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**PRODUCTION FUNCTIONS: A THEORETICAL  
AND EMPIRICAL STUDY**

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Ph.D. thesis submitted to the Department of Economics at the University of Warwick. The research reported here was conducted mainly within the Centre for Industrial Economic and Business Research at the University of Warwick and submitted for examination in June 1976.

## Chapter (1)

### Introduction, Aims and Scope

The aim of this study is to investigate whether there exists a realistic theory of production. One can hardly claim that economists have ignored the study of production functions. The published literature has already reached considerable proportions and is still growing rapidly. Despite the widespread interest, (or perhaps because of it) the area lacks a consensus of opinion. Yet an understanding of the technology of production is crucial to the development of realistic theories and to the formulation of a wide range of policies. Of all the areas that we might distinguish, it probably has the most widespread and important implications for other economic theories.

In arriving at the present state of the art, the economic theory of production has travelled a long and tortuous path. The bulk of empirical work still rests on functional forms which are direct descendants of the work of Cobb and Douglas (1928). Yet at the present time there is an under-current of distrust of the traditional (neoclassical) theory. The good empirical performance of the neoclassical estimates is in stark contrast with the widely held belief that, at best, the theory is a gross simplification of the real world. This tension has been intensified by the results of work on aggregation - such as those by Fisher (1969, 1971) - which have emphasised the stringency of the conditions which must be fulfilled before rigorous aggregation is possible, and by the growth of rival theories descended from the vintage models of Johansen (1959) and Salter (1966).

The tension that has developed has induced the protagonists in the debate to search for evidence that one or other of the alternative theories is correct. It is probably true to say that the vintage theories have not

yet been properly tested. This is mainly a consequence of the empirical success of the neoclassical functions, but partly due to the demanding data requirements of the more recent theories. The inability to provide conclusive empirical evidence has led to a number of attempts to reconcile the alternative schools of thought. The most recent of these is by Johansen (1972) and is based on the thesis that different theories are relevant to different levels of aggregation and over different periods of time. Johansen's intention was to construct a number of related economic boxes into which existing theories can be meaningfully slotted, rather than to replace the existing body of knowledge. The end-product is an integrated theory of production drawing on the more important strands of the present theories. An attempt to reconcile the opposing factions seems timely given the disarray of the current theory and given the central role that production functions play in economics.

The concept of an integrated production schema appears to be an important theoretical advance, and it is adopted as a framework for organising the research undertaken here. In this way, it is possible to ask formally whether the theory adds significantly to our present understanding of the production process and, in particular, whether it makes the existing theories any more tenable. Such an assessment must include a critique of the schema as a theoretical construct as well as tests of the more important concepts integrated within its framework. Although the work reports estimates of the more important production functions, particular attention is paid to vintage models, where little empirical work has so far been carried out.

A brief summary of the Johansen schema is provided in Chapter 2. It is argued that it is not an additional theory of production, but it represents a number of related boxes into which existing (or new) theories can be placed. It is demonstrated that the schema can incorporate both the fixed coefficient and neoclassical extremes. Consistency between these is reached by varying the level of aggregation and the time horizon. Chapter 2 also reviews some



of the more important generalisations that Johansen has added to the theory. It is argued that the concept of an integrated production system is extremely flexible in the sense that it can be made relevant to a wide variety of industrial situations. An unfortunate conclusion however, must be that the more sophisticated versions of the theory are unlikely to be empirically testable given the crudity of the available data. The final section of Chapter 2 looks at the implications of the schema for one particular area of economics. Tracing the implications through all of the various areas would be a mammoth task, and therefore this study limits itself to considering the schema as a vehicle for reconciling the opposing factions in the manpower planning debate (i.e. the 'manpower requirements' and the 'rate of return' schools).

After reviewing the existing theories of production one almost automatically feels a sympathy for anything as refreshing and well thought out as Johansen's work. Chapter 3, however, attempts to expose a number of important deficiencies associated with the construct and with the theories that Johansen chooses to slot into the various economic boxes. The first is the relevance of vintage theory to production in engineering and, in particular, the nature of technical change (i.e. whether it is embodied or disembodied and whether pure vintages can be distinguished). An important questionmark is placed against all of the existing works which attempt to reconcile the alternative theories of production. In the case of the Johansen model the dispute is not with the theoretical rigour of the explanation of the existence of substitution possibilities in macro functions, but with any attempt to extend this result to explain the good empirical performance of neoclassical functions found in the published literature. A further problem with vintage theory is associated with the restrictive assumption of profit maximisation. It is demonstrated in the final section of Chapter 3 that only limited elements of the new managerial and behavioural theories can be introduced into the theoretical framework.

The weakest link in vintage theory appears to be the ex ante function and Chapter 4 explores some of the avenues which promise to make this function a much more useful concept. It is argued that an ex ante region is more realistic than a unique function that is technically given at each point in time. If the concept of a region is adopted then it becomes necessary to find some means of handling movements within its boundaries if it is to play a practical role within a production schema. Here the work of Nordhaus (1969, 1973) appears to be particularly useful.

The empirical tests of the theories were based on data drawn from the U.K. engineering industries because engineering is a key sector of the economy and probably has the most comprehensive data of all U.K. industries. Important data deficiencies nevertheless existed and, on a number of occasions, it proved necessary to construct new data or to reformulate the functions so they could be estimated using existing data supplies. A major benefit of attempting research of this kind is the insight that it yields about data deficiencies and the ways in which they can be overcome. Appendices are provided which indicate how some of the gaps that exist in current U.K. official statistics (i.e. estimates of investment and fuel consumption by minimum list heading (MLH)) can be filled.

Given the availability of data and the nature of the functions themselves, cross-sectional estimates were considered to be most useful. The basic units of observation were minimum list heading industries (MLHs) within the U.K. engineering sector. This is the most detailed information one can expect from published sources: greater disaggregation would involve collecting data from individual firms along the lines of Layard, et al. (1971). Despite the level of aggregation, information was available that enabled results to be obtained for a time series of cross-sections. Nerlove (1966) and Nelson (1973) see this two dimensional pool of information as the key

to obtaining meaningful estimates of the underlying technology of production.

A variety of production functions are estimated at different levels of aggregation. Chapter 5 investigates the most detailed level which is concerned with individual production processes such as welding and turning. Some useful links are drawn here with the work being undertaken at the Science Policy Research Unit (Sussex University) whose qualitative approach is largely consistent with the simplest Johansen schema. Relationships at this level had to be estimated in employment function (as opposed to production function) form because of severe data constraints. The functions perform reasonably well and give important insights about the underlying ex ante technologies.

The success of the micro study indicated that it might be worth estimating putty-clay functions at a higher level of aggregation, where the loss in theoretical rigour appears to be offset by the existence of more complete and reliable data. Despite the wealth of employment and production function studies, remarkably little work has been carried out using vintage models. In order to provide a basis for empirical research, Chapter 6 develops a putty-clay model consistent with the Johansen schema and with the general body of vintage theory, that can be estimated on the basis of the U.K. data. The functions, which still distinguish particular labour skills and capital types, perform reasonably well, giving some support for the vintage approach.

Chapter 7 looks at the greatest level of aggregation used in this study (i.e. across NLHs for all production activities in combination). A number of production functions are estimated, which are drawn from the constant elasticity of substitution (CES) class of technologies. The results are largely consistent with the existence of substitution possibilities according to the simplest of the CES models. An attempt is

made to find if there are differences in the results between industries within engineering. Chapter 7 demonstrates that although aggregate neoclassical production functions appear to perform well empirically, the results give some cause for concern. Their empirical performance is much less acceptable when labour productivity appears as the dependent variable in place of output.

In Chapter 8 consideration is given to the arguments that have been put forward to explain the fundamental contradiction between functions thought to be intuitively plausible (based on vintage theory) and those that appear to arise from empirical research (based on neoclassical theory). It is argued that none of the current approaches (including Johansen's) is a complete explanation of the phenomenon. An alternative avenue is investigated that involves the construction of a 'surrogate' production function. Although the 'Cambridge controversy' has so far added little to our understanding of the empirical results, it is pointed out that certain parts of it can be salvaged to yield valuable insights about the contradiction. The explanation put forward indicates that the traditional (neoclassical) estimates are only indirectly related to the underlying technology of production. Other factors, not considered by the main body of neoclassical theory, are crucially important.

Chapter 9 gives a more detailed discussion of a number of other factors which influence the production process. In order to test the relevance of these factors to the explanation of variations in labour productivity, 'quality' and 'performance' variables are included in the regressions along the lines of Griliches and Ringstad (1971). Not all of these additional variables fit neatly into the neoclassical framework: some relate to aspects of the technology that have been given scant coverage in the literature and others relate more to market structure and industrial performance than to the technology of production. While the traditional capital and labour variables nevertheless prove to be significant explanatory

variables, the success of the 'performance' variables suggests that the explanation of labour productivity requires a more dynamic approach that does not only draw on the theory of production.

Chapter 10 ties the remaining loose ends together and a number of conclusions are drawn. The most important of these is that only now is a realistic theory of production beginning to emerge. This new theory is much more flexible than its traditional counterparts: different technologies are associated with different production situations. The bulk of existing theory is dismissed as largely misleading as a description of the underlying technology of production. It is argued that, if we wish to say anything about the production function, data of a much more detailed kind must be used. Nevertheless, the more traditional neoclassical results do not arise by chance, but because competitive forces constrain the set of economically viable technologies to lie on a function of this type. There is no reason to suppose that such models will be any the less useful for forecasting or modelling so long as the causes of the relationship are understood. This conclusion obviously is important for the theoretician, but it is also relevant to industry and government planners who have used neoclassical theory as a basis for modelling and forecasting without fully understanding the tools they have employed.

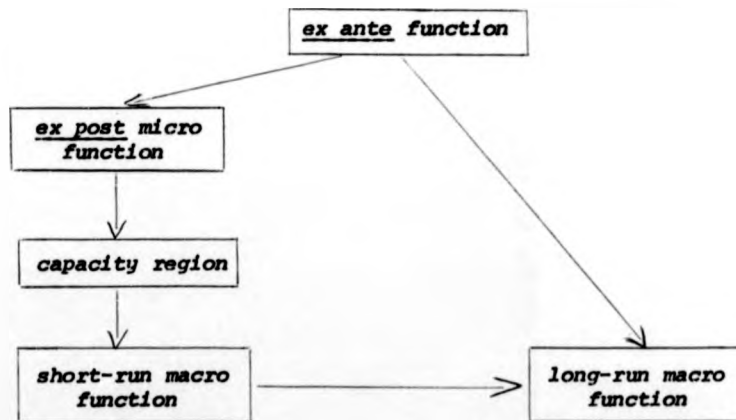
## Chapter 2 The Johansen Schema

### A. Introduction

The aim of this chapter is to provide an understanding of the nature and implications of the Johansen schema. The schema has been adopted as a framework for organising the research reported in later chapters and it is therefore essential to grasp the concepts that are used and the ways in which they are linked with one another. The most expedient way in which this can be achieved is by reviewing the theory contained in Johansen (1972). This chapter continues by introducing the concept of an 'integrated production system'. Sections B and C review the basic Johansen concepts and a number of generalisations. Finally, Section D records some of my own thoughts about the implications of the schema for manpower planning.

The most informative way in which to introduce the concepts is with the aid of a flow chart, shown as Diagram (2.1) below.

Diagram (2.1) The Integrated Production System



The chart is instructive in so far as it demonstrates the links between the

models and emphasises the importance of the ex ante function (which, is a relatively neglected part of the model).

The schema is entirely consistent with and closely allied to the putty-clay theory of production proposed in the pioneering works of Salter (1966) and Johansen (1959). The ex ante function represents the 'putty' state, while the ex post micro function (in its simplest form) reflects the 'clay' aspect of the model. While the ex post production units can be aggregated to a form easily recognised by the vintage theorist, Johansen (1972) chooses an alternative approach to aggregation to obtain the short-run macro function.

The schema is a much more flexible description of the technology of production than its predecessors. In distinguishing alternative levels of aggregation and time horizons, the model integrates even the extreme theories of production (i.e. fixed coefficient and neoclassical). In addition, the short-run macro function can itself take a variety of forms (depending on the size and distribution of established production units in the input-output space), including CES functions. It is somewhat misleading, therefore, to associate a given Johansen concept with any particular existing theory of production. It is probably more appropriate to consider the schema as a set of related economic boxes, into which alternative theories of production can be slotted. Diagram (2.2) outlines the theories that Johansen has chosen to place in these boxes.

Diagram (2.2) Some Links with Existing Theories of Production

Johansen Concept Existing Theory	Ex-Ante	Ex-Post Micro	Feasible Region	Short Run Macro	Putty- Clay	Long Run
Neoclassical	✓			✓	✓	✓
Leontief		✓		✓		
Linear Programming			✓	✓	✓	

The traditional theories of production tend to take up extreme positions: at one end of the continuum there is the fixed coefficient (or Leontief) model, which denies the possibility of substitution between the various inputs; at the other extreme is the neoclassical theory of production, where all inputs tend to be direct substitutes for one another. Diagram (2.2) demonstrates that, in certain industrial situations, it is possible to conceive of a broader theory of production which is reconciliatory in the sense that it incorporates the extreme positions, but relegates them to more subordinate roles. The extremes appear at opposite ends of the spectrum in terms of the level of aggregation and the time horizon.

#### B. The Johansen Concepts

In this section, consideration is given to each of the concepts that Johansen incorporates in his integrated system of production. The most straightforward form of the schema is considered here and the generalisations which have been proposed are left until the next section. During the outline of the various concepts, an attempt is made to emphasise the links between them.

#### The Ex Ante Function

The connection between the schema and putty-clay theory has already been noted. This theory allows substitution to take place ex ante, i.e. before the investment decision is finalised. The ex ante function attempts to summarise the alternative technologies that are available to the firm when making its investment decision. Salter (1966, p.15) has defined it as the "production function which includes all possible designs", and as such it is "purely technically determined" and "free from the influence of factor prices." The function is, in effect, the efficient envelope of all the



alternative production technologies conceived of by designers and available to managers at the time when the investment decision is made.

The exact nature of the function is still in some doubt.

Salter (1966) adopted the early Robinson view that substitution was relevant only in an ex ante sense. The Robinson (1971) view seems to have hardened, and the ex ante possibilities available to the firm at a given time are represented by a single point in the input-output space. Harcourt (1972, pp.55-6) argues, however, that although we might dismiss the ex ante function that spans national boundaries, we might at a national industry level concede the existence of a "small arc of 'best-practice' possibilities". Johansen (1972, p.196) adopts this position and argues in favour of a unique ex ante function with 'classical' properties<sup>1</sup> at any given point in time.

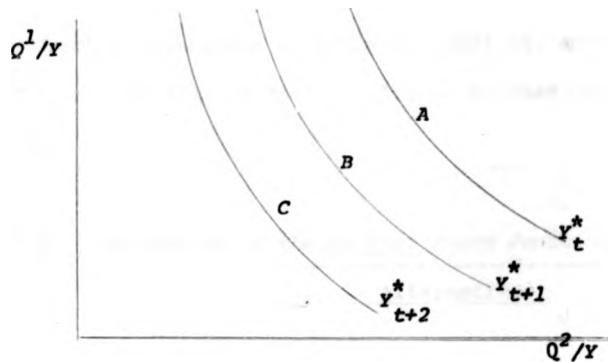
The Johansen and Robinson views are not inconsistent. Johansen (1972, p.9) argues that, in general, very few of the points from any particular ex ante function are observed, because most are associated with technologies which nobody wants and are therefore not developed. He argues however, that all points potentially exist and could be developed given research and development (R & D) effort (we leave the problems of introducing R & D into the analysis until Chapter 4 below). Johansen (1972, pp. 7-8) argues that different points may be associated with different pieces of capital. If a suitable common measure of heterogeneous capital goods cannot be found then the model collapses to the Robinson form. If such a measure can be found, the 'as if' ex ante function may become appropriate. For simplicity of exposition, in this section all factors of production are assumed to be current inputs. Johansen's treatment of capital is discussed in greater detail in Chapter 3. The assumption of a unique ex ante function with 'classical' properties is obviously a gross simplification.

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1. Johansen (1972, p.6) states that the functions have "such properties as are usually attributed to traditional production functions". Allen (1968, p.44) describes these properties in some detail and they are reviewed below when the necessary notation has been introduced.

Diagram (2.3) illustrates the ex ante function at various points in time  $t, t+1, \dots$ . For simplicity, the isoquants associated with the optimal scale of output,  $Y^*$ , are shown in the input per unit of output space.

Diagram (2.3) The Ex Ante Function at Various Points in Time



The points A, B, C .... describe the technologies actually drawn from the ex ante space and hence observed. The diagram shows just two axes from what may, in practice, be a multidimensional space. We may write the equation

$$Y_t = f_t(Q_t^1, Q_t^2) \quad \dots (2.1)$$

for the two input cases, where  $f(\ )$  has 'classical' properties.<sup>2</sup> Normally, the variables drawn from the ex ante function will be associated with full capacity working  $\bar{Y}_t = f_t(\bar{Q}_t^1, \bar{Q}_t^2)$ , where  $\bar{\ }$  denotes full capacity.

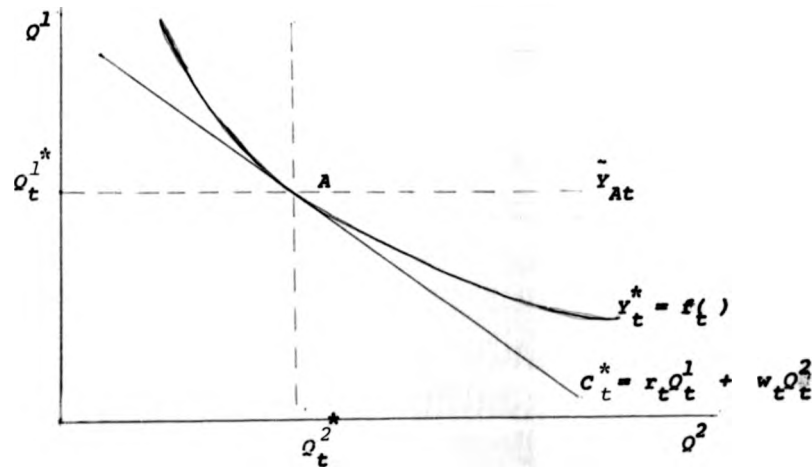
#### The Ex Post Micro Function

The ex post function is relevant once the investment decision has been finalised. It is derived directly from the ex ante function given prevailing factor prices. Diagram (2.4) shows firm  $F_A$ , faced by factor prices  $(r, w)$  and wishing to establish production facilities at the optimal scale of output,  $Y_t^*$ . Its cost-minimising choice of technology

2. Very briefly, this implies that  $\frac{\partial f}{\partial Q^1} > 0$  and  $\frac{\partial f}{\partial Q^2} > 0$  while  $\frac{\partial^2 f}{\partial (Q^1)^2} < 0$  and  $\frac{\partial^2 f}{\partial (Q^2)^2} < 0$ . In addition,  $Y \rightarrow \bar{Y}^1$  and  $Y \rightarrow \bar{Y}^2$  as  $\frac{\partial f}{\partial Q^1} \rightarrow 0$  and  $\frac{\partial f}{\partial Q^2} \rightarrow 0$  respectively, where  $\bar{Y}^1$  and  $\bar{Y}^2$  denote asymptotic values of output.

is  $(Q_t^1, Q_t^2)$ , where one of the isocost lines ( $C_t^* = r_t Q_t^1 + w_t Q_t^2$ ) is tangent to  $Y_t^* = f_t(Q_t^1, Q_t^2)$ . Once  $F_A$  has established the new production unit, the production function collapses to a single point, A, and the associated isoquant is denoted  $\bar{Y}_{At}$ . Diagram (2.4) is analogous to that used by Svernilson (1964, p.111).

Diagram (2.4) Derivation of the Ex Post Micro Function from the Ex Ante Alternatives



The most promising way of defining the relationship in the Johansen world is for a given vintage of capital used in a given process in a known firm:

$$Y_{jrv} = \frac{1}{\xi_{jrv}^1} \cdot Q_{jrv}^1 = \frac{1}{\xi_{jrv}^2} \cdot Q_{jrv}^2 \quad , \dots (2.2)$$

where all of the variables and parameters are of the same time period. The variables  $Y$ ,  $Q^1$  and  $Q^2$  take values such that  $0 \leq Y \leq \bar{Y}$ ,  $0 \leq Q^1 \leq \bar{Q}^1$  and  $0 \leq Q^2 \leq \bar{Q}^2$ ,  $\xi^1$  and  $\xi^2$  are constants. The subscripts  $jrv$  denote the  $j$ th firm,

( $j=1, \dots, m$ ), the  $x$ th production process, ( $x=1, \dots, s$ ), and the  $v$ th vintage of capital, ( $v=1, \dots, \tau$  - where  $\tau$  denotes the oldest vintage still in use). The new production regime corresponds to the traditional Leontief form. While the translation from a function exhibiting substitution possibilities to one with technical rigidity is the key feature of putty-clay models, what distinguishes the Johansen (1972) approach from the bulk of vintage growth theory is the level of aggregation at which the relationship is expected to hold. Johansen argues that it is most realistic to assume that the relationship characterises a production unit, where this may be a single piece of equipment or possibly a section of a plant.

