bar{\epsilon} = \frac{\tau_1 (v_1 - \bar{v}) + \tau_2 (v_2 - \bar{v})}{v_1 \tau_1 + v_2 \tau_2} \qquad (34)$$

In general, the range from the point where T is measured to the minimum sounding point may be broken up into m segments, each of duration τ, and a ship's speed v_m, assigned to each segment, The error in the average speed is then

$$\bar{\epsilon} = \frac{\sum_{i=1}^{m} (v_i - \bar{v})}{\sum_{i=1}^{m} v_i} = \frac{\sum_{i=1}^{m} v_i - m\bar{v}}{\sum_{i=1}^{m} v_i} \qquad (35)$$

By definition, the numerator of (35) is zero. Therefore the two-point method measures the average speed over the highlight.

In the three-point calculation method, the measurement period is of the length of 3 soundings, so that, practically, each measurement may be considered to be one of instantaneous speed.

(b) Constant Course

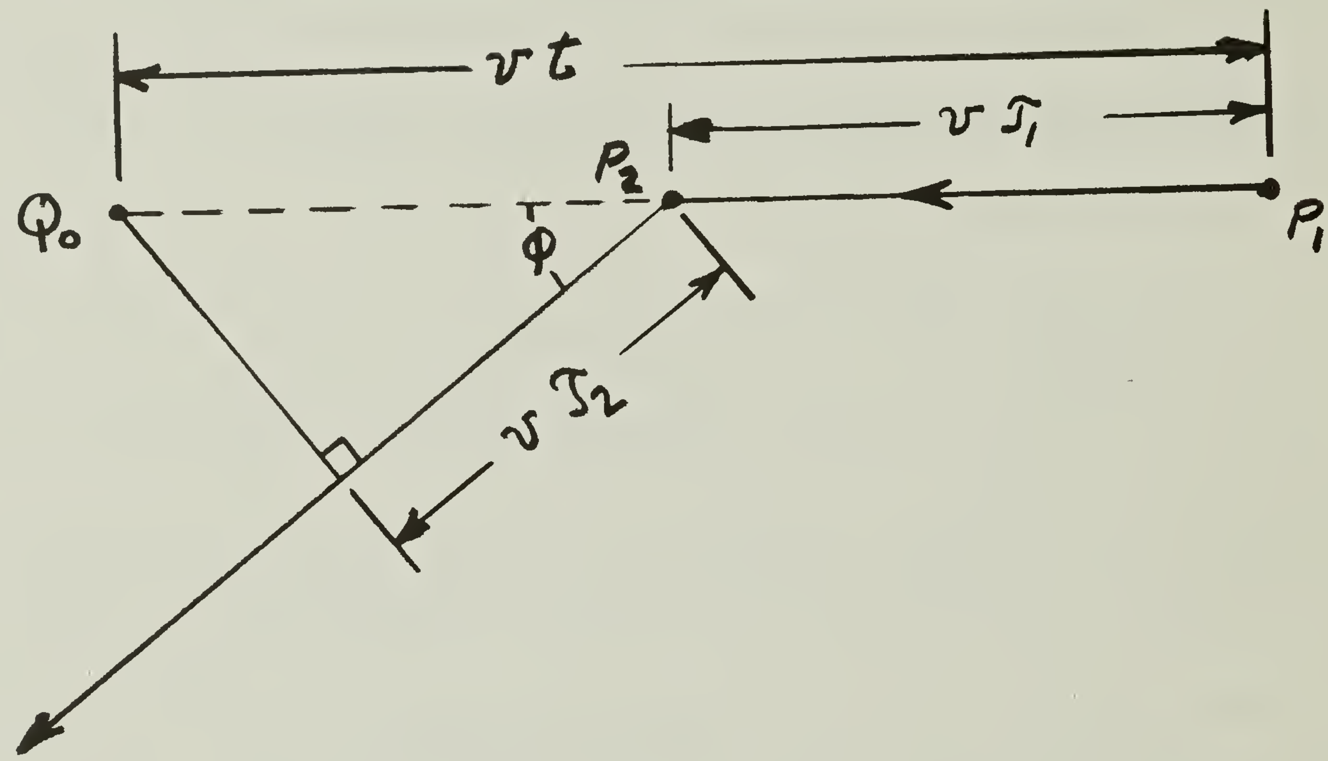
In the two-point calculation method, a change in ship's course with respect to the ground during a highlight crossing introduces error in the value of t. In Figure 5, suppose that a ship, traveling at constant speed v, records a travel time T at P_1 from a highlight at Q_0. Assume the recording is made at range t from the highlight; that τ_1 seconds later the ship's course is changed by ϕ degrees; that τ_2 seconds after changing course, the minimum sounding is recorded. The measured range, t_a, will then be $\tau_1 + \tau_2$. The value of t corresponding to the measured value of T will be $\tau_1 + \tau_2 \sec \phi$. The error will then be

$$\epsilon_\phi = \frac{1 - \cos \phi}{1 + \frac{\tau_1}{\tau_2} \cos \phi} \qquad (36)$$

For example, if $\phi = 10^\circ$ and $\tau_1 = \tau_2$, the error is about 0.75%.

If the value of T recorded at P_2 is used in the calculations, the range is τ_2 and there is no error due to the course change.

