Optimal Siting of Sub-Urban Air Mobility (sUAM) Ground Architectures using Network Flow Formulation

Nikhil Venkatesh*, Alexia P. Payan †, Cedric Y. Justin‡, Ethan Kee§,and Dimitri N. Mavris¶
School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, Georgia, 30332, U.S.A.

Air Mobility (AM) operating models have steadily made their way into public conscience over the past decade due to increased research activity pioneered by large technology corporations such as Uber and Amazon. Estimates concur that there are around 250 startup businesses with 22 major players working on such technologies with over $25 billion dollars in venture capital funding in 2017[1]. Given the meteoric rise of Air Mobility as one of the leading 21st century disruptive technologies, research effort across the spectrum of functions that can make AM concepts a reality are burgeoning - ranging from vehicle design to operations planning. More specifically, research efforts within the operations planning space deal with service route identification, ground infrastructure (such as charging stations and ports) placement and others. To this effect, the present study seeks to evaluate the feasibility and tractability of a formalized optimization method towards the siting of "vertiports" - ground infrastructure that aids the embarkation and disembarkation of AM commuters - as applied to a Sub-Urban Air Mobility (sUAM) operating model. Mixed-Integer Programming (MIP) formulations offer qualified benefits over other heuristic methods and the authors are confident of their relative performance given the proven track record of such methods in solving generalized facility location problems (GFLP). In this study, two optimization problems were considered: capacitated vertiport siting, where any vertiport considered would need to adhere to capacity constraints; and uncapacitated vertiport siting, where any vertiport considered does not have any capacity limit and can service unlimited demand. Results indicate that a network flow formulation using an MIP methodology is able to adequately place vertiports for sUAM business operations to satisfy demand flows associated with home-work commute.

Nomenclature

C_{j}	Capacity of Embarkation Vertiport for a Given Vertiport Pair
d_{il}	Demand per Home/Work Block Pair
i	Home Census Block
j	Embarkation Vertiport Facility
k	Disembarkation Vertiport Facility
l	Work Census Block
T_{ij}	Summed ground leg times
t_{ij}	Ground travel time from Home Census Block to Embarkation Vertiport Facility

*Graduate Research Assistant, Aerospace Systems Design Laboratory, School of Aerospace Engineering, 270 Ferst Drive, Atlanta, GA, 30332-0150, AIAA Member.

†Research Engineer II, Aerospace Systems Design Laboratory, School of Aerospace Engineering, 270 Ferst Drive, Atlanta, GA, 30332-0150, AIAA Member.

‡Research Engineer II, Aerospace Systems Design Laboratory, School of Aerospace Engineering, 270 Ferst Drive, Atlanta, GA, 30332-0150, AIAA Member.

§Graduate Research Assistant, Aerospace Systems Design Laboratory, School of Aerospace Engineering, 270 Ferst Drive, Atlanta, GA, 30332-0150.

¶S.P. Langley NIA Distinguished Regents Professor, Boeing Regents Professor of Advanced Aerospace Systems Analysis, School of Aerospace Engineering, Director Aerospace Systems Design Laboratory, 270 Ferst Drive, Atlanta, GA, 30332-0150, AIAA Associate Fellow.

t_{kl} Ground travel time from Disembarkation Vertiport Facility to Work Census Block

x_{ijkl} Routing Decision Variable

y_{jk} Vertiport Pair Locating Decision Variable

I. Background

Air mobility (AM) has experienced large growth over the last decade and some of the key contributing factors for this growth is due to the current state of urban transportation systems. Congested roadways, especially during times of home-work travel, is a growing concern among commuters and transportation authorities alike especially in fast-urbanizing and urbanized economies. Currently, the U.S. contains 34 large metro areas with an excess of 1 million people that have undesirable levels of congestion measured using the Roadway Congestion Index (RCI): an RCI of 1.0 or more constitutes an undesirable level of congestion. Figure 1 shows the spread of congestion indices across major U.S. metros for the year 2011 [2].

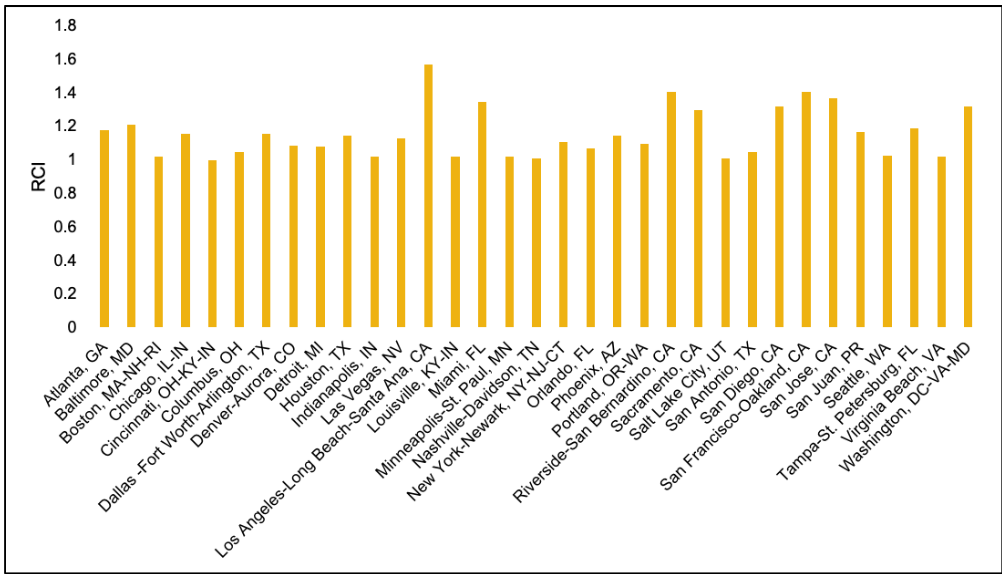


Fig. 1 RCI Spread across major U.S. cities [2]

Additionally, the current portfolio of path-based public transportation [3] schemes that include bus and train services are prone to over-crowding and delays. These commute delays stem from the nature of scheduled path-based transportation which execute multiple stops sequentially. Hence, AM provides qualified benefits over the traditional commuting schemes which include a significant reduction in travel time due to a node-based operating model, whereby passenger carrying vehicles operate along direct high demand, point-to-point routes rather than following predefined stops. This characteristic also yields improved resiliency to delays. In addition, it is hoped that AM offerings can be harmonized into the existing transportation service portfolio leading to reduction in ground traffic and improvement of urban congestion situation as more commuters, especially commuters-for-work, transition over to an AM offering.

The outlined benefits are such that AM can actually work as a disruptive technology/service model combination in a variety of markets not least commuter transportation. Last mile package delivery and emergency health services are adjacent industries which have shown interest in adopting AM solutions due to the benefits of reliability and low latency delivery. However, the present study seeks to focus on the "Air Metro" mode of operation. This mode most closely resembles current public transportation systems and will be used as a use case. AM not only has a variety of different use cases but can also operate on varying scales. Most commonly, AM operations can be further categorized into three main levels - urban, suburban, and regional. Each scope of operation differs from one another in the service range of the AM activities. Urban Air Mobility (UAM) concept-of-operations (CONOPS) are mainly concentrated around city limits (less than 20 miles from city center). On the other hand, suburban Air Mobility (sUAM) CONOPS expand to greater metro areas with service radii of around 50 miles. Finally, regional AM CONOPS function on the inter-city travel scale

(50 - 300 miles). Again, the primary topic of this study is an air metro application within an sUAM scope. Figure 2 concisely summarizes the various scopes of AM today.