The form of equation (2.2) implies that there are constant returns to scale in the use of production units. The more general case, where  $\xi^i = \xi^i(Q^i - Q^i)$ , is considered by Johansen (1972, pp.46-50) and later in this chapter. No time subscripts have been added to the variables or coefficients, but in the real world such factors as disembodied technical change and learning effects will affect  $\xi_1^1$  and  $\xi_2^2$  or  $Q^1$  and  $Q^2$ .<sup>3</sup> Although a number of studies have looked at the adjustment of input-output coefficients their work has tended to be very aggregate.<sup>4</sup>

#### The Capacity Region

Johansen attempts to develop a theory that adequately describes the micro situation. The micro 'building blocks' are used as the basis for constructing more aggregate models. Approaching the problem in this way enables the researcher to 'stop off' at any particular level of aggregation (whether it is at the level of a section of a plant, a plant, a firm or an industry). It is to the problem of aggregating individual

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3. If we measure  $Q_1$  and  $Q_2$  in physical units  $\xi^1$  and  $\xi^2$  change - but if we measure them in effective units  $Q^1$  and  $Q^2$  change.
  4. See for example Phillips (1955), Ghosh (1960), Stone (1963), Sevaldson (1963) and Wigley (1970). These and a number of other authors' works are reviewed by Bosworth and Evans (1973).

production units that we now turn. Given that the underlying micro technologies are adequately summarised by equation (2.2), if we overlook the existence of different sections of a firm, then the capacity (or feasible) region can be represented by the space defined by the following constraints,

$$\sum_j \sum_v \xi_{jv}^i y_{jv} \leq S^i \quad \dots (2.3)$$

$$y_{jv} \leq \bar{y}_{jv} \quad \dots (2.4)$$

$$0 \leq y_{jv}, 0 \leq S^i \quad \dots (2.5)$$

where  $i$  denotes the  $i$ th input ( $i=1, 2, \dots$ );  $\xi_{jv}^i$  refers to the input of the  $i$ th factor per unit of output used by the  $v$ th vintage of the  $j$ th firm; and  $S^i$  denotes the supply of the  $i$ th input to the industry.

The first inequality (2.3), defines the set of available production units that can be utilised given sufficiently large supplies of the factors of production. Each production unit is defined by inequality (2.4) to operate in the region at or below full capacity. Finally, inequality (2.5) states that only positive levels of inputs and outputs are relevant.

In order to carry out the theoretical analysis expediently, Johansen chooses to work in continuous terms. In order to do this he defines the capacity distribution,

$$f(\xi^1, \xi^2) \Delta \xi^1 \Delta \xi^2 \quad \dots (2.6)$$

which is the output potential of production units the technology coefficients of which fall in the region  $\xi^1$  to  $\xi^1 + \Delta \xi^1$  and  $\xi^2$  to  $\xi^2 + \Delta \xi^2$ . This continuous analogue to the linear programming case has its capacity region wherever  $f(\ ) > 0$ . (Subscripts -  $j, x, v$  - can be added to the coefficients as required).

The Short-Run Macro Function

The imposition of some form of managerial behaviour (for instance, output maximisation) on the linear framework of the capacity region enables a list of preferences about the order in which production units are utilised to be derived. In linear programming form, the production problem can be written, maximise  $Y = \sum_{j=1}^m \sum_{v=1}^T Y_{jv}$ , subject to equations (2.3) - (2.5) inclusive. We can write out the linear programming problem and its dual in the two-factor  $m \times T$  production unit case as follows

Linear Programme: Primary Problem

(Dual variables)

$$\begin{aligned}
 \text{Maximise } Y &= \sum_{j=1}^m \sum_{v=1}^T Y_{jv} \\
 \text{Subject to } \xi_1^1 Y_1 + \dots + \xi_{mT}^1 Y_{mT} &\leq S^1 & q^1 \\
 \xi_1^2 Y_1 + \dots + \xi_{mT}^2 Y_{mT} &\leq S^2 & q^2 \\
 Y_1 &\leq Y_1 & z_1 \\
 &\vdots & \vdots \\
 Y_{mT} &\leq Y_{mT} & z_{mT} \\
 Y_1 \geq 0; \dots; Y_{mT} \geq 0; S^1 \geq 0; S^2 \geq 0 & \dots & (2.7)
 \end{aligned}$$

Linear Programme: Dual Problem

$$\begin{aligned}
 \text{Minimise } q^1 S^1 + q^2 S^2 + \sum_{j=1}^m \sum_{v=1}^T z_{jv} Y_{jv} \\
 \text{Subject to } q^1 \xi_1^1 + q^2 \xi_1^2 + z_1 &\geq 1 \\
 &\vdots \\
 q^1 \xi_{mT}^1 + q^2 \xi_{mT}^2 + z_{mT} &\geq 1 \\
 \xi_1^1 \geq 0; \dots; \xi_{mT}^1 \geq 0; \xi_1^2 \geq 0; \dots; \xi_{mT}^2 \geq 0 & \dots & (2.8)
 \end{aligned}$$

The dual is obtained by transposing the coefficient matrix in (2.7) and using the coefficients 1 of the objective function ( $Y = \sum_{jv} Y_{jv}$ ) as constants in the right hand side of the inequalities in (2.8).

The values  $q^1$  and  $q^2$  can be interpreted as shadow prices of the inputs in terms of units of output. Whether a production unit is operated or not depends on the value of  $q^1 \xi_{jv}^1 + q^2 \xi_{jv}^2$ . Where this value exceeds unity ( $z_{jv} < 0$ ) the unit is laid up; and where it is less than unity ( $z_{jv} > 0$ ) it is operated. We interpret  $z_{jv}$  as the quasi-rent of the  $v$ th vintage of the  $j$ th firm.

Summation of output and inputs over production units that earn non-negative quasi-rents yields the aggregate variables  $Y$ ,  $Q^1$  and  $Q^2$ . The short-run production function at the macro level is defined by,

$$Y = F(Q^1, Q^2) \quad \dots (2.9)$$

Johansen (1972, pp 37-40) again finds it useful to translate this function into continuous terms. In order to do this, he (*ibid.*, pp 37-40) defines the set of points  $(\xi^1, \xi^2)$  that earn a non-negative quasi-rent as,

$$G(q^1, q^2) = \{ (\xi^1, \xi^2) \mid \xi^1 > 0, \xi^2 > 0, q^1 \xi^1 + q^2 \xi^2 < 1 \} \quad \dots (2.10)$$

The capacity distribution for a small cell was summarised by equation (2.6). Integration over all such cells, where there are production units that earn non-negative quasi-rents yields the aggregate variables,

$$Y = \iint_{G(q^1, q^2)} f(\xi^1, \xi^2) d\xi^1 d\xi^2 \quad \dots (2.11)$$

$$Q^1 = \iint_{G(q^1, q^2)} \xi^1 f(\xi^1, \xi^2) d\xi^1 d\xi^2 \quad \dots (2.12)$$

which enter equation (2.9), the short-run macro function.

Johansen (1972, pp 55-62), with full mathematical rigour, demonstrates that the marginal products of the macro function are,

$$\frac{\partial Y}{\partial Q^1} = q^1 \quad \text{and} \quad \frac{\partial Y}{\partial Q^2} = q^2 \quad \dots (2.13)$$

and that the slope of the isoquant is given by,

$$\left. \frac{dQ^2}{dQ^1} \right|_{dY=0} = - \frac{q_1}{q_2} \quad \dots (2.14)$$

Returns to scale (*ibid.*, pp 64-7),  $e$ , depend on the distribution of capacity in the input per unit of output space, but  $e < 1$  in all cases. The elasticity of substitution between factors,  $\sigma$ , (*ibid.*, pp 67-72) depends on: the variety of technologies at the margin; the absolute level of operations; and the amount of production capacity located in a small strip close to the zero quasi-rent line. The form of  $\sigma$  implies that a wide range of elasticities can be incorporated, including CES functions. Johansen (1972, pp 81-5), for example, demonstrates that, given appropriate assumptions, a Cobb-Douglas function can be obtained.

#### The Putty-Clay Function

Although Johansen does not discuss a putty-clay formulation in the context of the simplest schema, the functions outlined above form the basis of a putty-clay equation,

$$Y^* = \sum_{jv} \frac{1}{\xi_{jv}^1} U_{jv} Q_{jv}^1 = \sum_{jv} \frac{1}{\xi_{jv}^2} U_{jv} Q_{jv}^2 \quad \dots (2.15)$$

In this simple case, each production unit is assumed to exhibit constant returns to scale.  $U$  denotes the degree to which capacity is utilised, and we might expect, in the simple case,

$$U = \begin{cases} 0 - \text{ve quasi-rent} \\ 1 + \text{ve quasi-rent} \end{cases} \quad \text{where the number of units with zero quasi-rents}$$

is assumed to be negligible.



Superimposed on this static model is the tendency for the system to modify itself over time--in particular, by scrapping obsolete technologies and by introducing new technologies chosen from the ex ante function. More complete descriptions of empirically testable putty-clay models based on this sort of approach are outlined in Chapters 5 and 6.

#### The Long-Run Macro Function

The long-run macro function described by Johansen (1972, pp 19-25) is a highly theoretical construct. It appears to correspond to the perfectly efficient production technology (which fully utilises available supplies of resources) towards which traditional (average) functions are assumed to move. In a sector where the ex ante technology is continually improving and/or factor supplies are changing over time, the long-run macro function is unlikely to be static. Hence, the average production function is unlikely to catch it up. The function is constructed on the basis of factor supplies,  $S^i$ , available to the sector at a particular point in time. It is assumed, however, that the stock of capital available can take any form desired. The form chosen is that that maximises the output of the sector for the existing factor ratio, and it is fairly obvious that this information will be isolated from the most up-to-date ex ante function.

Consider an ex ante function that exhibits first increasing, and then decreasing, returns to scale. In this case there will exist an optimal size of production unit the factor inputs of which will be drawn from the optimal ex ante function associated with this factor ratio. We can write

$$\bar{Q}^i = \bar{Q}^i(S^1, S^2, \dots, S^n) = \bar{Q}^i(\dots) \quad \dots (2.16)$$

for the  $i$ th factor,  $i = 1, 2, \dots, n$ ; where  $\bar{Q}^i$  and  $S^i$  denote, respectively, the demand for and supply of the  $i$ th factor. The values,  $\bar{Q}^i$ , must obviously

satisfy the ex ante function,

$$y^* = f(\bar{Q}^1(\cdot), \bar{Q}^2(\cdot), \dots, \bar{Q}^n(\cdot)) = f(\cdot) \quad \dots (2.17)$$

Writing the input-output coefficients,

$$\xi^1 = \frac{\bar{Q}^1(\cdot)}{f(\cdot)}, \dots, \xi^n = \frac{\bar{Q}^n(\cdot)}{f(\cdot)} \quad \dots (2.18)$$

we have  $n$  equations in  $n$  unknowns, (i.e.  $\xi^1, \dots, \xi^n$ ), and hence we can isolate the "technique relation",

$$g(\xi^1, \xi^2, \dots, \xi^n) = 1 \quad \dots (2.19)$$

which is the unit isoquant of the long-run macro function. In addition, however, because all firms are the same size and have the same input ratio, it follows that

$$\xi^i = s^i/Y \quad \dots (2.20)$$

where  $Y$  is the industry output from all production units established.

Thus, we can write the more general form of the "technique relation",

$$g^* \left( \frac{s^1}{Y}, \dots, \frac{s^n}{Y} \right) = 1$$

with the implied production function,

$$g^*(s^1, \dots, s^n) = Y \quad \dots (2.21)$$

exhibiting constant returns to scale.

The unit isoquant of the short run macro function corresponding to each level of output coincides in a unique curve in the input-output space. Those of an ex ante function exhibiting first increasing and then decreasing returns to scale form a family of curves to the north-east of the isoquant associated with the optimal scale of output. Only the most efficient ex ante unit isoquant and the technique relationship coincide. It is fairly obvious that the family of ex ante isoquants will generally

fail to correspond with those of the long-run macro function except at the optimal level of output of the ex ante function. Where the ex ante function exhibits constant returns to scale, the isoquants of the two functions at any given level of output will coincide. In the case where the ex ante function is homothetic, the isoquant maps will look alike but isoquants that coincide will be associated with different levels of output except at the optimal<sup>5</sup> ex ante output level. The maps will look entirely different in the case of the non-homothetic ex ante function except at the optimal level.

### C. Some Suggested Generalisations to the Schema

Johansen has made a number of suggestions about the ways in which the schema can be generalised. It is unnecessary to document all of these and the following section merely records some of the more interesting changes.

#### The Ex Ante Function

Johansen (1972,p.7) has argued that it may be necessary (particularly when undertaking empirical work) to distinguish shift-working

- 5 This can be demonstrated quite easily,  $Y^*$  is the optimal level of output and the ex ante function has the form,

$$\bar{Y}^* = f(\bar{Y}^* \xi^1, \dots, \bar{Y}^* \xi^n) = \psi(\phi(\bar{Y}^* \xi^1, \dots, \bar{Y}^* \xi^n))$$

where  $f(\ )$  is homothetic;  $\phi(\ )$  is homogeneous of degree 1 and  $\psi$  is an increasing function of one argument. Using the inverse-function rule and the property of homogeneity we can write

$$Y = \frac{\bar{Y}^*}{\psi^{-1}(Y^*)} \phi(S^1, \dots, S^n). \text{ By appropriate redefinition}$$

of  $\psi(\ )$  and  $\phi(\ )$  the constant,  $\bar{Y}^* / \psi^{-1}(Y^*)$  can be set equal to unity. The optimal ex ante function still forms the kernel of the long run macro function.

or overtime working in the ex ante function. The simplest generalisation is the case of a shift-system characterised by two identical shifts:

$$Y = 2 \cdot f(K,L) \quad \dots (2.22)$$

A report by the National Board for Prices and Incomes (1970, pp.1-3) suggests the existence of a diversity of shift systems in the U.K. and empirical evidence in the Ministry of Labour Gazette (1954, 1965) indicates that different systems operate alongside one another even in the same industry. Senker, et al. (1975, p.74) note that in the toolmaking industry sophisticated equipment tends to be worked on a multiple shift system, but traditional capital is employed only on the main shift. Equation (2.22) is therefore an oversimplification, but in principle there is no reason why the ex ante function should not be appropriately generalised.<sup>6</sup> It should be noted, however, that the cost function will also be more complicated.

Johansen (1972, p.8) argues that it may prove difficult to justify a single ex ante function that summarises the technical knowledge (at the intensive margin) that is available to everyone everywhere without cost. While normative theory will be interested in the efficient (boundary) ex ante function constructed from all of the functions perceived by potential investors, a positive theory would require each function seen by the investors to be separately distinguished. Johansen (1972, p.9) also realised that it was extremely unlikely that all of the technologies from a particular ex ante function would be needed. If they are not, it is unlikely they would be developed and thereby observed. Johansen argues that such points potentially exist and would appear as the result of R & D effort, but he seems uncertain about how this new dimension can be introduced into the theory. This problem is reviewed in Chapter 4, and the work of Nordhaus (1969, 1973) is used to provide an answer.

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6. The impact of this generalisation on the ex post relationships is discussed on pages 2.16-17.

In an earlier article, Johansen (1959, p. 160-1 and 166) suggested that established capacity might be included as an argument in the ex ante function. This variable would be related to the pressure on natural resources or the availability of external economies or diseconomies<sup>7</sup>. More generally still, the form of the ex ante function might change with the size of established capacity (i.e. through learning effects).

#### Ex Post Micro Functions

Despite the assumption of technical rigidity, Johansen (1972, pp. 10-13) considers it likely that there will be "some scope" for ex post substitution at the micro level. Vintage models have been developed that allow ex post substitution. Solow (1960) and Phelps (1962), for example, developed putty-putty models where the ex post substitution conformed with the ex ante technology except that capital was assumed given at its end of the period value (modified only by disembodied changes and deterioration). Park (1966) follows this approach, but allows ex post substitution according to a more or less restricted part of the ex ante isoquant map. Johansen (1972, p.11) introduced the notation necessary to deal with even more general cases, but does not use it in his analysis. He claims (*ibid.*, p.13) that the simple formulation can, with some reservations, be treated as a good approximation to reality in many cases.

These reservations are linked with the important generalisations which Johansen (1972, pp. 46-53) introduces. The method of dealing with shift-working and overtime in the ex ante function, for example, is carried over directly to the ex post micro technologies. Each shift is allowed to have its own  $(\xi^1, \xi^2)$  points; similarly, normal hours may have one  $(\xi^1, \xi^2)$

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7. See Johansen (1972, p.6)

combination while over-time hours have another. A further generalisation allows the input-output coefficients to vary as output varies in the range,

$$0 < Y < \bar{Y}$$

according to,  $Q^1 = Q^1(Y) \quad dQ^1/dY > 0 \quad d^2(Q^1)/dY^2 \geq 0$   
 $Q^2 = Q^2(Y) \quad dQ^2/dY > 0 \quad d^2(Q^2)/dY^2 \geq 0$  .... (2.23)  
 The earlier

assumption of constant returns to scale ensured the existence of a single point in the input-output space, but now the capacity distribution appears along a given curve, defined by equations (2.23) above and written,

$$\xi^1 = h(\xi^2) \quad \dots (2.24)$$

where  $h$  is an increasing function. In practice, this makes no difference to the theoretical analysis as marginal production units are utilised up to the point where the marginal costs of production (valued on the basis of shadow prices) are equal to each other and to unity providing, of course, that the expansion curves intersect the zero quasi-rent line. The capacity of production units in the space  $\xi^1$  to  $\xi^1 + \Delta\xi^1$  and  $\xi^2$  to  $\xi^2 + \Delta\xi^2$  can still be represented by  $f(\xi^1, \xi^2) \Delta\xi^1 \Delta\xi^2$  for a small cell, and the theoretical analysis outlined by Johansen (1972, sections 31-4) still applies. Johansen adds, however, that for empirical analysis this generalisation complicates matters considerably.

Where marginal input requirements are decreasing over the whole range of output levels up to  $\bar{Y}_j$ , production units will either be fully utilised or not utilised at all. Production units are now represented in the calculations by their input requirements at full capacity utilisation ( $\xi^1, \xi^2$ ). Where marginal input requirements are first decreasing and then increasing as  $Y \rightarrow \bar{Y}$  no production units in the range  $0 < Y_j < Y_j^*$  will be employed, where  $Y_j^*$  denotes the level of output associated with the

minimum input requirements per unit of output. Production units can then be represented by increasing curves (such as equation (2.28)) and such units will be employed at a level of output  $Y_j$ , such that  $Y_j^* < Y_j < \bar{Y}_j$ , where their marginal costs are again equal to each other and equal to unity. It should again be possible to summarise established capacity in a small cell by  $f(\xi^1, \xi^2) \Delta \xi^1 \Delta \xi^2$  and the theory remains the same.

#### Short-Run Macro Function

The final generalisation concerns the efficiency assumption which underlies the short-run macro function. A positive theory where the short-run macro function is based on less than perfect efficiency is likely to be more realistic. If we write this function as,

$$\begin{aligned} Y &= \iint Uf(\xi^1, \xi^2) d\xi^1 d\xi^2 \\ Q^1 &= \iint \xi^1 Uf(\xi^1, \xi^2) d\xi^1 d\xi^2 \end{aligned} \quad \dots (2.25)$$

and rewrite the quasi-rent line as

$$z = \frac{p - q^1 \xi^1 - q^2 \xi^2}{p} = 1 - \frac{q^1}{p} \xi^1 - \frac{q^2}{p} \xi^2 \quad \dots (2.26)$$

where  $p$  denotes final output prices, then, for perfect optimisation,

$$U(z) = \begin{cases} 0 & \text{when } z \leq 0 \\ 1 & \text{when } z \geq 0 \end{cases} \quad \dots (2.27)$$

where  $U$  denotes the degree to which productive capacity and inputs are utilised.