FIGURE 5

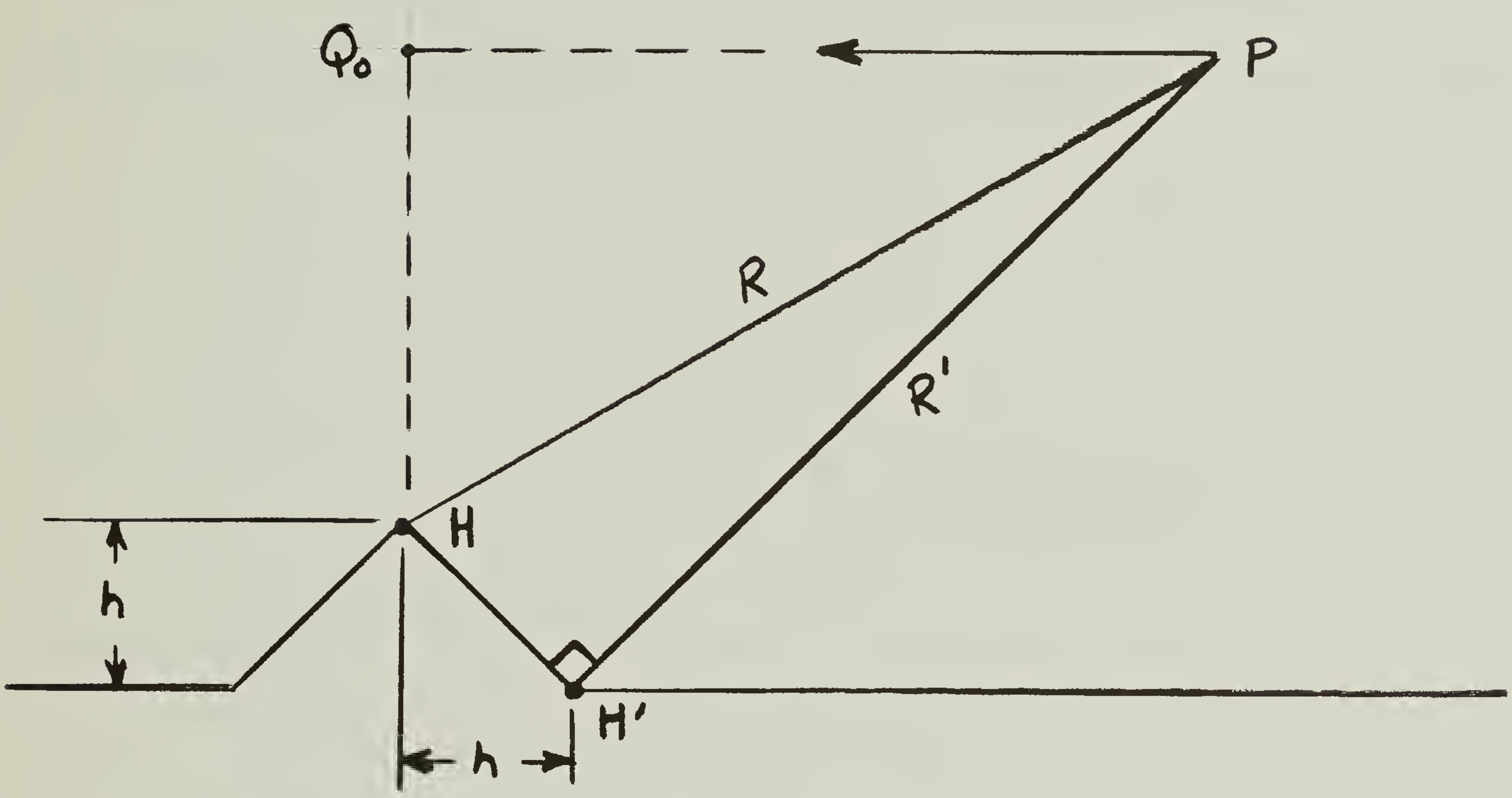


In the three-point method, the measurement again may be considered instantaneous relative to small, slow changes of course.

(c) Highlight Dimensions

The reflecting area of the highlight has been assumed to be of small dimensions compared to the depth. If the dimensions of the highlight are large enough so that reflections from various parts of its surface are received first as the ship passes by, an error results in the sounding T. We may estimate this error by considering an idealized example. In Figure 6, the highlight is assumed to be a circular conical hill of base radius h and altitude h, projecting up from a flat bottom of depth Ro + h. The ship, at point P, is

FIGURE 6



approaching Q_0. The first arrival on the sounding trace is from point H' rather than H, so that the sounded depth is R' instead of R. For this example, we ignore the ship's movement during the sounding time, i.e. we use the hyperbolic approximation. We have then, from Figure 6

$$R'^2 = R^2 - 2h^2$$

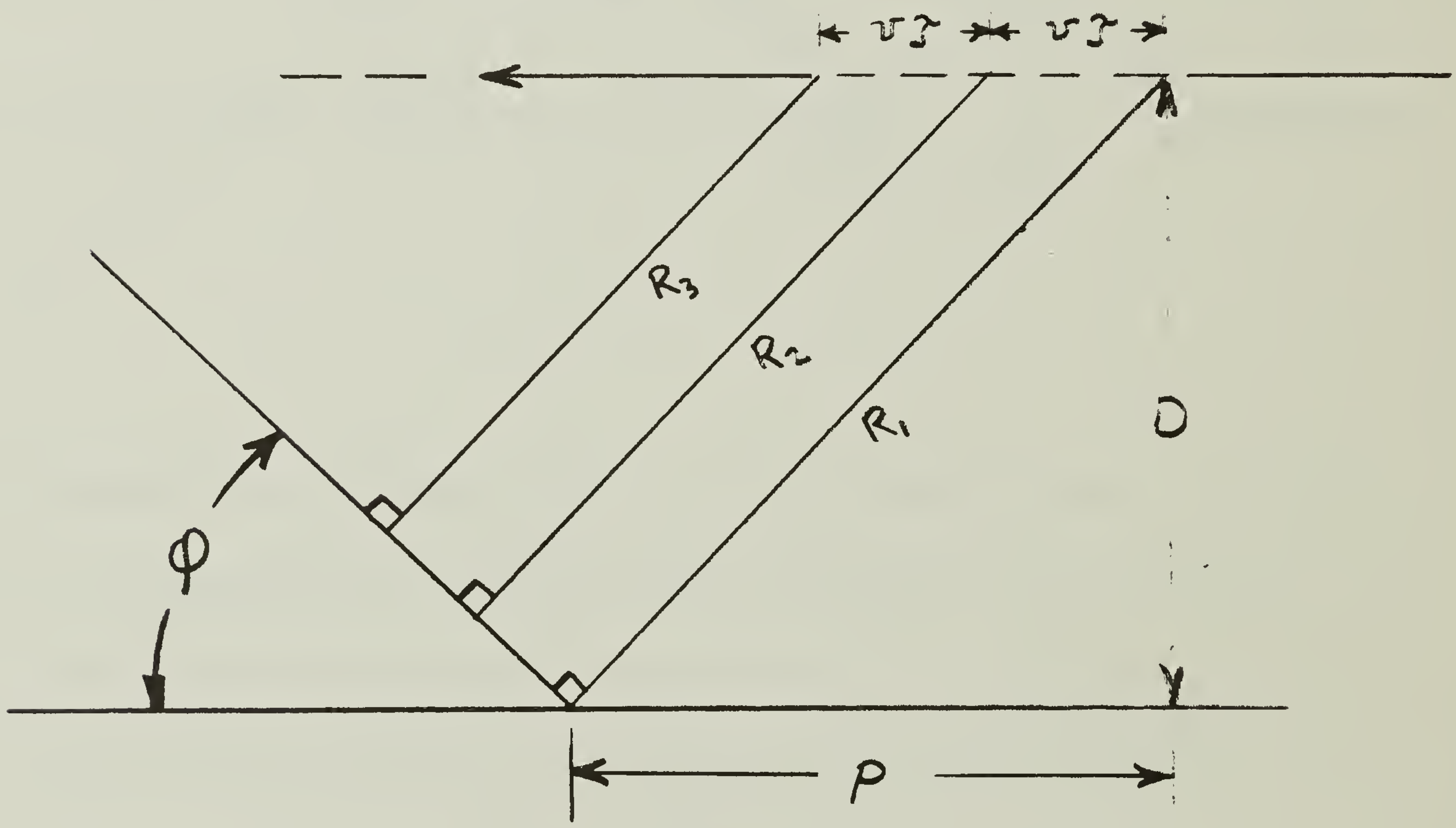
and the error in R, which is approximately the error in T, is given by

$$\epsilon_T = \left(\frac{h}{R}\right)^2 + \frac{1}{2}\left(\frac{h}{R}\right)^4 + \cdots \qquad (37)$$

In the two-point method for a 0.1% error in T, which is approximately a 0.1% error in v, R must be about 30 h. If R is only 10 h, a 1% error is made in v.