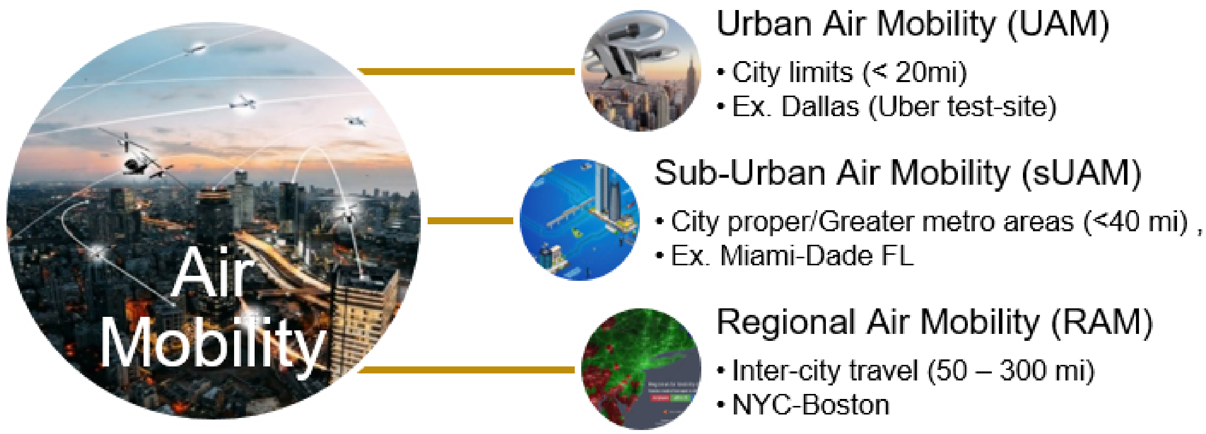


Fig. 2 Three-pronged scope of Air Mobility operations

No matter the chosen AM scope or use case, multiple functions such as certification and requirements analysis, vehicle design, operations planning, testing and finally roll-out are all equally important for a successful realization of an sUAM project. sUAM ecosystems require coordination across a large number of diverse entities (such as commuters, vehicles, and maintenance crew), and a rigorous operational setup is required to leverage the benefits of a sUAM air metro application. More specifically, the identification and placement of ground infrastructure such as "vertiports" - embarkation and disembarkation locations for sUAM commuters - is a key planning decision for an sUAM air metro endeavor.

Optimal placement of vertiports is of prime importance for successful sUAM air metro operations. Not only is this task crucial for streamlined services but ground infrastructure construction also demands heavy project capital investment upfront. Ground infrastructure and planning decisions at this stage of the project development carry significant project risk, and hence, decision makers and stakeholders need to be able to make well-considered business and operations decisions. To this effect, there has been considerable research focus on the optimal vertiport placement problem, or more generally ground depot placement problem. This problem has many parallels to the well-studied facility location problem (FLP), a classic combinatorial optimization problem in operations research [4]. Solutions to such optimization problems can be percolated to a level of business significance and are commonly used to generate valuable data-driven business insights. Hence, the authors believe that the use of a structured optimization method that can appropriately address and feasibly solve the optimal vertiport placement problem can greatly benefit parties involved in planning and implementation of a successful sUAM air metro service for home-work commuters.

II. Past Research

There exists a number of competing location models that have proven capabilities towards solving a FLP applied to diverse domains such as food delivery services and distributor portfolio management [4]. More generally, such location models can be largely differentiated based on whether they are continuous or discrete. A continuous model assumes that the facility locations which are meant to serve demand points (i.e. in the sUAM case, places between which people currently travel to go to or from work) can be placed anywhere on a continuous grid, whereas discrete models constrain the placement of facility locations to a fixed predetermined set of places. Given the theme of the problem the authors seek to solve, it is prudent to solely focus on the utility of discrete location models for the optimal vertiport placement. Such location models have the ability to solve two different sub-classes of the FLP which are capacitated facility location problems (CFLP) and uncapacitated facility location problems (UFLP). CFLPs place restrictions on the amount of demand that a given facility is able to meet, whereas UFLPs do not place such restrictions and allow unbounded demand satisfying a given set of characteristics for any given facility considered.

Within the umbrella of discrete location models, there exist further divisions based on whether the model logic is heuristic or optimization-based. Heuristic methods are learning-based methods that seek to find time efficient but not necessarily optimal solutions to problems in cases where optimal solutions may be computationally difficult to achieve. Wei et al. [5] studied the utility of the p-median location model, a type of heuristic approach, to optimally place vertiports for an sUAM application in southern Florida counties. The p-median model is an iterative approach that reads in demand point data and candidate facility locations and uses a k-means ++ clustering technique to initialize candidate facility locations to demand points. The k-means ++ algorithm is applied repeatedly to reassign demand points to new facility locations until a satisfactory objective value for the objective function is achieved. The objective function for the p-median model seeks to minimize the demand-weighted distance of all the facilities located. The results of the aforementioned study confirmed the applicability of the p-median model as a viable and scalable approach for the vertiport placement problem but do possess some hindering assumptions. Firstly, a choice regarding the number of facilities (i.e. vertiports) to place is made a-prior; this assumption can prove to be an obstacle in situations where infrastructure investment decisions have not been made or planners wish to determine the number of facilities to construct in an organic fashion. Secondly, the p-median model is a simplified model when it comes to the distance measurement between demand points and facilities. Euclidean distances are used to calculate distance-weighted coefficients for the objective function which most often do not realistically estimate the actual path distances between demand points and vertiport facilities.

III. Literature Review

Classical network models offer an alternative to the aforementioned p-median model with equally good if not superior solution quality. Network models consider demand points and facilities as nodes in a connected network graph and are modeled and solved as Mixed-Integer Programming (MIP) problems. In most cases, MIP formulations seek to minimize the overall cost of constructing and maintaining facilities to serve demand centers. Network models using MIP do not make a-priory assumptions on the number of facilities to place and neither do they require any initialization which reduces the solution variability. Network models present another significant advantage over p-median models in that they are able to leverage powerful industry-grade MIP solvers such as Gurobi [6].

Optimization-based studies have gained significant traction across various elements of AM/UAM operations such as vertiport capacity studies, UAM traffic flow management, as well as veriport placement. One such study that demonstrates the benefits of a network flow model is Vascik's formulation of an Integer Programming (IP) formulation based on a multi-commodity network flow model for vertiport operations to estimate UAM vertiport capacity envelopes [7]. A collection of nodes and arcs represent the physical components of a vertiport and its airspace through which aircraft arrive, transition, and depart to other destinations. The multi-commodity flow formulation describes the movement of aircraft through a given vertiport network. The decision variables within the IP are integer variables that specify the number of aircraft of a given type that travel along arcs linked to a vertiport. The objective of this IP formulation is to maximize the total reward value for aircraft arrivals and departures through a vertiport.

Previous IP formulations of the vertiport placement problem have been conducted. One such study is the vertiport placement for serving daily commutes of passengers around urbanized regions using eVTOL aircraft. Daskilewiscz et al. [8] undertook a binary IP formulation for vertiport placement whereby an uncapacitated vertiport (i.e. a vertiport with unlimited commuter capacity) can be placed in each census tract (a geographical unit in the U.S. Census data). The binary decision variables in this formulation stem from whether census tract pairs between which commuters travel can be served by eVTOLs operating between the corresponding vertiports. The objective of this IP formulation was to maximize the cumulative time saved in commuting for the entire network of census tracts considered.

With an intent to add to the aforementioned body of optimization studies conducted in the AM planning space, the authors hope to study the ability of MIP formulations to represent the network of census blocks considered, as well as potential vertiport locations to optimally place these vertiports given demand and travel time related constraints. In turn, the results of this study can hopefully lead to an improved set of mathematical modeling capabilities to study such classes of operations research problems encountered during the planning phase of an AM endeavor.

IV. Proposed Approach

A data-driven optimization framework as shown in Figure 3 is followed for the construction and solving of the aforementioned MIP-based network model. This approach allows for separated data and modeling environments. This characteristic is desirable as it leads to a good degree of modularity and flexibility whereby the optimization can easily be applied to different datasets.

The framework outlined in Figure 3 consists of four main steps. The *data layer* where different demand and location datasets are parsed into tractable forms for use. The *physical system modeling layer* where sUAM entities are modeled in the code. The *optimization problem formulation layer* where the objective function, constraints, and decision variables are constructed. Finally, the *solution layer* where the optimization results are compiled and derived into useful insights.