Johansen (1972, pp. 50-3) considers the more general case where  $U(z)$  is a non-decreasing function, such that  $0 \leq U(z) \leq 1$ . The short-run

macro function is now written not as  $F(Q^1, Q^2)$ , but as  $F(Q^1, Q^2, U)$  where,

$$F(Q^1, Q^2, U) < F(Q^1, Q^2)$$

except where  $U$  has the perfect optimisation form. Even more general forms of  $U$  can be envisaged, for example,

$$U = U(x, \frac{Y}{Y})$$

where profitability considerations are, to some extent, pushed into the background. (One example might be where there are consumer-buyer links not wholly dependent on prices).

#### D. The Implications of An Integrated Production

##### Schema for the Manpower Planning Debate<sup>8</sup>

The central role that the theory of production plays in economics derives mainly from the large number of areas that it permeates: growth theory, welfare economics, manpower planning and the theory of the trade cycle all to a great extent rely on the characteristics of the underlying technology of production. Equilibrium in the labour market, for example, is the result of balancing the demands for various skills with the corresponding supplies, where the demand relationships are based on the production function (which defines the possibilities of substitution between capital types and labour skills).

This section attempts to illustrate the importance of a sound knowledge of the production process when assessing the role to be played by manpower planning in establishing equilibrium in the labour market. Evidence about the ease of substitution between factors would be invaluable in resolving the debate about how much and what type of manpower planning should be undertaken. Little further has been said since Blaug's (1970) authoritative work, and yet the theory of production has been developing in a way which has important implications for manpower planning.

8. A more detailed discussion of the link between the technology of production and manpower planning is given in Bosworth (1974a, 1976a).



If any conclusion at all can be drawn from the preceding sections, it is that ideally we require a theory of production that is able to encompass all of the alternative situations: short-run and long-run, macro and micro. This is the direction in which economic theory has been moving and Johansen's schema can be seen as the end product of this trend. The schema reconciles the extreme theories of production by making different functional forms relevant to different production situations. It is this aspect of the Johansen model that could prove valuable in delimiting the areas where the 'manpower forecasting' and 'rate of return' analyses should be used and in providing insights about how the two approaches may be combined. Ahamad and Blaug (1973, pp.322-3) argue that manpower planning should develop by combining the two approaches.

#### The Manpower Requirements - Rate of Return Debate

The integrated production schema assumes that substitution is possible between different factors of production in the long run and that if factor prices are flexible they are able to play an allocative role. Under these conditions neoclassical production functions can be used to yield estimates of marginal products for use in rate of return calculations. In deciding about optimal educational strategies, the conditions will be ideal for 'rate of return' analysis. In other words, it should be possible to make broad comparisons between the benefits of extra education or training (for instance, the cumulative discounted marginal products or wages) and its costs.

In the short run, however, the production system indicates a world that is much less flexible and that corresponds broadly to a fixed coefficient technology. In a world where substitution between skills is more limited, comparisons of the sort outlined above are no longer a sound basis for making decisions about the amount or nature of training. It would

appear that the appropriate approach in this case would be to predict the magnitude of employment by skill that is just sufficient to produce the desired level of output and then, for members of each skill category, rate of return comparisons can be made that should enable the optimal level and type of training to be found for each group. As the factor requirement calculations are based on a linear programming analysis, shadow prices for each type of input can be estimated directly and (as these are analogous to the marginal products of neoclassical theory)<sup>9</sup> they can be used in the rate of return calculations.

The use of linear programming analysis in the short run enables labour-market rigidities to be built into the model. Where it is considered to be useful, the solution to the linear programming problem can be made contingent upon given factor supplies. As the planning horizon lengthens, we expect the rigid labour supply functions to give way to more elastic functions. So far, however, the traditional neoclassical employment relationships (models of the demand for labour) have not been introduced into simultaneous systems alongside labour supply functions.

The use of an integrated production system could help to avoid many of the criticisms that have been aimed at the manpower forecasting approach. First, it avoids assuming the existence of an unchanging relationship between inputs, which in the past has been the hallmark of the manpower requirements approach. Second, although the schema is demand oriented, its links with linear programming analysis allow supply features (via the constraints) to play some role. Third, using the simulation capabilities of the linear programming approach, it should be possible to make short-and medium-term, multivalued forecasts (contingent on output growth, investment behaviour and factor supplies). This should avoid the

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9. See Baumol and Quandt (1963).

important criticism of past forecasts made by Blaug (1970, pp. 159-66), that they could not be evaluated because they were single valued predictions based on assumptions that, in retrospect, proved to be false. Fourth, the relative prices of inputs (as in all putty-clay models) play a key part in determining (a) the characteristics of new technologies chosen and (b) which of the existing technologies are utilised. Finally, despite the demand oriented approach that the schema implies for the short-to medium-run, rate of return analysis can still be used to yield a picture of optimal short-run training strategies for particular skills. Having already predicted the absolute levels of demand for these skills using a manpower requirements approach, the costs and benefits of various training strategies can be compared using rate of return analysis. This is a step in the direction of integrating the two approaches, which Ahamad and Blaug (1973, pp. 319-20) deem to be desirable.

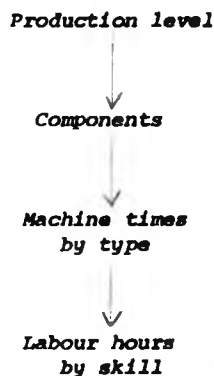
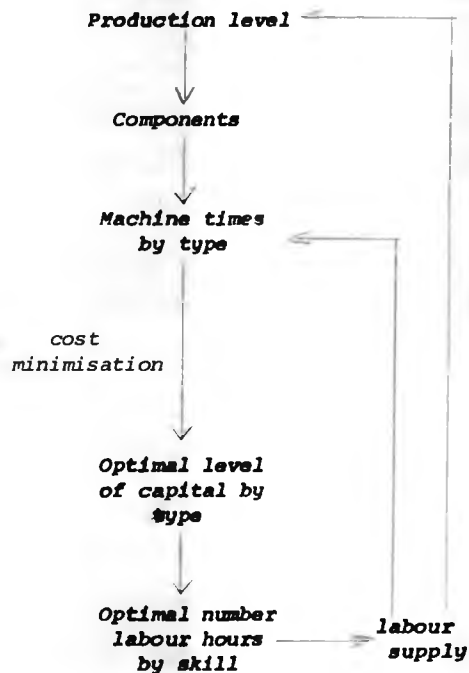
#### The Company-Industry Planning Debate

One final way in which the Johansen schema may contribute to reconciling various factions in the manpower planning arena is by removing any conflict between firm and industry level planning. The means of reconciliation is through the integration of functions which are relevant at different levels of aggregation.

Johansen (1972, p.4) feels that the company planner is likely immediately to recognise the relevance of the micro theory, which forms the foundation on which the more aggregate models are constructed, and thereby gain an appreciation of the macro models. In turn any attempt by the industry planner to construct realistic macro models will induce an interest in factors which are highly relevant at the company level and in the problems of aggregation. The gap between the industry and company planner may be partly (if not wholly) eliminated by the adoption of an

integrated system of production.

The Johansen micro theory may well be valid for a variety of industries at the plant or company level. The micro theory is concerned with the short-run problem of translating a desired level of production into a consistent set of factor demands. Diagram (2.5) shows the simplest sequence of events. Here, the level of output is assumed to be given and this level is translated into desired levels of output of each component part. In this simple illustration, each component can be produced only in one way and this defines the set of machine times (by type) and labour hours (by skill). Diagram (2.6) however, reports a slightly more general model. The level of production again defines the desired level of output of components but the various parts can now be produced by alternative means for instance, a hole may be broached rather than bored; a part may be cast rather than fabricated) and, in this way, a limited element of substitution is introduced. The optimal sets of labour and capital hours are chosen by cost minimisation, although (through the linear programming formulation) the choice is limited to sets consistent with the supply constraints. Alternative responses to supply conditions can be incorporated into the schema: for example, the entrepreneur may revise the desired level of output of the final product or attempt to expand recruitment or training. It is this sort of model that underlies the work on employment carried out by Bell (1972) and Senker and Huggett (1973)

Diagram (2.5)Desired Production and  
Factor DemandsDiagram (2.6)Desired Production and Factor  
Demands: A more complex view

The short-run micro theory is largely consistent with this sort of decision and revision process. The production manager, in particular, should be fairly happy with the Johansen picture although he may to some extent debate the cost minimising hypothesis and replace it by 'satisficing' behaviour.<sup>10</sup> The personnel and general managers may find the model outlined above very limited because manpower plans are only a part of the overall corporate strategy and the schema in its current form, is relevant to only a part of the manpower planning problem. In the taxonomy

10. Because of the large amount of data involved and the complexity of decision making.

outlined by Chadwick (1969, p.51) it is most relevant to the first phase: the "overall planning" of total numbers, skill groups, organisational groups, costs, etc. "Individual planning" is not a central concern and little has been said about career planning, management succession, recruitment or training. The organisation of data for the schema, however, would put human accounting on a formal basis and many of those interesting questions could be analysed.

Despite the recent dominance of the neoclassical theory of production and the rise of the 'rate of return' approach, new developments in the theory of production suggest that the 'manpower requirements' approach still has an important role to play. The Johansen schema can be viewed as an important advance in the theory of production. Empirical work based on the Johansen model should provide the information necessary to delimit the situations in which each of these approaches is appropriate. It should be remembered, however, that manpower planning is only one of a number of possible weapons. Any decision to make use of it will depend not only on its potential benefits exceeding its costs, but also on its efficiency relative to other weapons. As for the other weapons, the current and future role of technical change should perhaps be of at least equal interest to planners. There is, of course, no reason why the schema should not be modified so that it can be used to investigate the implications of technical change for manpower requirements, or the technical changes required to balance movements in manpower supplies.

### Chapter 3: An Integrated System of Production: Some Comments and Criticisms

#### A. Introduction

Several aspects of the schema can be criticised and most of these criticisms also apply to the more general body of vintage theory. The list of topics covered is not exhaustive, but a number of important questions are raised. Nothing is said here about the ex ante function, but the whole of Chapter (4) is devoted to this key link in putty-clay theory.

One potentially important deficiency of both the theoretical and empirical research to date concerns the treatment of ex post modifications to the capital stock, i.e. alterations made to machines already established and in use. What little information exists about modifications is reviewed in Section B. The traditional putty-clay and putty-putty models fail to deal adequately with this aspect of technical change and it presents some problems for the Johansen schema. Some comments are also made about problems caused when the machines purchased are not of the latest vintage. Section B also investigates the suggestion that the vintage effect is not an empirically significant factor in technical change. A brief review is given of the evidence which draws on direct observation of technologies introduced at the intensive margin and the evidence of the empirical performance of 'average' production functions. The body of evidence associated with traditional neoclassical production functions is dismissed as largely misleading and it is argued that the Johansen schema does not provide an explanation of the empirical success of such functions. Section C looks at the need to expand the schema beyond the restrictive assumptions of profit maximisation and perfect competition.

#### B. The Embodiment Hypothesis

The idea that technical characteristics are embodied both in

inputs and in outputs is fairly widely accepted and has formed the basis from which the vintage theory of production and the work on hedonic indices have been developed. While the work on embodiment has not concentrated on the capital input to the exclusion of all other inputs, capital is nevertheless a prime example of a bundle of technical characteristics (where different types of capital are represented by broadly different bundles). The concept of skills in the theory of human capital is based on a similar hypothesis, but the analogy should not be taken too far.<sup>1</sup> Capital, more than any other input, is open to the assumption that the technical characteristics become fixed once they are embodied, and it is for this reason that vintage theory has tended to concentrate on capital rather than labour (whose skills may be altered by learning by doing or by more formal training).

#### Modification and Technical Change

The fact that capital may be modified (with some investment) has largely been overlooked in the vintage approach, particularly in empirical research. Svernilson (1964, p.100) notes that, "The 'fixation' of the technological characteristics is in fact not absolute. Machinery may to some extent be modified and improved in order to increase its efficiency ..... Besides, equipment may be rebuilt and combined with new parts and such 'modernisation' will require new investment ....." Johansen (1972, p.4-5) recognises modifications as a possible stumbling block that may render parts of his study irrelevant. His fear was that "certain basic structures already acquired may facilitate the installment of new pieces of equipment, or that modernisation or reconstruction of old equipment may be more profitable than either operating the equipment in its old shape or acquiring new equipment. We would then get equipment of mixed vintages which

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1. See Blaug (1970, pp.1-8).



are different from equipment of any well defined vintage, and it may be necessary to trace the whole history of individual pieces of equipment". Even if such forms of technical change are empirically important, however, the static aspects of the integrated production schema will remain valid, and only the dynamic aspects will need modifying.

The importance of this form of technical change probably differs considerably between firms, although Svernilson (1964, p.110) states that "Detailed industry studies of the machinery equipment of the engineering industry carried out at the Swedish Institute of Industrial Research<sup>2</sup> indicate that the reconditioning and rebuilding of equipment play an important part". Examples of particular modifications can be found. Some existing types of machines have been modified to incorporate numerical control facilities, but, because numerical control demands technical standards which are generally lacking in the original machines, this is far from commonplace. Senker, *et al.* (1975, p.51) found evidence that digital readouts had been added to machines already installed for use in toolmaking activities. In addition, there is some evidence of refurbishing of grinding machines where the old hydraulic systems have been replaced by electric feeds.

There is an urgent need for research into the empirical importance of modifications to capital equipment. There is little UK published information relating to this question. The data reported below are drawn from the Census of Production (sales of principal products) and Business Monitor. Tables (3.1) and (3.2) report the available data for sales of parts relative to sales of complete metalworking machines by UK firms. Both series include an element of exports and no information is given here about import activity. In so far as the purchase of new machines and the repair of existing machines are substitute activities, the comparison of current purchases is meaningful. A more useful comparison would perhaps be that between the sale

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2. See Wallander (1962).

Table (3.1) CensuS of Production: Sales of Parts (£000)

Column	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Year	Sales of Metal Cutting Parts	Sales of Metal Cutting Machines	Ratio (1)/(2) (%)	Sales of Metal Forming Parts	Sales of Metal Forming Machines	Ratio (4)/(5) (%)	Sales of all Parts	Sales of all Machine Tools	Ratio (7)/(8) (%)
1946 <sup>a</sup>	2317	23751	9.88	901	4050	22.40	3875	32198	12.03
51 <sup>a</sup>	3607	34788	9.39	1445	8431	17.13	5885	45576	12.91
54 <sup>a</sup>	5613	47722	9.76	1766	10531	16.76	8108	62804	12.91
63	-	-	-	-	-	-	14530	108757	13.36
68	15775	117119	11.87	4473	31225	14.36	20248	148344	13.64

a. The columns associated with metal cutting and with metal forming machines include those machines that are easily allocated to the appropriate heading.

Table (3.2) Business Monitor: Expenditure on Parts (£000)

Column	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Year	Metal Cutting Parts	All Metal Cutting	Ratio (1)/(2) (%)	Metal Forming Parts	All Metal Forming	Ratio (4)/(5) (%)	All Parts	All Machines	Ratio (7)/(8) (%)
1967	5408	115075	4.69	1226	33877	3.61	6634	148952	4.45
8	6083	114034	5.33	1880	31120	6.04	7963	145154	5.48
9	7792	126443	6.16	2046	35626	5.74	9838	162069	6.04
70	9767	147971	6.60	2709	38237	7.08	12476	186208	6.70
1	9404	132271	7.10	3465	43895	7.89	12869	176166	7.30
2 <sup>a</sup>	17118	106460	16.07	6704	43196	15.51	23822	149656	15.91
3	20439	120551	16.95	8825	49744	17.74	29264	170295	17.18
4 <sup>b</sup>	12562	68742	20.68	4332	26655	16.25	16894	87397	19.33

a. Series not compatible with earlier years.

b. Based on information drawn from the first two quarters of 1974

of parts and the stock of metalworking machines, but unfortunately this is not possible, compatible information is not available. The observed ratio of parts to total obviously will change over time: with the size and age of the stock of machine tools and, less meaningfully, with the level of industrial concentration in the engineering industry (which affects the ratio of inter- to intra-firm transactions).

With regard to the information given. The two series differ considerably from one another. While the Business Monitor figures indicate a very strong upward trend in the ratio of parts to complete machines, the Census of Production shows a stable ratio of over 10%. The Business Monitor data are subject to important discontinuities where the coverage of firms or the classification has changed. The main change came in the fourth quarter of 1971 and this makes the later data very difficult to compare with the pre-1972 figures. The information recorded in Table (3.2) is taken straight from Business Monitor. Although information was available on both the new and the old basis for the fourth quarter of 1971, no attempt was made to adjust the pre-1972 series because the changes were considered too fundamental: numerically controlled machines were omitted,<sup>3</sup> there was a change in the coverage of firms, and sales, rather than deliveries, were recorded. Although the aggregate information about complete machine tools was roughly consistent in both the Business Monitor and Census of Production, the information about parts (and hence the ratio of parts to total sales) was not.

Even information about purchases of parts is, for a number of reasons, seriously deficient for our purposes. First, new parts may simply

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3. If numerically controlled machines are added back into the post-1971 totals the parts/total ratios become 12.89, 13.71 and 15.11<sup>4</sup> (where they were formerly 13.73, 14.66 and 16.22). The new ratios are not very different to the old even though they assume that the purchase of parts for numerically controlled machines is zero (because no information is available). The main changes are from the other two sources.

be replacements for identical parts that have become worn or broken. This type of change will alter the decline of the machine along its depreciation curve, but it will not alter the vintage of the machine unless the part is technically superior to the one it replaces. Second, metalworking machines form only a part of the capital stock. Third, the figures are only for inter-firm transactions although the major modifications may be designed and incorporated by the firm itself. Finally, these figures overlook the case of wholly rebuilt machine tools.<sup>4</sup>

What little evidence exists about rebuilt machines is reported in Table (3.3). The data relate to the output of a part of the machine tool industry (MLH 332), and, again, activities outside of this industry and intra-firm activities are not covered. Also, nothing is known about the completeness of the rebuilding - whether the machines are simply repaired, are slightly modified or are changed entirely so as to conform to the latest specifications. The rather low percentage of activity in rebuilding machines cannot be taken as conclusive evidence that it is not important. The declining percentage of rebuilt machines to total sales will almost certainly reflect the level of industrial concentration in the engineering industry (i.e. the ratio of inter- to intra-firm activities) just as much as any change in the propensity to rebuild machines.

There is also the question of cutting tools for the machines. Here is a case where a part of the machine is designed to be changed regularly. Senker, *et al.* (1975, p.6) point out that different tools enable general purpose machines to produce different products. Toolmaking is certainly a major industrial activity, but data deficiencies would not allow Senker, *et al.* (1975, p.21) to be more exact than to estimate

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4. A number of companies have special divisions wholly concerned with rebuilding machines

the combined 'in house' and 'contract' toolmaking net output at between £60 million and £200 million per annum. For most types of machines such tools have not remained unchanged over time. Bell (1972, p.74) and Senker, *et al.* (1975) record some of the changes that have occurred in the tools.

The technical changes can be classified according to degree. They range from simple replacement of a worn-out part (with no change to the remainder of the machine and probably, no radical impact on the machine's productivity) to the other extreme of a complete rebuilding (where every out-of-date part is replaced, resulting in the machine being as efficient as the very latest vintage). The two extremes represent situations that can fairly easily be integrated within the schema. Intermediate stages, where mixes of vintages result, are likely to pose more fundamental problems.

Table (3.3) Census of Production: Reconditioned Metal Working Machines (£000)

Column	(1)	(2)	(3)
Year	Reconditioned Machine Tools	(a) All Machine Tools	Ratio (1)/(2) (%)
1948	1544	32198	4.80
51	2992	54453	5.49
54	2897	73809	3.92
58	2478	79632	3.11
63	3808	123287	3.09
68	4017	172609	2.33

a. All metal cutting and forming machines, except welding and physio-chemical.

Acquisitions and Disposals of Second-Hand Plant and Machinery

Traditional vintage theory has assumed the ex ante function to be the locus of points that are the most technically efficient at the time the investment is made. Johansen suggested that a positive study may have to acknowledge the existence of a number of ex ante functions, but that the efficient envelope of such functions may still broadly correspond with the accepted view of the relationship.

UK investment data from the Census of Production indicates that in practice the ex ante choices may be complicated. The disposal data reported in Table (3.4) show that UK engineering industries sell some proportion of their capital stock. While there is no direct evidence that they purchase second-hand plant and machinery, this seems likely to be the case. The available data also suggest that trading in older vintages is likely to be more important for certain types of capital than for others (for example, relatively more important for vehicles than for plant and machinery).

Activity at the intensive margin will involve reviewing the ex ante functions of all vintages, although an adjustment of the functions to account for depreciation will play an increasingly important part the older is the vintage under consideration. Gaps in the function will appear where vintages with certain characteristics are not available. The impact of second-hand trading on the other Johansen concepts is much less marked. The short-run macro function, for example, depends only on the stocks of various production units held by the industry at a particular time and not on the transition from one stock to another.