We may estimate the three-point method error due to highlight size by considering another idealized example. In Figure 7, suppose that successive soundings are made on a highlight surface of slope ϕ rising from

FIGURE 7



a flat bottom of depth D. The soundings are assumed to be made instantaneously (hyperbolic assumption) and at equal intervals τ . The speed for this case is given by

$$v = \frac{1}{\mathcal{T}}\left[\frac{R_1^2 - 2R_2^2 + R_3^2}{2}\right]^{\frac{1}{2}} \tag{38}$$

This may be easily derived from (19) by neglecting all T compared to $\mathcal{T}$. For the present case, sounding a slope, we have

$$\left.\begin{aligned} R_1^2 &= D^2 + p^2 \\ R_2^2 &= (D - v\mathcal{T}\sin\phi\cos\phi)^2 + (p + v\mathcal{T}\cos^2\phi)^2 \\ R_3^2 &= (D - 2v\mathcal{T}\sin\phi\cos\phi)^2 + (p + 2v\mathcal{T}\cos^2\phi)^2 \end{aligned}\right\} \tag{39}$$

where v is the true ship's speed. Let v' be the speed calculated from (38). Then

$$v' = v\cos\phi \tag{40}$$

The error in the calculated speed due to the slope is then

$$\epsilon' = \frac{\phi^2}{2!} - \frac{\phi^4}{4!} + \cdots \tag{41}$$

For 0.1% error, ϕ is about 3°. For ϕ = 8.4°, the slope error is 1%.

E. Hyperbolic Assumption

The errors in the hyperbolic formula for ship's speed may be estimated by expanding (25)

$$\epsilon_h = \frac{1}{2}\left(\frac{T}{\Delta t}\right)^2 + \frac{1}{8}\left(\frac{T}{\Delta t}\right)^4 + \cdots \qquad (25a)$$

Considering the first term of (25a) only and using the numerical values from the previous example: at the maximum range T = 2.3 seconds, Δ t = 1110 seconds at 3 knots, 372 seconds at 9 knots and 162 seconds at 20 knots. The error is only .01% at 20 knots and entirely negligible at lower speeds.

If the effective half-angle beam width is reduced from 30° to 15°, the value of T_{max} is 2.07 seconds and the values of Δt are 257 seconds at 3 knots, 86 seconds at 9 knots, and 37 seconds at 20 knots. The error in the hyperbolic formula is then .03% at 20 knots.

The error in the use of the asymptote formula may be estimated by expanding (26) and considering only terms of the expansion of T_m/T since the terms T/t are negligible in comparison. The error is approximately

$$\epsilon_a = \frac{1}{2}\left(\frac{T_m}{T}\right)^2 \qquad (26a)$$

In the above examples, the error is 37.8% for the 30° beam angle and 46.7% for the 15° beam angle. For a 1% error in the asymptote formula, the effective half-angle of the transducer must be 82° or more. The range to the highlight in this case is about 5600 fathoms.

F. Summary of Errors

In review of the preceding analysis, it may be stated that topographic highlights may be used for ground speed measurement to 1% accuracy under the following conditions:

(a) Two-point method

1. A Precision Depth Recorder is required with a scan rate of 38 inches per second and a traverse feed rate of 192 inches per hour. For the numerical example above, the timing error in $\overline{T}$ is .01%, in ΔT is .08%, in t is .38% and the total timing error is 0.57%.

2. A sounding equipment is required with an effective half-angle transducer beam width of 30°.

3. The ship's course must be held steady about an average to within a few degrees.

4. The highlight depth must be at least 30 times as great as its relief. (The error in this case is .1%.)

Allowing 0.2% error in the velocity of sound, the total error is 0.87%.

(b) Three-point method

1. This method is generally useful for speeds of 20 knots or more if a timer with an accuracy of 1 microsecond is available. With this accuracy, the timing error is about .5%.

2. A beam angle of 30° is required.

3. The slope of the highlight reflecting surface must be 3° or less for a 0.1% error.

Allowing 0.2% error in the velocity of sound, the total error is about 0.8%.

The hyperbolic two-point formula may be used with negligible error with beam widths of 30°. The asymptote formula is generally inapplicable unless special sounding equipment is used which can record echoes from highlights at great ranges and very large beam angles.