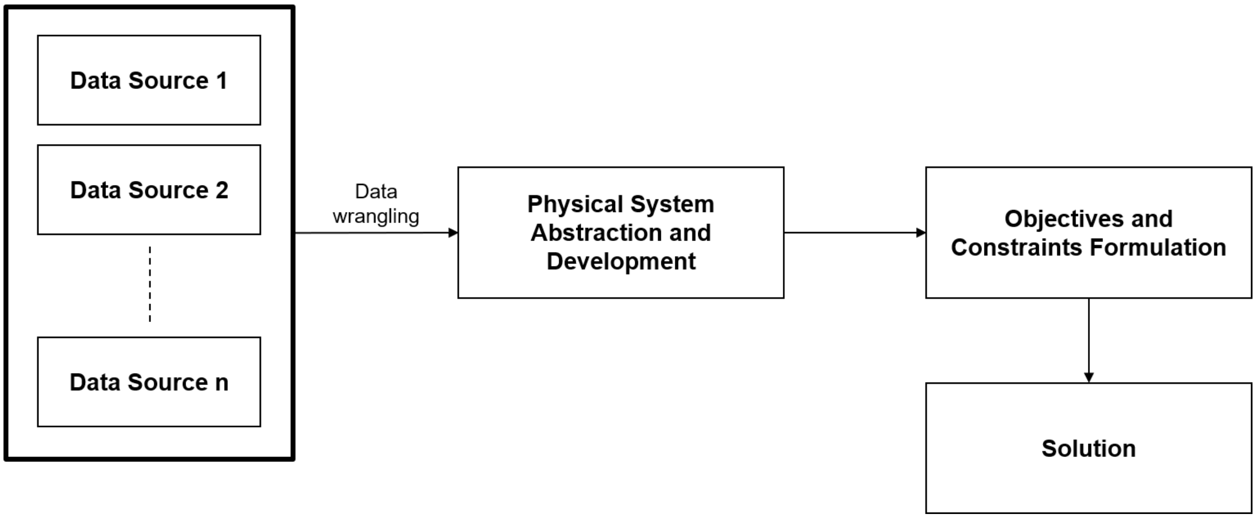


Fig. 3 Data-driven optimization approach

A. Data Layer

The South Florida region has seen considerable urban sprawl over the past years due to a remarkable southward population shift. This shift can mainly be attributed to tourism growth, favorable climate, and strong financial prospects [9]. For this reason, AM market makers consider this cross-section of the Florida population to be strong candidate users of future AM services like an sUAM air metro service. In fact, Wei et al. [5] considered the South Florida counties of Broward, Miami-Dade, and Palm Beach as a case study for the implementation of the p-median model for optimal vertiport facility placement as mentioned in Section II. Hence, the authors choose to work on the same South Florida counties in order to provide opportunities for direct comparison between the proposed optimization-based network model approach and the heuristic-based p-median model.

In order to describe the sUAM system entities in representative detail, certain key pieces of information are required: population centers or locations where people reside and work; directed demand or the number of people travelling from home to work and vice versa; and commute times or the time it takes commuters to travel from home to work and vice versa using current ground transportation means. Socio-economic information of this kind and scale can be almost wholly derived from publicly available and anonymized census data sources that track community social and economic indicators. The U.S. Census [10] and the Longitudinal Employer Household Dynamics (LODES) [11] datasets are stratified at varying degrees of geographical granularity from state-level to census block-level as shown in Figure 4. The use of data at higher granularity levels leads to richer models but can compromise model complexity and computational speed. Hence, the choice of a geographical granularity for the census data is a key model design decision. Given that the optimization environment is paired with Gurobi, "the most powerful mathematical optimization solver out there" [6], the authors have sought to use census block-level data that is consistently available across multiple datasets. Table 1 provides a compilation of all the data sources used as part of the modeling environment. The locations where vertiport infrastructures can be constructed is derived from a previous study [12].

B. Physical System Abstraction

1. sUAM Routings

The sUAM ecosystem is represented by groups of ordered nodes in a network graph. Each group of ordered nodes represents a routing option as shown in Figure 5 and is the building block of the network model described herein. Each routing option represents the potential movement of sUAM commuters from a home block to a work block, or vice versa, through a pair of two vertiport facilities. Looking at Figure 5, node *i* represents the home block, node *j* represents

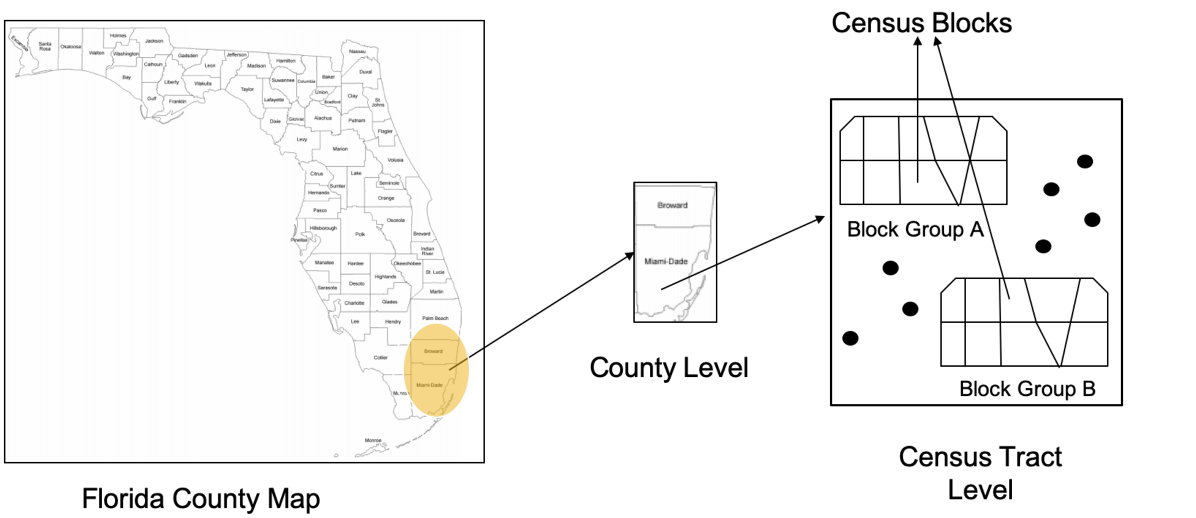


Fig. 4 Graphical representation of the U.S. Census data stratification

Table 1 Table summarizing the Florida state specific datasets used and their respective geographical granularity

Information Needed	FL State Datasets	Granularity
Population Centers	LODES Workplace/Residence Area Characteristics	Block-level
Demand	LODES O-D pairs	Block-level
Time	ACS 2016 5-year survey Travel Time	Census Block Group-level
Vertiport Opportunities	Previous Study Dataset	Census Tract-level

the embarkation vertiport, node k represents the disembarkation vertiport, and node l represents the work block. Time t_{ij} represents the ground leg commute time from a home block to an embarkation vertiport, t_{jk} represents the sUAM vehicle travel time between the vertiport pair, and finally t_{kl} represents the ground leg commute time from a home block to a disembarkation vertiport. The summation of the aforementioned leg times for a given routing form an important weighting coefficient for the operating cost objective function explained in Section IV.C.

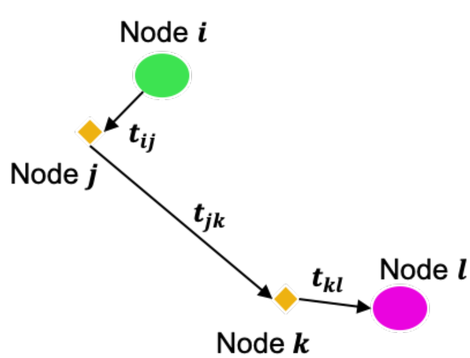


Fig. 5 Graphical representation of a routing option

A compiled set of potential routing opportunities to account for commuter demand across the 35,000 non-zero home-work census block pairs is compiled and filtered using a selection heuristic. The selection of "good" routing options prior to the optimization process reduces model complexity and allows the experimenter to tune the network model to specific requirements resulting in models of varying complexity. Firstly, the implemented selection heuristic only associates vertiport pairs with home-work block pairs in a routing option if the corresponding embarkation and disembarkation vertiports are within a 3 mile radius of the home block and work block respectively. This ensures that only close-proximity vertiport pairs are able to service commuter demand. Secondly, sUAM air metro opportunities as mentioned in Section I are best realized for trip distances in excess of 20 miles. As such, routings whose home-work block pair distances exceed 20 miles are included in the "good" options set. A simplified graphical representation of this selection heuristic is shown in Figure 6.