**Table (3.4) Census of Production: Acquisitions and Disposals of Investment Goods (million)**

<u>Industry Group</u>	<u>Plant and Machinery</u>				<u>Vehicles</u>			
	1963		1968		1963		1968	
	A	D	A	D	A	D	A	D
<i>Mechanical and Instrument Engineering</i>	75.3	5.6	123.0	10.0	10.7	4.1	19.3	6.8
<i>Electrical Engineering</i>	53.4	5.2	74.4	3.9	5.3	1.7	7.2	2.6
<i>Motor Vehicles</i>	63.8	1.3	71.8	2.9	2.9	1.2	3.8	1.7
<i>Aircraft</i>	10.9	2.2	26.3	1.2	0.8	0.2	1.7	0.3
<i>Other Vehicles</i>	2.3	0.4	3.6	0.4	0.4	0.1	0.4	0.1
<i>Metal Goods Not Elsewhere specified</i>	32.7	2.0	49.7	3.1	7.6	2.8	8.6	3.3
<i>Shipbuilding and Marine Engineering</i>	11.2	1.5	11.9	0.6	0.9	0.3	0.6	0.2

A: acquisitions; D: disposals

Evidence of Technical Change at the Intensive Margin

The implication of the simple embodiment and 'fixity' hypotheses appears to be that new capital (ie. the most modern) should be more efficient than the capital already in use. Johansen (1972, p.9) points out that such a situation assumes the existence of a single ex ante function where know-how is freely available everywhere to everyone who is interested. Where this is not the case, there will exist a whole nest of ex ante functions, and observations drawn from the real world about the intensive margin will be characterised by a variety of technical efficiencies. Of course, some of these production units may not survive



in the long-run, but can exist in the short-run.

Evidence about the vintage hypothesis is scarce. Verdoorn's law, for example, gives some support for the vintage hypothesis but it is not direct proof. The positive relationship between the rate of growth of productivity and the rate of growth of output may be caused by high levels of investment in the latest vintage by rapidly growing industries. Continuing in this vein, Wabe (1974, p.38) has suggested that an industry with no growth (or even negative growth) in output will experience productivity change from replacement of obsolete equipment (or scrapping of obsolete equipment with no replacement). The vintage hypothesis thereby suggests the possibility of a U shaped relationship for the Verdoorn regression. So far, only Helps (1974) has tested this hypothesis and has provided tentative empirical evidence of a non-symmetrical U shaped function. However, the evidence is in many ways poor, because, if the vintage hypothesis is correct, the Verdoorn relationship is misspecified.

Few studies have looked into the possibility of estimating vintage functions where capital of different vintages appear directly in the function. One reason is that there rarely exists information that is sufficiently detailed for a function of this type to be estimated. A general reaction is to attempt to make use of rather aggregate published investment data. Heathfield (1972b) has isolated a model where aggregate output per head is a function of cumulative past investment, capital usage and a time trend. Fair (1971) has isolated a model, estimated as an employment function, where the change in full capacity employment is a function of the change in full capacity output, the wage-rental ratio and a time trend. The estimated function is again obtained by

aggregation over vintages and, although the results relate to two fairly detailed industry groups, they appear a long way removed from the micro vintage models.

The most incisive evidence is likely to come from direct observation of 'best-practice' and 'average' production units. Salter (1966 pp.48-50 and 95-99) summarises some information of this type, but the evidence he provides is suggestive rather than conclusive. Only his Table ( 9 ) in (*ibid.*, p.96) which relates to the beet and cement industries, links productivity to the age of plant. Even so nothing is known about the role played by increased capital intensity, which might be independent of vintage effects. The remaining data is even less conclusive. While Salter's Table (11) (*ibid.*, p.98) indicates that productivity growth was faster in industries which undertook important modifications to their capital stock, it again fails to distinguish growth in the capital-labour ratio from vintage effects. Tables (5) and (10) in Salter (1966, pp.48 and 97) give information about 'best-practice' and 'average' production units. Although it is tempting to interpret the difference in efficiency between the 'best-practice' and the 'average' production units as the result of lags in the adoption of up-to-date technologies, this has not been proven.

All of the Salter evidence points towards the existence of embodied technical progress (although the phenomenon would appear to be more important in some industries than others), but the evidence is only suggestive. Gregory and James (1973) attempt to provide a more substantial test of the vintage hypothesis. Their results, which are the most important to date, are not favourable to the vintage hypothesis. Their evidence, drawn from Australian manufacturing industries in the mid-1950s, involved a comparison of the efficiency of new factories with the average efficiency of existing establishments in the same industry.

They concluded that the average age of factories is not important in explaining variations in value-added per worker between establishments within a given industry, with the implication that it is not possible to automatically associate new factories with best practice techniques. In addition, the results indicate that vintage effects are quantitatively insignificant as an explanation of cross-sectional productivity dispersion in most industries (or indeed over time for a given industry) and that, although vintage effects may be important in a few industries, the number of such industries is likely to be small.

Haig (1975, pp. 378-9) argues that the Gregory and James results are by no means conclusive. The ratio of average labour productivity of new to existing plant is about 1.10 to 1.18 (depending on the approach adopted). The 95% confidence intervals for the 1.10 ratio are 0.98 and 1.23, which range from an almost zero to a strongly positive vintage effect. Haig notes that other sources of evidence suggest an average lag of seven years between the best-practice and industry average. This implies a productivity difference of about 25%, which is not very far removed from the upper boundary of the confidence interval.

Gregory and James (1973, p.1144) felt that there was little enough evidence that new factories were more efficient than established production units, but said that what little differential there was could be partly explained by the greater size of new factories (quite independently of vintage effects). Haig (1975, p.379) criticised the use of value-added as a measure of size. He favoured a measure of firm size based on the number of employees and found that, using the same functional form, the coefficient on size became insignificantly different from zero.

However, Gregory and James (1973, pp. 384-6) pointed out that: the main results do not depend on the proof of this relationship; value added is a theoretically more valid measure; because larger firms are more capital intensive, employment measures of firm size bias the slope coefficient downwards; and, even if an employment based measure is used, the relationship remains significant if a log-linear or semi-log formulation is adopted.

One final point that Haig (1975, pp. 380-1) dealt with was the possibility that the prices of output from new factories are forced downwards in an attempt to break into a market controlled by existing production units, thereby reducing value added, but, as Gregory and James (1973, p. 387) point out, no evidence exists to support this hypothesis or to refute it.

With regard to existing production units, it is important to note that, in addition to the embodied technical change that they experience when replacing and adding to their capital stocks, they experience disembodied technical change through which they accumulate gains in productivity in the gestation period between the date when the new plant is planned and the date at which it achieves its planned performance. Both Salter (1966, pp.83-99) and Svernilson (1964) have stressed the coexistence of embodied and disembodied technical changes. The date at which plans are formalised may precede the opening of the factory by a number of years and the associated threat of new competition may induce existing units to search for and to introduce changes in order to remain competitive or, to remove the new source of competition.

The Gregory and James study looks at a single snapshot from the dynamic process of technical change, embodied changes may dry up at any time. Work on demand induced technical change—see, for example, Schmookler (1966)—suggests that expectations or experience of a recession may cause a cut back in R & D in investment goods industries. In

addition, other authors - see Ruttan(1959)-have argued that major innovations or clusters of innovations may follow one another after quite long intervals and these changes may be associated with specific pieces of equipment such as computers and machine tools. Under these conditions, embodied changes may, at a given time, be quantitatively insignificant compared with disembodied changes, but this may not be true at all times.

There are a number of other problems that neither Haig nor Gregory and James considered. The first is that a cross-industry regression is unlikely to yield any insights about the empirical validity of the concept of increasing returns to scale. Optimal firm size may well differ significantly between industries and the question of returns to scale should be considered by comparing plants of different sizes within a given industry. The second point is that the linear formulations which both Haig and the authors adopt may give misleading results. Unless returns to scale are ever-increasing, a quadratic function might prove more realistic. If the actual function has quadratic form, then it is by no means certain that the large average size of firms in the sample works in favour of higher productivity. Indivisibilities of plant and high levels of industrial concentration (the latter may give rise to the possibility of goals other than profit maximisation) may result in greater than optimal firm size and lower than maximum productivity. Such factors may off-set even potentially large gains from embodied technical change. This might also be one explanation why in about one-half of the new factories labour productivity was lower than the industry average.

The authors had insufficient data to estimate their regression equation for each industry in turn. The 1969 Australian census data, however, enables a simple comparison of firm size and productivity for different industries. For existing factories, productivity is often not a monotonically increasing function of firm size. Some examples are given, in Tables (3.5), (3.6) and (3.7), where the largest group reported

Tables (3.5) - (3.7) Average Labour Productivities by Firm Size and Industry Group

Table (3.5) Cement Products <sup>a</sup>

<u>Industry Group</u> \ <u>Firm Size</u>	10 - 19	20 - 49	50 - 99	100+	Total
Ready mix concrete	9.97	-	-	-	12.87
Concrete pipe	7.95	8.25	8.33	7.15	7.81
Concrete products (excluding pipes)	5.88	6.90	7.48	7.16	6.36
Asbestos cement products	-	-	-	-	7.48
All industries shown	-	-	-	-	-

Table (3.6) Industrial Chemicals <sup>b</sup>

<u>Industry Group</u> \ <u>Firm Size</u>	10 - 19	20 - 49	50 - 99	100+	Total
Chemical fertilizers	-	6.90	12.40	11.81	11.77
Industrial gases	-	-	-	-	-
Synthetic resins and synthetic rubber	11.69	16.32	12.28	16.12	15.25
Organic industrial chemicals not elsewhere specified	-	9.98	17.08	11.53	13.99
Inorganic industrial chemicals not elsewhere specified	-	-	-	-	-
All industries shown	13.57	9.53	13.73	12.00	12.24

Footnotes to the tables:

<sup>a</sup> Cement itself is omitted leaving the Australian Standard Industrial classification order sub-groups 2832-5

<sup>b</sup> Taken to be basic chemicals; Australian Standard Industrial classification order 2711-5

Table (3.7) Motor Vehicles<sup>c</sup>

Industry Group	Firm Size				Total
	10 - 19	20 - 49	50 - 99	100+	
Motor vehicles	-	7.21	-	8.22	8.19
Truck & bus bodies etc.	4.48	4.80	5.03	4.67	4.68
Motor vehicle instruments & electrics	-	4.20	-	4.31	4.29
Motor vehicle parts not elsewhere specified	4.90	4.81	5.05	5.42	5.28
All industries shown	4.66	4.99	5.46	7.15	6.86

## Footnotes to the tables:

- c Taken to be total motor vehicles and parts: Australian Standard Industrial Classification order 3211-4
- Indicates that the information for the corresponding cell is not published individually, although it appears in the SIC total.

Source: "Manufacturing Establishments: Selected Items of Data Classified by Industry and Employment Size", Economic Census 1968/9, Commonwealth Bureau of Censuses and Statistics, Canberra.

100 employees or more, but experience a lower productivity than smaller factories in the same industry and sometimes lower than the industry average. Concrete pipes, organic chemicals and the truck and bus bodies industries all have productivities in the 100+ groups which are lower than their industry averages. In addition, in five out of the seven cases shown here where information was given about the 50-99 and 100+ groups, productivity was lower in the largest group than in the second largest. Added to this is the tendency for larger firms to be more capital intensive. If this was taken into account, the variations in labour productivity attributable to vintage effects would be even lower for larger firms

Despite the authors' claim that the industry breakdown was detailed - Gregory and James (1973, p.1140) - a further problem concerns the aggregate nature of some of the industry groups. Tables (3.5), (3.6) and (3.7) can be used to illustrate the fact that the census data conceals a wide range of labour productivities within a given industry group even for firms of the same size, caused by intra-group variations in the nature of products and production technologies, even if the sample is limited to firms employing more than ten workers - see Gregory and James (1973, p.1139). The range of variation is increased even further if we look across establishments of different sizes as well as over product types within an industry group.

It appears from the results summarised in these three tables that all of the industries experience wide ranges of productivity. Expressing these ranges as the ratio of the highest to the lowest productivities, the ratios calculated from Table 1 in the Gregory and James paper (for the new factory sample) are 3.96 and 7.20 for cement products and industrial chemicals respectively, (the range for motor vehicles was not



available). The productivity ranges calculated for existing factories from Tables (3.5), (3.6) and (3.7) are 2.18 for cement products, 2.80 for industrial chemicals and 1.96 for motor vehicles. Even if firm size is held constant, the variation is still large: with a maximum ratio of 1.70 for cement products (10-19 size group); 2.68 for industrial chemicals (20-49) and 1.91 for motor vehicles (100+). Close to one-half of the variation in productivity isolated by the authors may be accounted for in this way, even though the industry and firm size categories reported here are still aggregates. In addition, no attempt has been made to account for differences caused by regional conditions. If new plants are established in both relatively high and low labour cost areas, this will affect the range of productivities experienced.

The review of the existing literature contained in this chapter indicates that there is an urgent need for further empirical research. Perhaps the most incisive evidence will prove to be direct observation of new and old factories. Given a more appropriate sample of new firms than could be constructed from the Salter data, with some improvements, the approach adopted by Gregory and James could prove a useful first step in establishing which industries are characterised to any extent by vintage effects. The criticisms outlined here suggest that the sample should be based on more disaggregated industry groups, be stratified by region and have sufficient observations to enable the nature of 'returns to scale' to be established for each industry. With such a sample, the difficulties of isolating the importance of vintage effects would be greatly reduced.

#### Evidence of the 'Average' Production Function

A major body of evidence often used against the vintage hypothesis

concerns the fact that neoclassical production functions invariably explain the empirical 'facts of life' so well that the alternative, (vintage) formulations are redundant. The work of Layard, et al. (1971) is a particularly relevant example of this type of evidence (as it is a cross-sectional study of plants in the electrical engineering industry). Their results indicate that neoclassical functions fit well and are characterised by a high degree of substitution between factors (ie. the isoquants appear to be almost straight lines). From this evidence they (*ibid.*, p.20) draw important normative conclusions about the role and intensity of manpower planning.

There have been a number of attempts to incorporate elements of vintage theory in the traditional neoclassical framework. Griliches and Ringstad (1971, p.29), for example, on the basis of establishment data, attempted to pick up embodiment effects using a traditional CES function but with dummies for each of the years when establishments were founded. They concluded (*ibid.*, p.67) that, "No clear pattern emerges for the 'year of establishment' variables. Given our data we are unable to detect any 'embodiment' effects". Many of the comments made about the Gregory and James study, however, apply equally to this study and it is not too surprising that no pervasive pattern was found. A study by Hildebrand and Liu (1965) was more favourable to the vintage hypothesis. They weighted the capital variable by the ratio of net to gross book value of capital to account for variations in depreciation and age. Where investment causes the ratio of net to gross values to differ, the capital elasticity is considerably larger and also the returns to scale. This is taken to indicate that the modernisation of equipment plays an important role. Although some evidence can be found to support the vintage hypothesis, this approach lacks theoretical rigour and the

variables which summarise the vintage effects have not always performed well. The results obtained have failed to give any real impetus to empirical work based on more theoretically rigorous vintage models.

In the light of the apparent success of the neoclassical models, it is interesting to ask whether the Johansen approach can provide an explanation for the body of empirical results which exist. Substitution in the schema arises from the diversity of tasks which any particular piece of equipment can undertake. In this way there exists the possibility of substitution, but of a much more limited kind than is assumed in the neoclassical models. The universe of machine tasks might, for example, include sets denoted by A, B and C which are performed by three different machines (ie. milling, planing and broaching). The size of any particular set relative to the universe of tasks is an indication of the degree of specialisation of the machine. In so far as the sets intersect, there may exist a sub-set of tasks which can be performed on any of the machines. In this sense, a limited degree of substitution is possible between the machines. The implication of the simplest form of the Johansen schema is that we may analyse substitution between machines of different types if we reduce the set of operations from the universe as a whole to a sub-set which is common to all machines in our sample.

The Johansen (1972) schema can generate two-input short-run

macro functions with neoclassical properties. Inputs are measured in real terms and disaggregated into sufficiently detailed groups to be internally homogeneous.<sup>5</sup> If a meaningful common measure of different types of capital could be found, then it is possible to treat capital as a current input. In the two factor case, the ex ante function can be written

$$\bar{Y} = f(\bar{K}, \bar{L}) \quad \dots (3.1)$$

and for simplicity, omitting the vintage subscripts the ex post functions appear as,

$$y_j = \frac{1}{\xi_j^K} K_j = \frac{1}{\xi_j^L} L_j \quad \dots (3.2)$$

The short-run macro function can now be summarised in linear programming form as,

$$\max \quad Y = \sum_j Y_j$$

$$\text{subject to } S^K \geq \sum_j \xi_j^K Y_j$$

$$S^L \geq \sum_j \xi_j^L Y_j$$

$$Y_j \geq 0$$

$$\text{and } K_j, L_j, S^K, S^L \geq 0 \quad \dots (3.3)$$

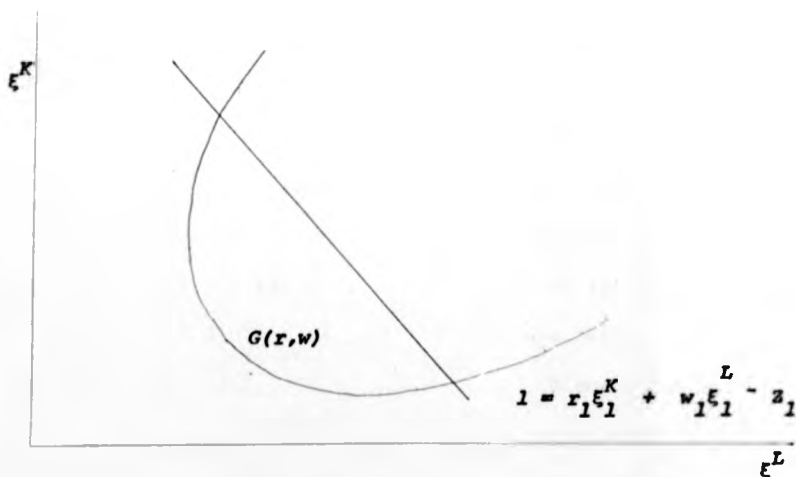
5. Hahn and Matthews (1964, p.110) point out that "As far as pure theory is concerned the 'measurement of capital' is no problem at all because we never have to face it if we do not choose to. With our armchair omniscience we can account for each machine separately".

Where,  $Y_j > 0$  implies that  $K_j > 0$  through the assumption of constant returns to scale and the upper limit,  $\bar{Y}_j$ , is now subsumed in the first inequality.

The essence of the Johansen proof of the nature of the short-run macro function for this two input case,  $Y = F(K,L)$ , can be demonstrated in diagram form.  $G(r,w)$ , shown in Diagram (3.1), represents the set of established, technically feasible and economically viable points.  $Z$  represents the zero quasi-rent line. The total (industry) output of all production units in this region,  $Y_1$ , is found by double integration (over  $\xi^K$  and  $\xi^L$  consistent with the non-negative quasi-rent condition). Total input demands for the industry,  $K$ , and  $L$ , are found in a similar way (i.e. by integration of the input-output coefficients weighted by the level of output).

Diagram (3.1)

The Utilised Region,  $G(r,w)$



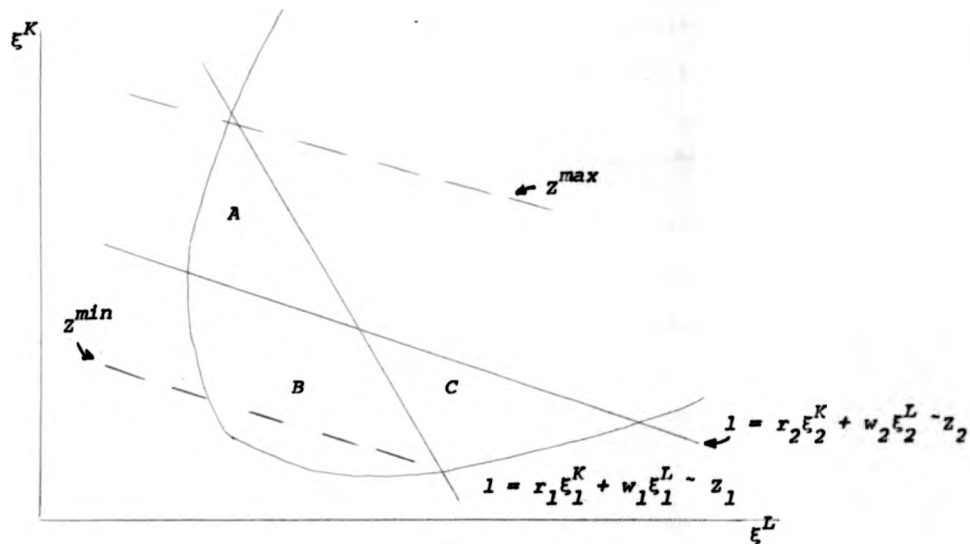
The situation we are interested in concerns what happens to total

input demands where relative factor prices change (ie.,  $\frac{r}{w_1} + \frac{r}{w_2}$ )

while output is maintained at the level  $Y_1$ . Diagram (3.2) illustrates the possible outcomes.