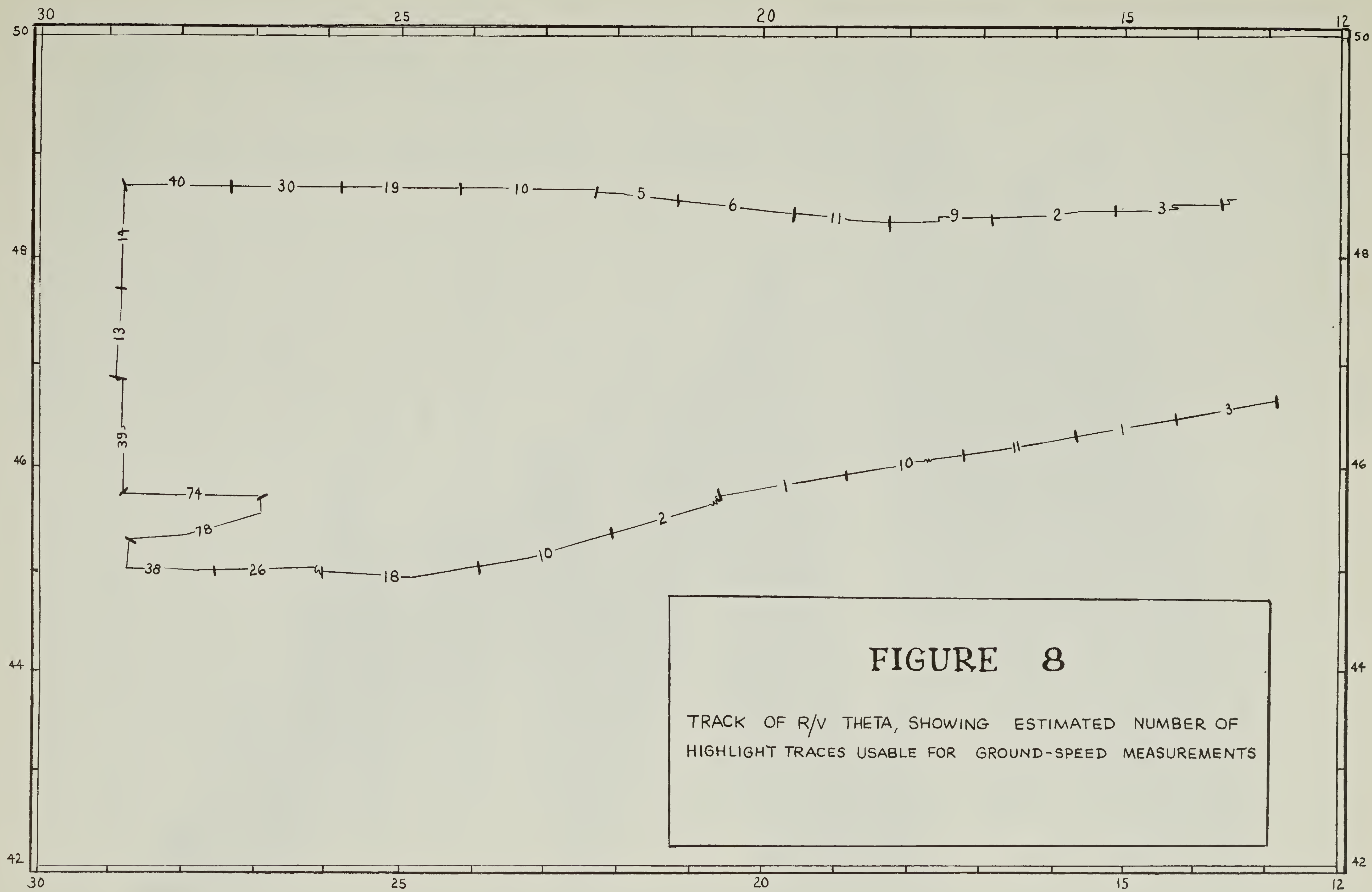


FIGURE 8

TRACK OF R/V THETA, SHOWING ESTIMATED NUMBER OF HIGHLIGHT TRACES USABLE FOR GROUND-SPEED MEASUREMENTS

HIGHLIGHT POPULATION

An important consideration in the applicability of highlight ground speed measurements is the frequency of occurrence of highlights. In rough submarine terrain, an expanded scale sounding record of a few hours duration often shows hundreds of features which have highlight-type traces. For speed measurement purposes, however, only a small percentage of these features are usable.

Some of these features have too great a relief compared to their depth for accurate speed measurement. Some features are poor reflectors and give sounding traces which are too faint. The great bulk of these features which are unusable, however, are so because of the interference of their sounding traces with those of nearby features. In some cases, the echoes from two or more features arriving with small time separations cause asymmetry in the shape of the trace. More usually, the traces are submerged in the total echo return. With a high-power, short-pulse sounding equipment, many more usable highlight traces could be obtained. The shorter pulses would allow individual traces to be seen clearly when the echo-time separations are short.

A rough estimate of the highlight population has been made from the examination of records made by R/V THETA over 1600 miles of track in the Eastern Atlantic (see Figure 8). The track was divided into convenient intervals and the number of usable highlight traces counted for each interval. This procedure is obviously subjective and depends greatly on the judgment of the observer. In Figure 9, for example, it was judged