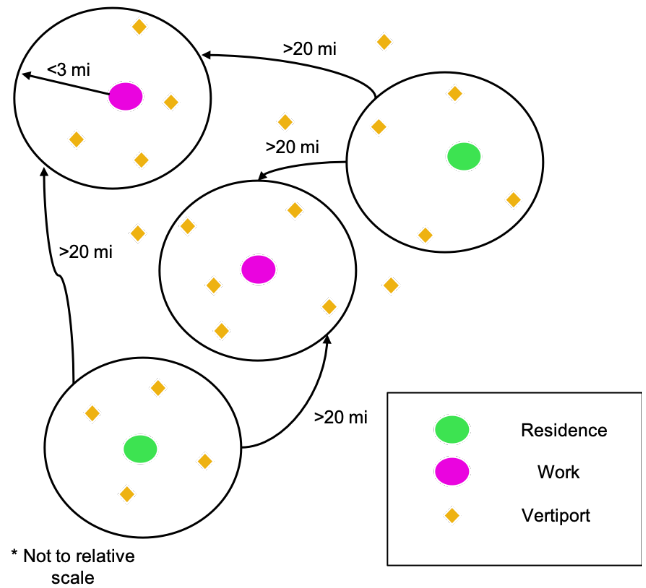


Fig. 6 Graphical representation of routing options selection heuristic

2. UAM Vehicle Mission Profile

In order to most accurately capture the performance and operational envelopes of a potential sUAM vehicle that will serve the aforementioned sUAM routings, a mission profile characterizing the various mission segments of the vehicle was constructed using a nominal vehicle requirements summary provided by Uber [13]. For modeling tractability, a simplified version of the proposed mission profile was created as shown in Figure 7. The symmetric mission profile consists of five primary segments: taxi-out, acceleration and climb, cruise, deceleration and descent, and taxi-in. The taxi-out and taxi-in phases increment sUAM trip time by a fixed amount set to 3 minutes to account for boarding/de-boarding and taxiing from and to the gate. The remaining three flight segments each consist of two properties: an averaged velocity at which the vehicle is operated and a fraction of the total trip distance during which the vehicle is operating within a given flight segment. As expected, the cruise segment comprises the majority of the flight time (80%) and the cruise speed for this vehicle is set to a nominal 150 mph. This speed value is in accordance with industry-projected estimates. A mission profile additionally affords the experimenter with additional degrees-of-freedom with which to study the effectiveness of the sUAM network to changing performance and range characteristics of the vehicle under consideration.

3. Ground Vehicle Travel Times

One of the primary capability gaps identified in previous efforts made to address the problem of vertiport siting was the ability to accurately estimate ground travel time during peak traffic hours. The potential to ease traffic congestion and reduce daily commute times is one of the primary drivers behind the significant investments into the development of urban air mobility. Therefore, it is important to develop an accurate way of estimating current daily commute times from origin to destination both to act as a baseline for comparative advantages of utilizing air mobility as well as to determine time losses due to traffic congestion during the commute to and from potential vertiport locations. The U.S. Census Bureau LODES data gives significant insight into this problem by providing information on the daily commuting

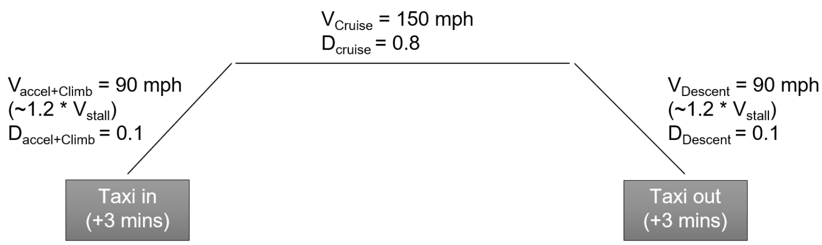


Fig. 7 Nominal mission profile of sUAM vehicle

patterns for the civilian workforce. This information is broken up into the number of people commuting on a daily basis between an origin census block and a destination census block. Since census blocks are generally sized less than half a kilometer square and in densely populated areas can be less than an acre in size, this provides highly granular data from which an accurate surrogate model can be developed. The next challenge is to accurately calculate travel times between origin and destination blocks. This was done using Microsoft Bing Maps Routes API which calculates the shortest driving route between an origin point and a destination point. The API also provides the option to pass in a specified time of day from which historical traffic data is used to provide the actual travel time with given traffic conditions as well as the predicted travel time without traffic. Since the LODES data provided by the U.S. Census Bureau includes on the order of several million origin and destination commuting block pairs, it would not be computationally feasible to individually calculate commute times for each block pair in an optimization problem designed to run in a few minutes. In addition, API restrictions limit the number of calls that can be made in a 24-hour period to 50,000, which is well below the potentially tens of millions that would be needed. For this reason,a surrogate model was built using real world traffic information that could accurately estimate the travel time for a wide range of commutes with a high degree of accuracy. In order to develop the surrogate model, a uniform random subspace of 250,000 origin and destination block pairs was selected from the initial set of roughly two million LODES data points for the south Florida metro area. Peak rush hour traffic commute times along with no traffic commute time estimates were calculated using the Bing Maps Routes API and tabulated. A multi-variable regression model was then developed using geo-localized travel time and distances to predict commute times for any combination of origin and destination blocks during peak rush hour times. The resulting actual vs. predicted plot and residual errors from the South Florida Metro area-based model are shown in Figure 8.

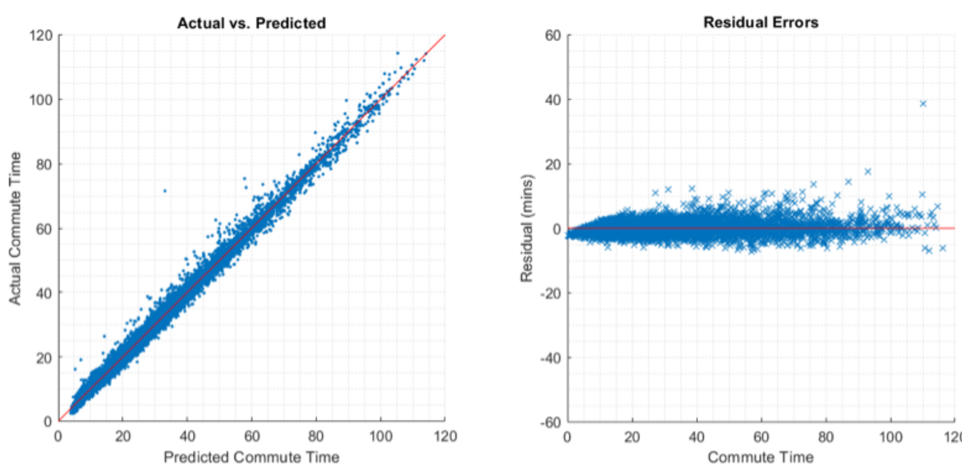


Fig. 8 Surrogate model performance actual vs. predicted (left), and residual error (right)

The relatively tight grouping of the actual vs. predicted plot indicates that the surrogate model performs well as an accurate means of estimating average commute time during peak traffic hours, with only a handful of outliers that do not represent a significant portion of the population. The plot of residual errors as well as the probability density function (PDF) of the modeling errors in Figure 9 show that the majority of cases were modeled with a low error. The cumulative distribution function (CDF) plot in Figure 9 further indicates that about 55% of the cases are modeled with less than one minute error in their overall commute time, and about 90% of the cases are modeled with less than two minutes error in their overall commute time.