Diagram (3.2)

Derivation of the Isoquant



If we suppose that relative factor prices are given autonomously, a change in relative prices from  $\frac{r_1}{w_1}$  to  $\frac{r_2}{w_2}$  is consistent with a wide range of zero

quasi-rent lines,  $Z$ , as the absolute level of prices is not fixed. It can be shown, however, that  $Z_2$  must lie between  $Z^{\min}$  and  $Z^{\max}$  if output is to remain at level  $Y_1$ . At  $Z^{\max}$ , no production units are lost but some are gained, while at  $Z^{\min}$  none are gained but a number are lost. In between, there should exist a function  $Z_2$  which creates a new half-space such that the output lost from the industry (from technologies in region A) is just compensated by the output gained (from production units whose technologies fall in region C). The technologies lost were relatively capital intensive and those gained were relatively labour

intensive. The aggregate input demands must have been affected in such a way as to lower  $K$  and raise  $L$ . The capital/labour ratio along the isoquant moves inversely with the rental/wage ratio - the key characteristic of neoclassical technologies.<sup>6</sup>

The way now seems open to find the distributions  $f(\xi^K, \xi^L)$  which are consistent with CES production functions. Unfortunately, a meaningful summary statistic of heterogeneous capital is not generally available. Johansen (1972, p.8) suggests using a value measure of capital. This does not give rise to any real problems in the two-current and one-fixed input case where substitution between current inputs is being analysed. Where capital is treated as a current input and measured in value terms, as the zero quasi-rent line changes so too does the valuation of capital and hence, the distribution  $f(\xi^K, \xi^L)$ . Under these circumstances the isoquant need not retain the desired neoclassical properties. The Johansen schema cannot therefore be used as an explanation of the neoclassical empirical results which have used a value measure of capital.

Johansen's very careful treatment of capital throughout his book indicated that he was very aware of the problems associated with the capital input. His preference was to treat every type of capital quite separately. Under these circumstances, the original representation of the schema, described by equations (2.7), is appropriate. If again, for simplicity, we omit the vintage subscripts, the linear programming problem appears as,

$$\begin{aligned} \max Y = \sum_j Y_j \quad \text{subject to } S^i &\geq \sum_j \xi_j^i v_j \\ \bar{Y}_j &\geq Y_j \geq 0 \quad \text{and } S^i \quad \xi_j^i &\geq 0 \quad \dots (3.4) \end{aligned}$$

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6. For the isoquant to exist quantities of vintages must be waiting idly for a price change in its favour.

The role played by capital is now summarised in the output inequality, because this can be expressed in terms of capital usage as,

$$\frac{1}{\xi_{jv}^K} K_{jv} > \frac{1}{\xi_{jv}^K} K_{jv} > 0$$

This rigorous treatment of the capital input means that changes in relative factor prices, while holding output constant, produce movements around the isoquant in the  $(\xi^1, \xi^2)$  plane concerned with current inputs, but a movement from plane to plane in the case of capital. In practice, we might expect changes in capital types to cause a movement from plane to plane in terms of current inputs. To take a well tried example, the skill and fuel requirements necessary to use a bulldozer are very different to those needed for a shovel. From an empirical point of view, however, theoretical rigour implies immense problems of measurement and estimation - see, for example, Pyatt (1964b, pp.24-5)

The current explanations of the neoclassical empirical results are reviewed in Chapter (8). An alternative explanation is provided and some further comments are made about the 'measurement of capital' problem.

C. The Importance of Perfect Competition and Profit Maximising Behaviour to the Johansen Schema: Some initial Thoughts on the Effects of Alternative Market Regimes and Managerial Behaviour.

Profit Maximization under Perfect Competition

The market regime which Johansen assumes to underlie all of the constructs is one of perfect competition in a single product market and the theory is valid in a centrally planned economy intent on using resources in the most efficient manner.

It is interesting to point out some of the features of the Johansen short-run macro function. The primal problem is to maximise output subject



to supply constraints and the condition of non-negative activity levels for each production unit. The choice of output maximisation appears unusual in a competitive model because it is not necessarily consistent with profit maximisation. It is appropriate in so far as the production problem is often phrased in terms of obtaining maximum output from given resources (a technically efficient relationship). More normally, however, we see the problem in the context of profit maximising or cost minimising behaviour in the light of a given technology.

Johansen avoids the potential inconsistency between the output and profit maximising results by assuming that factor rewards are equal to the shadow prices which appear in the solution to the dual problem. Full use is made of the link between linear programming and marginal analysis. In the output maximising model, units along the separating plane earn zero quasi-rents, ie. on the basis of imputed prices the marginal costs of production are set equal to the marginal revenues to which they give rise. This is the traditional profit maximising result.

The short-run macro function, however, relates to an industry or sector defined in terms of the product market. The industry may not, however, be an appropriate level of aggregation when analysing factor markets. If, for example, the labour market is wider than the industry, the going wage rate may be determined by market supply and demand and imposed on the industry. In so far as these rates differ from the shadow prices found in the solution to the dual problem, cost minimising and output maximising behaviour will give different results.

In such a world, each firm will attempt to maximise its profits in the light of current prices, subject to its own technical constraints and those industry level constraints which impinge on its performance. There is no simple, industry level objective function - for example, profit

maximisation by a large number of relatively small firms acting independently of one another is not generally consistent with a single objective function aimed at maximising total profits in the industry. We return to the case of perfect collusion, or monopoly, in the next section. Here we consider the result of each firm simultaneously deciding its output level at the point where  $p = MC$  (where  $MC$  denotes marginal cost), but at the aggregate level,  $\partial P/\partial Y < 0$ .

It is assumed that there is a large number and varied distribution of production units. The marginal cost curve for the industry is the horizontal sum of the same curves for individual firms and the equilibrium level of output is determined by supply equal to demand at the output level  $Y^C$ . At this level quasi-rents are zero on the marginal units. This is the worst position which can be tolerated: the whole of the revenue which accrues from operating the marginal unit is 'consumed' by the variable costs of running it.

As firms are induced to introduce their most efficient vintages first (and least efficient last), given constant price per unit of input, marginal cost is a non-decreasing function of output. At very low levels of output, the marginal unit will have the same characteristics as all other units presently in use, hence, marginal and average variable costs coincide. As output rises, less and less efficient units are brought into use and marginal costs increase. Average variable costs (AVC) are pulled upwards, but lag behind marginal costs (MC) because the units from which the average is calculated are all at least as efficient as the marginal unit and, in general, some will be more efficient.

So long as the units in use are characterised by more than one level of efficiency, the non-decreasing form of the marginal cost function opens up a gap between the marginal cost and average variable cost functions. If this gap is not filled by the average fixed costs at  $Y^C$  then firms in the industry can earn transitory profits. The equilibrium level of

output,  $Y^C$ , is assured whether short-run profits are positive or negative (i.e. whether the average total cost curve, (ATC), passes above or below MC at  $Y^C$ ) because some production units are earning positive quasi-rents and they are contributing to their sunk costs.

In the case of perfect competition, the output of the industry is simply the sum of all output activities carried out on production units earning non-negative quasi-rents in the light of the current demand situation.

#### Profit Maximisation under Monopoly

The case of the monopolist is more straightforward. Given the single aim of maximising profits, the equilibrium level of output,  $Y^m$ , is determined by the intersection of the marginal revenue (MR) and marginal cost (MC) curves. The price charged by the monopolist is higher and the output supplied is lower than in the competitive industry. It is an interesting result of the static analysis that the model predicts that the monopolist will tend to lay-up a greater part of his capital stock. Under price and factor cost expectations where these inefficient vintages are never expected to do better, the monopolist may scrap capital earlier and employ a smaller (though, on average, more efficient) capital stock than a group of competitive firms. The monopolist might, on the other hand, be willing to maintain excess capacity as a barrier to entry - see Spence (1974).

The programming problem is one of maximising profits subject to the technical constraints, non-negative activity levels and average revenue greater or equal to average variable costs (which is the condition under which production continues even though losses are being incurred). In the case of monopoly (or perfect collusion) we can again talk of an industry level model. There is a single, industry-level objective function of the

traditional constrained profit maximising type,

$$\max \quad \Pi = pY - \sum_j \sum_v \sum_i c_{jv}^i \epsilon_{jv}^i Y_{jv} \quad \dots (3.5)$$

subject to inequalities (3.4)

where  $p$  is the price per unit of output and  $c^i$  is the price per unit of the  $i$ th input. In this way the inputs,  $K$  and  $L$ , can be isolated which satisfy the short-run macro function,  $Y = F(K, L)$ .

The short-run macro function is, however, an industry level model and, at this level of aggregation, product prices will generally not be constant at different levels of output. As the industry constitutes the sole supply of the product, the demand curve may be assumed to be downward sloping to the right in the price-quantity space. In other words, the monopolist can affect his level of profits by restricting industry output and raising the price of the product. In a simple model of labour supply, whether the wage rate is constant will depend on the size of the industry demand relative to total demand for that factor. In so far as the industry constitutes an important part of total market demand for any particular input, increases in its level of activity will tend to bid up the price of inputs.

Changes in product and factor prices can be incorporated in the programming case quite easily. By making,  $p = p(Y)$  and  $c = c(Y)$  where,  $p'(Y) < 0$  and  $c'(Y) > 0$  (which are the first differentials with respect to output). We can include a downward sloping industry demand curve with an upward sloping supply curve for any particular input. This generalisation means that the objective function has terms  $Y.p(Y)$  and  $Y.c(Y)$  and the expression will generally be non-linear.<sup>7</sup>

<sup>7</sup> See Baumol and Quandt (1963, pp. 437-8)

Given knowledge of the demand and supply functions, an iterative procedure can be used to obtain a solution. At each level of  $Y$ ,  $p$  and  $c$  are both constants and the problem remains one of selecting a set of activity levels,  $Y_{jv}$ , which maximise profits.

#### Profit Maximisation Under Other Market Regimes

Generalisation of the market regimes to the intermediate states of oligopoly and monopolistic competition appears more difficult to effect. In such cases the essential problem is to find some means of determining the market shares of firms in the industry at given levels of output for the industry. Once this is known profit maximising behaviour can be used to allocate output among production units owned by each firm. The allocation of demand between firms in an oligopolistic market will depend on the type of oligopoly (i.e. collusive or non-collusive), the nature of competition (i.e. price or quality) and the manner in which each firm's product characteristics are decided upon (e.g. a process akin to game theory or price leadership). In principle some form of allocative model determining market shares can be devised for a particular regime and spliced onto the system which would then function broadly as before.

#### Alternative Theories of Managerial Behaviour

At this stage, however, it seems more important to assess the implications of managerial and behavioural theories of the firm for the schema. Managerial theories suggest that, given market power, the objective function may take a form other than profit maximisation. The most commonly cited is sales revenue maximisation. Behavioural theories, on the other hand, suggest that firms may be controlled by management intent on satisficing, which only under extreme conditions collapses to profit maximisation.

Managerial Theories. Given the static nature of the short-run macro function, the dynamic managerial theories, such as those put forward by Mueller (1967) and Marris (1971), do not appear particularly relevant to this part of the schema. Here, the analysis is limited to Baumol's (1959) sales revenue maximisation hypothesis. Although it is the least satisfactory of the managerial models, it is sufficient to give a flavour of the implications of alternative managerial theories for the equilibrium result. The assumption of sales revenue maximisation subject to a profit constraint results in a slightly more complicated programming problem.

Max  $pY$

Subject to inequalities (3.4)

$$\text{and} \quad \sum_j \sum_v \sum_i (pY_{jv} - c_{jv}^i Y_{jv} - AD_j) \geq \Pi^{\min} \quad \dots (3.6)$$

where  $AD$  denotes the level of advertising expenditure. Advertising plays an integral role in the revenue maximisation model. Baumol allowed advertising expenditure to affect the level of output, but not the price of the product. Under these conditions, it is possible to raise the level of output without reducing price: the extra expenditure on advertising induces additional consumption at the same price. Sandmeyer (1964) and Haveman and DeBartolo (1968) have modified the Baumol model in order to allow advertising to affect both price and quantity. This again further complicates the relationships underlying the objective function but, in principle, the solution can be found in the same way.

One case of particular interest concerns the equilibrium position where  $MC > AR > MR$ . Expansion after the point where  $MC = MR$  reduces profits but continues to expand revenue. Expansion after the point where  $MC = AR$

(anywhere up to the point where  $ATC = AR$ , i.e. profits are zero) is achieved by employing capital which, under a profit maximising goal, would be laid-up. The monopolist finances the losses made on operating inefficient and uneconomic capital from the pool of profits made from more efficient units.

#### Behavioural Theories

In the behavioural approach, the firm is seen essentially as a coalition of individuals and groups. Each group tends to be concerned with a different (although obviously related) aspect of the organisation and hence, has its own goals. For this reason, the behavioural theory has tended to specify a fairly large number of managerial goals. Cyert and March (1963), for example, distinguish five: production goal; inventory goal; sales goal; market share goal and profit goal. Cohen and Cyert (1965, p.338) also argue that this is the optimal number of goals to specify.

Because different parts of the organisation are associated with different goals, they will often be inconsistent. In attempting to eradicate inconsistencies (which the behavioural theory suggests will not be wholly successful) the various groups within the coalition will be brought into conflict. If decision making is to achieve a clearly formulated goal, such inconsistencies and conflicts must be eliminated. But, as Wildsmith (1973, p.24) points out, if this is true of "economic man" it is certainly not true of "organisational man". Cyert and March argue that the goals which evolve are imperfectly rationalised and tend to be stated in the form of aspiration levels rather than maximising constraints. The behavioural theory does not centre on a unique objective of the firm, and the objectives which are considered strategic are not formulated in terms of a maximisation hypothesis.

A major problem is that on dropping explicit maximisation from the analysis the possibility of deriving simple and elegant solutions which

describe the position of the firm at any particular point in time becomes more remote. Although the concept of the capacity region remains relevant, its derivation from the ex ante functions and the manner in which it collapses to the short-run macro function are no longer obvious. Failure to include an explicit maximand makes all of the concepts more difficult to handle. Profit maximisation was central to the Johansen thesis and behavioural aspects played little or no role. As Wildsmith (1973, p.34) points out, "Profit maximisation in such circumstances is not so much a behavioural assumption as a necessary condition for survival ... The problems arising from considering the firm as an administrative and social organisation are not relevant".

Although the behavioural theories drop the use of an explicit maximand, Baumol (1965, p.297) has argued that the limited, sequential, problem-orientated search procedure suggested by the theory may be consistent with an implicit form of optimisation of a deeper kind. Search is now seen as having a positive cost and being subject to diminishing returns. Baumol sees the firm as acting in an "optimally imperfect" manner if it seeks a point where the additional costs of change are just balanced by the additional advantages gained. A similar approach is adopted by Nordhaus (1973) and reported in Chapter (4) where an attempt is made to derive a unique ex ante function from the more realistic concept of an ex ante region.



## Chapter 4. The Ex Ante Function

### A. Introduction

The ex ante function represents an attempt to summarise the technologies from which a firm can choose when making an investment decision. The function plays an important role in vintage production theory and forms a key link in the Johansen schema (1972). The formulations of the ex ante function that have appeared in the literature have invariably been naive and, at best, only very crude approximations to the ex ante choices available in the real world. While the development of more realistic formulations may make the mathematics of vintage production and growth theories intractable, it is nevertheless important that better descriptions of the real world should evolve, in order that the simplifying assumptions used in the theoretical models are understood and appreciated.

### B. A Brief Review of the Literature

After a long history of neoclassical thought, and a brief excursion into the area of input-output analysis, two articles stand out as the turning point in the development of economic thought in the theory of production: the works of Johansen (1959) and Salter (1966). As Harcourt (1972, pp.54-5) points out, we might accord 'priority of invention' to Salter, whose contribution to the evolution of thought in this area stretches back to the research for his Ph.D. (published in 1955).

Salter's work (1966, pp. 17-21 ) suggested that the ex ante function could be summarised by,

$$Y_o = f(K_o, L_o, T_o) \quad \dots (4.1)$$

where  $Y_o$ ,  $K_o$  and  $L_o$  respectively denote the output, capital and labour associated with the most modern capital available.  $T_o$  denotes the length

of life of capital and is included because equipment with given  $Y_0$ ,  $K_0$  and  $L_0$  may vary in its profitability depending on the period over which these resources flow. Salter (1966, p.15) has defined the function as the 'production function which includes all possible designs,' it is 'purely technically determined' and 'free from the influence of factor prices.' The ex ante function is the efficient envelope of all of the alternative production functions conceived of by designers and available to managers at that time. This function has appeared in slightly different guise in the form of the 'innovation possibility curve' - see Ahmad (1966).

Salter assumed that the function had the well-behaved neoclassical properties. On the further assumption of constant returns to scale, Harcourt (1972, pp.56-7) writes this as,

$$\xi_0^L = f(\xi_0^K) \quad \dots (4.2)$$

such that

$$\frac{\partial \xi_0^L}{\partial \xi_0^K} < 0, \quad \frac{\partial^2 \xi_0^L}{\partial \xi_0^K^2} > 0 \quad \dots (4.3)$$

where  $\xi_0^L$  and  $\xi_0^K$  denote input per unit of output of labour and capital respectively, associated with the latest vintage. This function, equation (4.2), applies at only one point in time and is a summary of all of the best practice techniques that can potentially exist given the current state of technical knowhow. As new knowledge becomes available, the function shifts inwards - see Salter (1966, pp.21-3).

Salter (1966, pp. 19-20) recognised that capital was generally long-lived, hence the  $T_0$  variable, and that any attempt to pick the optimal amounts of  $K_0$  and  $L_0$  would involve some form of discounting procedure.

Harcourt (1972, pp.56-7) explains the simplest case where the quasi-rent per unit of output,  $z$ , is assumed to be constant over time,

$$z = (p - w\xi_0^L) \quad \dots (4.4)$$

The present value per unit of output,  $pv$ , is seen to be,

$$pv = [p - w\xi_0^K] B - \xi_0^K \quad \dots (4.5)$$

where,

$$B = \frac{[(1+i)^{T_0} - 1]}{i(1+i)^{T_0}} \quad \dots (4.6)$$

$B$  is the present value of the stream of one pound per annum for  $T_0$  years, assuming  $T_0$  is the constant and equal expected lifetimes of the techniques - see Allen, (1969, pp.32-3 and 448-9). Hence,

$$\frac{\partial (pv)}{\partial \xi_0^K} = -wB \frac{\partial f(\cdot)}{\partial \xi_0^K} - 1 = 0 \quad \dots (4.7)$$

$$\text{and} \quad \frac{\partial f(\cdot)}{\partial \xi_0^K} = -\frac{1}{Bw} \quad \dots (4.8)$$

$B$ , however, is the inverse of the expected annual payment on investment and, hence,  $Bw$  is the ratio of factor prices.  $\frac{\partial f(\cdot)}{\partial \xi_0^K}$  is the slope of the

ex ante function. The entrepreneur chooses a point where the isoquants are tangent to the isocost lines. Wigley, (1970, pp. 114-5) demonstrates the analogous result where the time series of quasi-rents generated by the various investments are seen in the context of exponentially changing factor and product prices - also see Harcourt (1972, pp.58-60).