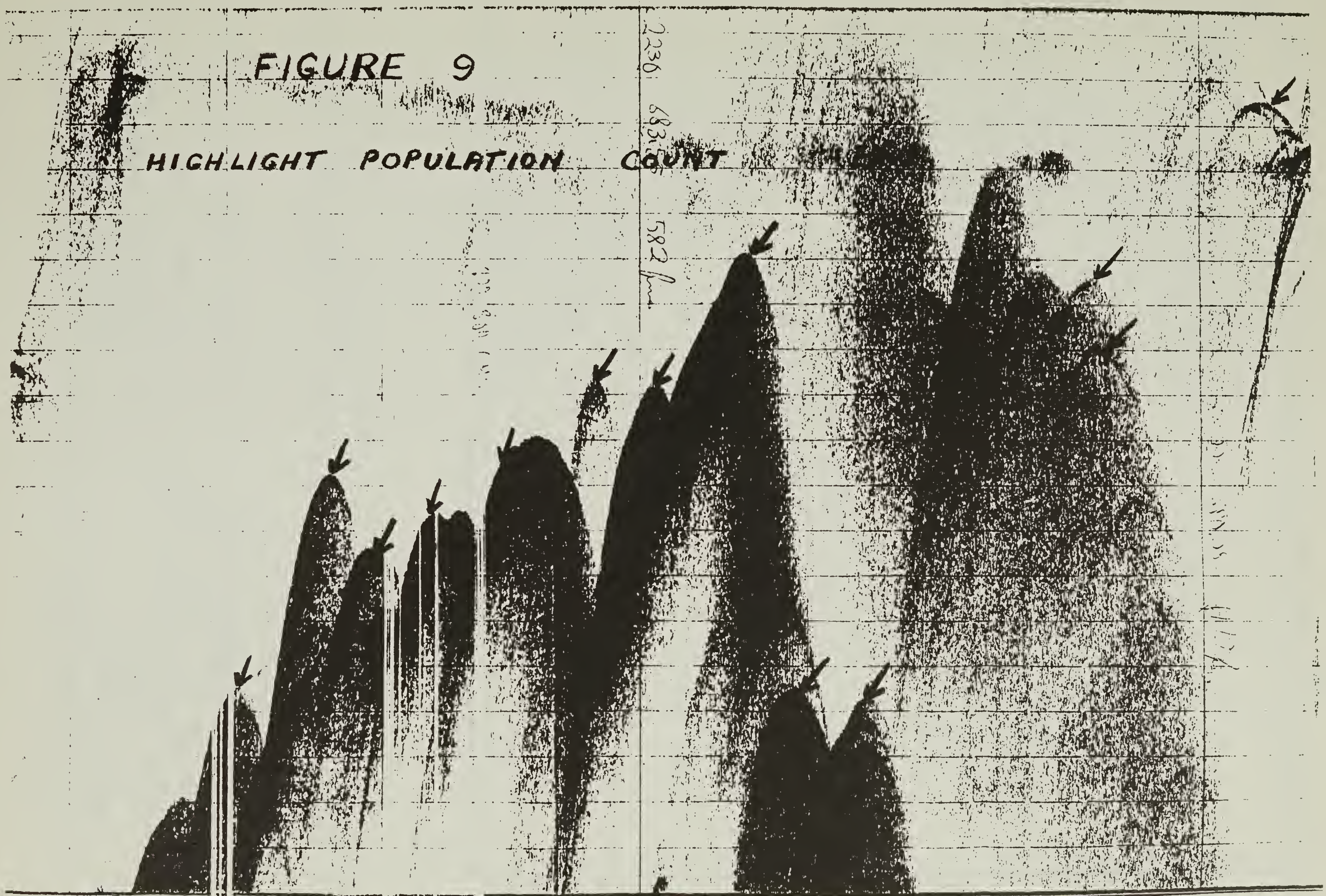


FIGURE 9
HIGHLIGHT POPULATION COUNT
2230 683 582 fm

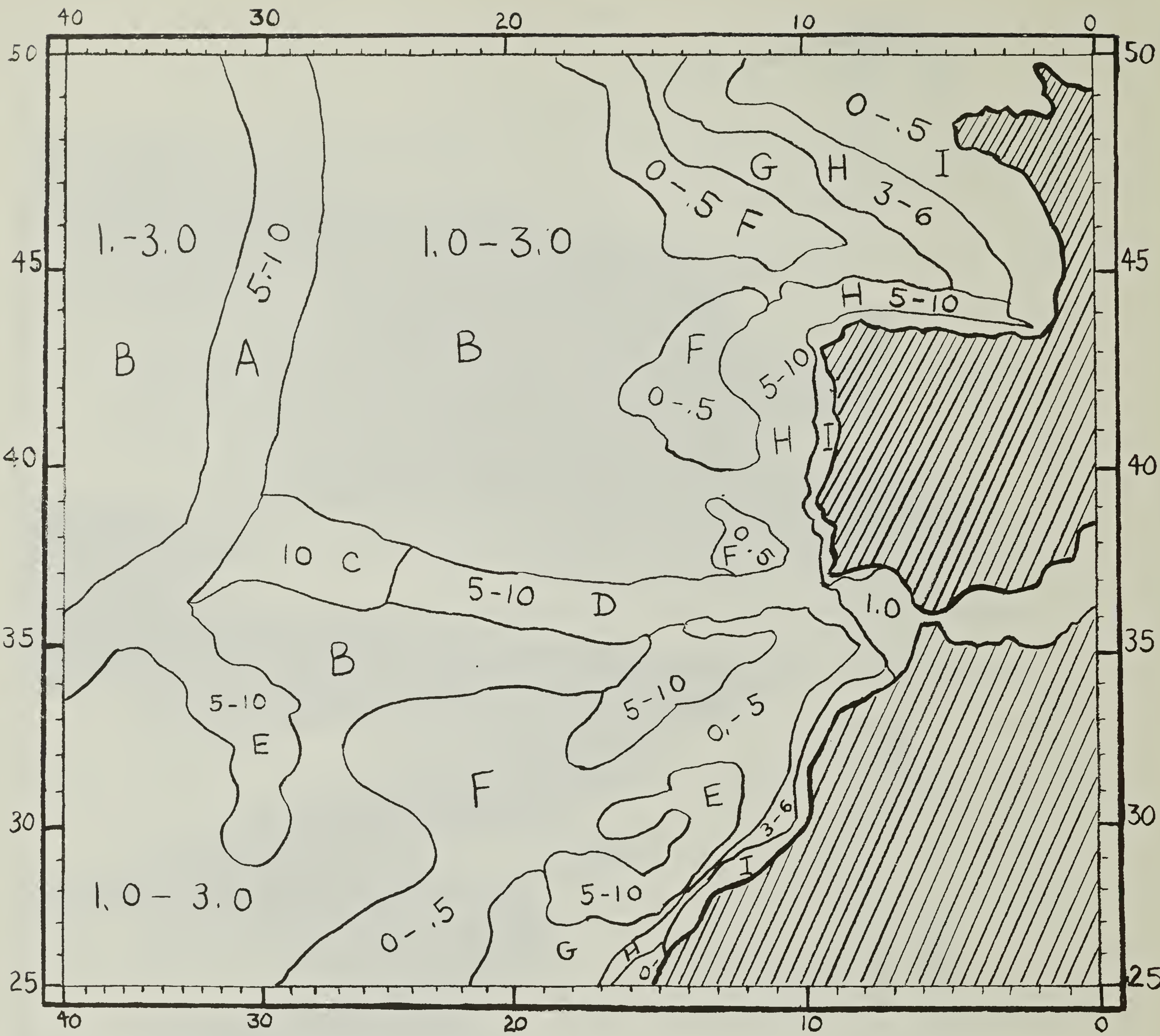


FIG. 10

Eastern Atlantic HIGHLIGHT POPULATION

NUMBERS INDICATE NUMBER OF USEFUL HIGHLIGHT TRACES PER 10 MILES OF TRACK

A. Crest of Mid-Atlantic Ridge
B. Flank of Mid-Atlantic Ridge
C. Azores Plateau
D. Iberian Plateau
E. Seamounts & Islands
F. Abyssal Plains
G. Continental Rise
H. Continental Slope
I. Continental Shelf

that the highlights marked with arrows are suitable for ground speed measurement with good accuracy.

The results of this estimate are illustrated in Figures 8 and 10. Figure 8 shows the number of highlights judged to be usable over the marked intervals. The track of Figure 8 passes through several topographic provinces which have been classified on the basis of relief and texture of the bottom. By extrapolating the results of the highlight count along the track so that similar topographic provinces have the same highlight population, the chart of Figure 10 is obtained.

The average highlight density for the region is about 3 highlights per 10 miles of track. The minimum density of 0 to 0.5 is obtained in the abyssal plain regions and the maximum density of 5 to 10 in the rift zone of the Mid-Atlantic Ridge. Even with this coarse estimate, we may reasonably conclude that a ship traveling at 10 knots would have an opportunity for a highlight ground speed measurement at least once per hour in all regions except the abyssal plains.

For certain special applications, it is possible to provide an artificial highlight by using an acoustic beacon. This is a small, self-contained, electronic device which can be dropped to the bottom in any depth of water to transmit acoustic signals either automatically or in response to a signal from a ship. The beacon is limited in operation by battery life but has the advantages of simplicity and economy. For oceanographic operations at a specific place where natural highlight features may not occur, the beacon may be dropped to serve as a point reflector.

The transponding acoustic beacon has several advantages over a natural highlight. It can transmit a stronger signal than a normal bottom echo. It can transmit an echo on a different frequency than the received pulse so that the bottom return on the ship's recorder may be omitted. It can be arranged to transmit at large beam angles so that the speed calculation accuracy is improved. The beacon highlight introduces no errors due to its size so that it can be used in shallow water. Finally, because of its small size, strong signal, and immunity from interference by other highlight echoes, it is especially adaptable to instrumentation which will automatically compute the ship's ground speed.