The significant accuracy improvement afforded to the ground transportation travel times between blocks through the use of a traffic API-based surrogate over an initially formulated distance and constant vehicle speed proxy stood reason enough to integrate this element within the overall data-driven optimization framework. The surrogate equation constructed is queried through the an input consisting of 2 pairs of coordinates representing the ‘origin’ and ‘destination’ blocks (which may be home/work, home/embarkation vertiport, disembarkation vertiport/work, or any other pair of locations).

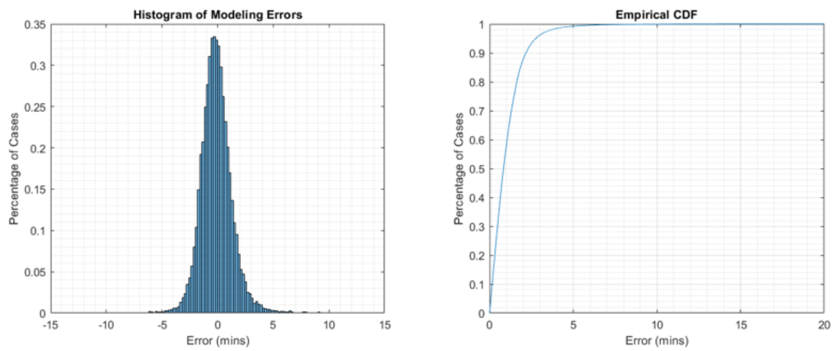


Fig. 9 Histogram (left) and CDF (right) of the ground travel time modeling errors

C. Objectives and Constraints

Generally speaking, the optimization problem is formulated as a hierarchical multi-objective MIP problem that finds the set of routings with minimized setup cost of vertiport facilities that can serve all demand block pairs and achieve maximum time savings for commuters. Consequently, the minimal set of vertiport pairs that can service the stipulated demand across the census blocks considered can then be derived from this routing set.

The MIP formulation is modified slightly for the uncapacitated and capacitated vertiport facility cases. For the uncapacitated case, two decision variables are created: x_{ijkl} which is a binary decision variable that represents a routing assignment, i.e. whether the vertiport pair (j,k) is assigned to the home-work block pair (i,l) and y_{jk} which is again a binary decision variable that represents whether or not a given vertiport pair is available to service demand or not. The uncapacitated problem has been modeled such that each demand block pair is singly-sourced, which means that a single vertiport pair is to serve any given demand block pair (i.e. no demand split across vertiport pairs for a given home-work block pair is allowed). This is apt as the vertiport facility is not volume-constrained and thus can accept any amount of demand, hence the use of binary decision variables for the routing assignments. The constraints for the uncapacitated problem are outlined in Eq. 3 and Eq. 4. Equation 3 states that across all routings associated with a given home-work block pair, the demand must be completely served. Equation 4 states that the demand can only be accepted/serviced by vertiports that are active. Finally, given that this is a multi-objective formulation, there are two objective functions that need to be satisfied concurrently. First, Eq. 5 seeks to minimize the total number of vertiport pairs used to serve the network demand. Second, Eq. 6 seeks to minimize the summed ground travel times to and from the vertiport facilities across the selected routings.

$$x_{ijkl} \in \{0,1\} \quad \forall j \in J, \forall k \in K \tag{1}$$

$$y_{jk} \in \{0,1\} \quad \forall j \in J, \forall k \in K \tag{2}$$

$$\sum_{(j \in J, k \in K)} x_{ijkl} = 1 \quad \forall (i,l) \in (I,L) \tag{3}$$

$$x_{ijkl} - y_{jk} \leq 0 \quad \forall i \in I, \forall j \in J, \forall k \in K, \forall l \in L \tag{4}$$

$$min \sum_{(j \in J, k \in K)} y_{jk} \tag{5}$$

$$min \sum_{(i \in I, l \in L)} \sum_{(j \in J, k \in K)} (t_{ij} + t_{kl}) x_{ijkl} \tag{6}$$

The capacitated problem formulation is different from the uncapacitated case in two aspects. First, the problem no longer assumes that the demand for each home-work block pair is to be singly-sourced (i.e. multiple vertiport pairs can fractionally satisfy the demand for a given home-work block pair together). Second, the amount of demand serviced by a given vertiport facility is now constrained by the commuter capacity of that facility. The multi-source aspect can be accounted for by modifying the routing assignment variable to be a non-negative continuous variable between 0 and 1 as shown in Eq. 7. A value of 0 corresponds to no demand being captured for a given home-work block pair, and a value of 1 corresponds to a complete capture of a given block pair demand. Any value between 0 and 1 corresponds to a fractional demand being served by a vertiport pair for a home-work block pair. The capacity constraint shown in Eq. 8 ensures that for any number of home-work block pairs serviced by a vertiport pair, its facility capacity is not exceeded by the aggregate demand served across the aforementioned block pairs. The objective functions for the uncapacitated case, the remaining constraints, and assignment decision variables are applied to this formulation again.

$$0 \leq x_{ijkl} \leq 1 \quad \forall (i,l) \in (I,L), \forall (j,k) \in (J,K) \tag{7}$$

$$C_j y_{jk} - \sum_{(j \in J, k \in K)} d_{il} x_{ijkl} \geq 0 \quad \forall (i,l) \in (I,L) \tag{8}$$

D. Implementation

The implementation of the data-driven optimization framework was done in Python 3.6 paired with Gurobi API for Python. Python excellent Pandas and DBF packages are well-suited to handle data wrangling activities of vast census datasets. An object-oriented programming (OOP) construct is used to represent the network model elements described in Section IV.B. OOP provisions such as polymorphism allow for high degree of code modularity and flexibility. An experimenter can leverage OOP to quickly create and solve different network model configurations. Coupled with OOP properties, the code also contains a high level of user-defined parameterization within the network model creation and routing option down-selection. User parameterization includes settings to tune home-work block pair time separations (i.e. only consider models with home-work block pairs with a ground time separation of 30 minutes or more for example), vertiport demand-handling capacity, and embarkation/disembarkation vertiport assignment radius (i.e. a certain number of mile radius within which to assign embarkation and disembarkation vertiport pairs to a given home-work block pair). Such user-defined parameters afford a high degree of model flexibility and allow users to conduct meaningful MIP experiments and "what-if" scenarios.

V. Results

A. Edge-case testing

The proposed Mixed-Integer Programming approach possesses the capability to solve uncapacitated and capacitated vertiport placement problems with key results being a minimal set of vertiport pairs that can service the home-work block pairs demand requirements with the shortest aggregate ground leg time (i.e. maximum aggregate time savings across the entire network). Prior to applying the modeling and optimization environment described in previous sections to business scenarios, two edge cases are run to test the robustness and validity of the algorithm and impart confidence to the experimenter that it produces results as expected. The first edge case is to assess the distributed demand servicing capacity of a single uncapacitated vertiport pair with a large catchment area (> 20 miles) that can potentially service many block pairs. This particular case allows the experimenter to test whether all the demand from spread-out blocks are accounted for by a single vertiport pair which is the expected result for a case without a leg time minimization objective and uncapacitated vertiports. Figure 10 shows the results for a case with a network size of 10 block pairs. As evidenced in the figure, the demand from each of the 10 home blocks across a large catchment area is assigned to a single vertiport pair, which strengthens confidence regarding the efficacy of the catchment radius capability of the model.