The Johansen (1959) contribution progresses along similar lines. A unique ex ante function of the general form,

$$Y_0 = f(K_0, L_0, Y, t) \quad \dots (4.9)$$

is assumed - see Johansen (1959, pp.160-1). The variables  $Y_0$ ,  $K_0$  and  $L_0$

are defined as before. The time trend,  $t$ , appears in an attempt to account for improvements in technical knowhow and new discoveries of natural resources over time.  $Y$  appears as an argument to cope with variations in the pressures on natural resources - an interesting innovation that was not carried over to Johansen's later work - see Johansen (1972). The expected direction of the influence of these factors on ex ante output can be summarised by the first order partial derivatives:

$$\frac{\partial Y_0}{\partial K_0} > 0 ; \quad \frac{\partial Y_0}{\partial L_0} > 0 ; \quad \frac{\partial Y_0}{\partial Y} \leq 0 ; \quad \frac{\partial Y_0}{\partial t} \geq 0$$

.... (4.11)

Wan (1972) has suggested that Johansen restricted the ex ante function to the case of infinite elasticity of substitution between  $K$  and  $L$ . Johansen (1959, p. 171) used this assumption only under duress, when a more general functional form would have caused the mathematics of his growth model to have become intractable - for example, in the case of capital with fixed, finite life. This particular form of the ex ante function can be written,

$$f(K_0, L_0, Y, t) = aK_0 + bL_0$$

where  $a$  and  $b$  are constants. Johansen (1959, p.160 and 166) considered,

$$f(K_0, L_0, Y, t) = Ae^{-g_0 t} \alpha_0 K_0^{\beta_0} L_0^{\rho_0} Y^{-\rho_0} \quad \dots (4.12)$$

to be "a rather satisfactory form of  $f$ " (where  $\alpha_0$ ,  $\beta_0$ ,  $\rho_0$  and  $g_0$  are constants) and that  $\alpha_0 + \beta_0 = 1$  is "possibly a realistic hypothesis".

Unique well-behaved ex ante production functions have been adopted in two important articles on 'putty-putty' theory. Solow (1960) and Phelps (1962) adopt the following family of functions,

$$Y_{vt} = A e^{gv} K_{vt}^{\alpha} L_{vt}^{1-\alpha} \quad \dots (4.13)$$

where  $v$  denotes the  $v$ th vintage of capital and,  $o$ , as previously, refers to the most up-to-date vintage. Technical change, at a rate  $g$ , occurs up to the point in time when the investment is made. No further advance takes place once the capital is installed. The growth problem is one of allocating labour across fixed amounts of capital of various ages, in the most efficient manner possible. This form of ex ante function is chosen because - as Allen (1968, pp.283-6) demonstrates - it is fairly straightforward to aggregate over vintages to obtain an overall or 'aggregate' production function. Fisher (1969) has demonstrated how equation (4.13) has special properties that allow aggregation to proceed and that usually aggregation is not possible for more general functional forms. Svernilson (1964, pp. 110-1) also chooses a Cobb-Douglas ex ante function with disembodied technical change, but with generalised returns to scale (ie.

$\alpha + \beta \leq 1$ ), for use in a putty-clay model.

The general area of neoclassical growth theory with induced technical change need not detain our attention for long. The general area is based on the approach shown in Samuelson (1965). The key variation on traditional growth theory is that the innovation possibility curve is introduced explicitly, in a simultaneous system rather than being given exogenously. However, Nordhaus (1973, p.120) argues that "the innovation possibility function is not a true case of induced innovation or invention. The true case of induced invention requires at least two productive activities, production and invention. If there is no invention, then the theory of induced innovation is just a disguised case of growth theory with exogenous

technological change." Another good reason for leaving two-sector growth models well alone is that the intractability of the mathematics of the derivation of an equilibrium growth path precludes the theoretician working with realistic theories of invention and production.

In a later paper Johansen (1972) briefly develops a more sophisticated model of the ex ante function. This particular treatment was concerned with the fact that, in the previous literature, knowledge was assumed to be freely available. Johansen recognised two dimensions to this problem. First, in a positive study, it may be necessary to distinguish a large number of ex ante functions - up to one for each firm - while, in a normative study, it might be possible simply to consider the efficient envelope of such functions. Second, the idea that, while not all of the ex ante technologies will be in use or even readily available, such technologies can be developed with the application of R & D resources.

Both Johansen (1972) and Salter (1966) are consistent in their belief that unobserved points on the ex ante function potentially exist and, indeed, would have been observed had slightly different factor price ratios existed. Salter (1966, p.14) explains that, "No engineer goes to the trouble and expense of developing techniques that he is certain will prove uneconomic." While Johansen (1972, p.9) argued that the intermediate technologies could have been developed with a certain amount of R & D effort, Salter (1966, p.24) tended to play-down the role of induced R & D. In Salter's analysis potential points on the ex ante function, as yet undeveloped, might be achieved by either finding new uses for existing technologies or rearranging existing technological knowledge into forms appropriate to the new factor prices. Salter's discussion of the question failed to recognise that research, development, drawing and design have costs. Johansen's recognition of the

role played by R & D was not translated by him into a realistic theory of ex ante choices.

The most promising line of development appears to be that discussed by Nordhaus (1973). In place of the unique, well-behaved innovation possibility function, Nordhaus has substituted a production region that stretches from known, but unused, to unknown technologies. There are costs of moving about within this region: costs of search, R & D, technology payments, etc. Despite the complexities that this approach threatens to bring with it, this does appear to be the potentially most rewarding avenue uncovered so far. We return to this question later in this chapter, but first, consideration is given to whether there are circumstances under which a 'well-behaved' ex ante function will appear to exist.

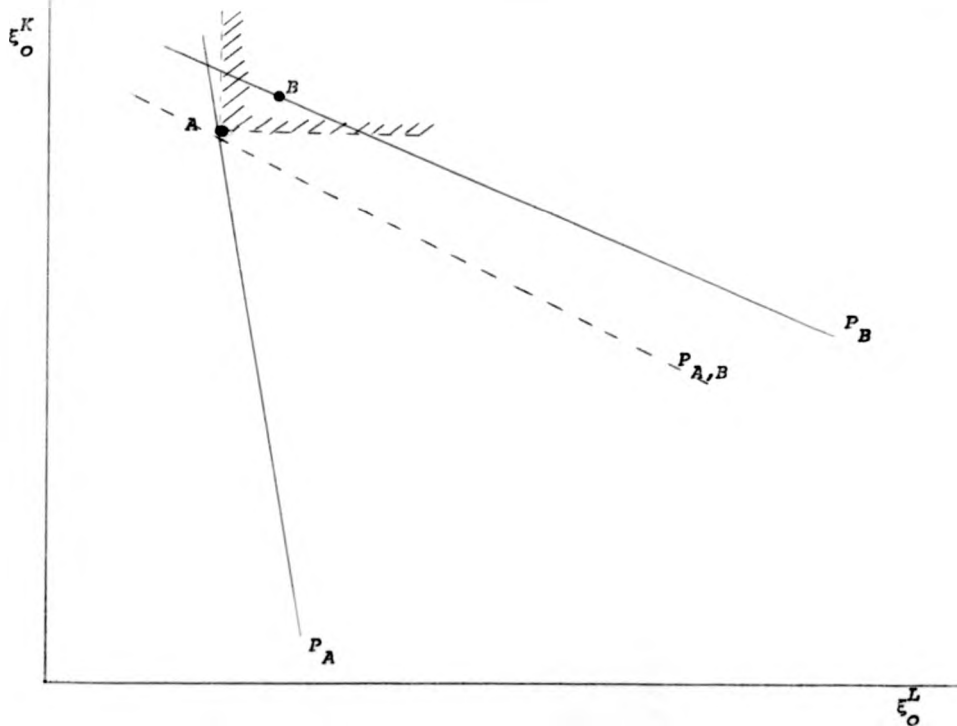
#### C. A Well-Behaved Ex Ante Function

This section investigates a special case where the ex ante function - i.e. the locus of points joining the new technologies chosen by firms - is 'well-behaved'. Consideration is given to an industry with a large number of firms, with a high degree of competition. Firms in the industry are assumed to have perfect knowledge about the complete set of ex ante possibilities. It is further assumed that, while some firms can establish their new plant in regions where factor prices are advantageous, a large number of firms are 'locked' into factor markets. If the firms that are locked in a particular market are to survive, a wide range of possible wage/rental ratios must exist in the final equilibrium situation. The investment in new plant is assumed here to be completely independent of the other operations of the firm. In other words, the new plant supplies an output that competes directly with the outputs from new plants owned by other firms in the industry.

Diagram (4.1) shows the initial adjustment procedure adopted by each of the firms in the industry. Point B represents a technology firm  $P_B$  had in

mind with its existing factor prices. Assuming, for a moment, that under perfect competition profits are zero, the price the firm would have charged is  $P_B$ .

Diagram (4.1) Adjustment to Superior Technologies or Cheaper Factor Markets



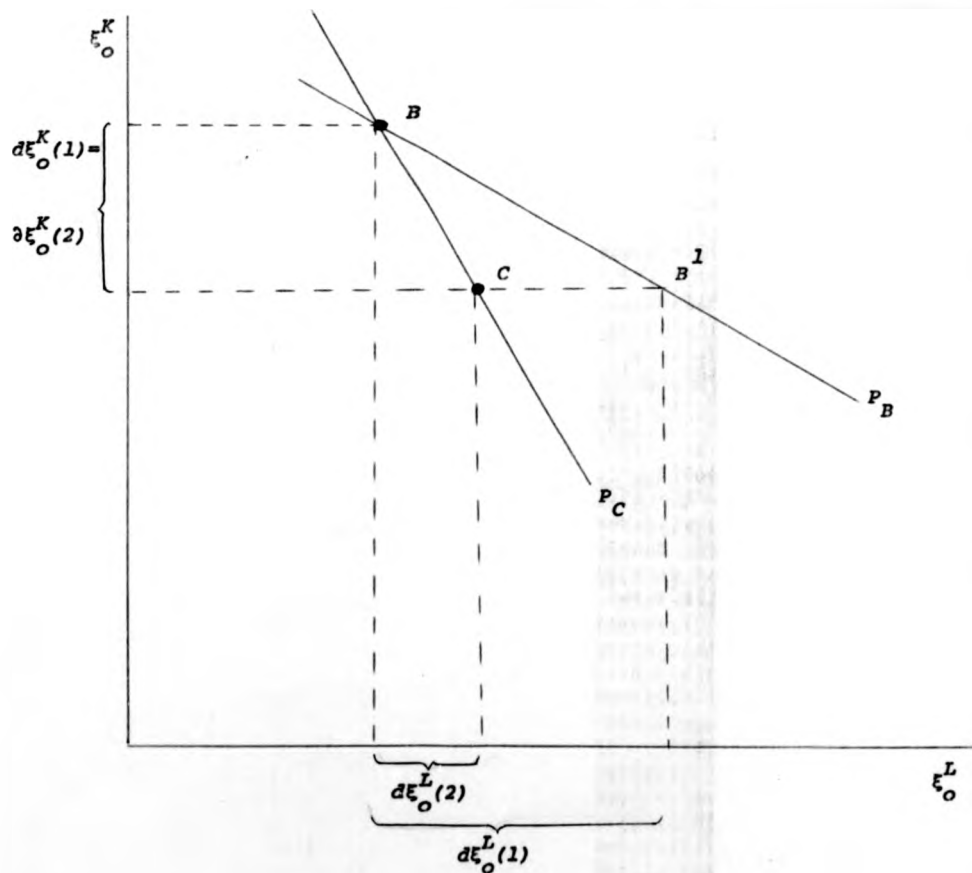
Firm A is situated in a region of higher factor prices, but had planned to introduce a relatively more efficient technology. Although  $F_A$  and  $F_B$  might, by chance, pick initial situations that would allow them to charge the same price,  $P_A = P_B$ , both firms could do better:  $F_A$  could establish its plant in  $F_B$ 's factor markets;  $F_B$  could adopt  $F_A$ 's more efficient technology. If there are no costs involved in the adjustment (i.e.,  $F_A$  does not force factor prices upwards in the new market and  $F_B$  does not incur any knowhow



costs in adopting  $F_A$ 's superior technology), both firms could achieve point A on a lower price line  $P_{A,B}$ .

Diagram (4.2) illustrates the next step in the argument. It is advantageous for  $F_B$ , currently considering point B, to adopt the technology represented by point C even though the adjustment involves increasing the labour input per unit of output. The adjustment allows a lower price line

Diagram (4.2) The Profitability of Adjustment When Increasing One Input.



to be reached because the associated fall in capital per unit of output,  $d\xi_O^K(2)$ , more than compensates for the associated increase in labour per unit

of output,  $d\xi_o^L(2)$ , at the factor prices experienced by  $F_B$ . We can demonstrate this quite easily, using,

$$p = w\xi_o^L + r\xi_o^K \quad \dots (4.14)$$

and totally differentiating,

$$dp = \frac{\partial p}{\partial w} dw + \frac{\partial p}{\partial \xi_o^L} d\xi_o^L + \frac{\partial p}{\partial r} dr + \frac{\partial p}{\partial \xi_o^K} d\xi_o^K \quad \dots (4.15)$$

$F_B$ 's factor prices are, however, assumed to be unchanging, i.e.

$$\frac{\partial p}{\partial w} dw = \frac{\partial p}{\partial r} dr = 0 \quad \dots (4.16)$$

and hence, from (4.15) and (4.16),

$$dp = \frac{\partial p}{\partial \xi_o^L} d\xi_o^L + \frac{\partial p}{\partial \xi_o^K} d\xi_o^K \quad \dots (4.17)$$

As we might expect, the movement from B to  $B^1$  produces a zero change in price. From (4.14) we find,

$$\frac{\partial p}{\partial \xi_o^L} = w, \quad \frac{\partial p}{\partial \xi_o^K} = r, \quad \text{and} \quad \frac{dL}{dK} = \frac{-r}{w} \quad \dots (4.18)$$

and hence, from (4.17)

$$dp = w\left(\frac{-r}{w}\right) dK + rdK = 0 \quad \dots (4.19)$$

Now compare the movement from B to  $B^1$ ,

$$dp(1) = \frac{\partial p}{\partial \xi_o^L(1)} d\xi_o^L(1) + \frac{\partial p}{\partial \xi_o^K(1)} d\xi_o^K(1) = 0 \quad \dots (4.20)$$

with the move from B to C,

$$dp(2) = \frac{\partial p}{\partial \xi_o^L(2)} d\xi_o^L(2) + \frac{\partial p}{\partial \xi_o^K(2)} d\xi_o^K(2) \quad \dots (4.21)$$

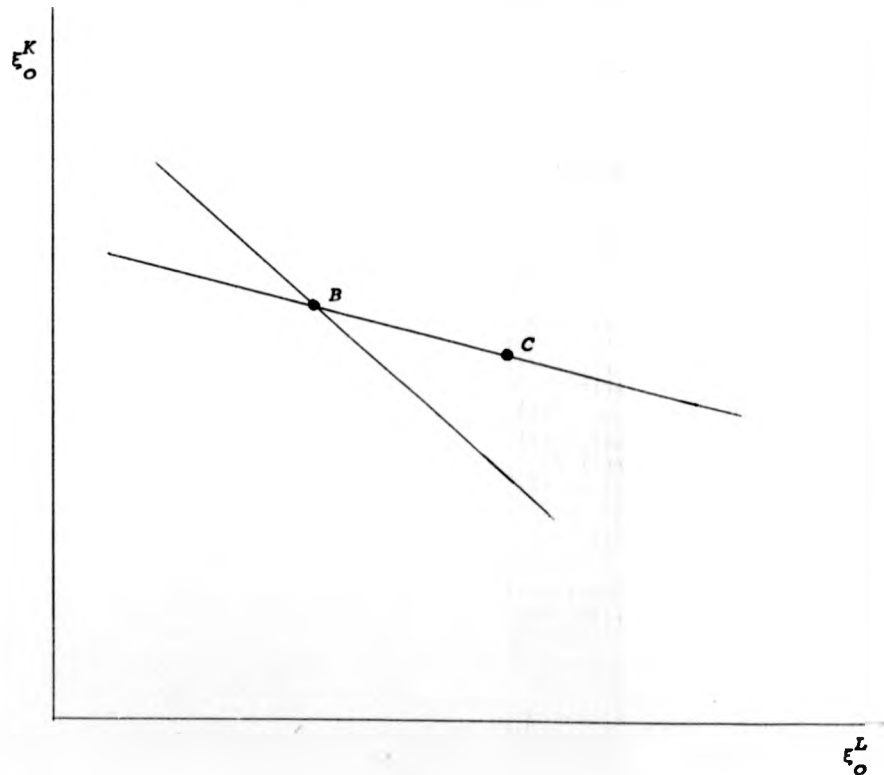
setting,  $d\xi_0^K(1) = d\xi_0^K(2)$  and knowing that  $d\xi_0^L(2) < d\xi_0^L(1)$ , it must

follow that if  $dp(1) = 0$  then  $dp(2) < 0$ . Hence, if  $F_B$  adopts the technology represented by point  $C$  (or indeed any other technology inside of  $B^1$ ) the going price per unit of output must fall. Only where point  $C$  lies outside of the line  $P_B$  will  $F_B$  find it unprofitable to adopt  $F_C$ 's technology.

This case is shown in Diagram (4.3).

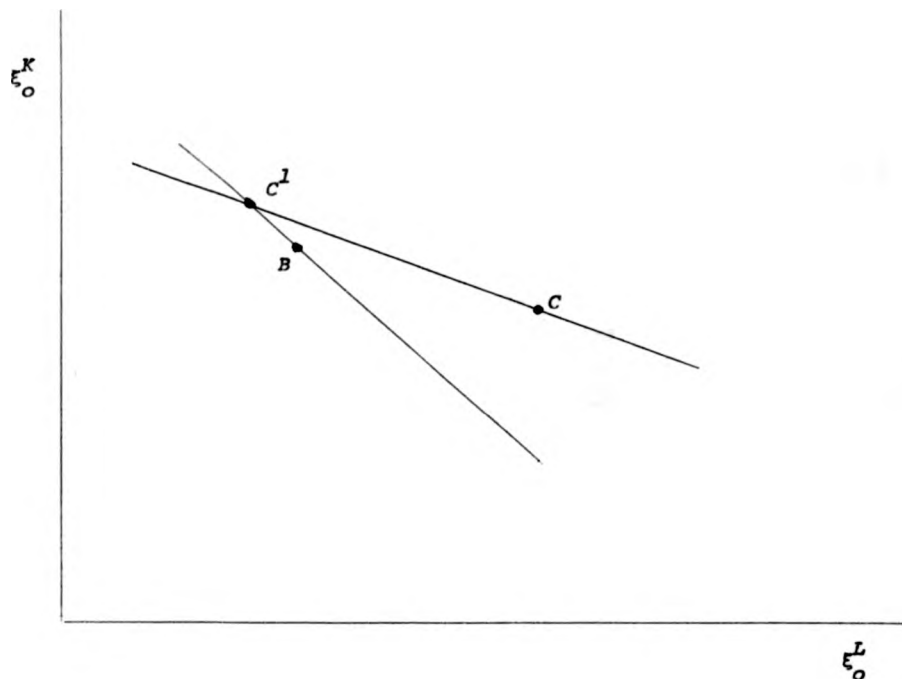
Diagram (4.3) A Case Where There is No Incentive For Firm B to Adopt Firm C's Technology.

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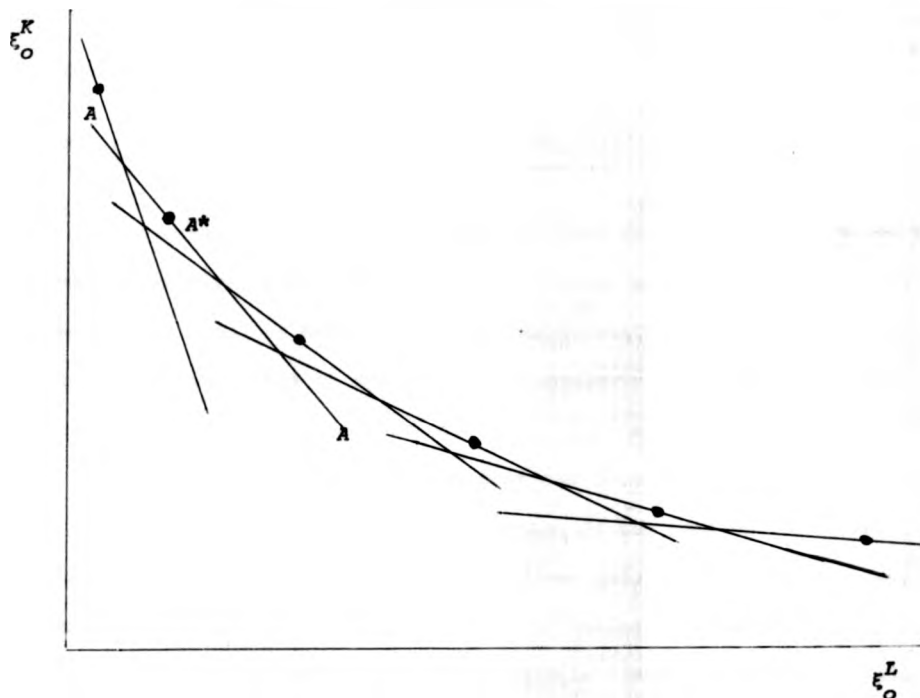
The final case considered is an extension of the arguments associated with Diagrams (4.2) and (4.3). Diagram (4.4) shows  $F_C$  having a price line with smaller negative slope than  $F_B$ 's price line, and  $P_C$  cuts  $P_B$  above  $B$ .