CALCULATIONS

In Figure 1, a group of highlights is shown which have been used for ground speed calculations on the Research Vessel VEMA. The record was made on a Precision Depth Recorder Mark V using an AN/UQN-1 B SONAR Sounding Set for transmission and reception of the acoustic pulses. The ground speed of the ship along the track was carefully measured by adjusting LORAN positions, dead reckoning information and coincidence of the track with an acoustic marker beacon position which lay on the bottom. The acoustic beacon has been described in a previous report and its use is considered in the next section (Luskin and Davidson, 1957). The ground speed along the track computed from the navigation fixes is 8.52 knots with an estimated probable error of ± .1 knots.

The highlight calculations are shown in Table 1. The mean calculated ground speed is 8.45 knots with a probable error of ± 1.5%, or ± .13 knots.

In making these calculations, a considerable amount of judgment is required both on the choice of highlights and on the portions of the highlights to be measured. For example, Highlight No. 1 is chosen because of its symmetry and smooth shape. Δt was measured since good sections of the approach and departure traces are available.

TABLE 1

High-light No.	T_m msecs	T msecs	msecs	Range used	Range secs	t_a	t_a	knots
1	1500	1525	25	Δt	96	47.25	48.75	8.23
2	1564.9	1589.9	25	Δt_d	48	47.19	48.78	8.41
3	1540.4	1564.4	24	Δt_a	45	44.23	45.79	8.69
4	1408.5	1459.6	51.1	Δt	130.5	64.50	65.96	8.47

Velocity of sound used 805.5 fathoms/second

Mean velocity 8.45 knots

Probable error ± 0.13 knots

On the approach trace, there is a distinct departure from symmetry for ΔT greater than about 30 milliseconds so that ΔT was chosen at 25 msecs. In Highlight No. 2, almost the entire approach trace is asymmetrical so that the departure trace was used for measurement. In Highlight No. 3, only a small portion of the departure trace is measurable so the approach trace was used. This approach trace, as in the first case, becomes asymmetrical for ΔT greater than about 30 milliseconds. Highlight No. 4 is an excellent one for measurement, having good symmetry for ΔT up to 60 or 70 milliseconds.

Figure 1 illustrates the need for high-power short pulse sounding equipment for the application of highlight measurements to ground speed determinations. With short pulses, the crossovers of the traces in rough topography could be distinguished more clearly and the traces measured over a greater part of their length. The use of shorter pulses, however, cuts down the signal/noise capabilities of the sounder unless the power output is increased.

The effective half-angles of the transducer for the series of measurements in Table 1 may be calculated from

$$\phi = \sin^{-1} \frac{2vt}{VT}$$

and are tabulated below.

TABLE 2

Highlight No.	1	2	3	4
	$6^{\circ}38'$	$6^{\circ}36'$	$6^{\circ}05'$	$9^{\circ}32'$

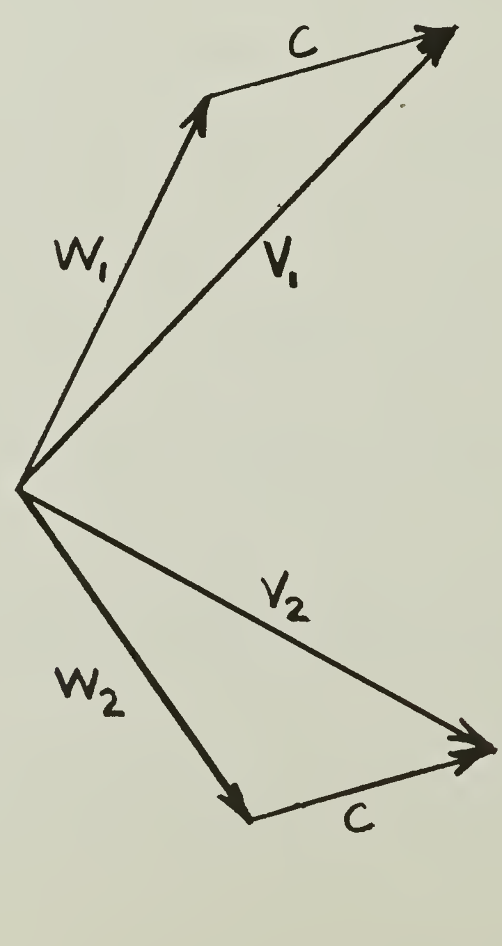
The effective beam angle for highlight speed measurements is much smaller than the effective angle for sounding applications. This angle may be increased by using a lower operating frequency or using several tilted transducers.

APPLICATIONS

A. Current Measurement

An important application of the use of topographic highlights for measuring true speed over the ground is in the determination of the ocean surface currents. For this application the true ground speed in two directions must be established. Instruments have been developed and are becoming available shortly which can measure the speed of the ship through the water in the direction of the ship's heading with great accuracy. The combination of such an instrument with a true ground speed measuring system allows the determination of the vector velocity over the ground and hence the surface current.

FIGURE 11



In Figure 11, the vector relationship between the various velocities is shown. W_1 and W_2 are the ship's water velocities on the two different headings. The magnitude and direction of these vectors are known. V_1 and V_2 are the respective ground velocities on the two different headings. The magnitude of these vectors may be measured by the use of highlight reflections. The vector C represents the velocity of the water over the ground and is the same on both headings of the ship. The magnitude and direction of C are both unknown.