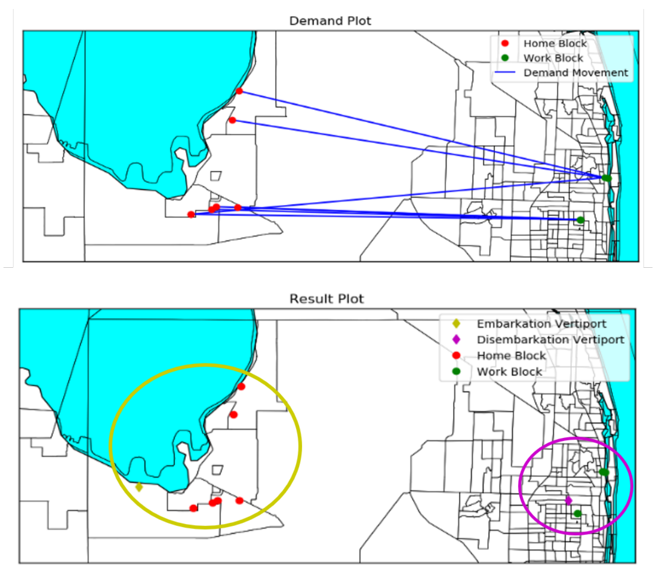
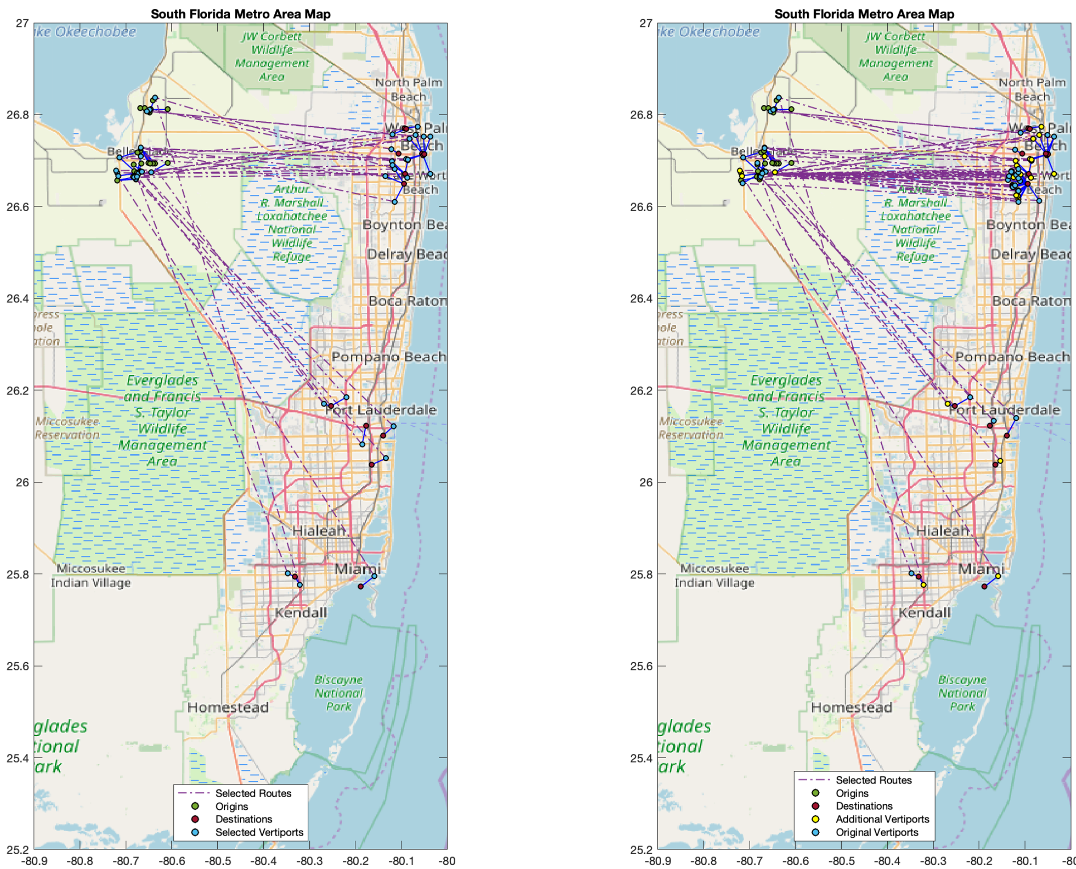


Fig. 10 Edge case 1: distributed demand servicing capability

The second edge case involves testing the demand servicing robustness characteristics of the network using capacitated vertiports in regions with abnormally high-demand block pairs such that multiple vertiport pairs are required to completely service the demand emanating from such block pairs. For this test exercise, an input network of 50 block pairs is selected with the following characteristics: 30 minutes or more ground time separation and catchment radius of 3 miles. This block-pair network is modified by multiplying the demand for the original highest demand block pair by a factor of 1000 in order to spur more vertiport pair routings to handle this inflated demand block pair. The demand for the block pair was initially 13 which is then increased to 13,000. In order to satisfy 13,000 customers, a minimum additional 33 vertiport pairs, each with 400 person capacity, would be required. This edge case is devised to test whether the minimum number of additional vertiport pairs (in this case 33) is actually identified and presented as part of the new optimal set of vertiport pairs by the optimizer. Figure 11 shows the results of this test case. Figure 11a represents the solution vertiport network and the corresponding routings for the original demand network whereas Figure 11b shows the additional vertiport pairs, represented in yellow, that are placed in the solution network to satisfy the single block pair with 13,000. This case requires an additional 24 unique vertiports to be placed in the network to service the single inflated demand.



(a)

(b)

Fig. 11 (a) Original demand network (b) Inflated demand network

Once the validity of the modeling and optimization environment is confirmed, two types of business scenarios are generated and tested. The first scenario is a vertiport and leg time minimization whilst meeting all network demand. The main purpose of this experiment is to understand the quality of the solution provided and compile optimal vertiport networks across various network sizes and other model control parameters. The second scenario is one that is of more potential business value to decision-makers whereby a predefined number of uncapacitated vertiports that an organization is willing to construct is specified to the optimizer and the percentage of demand that could be captured while maximizing time savings for a given network is tracked. This second experiment provides planners operating under financial constraints with the foresight to decide on a sub-optimal network of vertiport opportunities that captures a suitable demand percentage. The aforementioned two scenarios provide the authors with the opportunity to exercise the proposed parametric framework on realistic use cases.

B. Capacitated Problem

Table 2 shows the results of the MIP optimization for networks/input problems ranging from 10 to around 1700 block pairs within the South Florida region. The capacity of each vertiport is assumed to be homogeneous across the network and does not vary based on the relative location of the vertiport within the coverage area. The vetiports capacity is calculated as follows. First, we assume 5 hours of daily operations divided equally between the morning and evening hours to pick-up work commuter traffic. Then, we assume that, on average, an sUAM aircraft takes-off every 2 minutes and that these aircraft have a capacity of 4 passengers each [**?**]. This provides a single vertiport with the capability to handle 500 commuters per day. All the tabulated cases are run with some commonalities in run parameters such as an integrated mission profile with a 150 mph cruise segment, a non-zero minimum demand threshold between block pairs considered, a minimum 30 minute ground travel time between considered block pairs, and finally, a constant 1.5 mile catchment radius around home and work blocks. The initial dataset of home/work block pairs is filtered to only keep those that meet each of the aforementioned criteria. Among the filtered set, only a specified number of block pairs that exhibit the highest demands are down-selected for the analysis. Increasing values for this specified number are considered as shown in the second column of Table 2. The base run parameters were set constant across all the scenarios considered so as to allow for better network comparison between cases and isolate the effect of denser demand networks on the optimized network of vertiports placed to accommodate the corresponding demand.

Additionally, the LODES data that provides the demand statistics at a block pair level shows that only a few people typically travel between block pairs, meaning that the block-block flows of people are mostly rather small. Since the optimization is expected to identify and select sUAM routings that cater to block-block demand flows, there are limited opportunities for the capacity envelopes of the vertiports to be exceeded given the low demand flows between blocks. Hence, in order to allow for fractional demand satisfaction across routings to cater to a particular block-block flow, the demand for a subset of the scenarios considered such as cases 4 and 6 in Table 2 were artificially multiplied by an arbitrary factor of a 100. This then allows for some synergies among vertiports when addressing high demand block pairs that may exceed the capacity of any individual vertiport. When compared to heuristic model results, the proposed MIP formulation possesses a key benefit. Since the proposed MIP formulation is built on top of a network flow model, the optimization results provides a set of optimal ordered routings to satisfy the demand considered, not just an un-ordered set of vertiport placement locations. This is revealing as it allows decision-makers or planners to tie back the set of vertiport opportunities to their intended routings allowing for future schedule and route network optimization exercises. Figure 12 depicts the progression of the optimized network of capacitated vertiports addressing artificial demand requirements for 100, 1000, and 1742 block pairs within the South Florida region. Looking at Table 2 and Figure 12, a fairly constant network-aggregated time saving of approximately 65% is achieved for increasing numbers of block pairs considered in both cases of artificially-inflated and real-demand values. Additionally, the results line up with mathematical intuition in that artificially-inflated demand networks such as case 6 require a handful more vertiport pairs to distribute block pair demand amongst one another in order to totally satisfy demand constraints described in section IV.C.