Diagram (4.4) *The Profitability of Adjustment When Increasing the Capital Input*



Without the necessity of developing the mathematics again, it is fairly easy to demonstrate that  $F_C$  will find it advantageous to adopt the technology represented by point B, even though this involves a higher capital input per unit of output. In this case it is the lower labour input per unit of output that more than compensates for the increased capital. We can simply argue that  $F_C$  is indifferent between C and a technology (perhaps imaginary) represented by  $C^1$ , but  $F_C$  is known to prefer B to  $C^1$  given its factor price ratio, and hence must prefer B to C.

From the arguments developed above, it is plain that the only set of price lines consistent with equilibrium in this industry must correspond with a frontier analogous to Samuelson's (1962) surrogate production function, shown in Diagram (4.5).

Diagram (4.5) Technologies Consistent With Industry Equilibrium

There is, in principle, no reason why the adjustment process should not result in the existence of a single price line, such as AA, with a large number of firms all using the technology represented by A\*. Given the assumptions that have been made, this does not preclude that, by chance, a large number of lines, such as AA, exist that lie on the surrogate function.

If all the points on the surrogate function are consistent with normal profits  $\pi^*$  ( $= \frac{\pi^*}{Y}$ ), situations can be envisaged where it will be worth-while for firms inside this frontier to sacrifice some of their current profits - in the form of expenditure on R & D or marketed technical knowhow - in order to move closer to the long-run function in the longer term. In other words, from Diagram (4.1),  $F_B$  might be willing to pay  $F_A$  for knowhow about the technology associated with point A ( and  $F_A$  might be willing to

incur higher transport costs in order to establish its new plant in a region of relatively low factor prices). In the sections that follow, more detailed consideration is given to the knowhow costs of moving around the input-output space.

D. The *Ex Ante* Region and the T-isocost Family

The important conclusion that can be drawn from the discussion of the current literature is that the *ex ante* function, as it usually appears, is naive and can only, at best, be a crude approximation to the investment opportunities available to the firm. The analysis contained in Section C, on the possibility that the 'effective' *ex ante* isoquant is 'well-behaved', only helped to confirm the suspicion that the form of the function generally used in the literature is a poor description of the real world. The assumptions that *ex ante* functions are exogenously given, unique at any given point in time and characterised by 'classical' properties may greatly simplify construction of vintage growth models - see, for example, Johansen (1959) and Svernilson (1964) - but one would hardly like to defend them as realistic descriptions of the real world. Attempts to endogenise technical progress - see, for example, Kaldor (1957), Phelps (1966) and Conlisk (1973) - and the research aimed at establishing the nature of production functions associated with the output of inventions - see Machlup (1962), Schmookler (1966), Nordhaus (1969) and Kamien and Schwartz (1972) - both indicate that an *ex ante* region may be more appropriate than a unique function. If the concept of an *ex ante* region is to become acceptable, however, it is essential to establish the boundaries that envelop it and the manner in which movement within the boundaries occurs.

A Simple Model of R & D

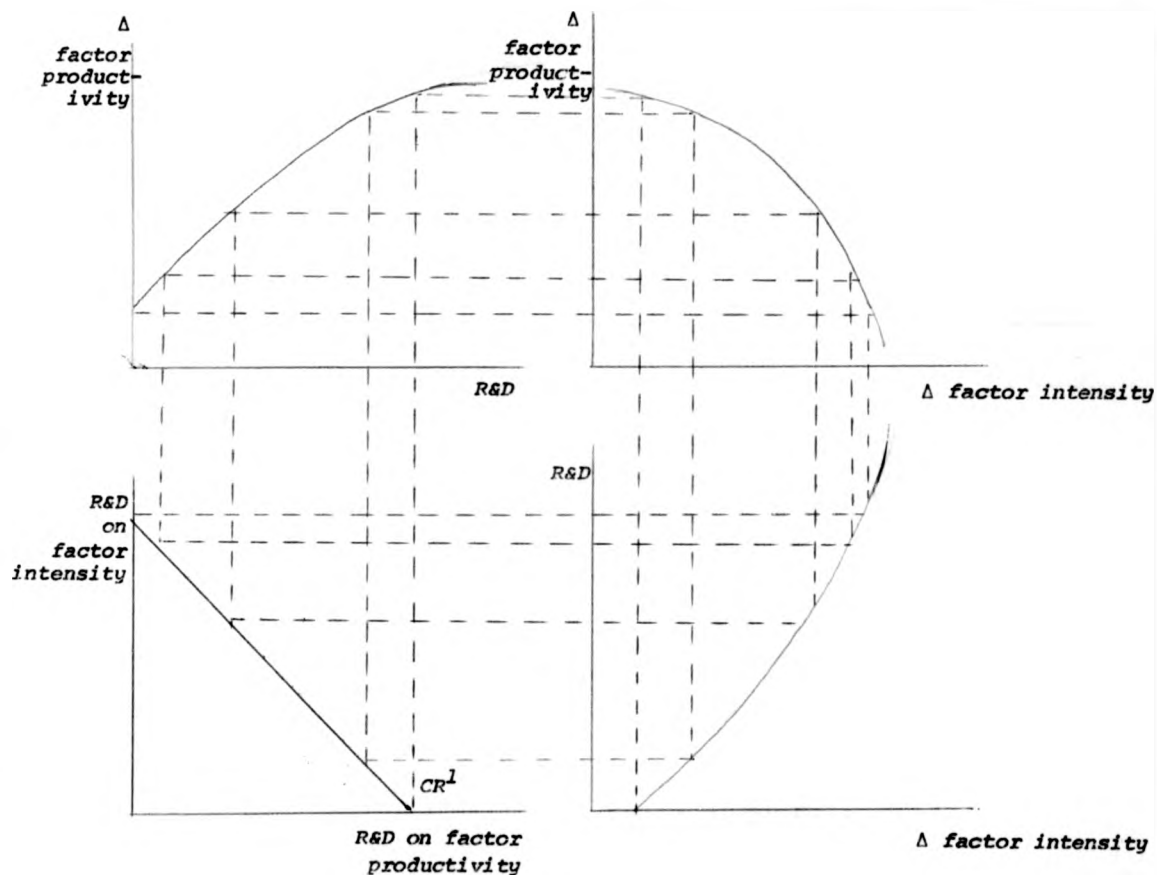
It can be argued, with a certain amount of conviction, that the 'inefficient' boundary of the region is the locus of points drawn through the most efficient technologies already in use within firms. Rationality appears to demand that firms do not choose technologies that are inferior to the best that they are already using. In this case, the inefficient boundary of the ex ante region is simply the best-practice boundary of the industry.

The 'efficient' boundary of the ex ante region is a much less precise construct. Its existence stems from a priori beliefs about the relationship between R & D effort, changes in factor productivity and changes in factor intensity. A simple model of R & D will be developed in an attempt to isolate this boundary and it will become apparent during the analysis that a model of this type underlies the Nordhaus (1973) concept of the T-isocost family.<sup>1</sup>

A simple model of the relationship between R & D inputs and outputs is shown in Diagram (4.6). This diagram is an attempt to summarise the nature of the innovations, and their associated costs, that can be produced by a single firm at a given point in time. An analogous argument could be applied to a capital goods producer considering his R & D activities for a particular firm that he supplies. Two important characteristics of the innovations are considered in detail: changes in factor productivity and changes in factor intensity.

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1. T-isocost is adopted in preference to C-isotech. Nordhaus (1973, p.211) defined the C-isotech in the following manner: "the set of techniques attainable with a given cost". As he intended the level of costs to be held constant (and not the level of technology) it would seem more appropriate to call the concept the T-isocost line.

Diagram (4.6) *The Factor-productivity/factor-intensity trade-off*

The underlying idea is that the further the new technology is from existing technologies, the more radical and the more costly will be the innovation. This appears to be an extension of morphological analysis - see Wills and Wilson (1972, ch.2.2). Morphological analysis is, according to Zwicky (1948) an attempt "to identify, index, count and parameterise the collection of all possible devices to achieve a specified functional capability". The functional capability in this case is to produce the type of product associated with the industry with plant whose input requirements per unit of output are defined by  $(\xi_o^K, \xi_o^L)$ .



Having defined all of the possible ways of achieving the desired functional capability, the alternative solutions can be measured in terms of their morphological distance - i.e. the ratio of the total number of features of the technology to the number of features that the technology has in common with known technology. Two important conclusions are drawn in the technological forecasting literature: (a) the probability of a technological break-through per unit of time will be a decreasing function of its morphological distance (b) opportunities available will be approximately proportional to the size of the occupied territory.

It is the first of these two findings that is adopted in a slightly modified form - the further is the desired technology from existing technologies, the less features the two will have in common and the more costly is the associated R & D. Risk is given cursory treatment: either it is, for the present, left to one side, or, it is considered to be appropriately subsumed in the costs of producing an innovation. In this second case it is assumed that, where a more radical innovation is envisaged, an initial failure of the research programme to yield a solution can be overcome by intensifying R & D. In this way, risks are translated into probable costs and the analysis can then proceed broadly along the lines suggested by Nordhaus (1973).

Diagram (4.6a) considers the relationship between R & D effort and the change in factor productivity that will result if this new technology is embodied in capital in which the firm invests. The existence of a limiting value of change in factor productivity may result from two constraints on the system. First, although we have not specified exactly the period in which we are working, the time horizon is finite and, although

technological potential may be boundless in the long-run, in the short-run it is almost certainly fixed. The shape of the curve reflects the fact that the more readily available and productive ideas will be used first, leaving the rest until higher levels of R & D intensity are reached. Second, at a given point in time, there will be limited resources that have a positive marginal product in R & D activities. The shape of the curve stems from the belief that the most efficient resources will be used first, leaving the less efficient to be introduced at higher levels of research effort.

A similar sort of relationship can be imagined to exist between R & D effort and the change in factor intensity that the firm can achieve. This relationship is shown in Diagram (4.6d). The underlying idea is that firms develop a stock of information and knowledge about the production process at the level (or levels) of factor intensity that they have experienced. Movement away from existing factor intensities will demand research effort. The greater the desired change, the greater will be the necessary effort. In addition, the argument about limited R & D resources and their quality is again pertinent in the short-run.

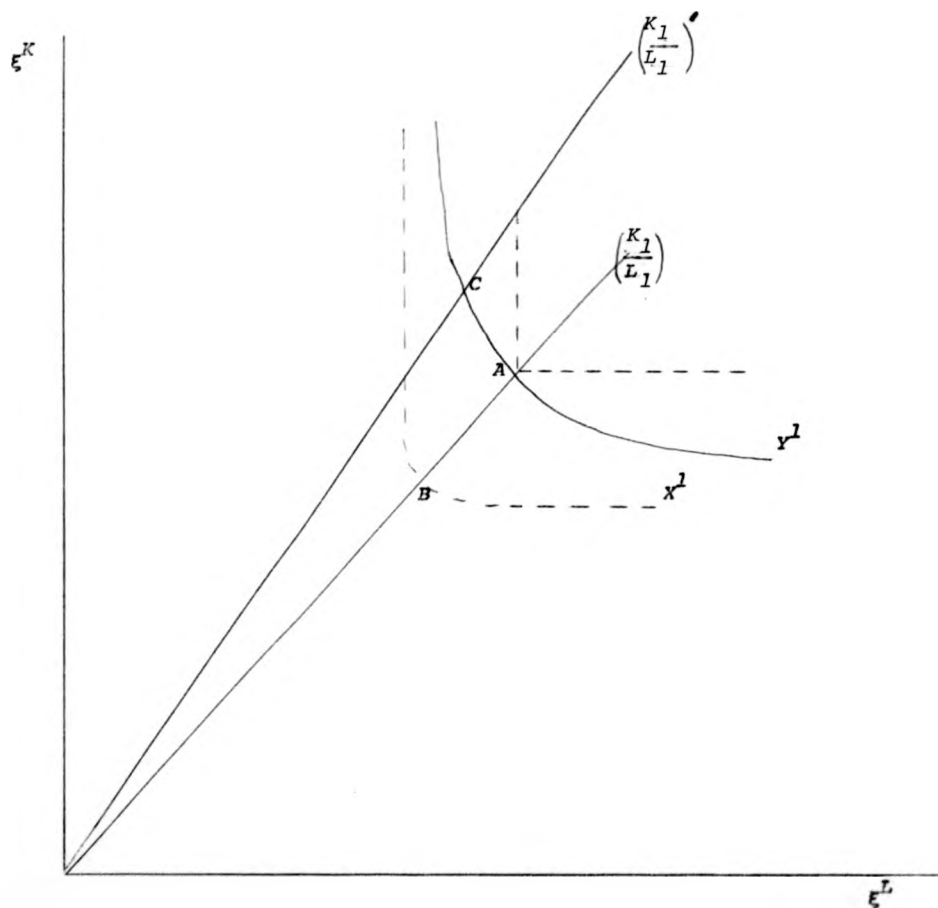
In the analysis presented in Diagram (4.6), the two directions of technical change are considered as being wholly independent of one another. In other words, the improvement of output per unit of input is equally difficult whether the firm is at a low or high capital intensity. The modification of this analysis is fairly straight forward and we return to this a little later.

It is Diagram (4.6c) that provides the essential link with the concept of the T-isocost line. Lines shown in this diagram correspond with given levels of R & D effort. At a given cost level resources can be purchased for work either on factor intensity or factor productivity. The line shown in the diagram illustrates the alternative allocations that can be made from a fixed R & D budget,  $CR^1$ , to either or both ends.

Given these three curves, (i.e. those shown in Diagrams (4.6a), (4.6c) and (4.6d) ), the trade-off between changes in productivity and factor intensity can be constructed directly. The trade-off curve is traced in Diagram (4.6b). Given invariant R & D production curves in Diagrams (4.6a) and (4.6d), different levels of R & D effort will produce alternative trade-off curves. Higher levels of effort will tend to shift the trade-off curve to the north-east, while lower levels of effort will shift it to the south-west. The ultimate efficient boundary, within the time period under consideration, is associated with the trade-off curve furthest to the north east, a level of expenditure  $CR^{\max}$ . This ultimate boundary may not be experienced because the profitability of R & D may fall below an acceptable level prior to  $CR^{\max}$  or, the firm may find itself faced by a constraint on the supply of R & D resources.

The trade-off curves for each firm can be drawn directly in the isoquant diagram, in a form that can be recognised as the T-isocost family described by Nordhaus. Diagram (4.7) shows the existing technology,  $Y^1$ , and the technologies that can be achieved with expenditure  $CR^1$ . The ray through the existing best practice point, A, for the firm is drawn as  $\frac{K_1}{L_1}$  and this denotes zero change in factor intensity. The size of productivity advance associated with zero change in factor intensity from Diagram (4.6b) is then plotted along the ray for this firm, B. Other rays, (eg.  $(\frac{K_1}{L_1})'$ ) can be plotted around the original ray, corresponding with various changes in factor intensity. The corresponding productivity changes are measured along their length towards the origin (i.e. CD). The resulting locus of points for the jth firm is represented by  $X^1$ . Each of these curves must be asymptotic to both a vertical and a horizontal line because all technologies to the north-east of any point on such a curve can be achieved simply by hoarding one or both inputs.

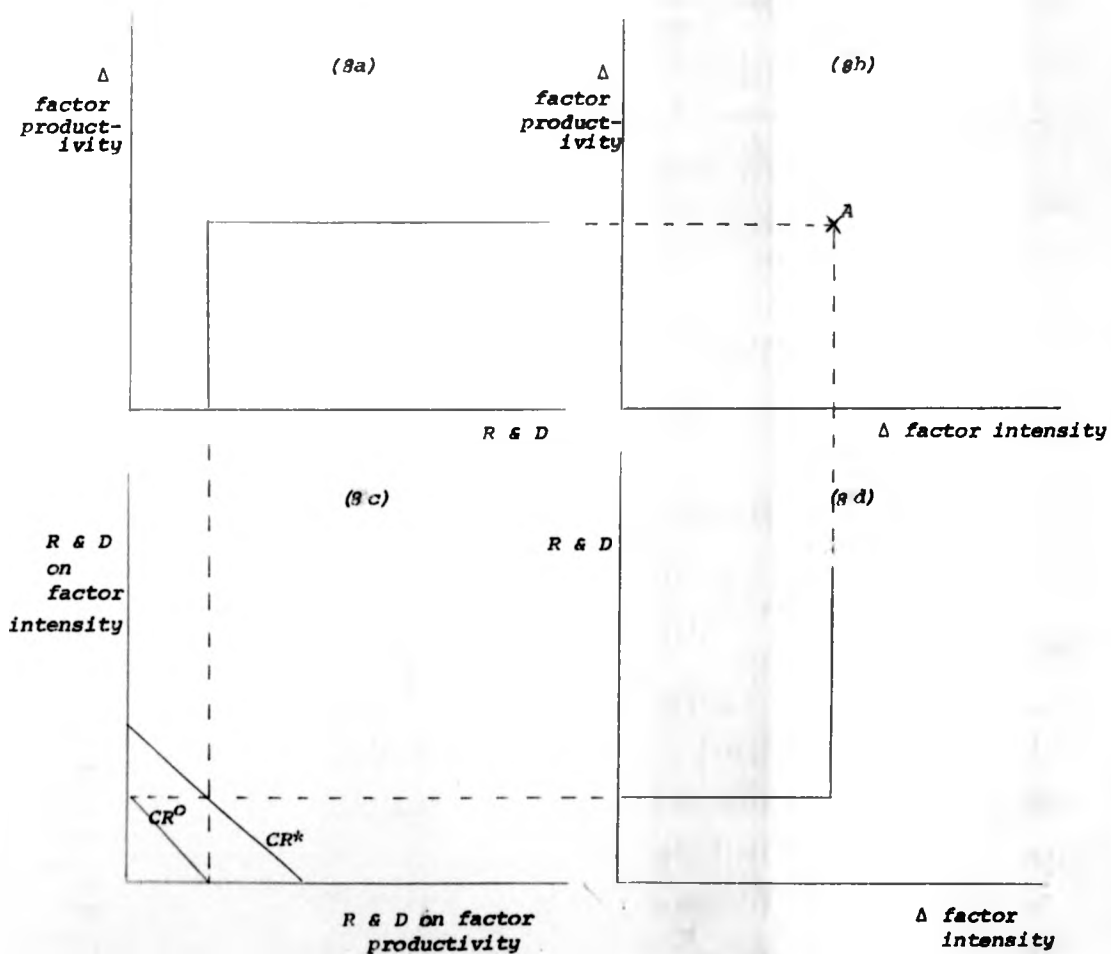
Diagram (4.7) Construction of the T-isocost Family in the Input-Output Space



Clay-Clay Technology - A Special Case

It is fairly easy to develop the special case of the clay-clay technology in the same diagrammatic form used in Diagrams (4.6). This special case is shown in Diagrams (4.8). At higher levels of expenditure

Diagram (4.8) Derivation of the T-isocost Family - the Clay-Clay Case



than  $CR^*$  no further gains in changes in factor productivity or changes in factor intensity occur. Given a level of expenditure  $CR^*$ , it is just possible to achieve the changes associated with point A in Diagram (4.8b).

At levels of expenditure lower than  $CR^*$ , but higher than  $CR^0$ , it is possible to achieve either the change in productivity or the change in factor intensity, but not both. At levels lower than  $CR^0$ , neither change can be brought about. One final possibility is that the vertical part of Diagram (4.8a) is missing and the horizontal part of (4.8d). In this case there would again be a clay outcome, but, in addition, technical change in the ex ante function would be exogenously given.<sup>2</sup>

Inter-Dependence of Changes in Factor Productivity and Changes in Factor Intensity - A More General Case

Under the assumption that the curve in Diagram (4.6a) is not independent of the curve in (4.6d) - or vice versa - a great variety of T-isocost lines can result. Under these circumstances, the 'well-behaved' T-isocosts are a special case. One example of a poorly-behaved case is shown in Diagrams (4.9). In this case, for whatever reason, it is argued that as the firm attempts to attain larger and larger changes in its factor intensity a larger pool of knowhow becomes available that shifts upwards the change in factor productivity - R & D curve in Diagram (4.9a). One explanation might be, for example, that, as it changes its factor intensity, this particular firm moves closer and closer to the technology of another firm in the industry. If some of the knowhow of this other firm is available then it may decrease the costs of R & D associated with the level of factor productivity.