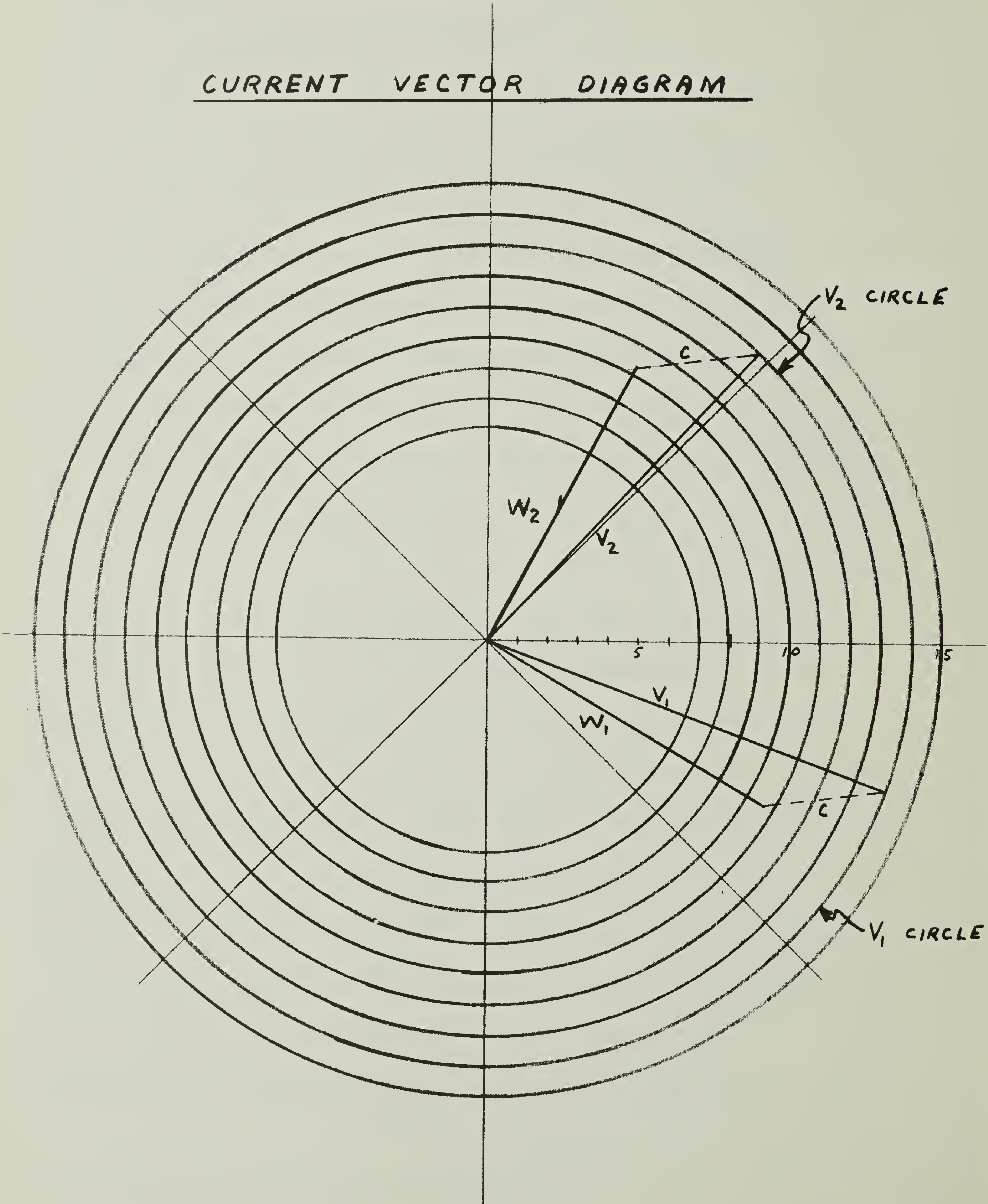
It is possible to obtain an analytical solution to this problem but the equations are complicated and extremely cumbersome for practical use. It is much simpler to obtain a graphical solution by trial and error. This may be done as follows:

In Figure 12, a grid of circles is laid out with the radii of the circles representing velocity magnitude. This may be done on a large chart to a scale of 1 inch per knot so that, say, 1/20th of a knot is easily readable. The vectors W_1 and W_2, established from Log and Compass measurements, are laid out on the chart. The magnitude of the vectors V_1 and V_2, established from ground speed measurements, is represented by the appropriate circles as shown in Figure 12. The starting points of the vector C are then known for the two cases. It is also known that the vector C must terminate on circles 1 and 2 respectively for the two cases. With a parallel guide and a scale, now, the vector C is found by fitting it appropriately to the vector diagram.

The collection of data on ocean surface currents is an important field in itself and has application to many problems of oceanography as

FIGURE 12

CURRENT VECTOR DIAGRAM



well as navigation. In particular, the measurement of the current is of great importance in the measurement of gravity from a submerged submarine. When making these measurements, the speed of the ship through the water is about 2 or 3 knots, so that strong currents may be of comparable magnitude. The magnitude of the velocity correction to the gravity measurement is given approximately by

$$\Delta g = 7.5\, S \cos \phi \qquad \text{milligals}$$

where S is the East-West component of the ship's ground speed in knots, and ϕ is the latitude. If the water speed log is used for supplying speed correction data, large errors may result.

For example, suppose the ship is making 3 knots through the water in a northerly direction at the equator and a 1 knot E-W current is present. The error is 7.5 milligals. Suppose, again at the equator, the ship is on an easterly heading at 3 knots and a 1 knot NW current is present. The error is 5.25 milligals. Actually, the usual procedure is to correct for the ground speed by adjustment of the ship's dead reckoning track with star and LORAN positions. However, these positions are not always available and are subject to considerable error. The use of highlight ground speed measurements with water speed log measurements offers a method of relatively great precision for gravity corrections.

B. Inertial Navigation

The U.S. Navy is at present developing an Inertial Navigation System whose primary purpose is to allow the precise location of guided missile platforms at sea. For this system, a highly precise determination of the velocity components of the ship's speed over the ground is essential.