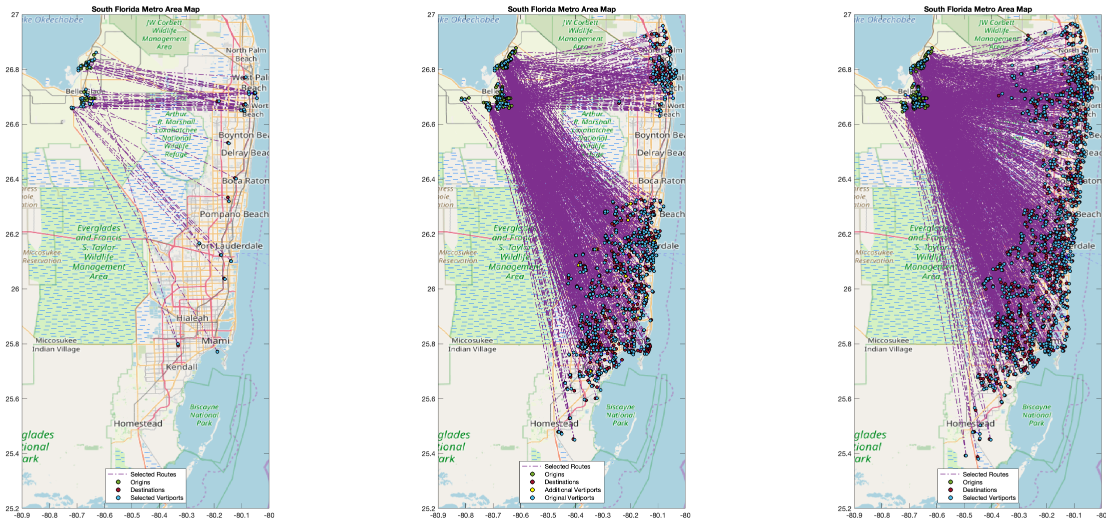
Table 2 Tabulated results across 6 scenarios for capacitated optimization problem

Case	# Block pairs	Demand/block pair inflation factor	% Time savings	# of vertiport pairs placed
1	10	1x	61.50	9
2	100	100x	74.73	63
3	1000	1x	65.74	789
4	1000	100x	65.59	805
5	1742	1x	69.82	1169
6	1742	100x	69.53	1209

C. Uncapacitated Problem

For the second exercise, a series of increasing network size/input problem cases with block pair counts ranging from 100 to 1742 were created. Each test case has a user-defined number of uncapacitated vertiport pairs to place in the network in order to maximize the demand capture and provide adequate time savings for each block pair considered. Similar to the capacitated results, a common set of run parameters were established for each of the runs presented in Table 3. These run parameters are an integrated mission profile with a 150 mph cruise segment, a non-zero minimum demand threshold between block pairs considered, a minimum 30 minute ground travel time between considered block pairs, and finally, a constant 1.5 mile catchment radius around home and work blocks. Once again, only a specified number of higher-demand block pairs that meet the above run conditions are selected for each test case.

In this context, two primary metrics are tracked: the cumulative % demand satisfaction across all the selected routings and the cumulative sum of the ground leg times for each routing in the selected routings set. Indeed, these two metrics allow the experimenter to weigh the tradeoffs between demand satisfaction and door-to-door travel time minimization. This is especially noticeable in cases 4 to 6 in Table 3. Indeed, compared to case 4, case 6 features an



(a) (b) (c)

Fig. 12 (a) Case 2: 100 block pairs; (b) Case 4: 1000 block pairs; (c) Case 6: Maximum number of block pairs

Table 3 Tabulated results across 6 scenarios for uncapacitated optimization problem

Case	# Block pairs	# Vertiport pairs to place	% Demand satisfaction	Cumulative ground leg time (minutes)
1	100	10	17.48	88.46
2	100	50	52.84	493.78
3	1000	100	16.93	996.88
4	1742	150	13.84	1348.14
5	1742	500	30.18	4506.87
6	1742	1000	57.70	9993.12

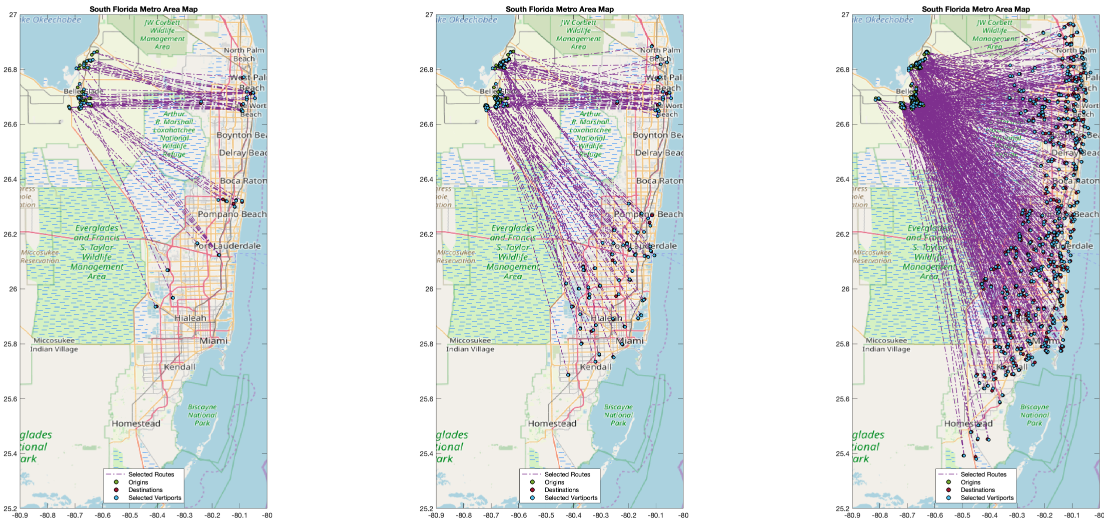
approximately 5.5 time increase in the number of vertiport pairs placed which yields 34% additional demand satisfaction and 10 times more cumulative travel time saving accrued across the larger set of selected sUAM routings. A visual representation of the optimized uncapacitated vertiport sUAM network with increasing network sizes corresponding to Case 2, 3, and 5 from Table 3 is depicted in Figure 13

D. Sensitivity Analyses

In addition to understanding optimized network responses to changing input problem sizes, a parameterized modeling approach also affords additional insights into the optimized network responses to other run parameters of interest. In this study, a set of three independent sensitivity analyses using the capacitated problem were conducted with the aim to uncover interesting trends in the overall network characteristics in response to variations in three parameters: the *relative tolerance* parameter which constrains the relative degradation factor of the primary objective whilst optimizing for the second objective (specific to hierarchical optimization setups such as this); the vertiport *catchment radius* of home and work blocks (which are assumed to be the same in this study); and the *block pair ground travel time separation* between the home and work blocks used to filter down block pairs of interest.

1. Sensitivity to the Relative Tolerance

A hierarchical MIP optimization approach is used to solve the uncapacitated vertiport siting problem that consists of solving to optimality two interrelated objective functions. The primary objective is the maximization of demand



(a)   (b)   (c)

Fig. 13 (a) Case 2: 100 block pairs with 50 vertiport pairs requirement; (b) Case 3: 1000 block pairs with 100 vertiport pairs requirement; (c) Case 5: Maximum number of block pairs with 500 vertiport pairs requirement

capture and the secondary objective is the minimization of the cumulative ground leg travel time for the selected routings/decision variables after the primary objective optimal value is attained.

Naturally, in the hierarchical optimization setup, there exists a balance between the solution quality of the primary objective in relation to the secondary objective specified. The workflow of the Gurobi optimizer is setup in such a way that the primary objective function is solved to optimality prior to solving for the secondary objective function. Additionally, the *relative tolerance* is a Gurobi parameter that specifies the allowable degradation limit of the primary objective value when searching for a solution to the secondary objective. This *relative tolerance* parameter can be thought of as a proxy for user/business preference for uncapacitated vertiport siting scenarios between demand maximization and time savings. A relative tolerance parameter value of 10% has been used for all the test cases presented in the previous sections, but it is valuable to understand the effects on the solution of varying *relative tolerance* values.

This sensitivity study sweeps through the *relative tolerance* parameter from 0% (no degradation in the primary objective value when optimizing for the secondary objective) to 50% (which allows up to 50% degradation in the attained primary objective value when optimizing for the secondary objective). This sensitivity study was performed on an uncapacitated problem with a size of 1000 block pairs separated by 30 minutes ground time, a catchment radius of 1.5 miles, and a vertiport pair placement requirement of 100.

Table 4 and Figure 14 summarize the changes in the percentage demand satisfied and the cumulative leg travel times when the *relative tolerance* parameter varies from 0% to 50%. Results shown that an increase in the *relative tolerance* parameter results in lower cumulative ground leg times across the selected routings, but also in lower demand satisfaction percentages.

2. Sensitivity to the Catchment Radius

The *catchment radius* parameter defines the localized distance around a block within which vertiport opportunities may be selected and synergized to satisfy demand flow. Larger *catchment radius* parameters result in a larger number of potential vertiport locations available to create routing options for a given block pair, thus increasing the number of routing opportunities to satisfy the demand flow for this block pair. This potentially also provides the opportunity for multiple blocks to share one or more identical potential vertiport locations in their routing options. When trying to minimize the number of vertiports placed to satisfy the demand flows for the corresponding block pairs, this would in turn reesult in a reduction in the overall number of vertiports chosen and potentially higher time savings.

This sensitivity study was performed on a capacitated problem with a size of 100 block pairs that are separated by

Table 4 Sensitivity of optimization results to the Gurobi relative tolerance parameter for uncapacitated problem

RelTol (%)	% Demand Met	Cumulative leg times across selected routings
0	17.89	1199.67
10	16.05	908.392
20	14.92	872.601
30	12.65	846.207
40	10.64	841.235
50	9.86	840.523

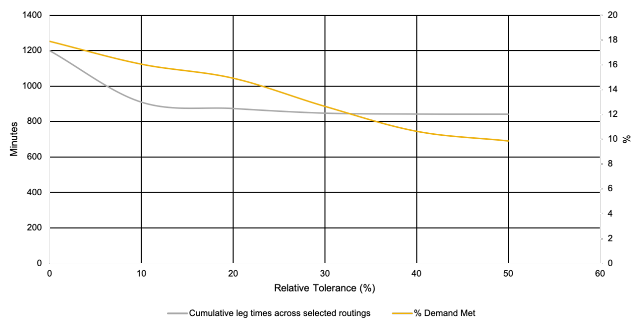


Fig. 14 Sensitivity study - relative tolerance parameter

at least 30 minutes of ground travel time. Table 5 and Figure 15 summarize the changes in the number of vertiport pairs placed and the network-aggregated time savings obtained when the *catchment radius* parameter increases from 0.5 to 2.5 miles. As inferred previously, Table 5 shows that larger *catchment radius* values for each block result in greater numbers of common routing opportunities available for consideration, which requires an overall lower number of vertiport pairs to satisfy the same demand. Naturally, larger *catchment radius* values promote greater synergies for vertiport combinations but might sway away from one of the goals of sUAM operations which is to provide easily accessible vertiport locations for customers both on the embarkation side and the disembarkation side.

Table 5 Sensitivity of optimization results to catchment radius parameter for capacitated problem

Catchment radius (mi)	% Time Savings	# of VP pairs placed
0.5	65.114	81
1	68.47	73
1.5	74.19	59
2	76.18	54
2.5	76.16	54

3. Sensitivity to the Block Pair Ground Travel Time Separation

The final sensitivity study is once again conducted on a capacitated problem with an upper-limit of a 100 block pairs separated by 30 minutes ground travel time, where each block is assigned a catchment radius of 1.5 miles. The objective of this sensitivity study is to analyze the changes in the vertiport network solution to a variety of input problems differentiated by the block-block ground travel time. Each test case considers block pairs that are separated by a ground travel time of between 20-30, 30-40, 40-50, and 50-60 minutes apart.

Table 6 and Figure 16 summarize the changes in the number of vertiport pairs placed and the network-aggregated time savings obtained when the *block pair ground travel time separation* parameter is within one of the four aforementioned

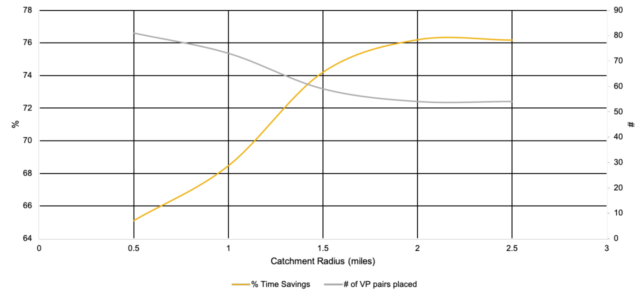


Fig. 15 Sensitivity study - catchment radius parameter

ranges of values, along with the corresponding numbers of block pairs considered as input to the optimization problem. Results show that the % time savings accrued across the network for block pairs between 20 and 30 minutes apart from one another is noticeably higher than that for the other scenarios. This may be attributed to the fact that the ground separation times are traffic-driven and hence shorter trips tend to be more affected by traffic impediments potentially due to smaller roadways and other city-related effects. On the contrary, longer trips may be associated with the utilization of highways and multi-lane roads with more favorable traffic conditions. Hence, the improvement in door-to-door travel time savings is not as significant as in the shorter trip case (20 - 30 minutes) for this particular problem setup.

Table 6 Sensitivity of optimization results to block pair ground travel time separation for capacitated problem

Distance (minutes)	# Block pairs in network	% Time Savings	# of vertiport pairs placed
20 - 30	98	89.35	23
30 - 40	23	72.33	14
40 - 50	100	66.75	78
50 - 60	100	75.76	56

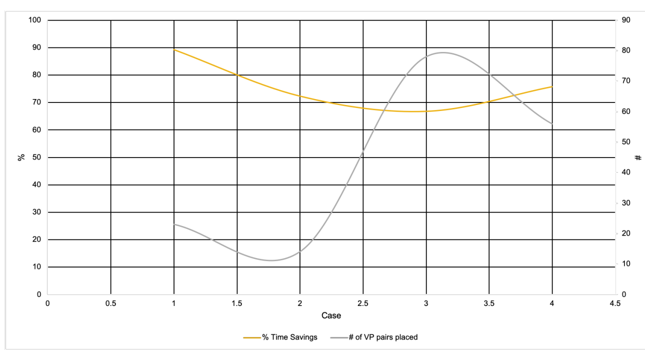


Fig. 16 Sensitivity study - block pair ground travel time separation parameter

VI. Conclusions and Future Work

In conclusion, a parametric and highly flexible MIP network flow formulation was developed for solving vertiport placement problems for Air Mobility operations. Using a highly granular network model at the census block level, the proposed environment is capable of mathematically representing and optimizing various concepts of operations for Air Mobility studies, and of providing valuable insight to decision-makers and planners with regards to various mobility, infrastructure, investment, and demand capture considerations. Future studies aim to explore the utility of a column generation (CG) methodology to enhance the optimization workflow. Such methods are especially useful in solving MIP problems such as the sUAM vertiport placement problem where there exist a large number of decision variables but a comparatively low number of constraints.

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