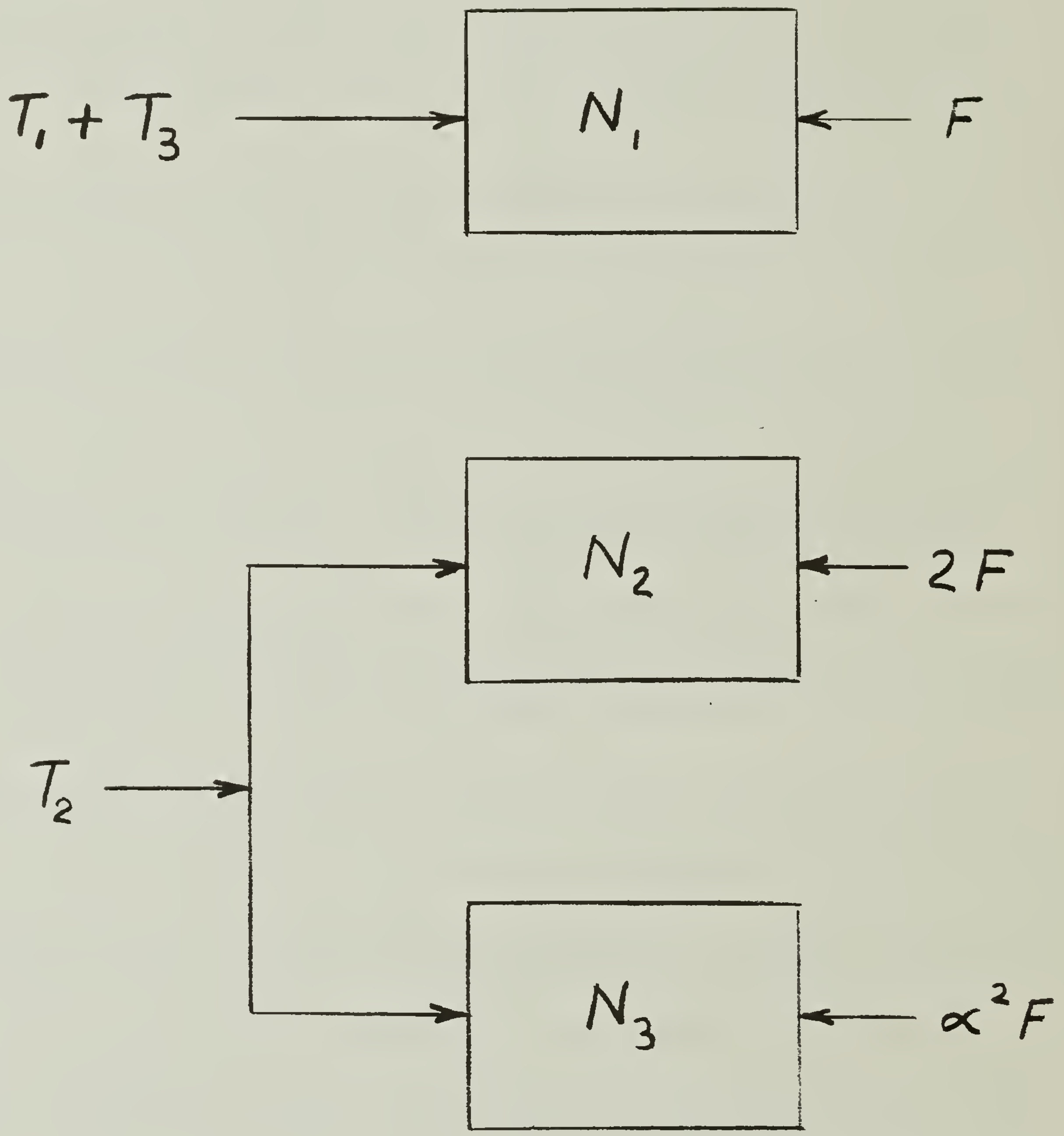
The highlight method of ground speed measurement has several disadvantages for this application although it is basically capable of producing the desired information. First, the highlight method is dependent upon the occurrence of suitable topographic features along the ship's track so that the measurements are necessarily intermittent. Secondly, the determination of the vector velocity requires that the ship alter course, which may be inconvenient. Thirdly, the ground speed determination requires data measurements and other operations which are not automatic so that the process of feeding the velocity data to the computers of the Inertial Navigation System is awkward.

However, if the navigation problem is restricted to the relocation of specific positions at sea, the highlight method offers attractive possibilities. Suppose that the guided missile launching sites are restricted to certain specific locations at sea, and that these positions are marked with transponding acoustic beacons. The beacons can be made secure, if necessary, by arranging that their circuits become active in response to special coded signals. These beacons would then fulfill the functions of position location as well as provide a means of determining ground speed precisely. By arranging several beacons to form a path, the velocity components may also be determined. The disadvantage of this system, of course, is its limited life. The outstanding virtues of this system are its immediate applicability, its simplicity, and its outstanding economy. The cost of an acoustic beacon with a one-year battery supply is considerably less than the price of one day's operation of a missile-launching ship.

This beacon-highlight system is also useful as an aid in the development of the Ship's Inertial Navigation System. The need for a precise, independent means of checking the operation of the SINS during its development can only be met by using landmarks, SHORAN, LORAC and similar systems. These methods are all restricted to short ranges. The radio navigation systems also require the installation and maintenance of complex electronic equipment both ashore and aboard ship. The beacon-highlight system, on the other hand, can be set up anywhere in the ocean, and requires only the use of an Echo-Sounder and Precision Depth Recorder. In particular, a beacon position may be selected in a convenient location which bears close resemblance to an actual launching site at some other place. The entire launching operation may then be tested with convenience under simulated operating conditions. The outstanding virtues of this application are its immediate applicability, its simplicity, and its outstanding economy. The cost of an acoustic beacon with a one-year battery supply is considerably less than the price of one week's operation of a LORAC system.

The use of a beacon-highlight system in a specific location eliminates the two disadvantages of intermittent operation and the necessity

FIGURE 13



SCHEMATIC COMPUTER FOR

3-POINT METHOD

When $N_1 - N_2 - N_3 = 0$

$$\alpha^2 = \frac{T_1 + T_3 - 2T_2}{T_2} \qquad \text{(EQ. 20)}$$

for changing course to determine the velocity components. In addition, it appears to be feasible to use an automatic computer with a transponding beacon and a special echo sounder. A schematic arrangement of a computer using the Three-Point Method is shown in Figure 13. The Echo-Sounder is arranged to operate as a transponder as well as the beacon. The three counters are gated so that the travel-time of the first and third soundings is measured by counter 1 as N_1 with input frequency F_1; the travel-time of the second sounding is measured by counter 2 as N_2 with input frequency $2F_1$; and the travel-time of the second sounding is also measured by counter 3 as N_3 with variable input frequency $\alpha^2 F_1$. If the counters are arranged in a computing loop which nulls the quantity $N_1-N_2-N_3$, the ship's speed may be determined automatically and in suitable form for the SINS computer input.

CONCLUSIONS

The traces of topographic highlight features made on an expanded-scale Precision Depth Recorder may be used to measure the speed of a ship over the bottom. With only minor development of existing sounding and recording equipments it is feasible to attain an accuracy of measurement of 1%.

When a complete trace of the highlight feature is available, the ship's ground speed is calculated from the simple formula

$$v = .7071\, V \left[\frac{\Delta T \quad \overline{T}}{t_a \; t_d} \right]^{\frac{1}{2}} \qquad (12)$$

where V is the velocity of sound propagation and the quantities ΔT, $\overline{T}$, t_a, t_d are determined from measurements of the trace. When only a partial sounding trace is available, not including the minimum sounding, the calculations of the ground speed is awkward unless special equipment is used. With a transponding echo sounder and a high speed counter (1 megacycle), the ground speed may be obtained from three successive soundings by

$$v = \frac{V}{2} \left[\frac{T_1 - 2T_2 + T_3}{T_2} \right]^{\frac{1}{2}} \qquad (20)$$

An estimate of the frequency of occurrence of natural topographic highlights indicates that, except for the abyssal plain regions, a minimum of one, and an average of three usable highlight traces can be obtained over a typical 10-mile track. For special applications, a transponding acoustic beacon may be dropped to the ocean bottom to serve as an artificial highlight.

Highlight ground speed measurements are applicable to the determination of ocean surface currents if combined with accurate water-speed measurements. By obtaining water and ground speed measurements on two headings, the ground velocity vector may be specified. One particular application of this technique is in the determination of E-W velocity for correcting gravity observations made from submerged submarines.

The use of highlight ground speed measurements is also applicable to the Ship's Inertial Navigation System being developed by the U.S. Navy. In particular, the use of an acoustic beacon and the highlight technique allows a simultaneous determination of position and ground speed at a given location. This beacon-highlight system is extremely simple, immediately applicable and extraordinarily economical.

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