Asymmetry

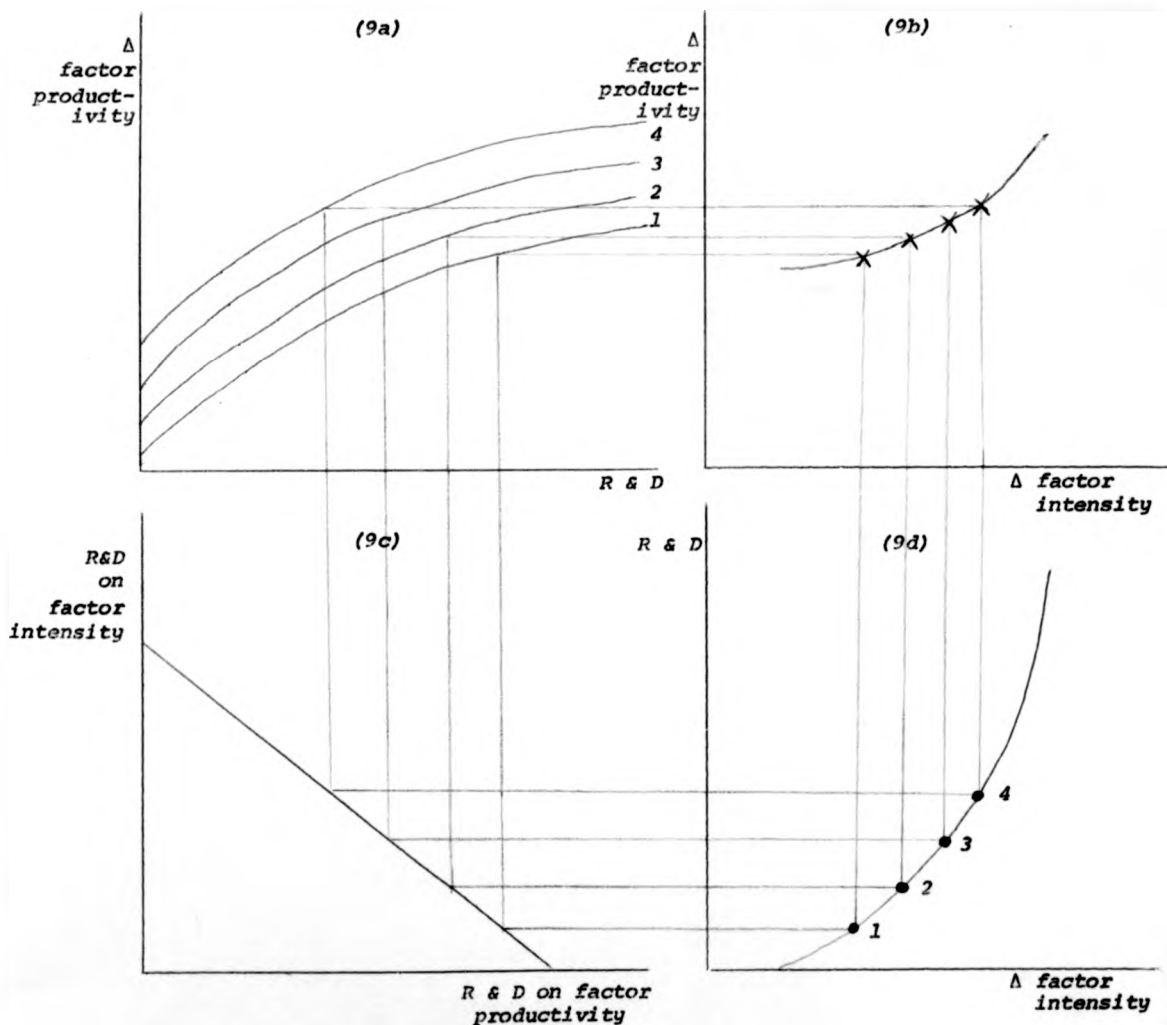
The way in which the model of R & D has been developed assumed that: first, factor productivity will always change in the direction of reducing input requirements per unit of output; second, the factor-intensity relationship with R & D is symmetrical around the existing factor ratio

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2. R & D in this case is obviously redundant.

There is no real reason why the factor intensity curve

Diagram (4.9) Interdependence of Innovative Outputs From R & D



should not be asymmetric, but the relaxation of the symmetry assumption would severely complicate the diagrammatic representation.

E. Endogenising R & D

This chapter has developed the idea of an ex ante region where movements within the region involve risks and costs, but also give rise to potential benefits. Higher R & D and technical knowhow payments may be more than off-set by savings in labour and capital costs. The R & D model developed here includes these characteristics and, in addition, makes the firm's activity at the intensive and extensive margins dependent on the nature of the firm's historically given capital stock and the R & D opportunities open to the firm.

The analysis concentrates on the cost-reducing innovations undertaken by firms and attempts to demonstrate that the 'well-behaved' ex ante function can be dispensed with as far as the individual firm is concerned. The model developed in this section is largely consistent with the work on aggregate neoclassical functions discussed in Chapter ( 8 ). In this later chapter, the firm is assumed to assess its overall performance in terms of key accounting variables: aggregate labour and capital inputs in the light of their average prices, aggregate output and profits. In this section, the firm is seen as judging the desired direction of technical change in terms of its aggregate inputs of capital and labour and their average prices. In addition, the overall performance of the firm - in terms of a key profit variable - is assumed to impinge on the activity of the firm at its intensive and extensive margins.

The model focuses on the defensive behaviour of firms. An adjustment mechanism is assumed to exist which is triggered when firms realise that their performance is inferior to the 'norm' in the industry. Defensive firms are seen as satisficers in the sense that they only focus their attention on improving their profit performance when this falls below some desired level and the driving force for improvement decreases as profit performance

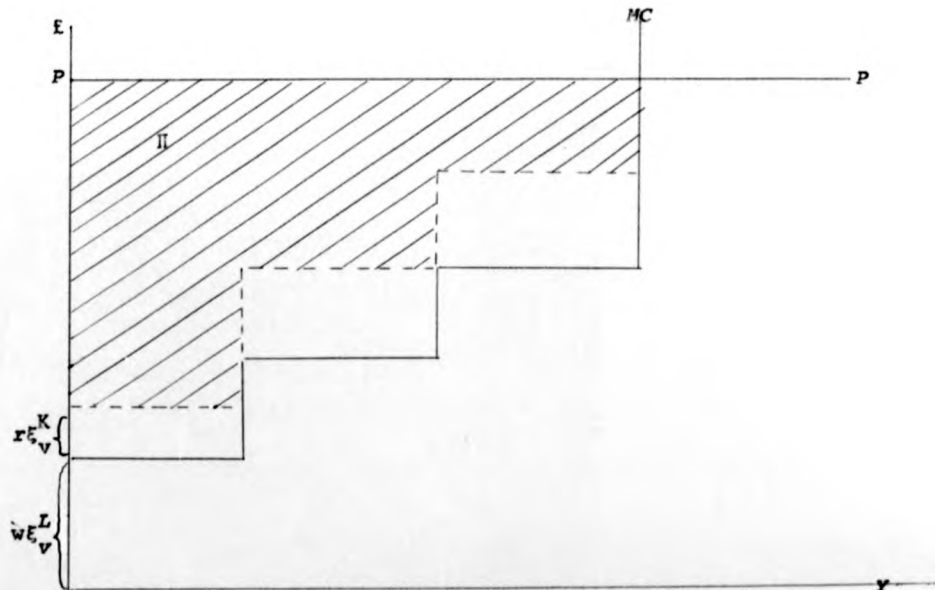


approaches the 'norm'. Once the drive mechanism is triggered, the firm is assumed to attempt to seek an efficient allocation of resources between R & D and investment. This type of behaviour could be argued to be untypical of a 'laggard' firm, but it will at least serve as a first approximation. In addition it may be assumed that the real 'laggard' - if it survives at all - is likely to be taken over or to experience a change of management that will result in greater efficiency and profits. The model of defensive firms is broadly consistent with the later work contained in Chapter ( 8 ), but, in addition, the model could be extended to look at the behaviour of market leaders and to include product as well as process change.

Changing Profits By Investment With No P & D

A useful starting point for developing a model that looks at the impact of R & D is to review the alternatives available to the firm, other than R & D, that enable the firm to raise its level of profits. The simple vintage model generally adopted in the literature exists within a perfectly competitive product market environment. The current position of a particular firm is demonstrated in Diagram (4.10). The whole area below the price line,  $P$ , forms

Diagram (4.10) Profits of the Firm With Various Vintages of Capital



the total revenue of the firm,  $pY$ . The area below the marginal cost curve,  $MC$ , is the wage bill,  $\sum_v E_{vt}^L Y_v = wL$  and the extended part of the steps represent

the annual capital cost of each vintage,  $\sum_v E_{vt}^K Y_v = rK$ . The rental is

equivalent to the annual repayment of sunk costs and the associated interest charge. Profits of the firm are the residual, shaded area, labelled  $\Pi$ .

The question arises as to whether the firm will make any sort of decision about investing or scrapping simply on the basis of current profits. In Chapter ( 8 ), for example, it is argued that the firm makes a decision on the basis of a comparison between expected profits and long-run potential profits. The long-run actual profits of the current capital are,

$$\Pi_{LR} = \int \int_{vt} \frac{(p_{vt} Y_{vt} - w_{vt} L_{vt} - r_{vt} K_{vt})}{(1+i)^t} dv dt \quad \dots (4.22)$$

Adopting some average life expectancy of its remaining capital,  $\bar{T}$ , the firm might calculate,

$$\Pi = \int_v \frac{\Pi_{LR,v}}{\bar{T}} dv \quad \dots (4.23)$$

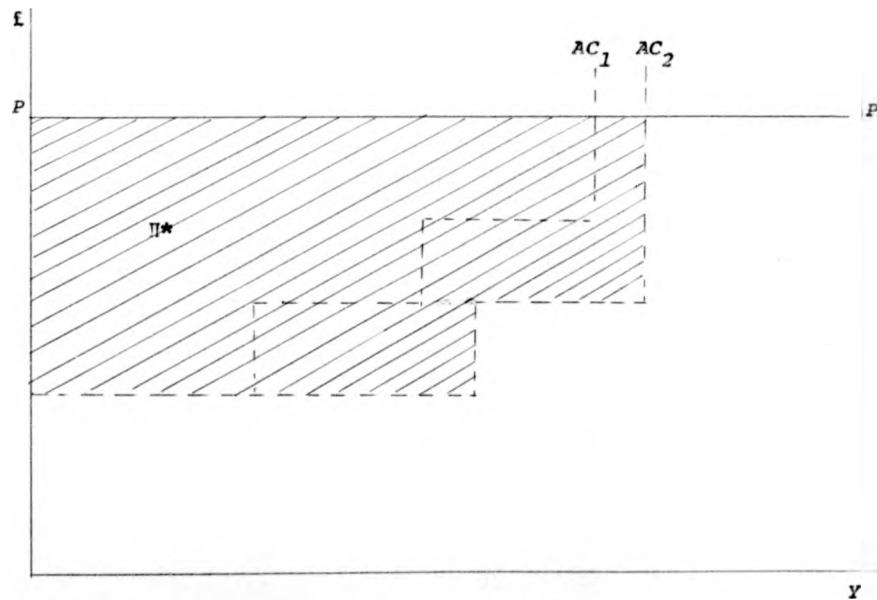
as being representative of its current profit situation. Diagram (4.10) is interpreted in this way.

The three separate vintages shown in Diagram (4.10) are all earning positive quasi-rents,  $(pY_v - wL_v)$ , and, hence, there is no immediate reason why the firm should alter its profit position by scrapping its least efficient vintage. As long as  $pY_v > wL_v$ , for any particular vintage, the firm is paying something towards sunk costs or adding to its profits. If, however,  $\Pi < \Pi^*$ , there is an inducement to invest in new capital. With no technical

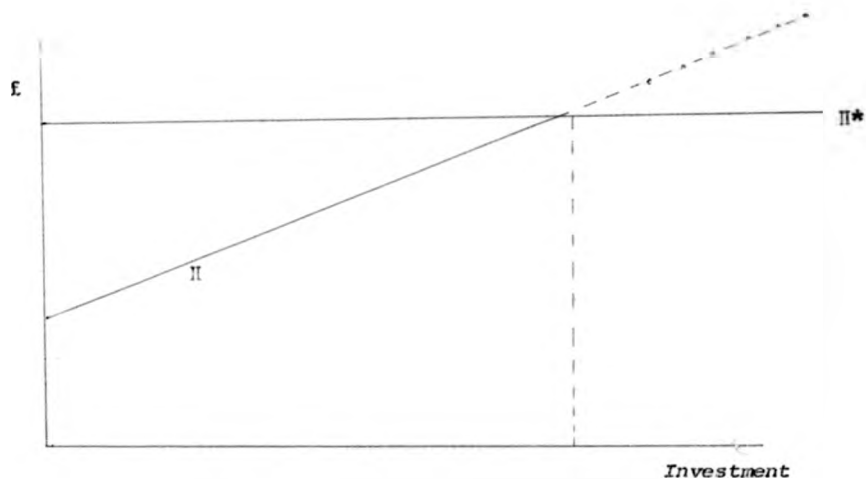
change in the latest vintage, the effect is to shift the cost curves horizontally to the right, as shown in Diagram (4.11). This will occur, in a world where managers rely on ICF techniques, as long as the extra investment involved,  $I_1$ , has the following properties

$$\int_{t=0}^{\infty} \frac{pY_1 - wI_1}{(1+i)^t} dt > p_K I_1 \quad \dots (4.24)$$

Diagram (4.11) Increasing Profits Through Activity at the Intensive Margin



The model is fairly unrealistic if  $p, w, p_K, i$  and the input-output coefficients  $(\xi_1^K, \xi_1^L)$  are all fixed. In this case the firm can carry on expanding its absolute value of profits, although  $\frac{\Pi}{Y}$  and  $\frac{\Pi}{K}$  will tend to limiting values. In our simple model of satisficing behaviour, see Diagram (4.12), profits of the firm expand until  $\Pi = \Pi^*$  at a change in capital

Diagram (4.12) Increasing Profits and Satisficing Behaviour

of the latest type,  $I_1$ .

This case changes quite dramatically where the firm faces constraints on its supplies of factors or a downward sloping demand curve. First, in the case of a downward sloping demand curve, marginal cost equal to marginal revenue implies a contraction of output below the profit maximising level which may involve laying up vintages earning positive quasi-rents. Second, the firm has to choose which set of cost curves it wishes to employ. The firm may, for example, find it profitable to scrap its older vintages and to invest in new capital, even though the older vintages are earning positive profits (i.e. quasi-rents net of interest payments and other sunk capital costs). This will occur where the new capital is so much more efficient than the old that the profit margin on the new capital covers the sunk capital costs still outstanding on the old capital, in addition to any profits that are lost, but net of the second-hand value of this capital. This can be written algebraically as,

$$\int_{t=0}^T \Delta Y_1 \frac{[p-w\tau_1^L]}{(1+i)^t} dt - p_K \Delta Y_1 \tau_1^K - \int_{t=0}^{T-\tau} \frac{r \Delta Y_1 \tau_1^K}{(1+i)^t} dt >$$

$$\int_{t=0}^{T-\tau} \Delta Y_1 \frac{(p-w\tau_1^L - r\tau_1^K)}{(1+i)^t} dt - p_{KS,\tau} \Delta Y_1 \tau_1^K \quad \dots (4.25)$$

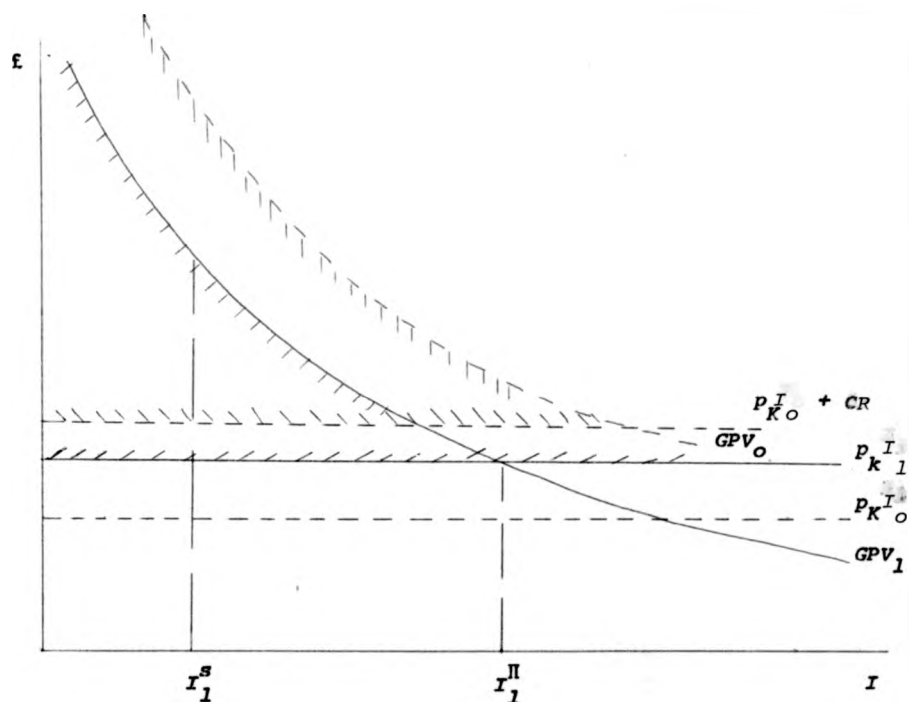
where  $(T-\tau)$  are the remaining years of active life of the oldest vintages and  $p_{KS,\tau}$  denotes the second-hand price of capital of  $\tau$  vintage. All of the gains and losses are given for a positive change in output,  $\Delta Y_1$ , at the intensive margin, compensated exactly by an equal negative change at the extensive margin.

#### Joint Investment - R & D Decision

While the firm may be able to improve its profit performance by investing heavily in the latest vintage, it might increase its profits in a more efficient way by jointly undertaking R & D and investment. Here R & D is assumed to alter the technical coefficients on the most efficient vintage available - in other words, R & D creates a new vintage which then appears as a further investment option open to the firm.

The general idea can be shown in diagram form. As one option, the firm might consider investing in the existing best-practice vintage. Diagram (4.13) illustrates the case where the gross present value, GPV, associated with each additional unit of investment,  $I$ , falls. In the traditional way, the present value of investment is maximised at the point where  $GPV = p_K I_1$ , i.e. the maximum area between the two curves. The point of maximum profits is labelled  $I_1^{\Pi}$ . The satisficer might add increments to the capital stock until  $I_1^S$  is reached, when the enclosed area (i.e. under  $GPV_1$  and above  $p_K I_1^S$ ) represents a present value

Diagram (4.13) Present Value of Investment in Physical Capital and in R & D



which, when averaged over the expected life of the new capital, raises  $\Pi$  sufficiently to achieve  $\Pi^*$ .

The same diagram also shows how R & D may modify this situation. The firm is now assumed to undertake a small amount of R & D,  $CR$ , which has a direct impact on the values of the input-output coefficients. R & D creates a new vintage characterised by  $(\xi_0^K, \xi_0^L)$  where, in general,  $\xi_0^K \neq \xi_1^K$  and  $\xi_0^L \neq \xi_1^L$ . There are likely to be two effects of R & D - the first attempting to substitute capital for labour (or vice versa) and the second attempting to reduce input requirements for each factor per unit of output.

It is assumed in Diagram (4.13) that, whatever substitution effects exist,  $\xi_0^K$  and  $\xi_0^L$  both fall as the level of research is increased. This has an impact on both of the curves shown in the diagram. First, the horizontal  $p_{K^I_1}$  line falls to  $p_{K^I_0}$ , but, in addition there is a cost of doing research that must be added in. The new investment line is shown as  $p_{K^I_0} + CR$ .<sup>2</sup> In addition, as the labour required per unit of output is also assumed to fall,  $(p-w\xi_0^L)$  increases with the level of research - in other words, the  $GPV_1$  line shifts outwards to  $GPV_0$ . If for every level of  $I$ , the distance between the  $GPV$  and investment (including research) curves increases with research, then, the satisficer will find that  $\Pi^*$  can be achieved with a smaller investment in physical capital and the maximiser will find his total long-run profits have increased.

The exact distribution of a given research budget between decreasing  $\xi^K$  and  $\xi^L$  will depend on: the productivity of  $R \ \& \ D$  in these two directions; the relative costs of capital and labour; and the costs per unit of research effort. For any given research expenditure, the profit maximiser will choose the allocation between  $\xi^K$  and  $\xi^L$  that produces the greatest net present value. In addition, the profit maximiser will search for the level of research expenditure  $CR^*$  that gives the largest area possible between the  $GPV$  and  $p_{K^I} + CR$  curves.

#### A Mathematical Exposition For The Profit Maximiser

The present value equation appears as,

2. The  $CR$  curve has to be constructed as  $\frac{CR}{I}$  and total research costs are again the sum of the area under the  $CR$  line but above the  $p_{K^I_0}$  line. In this case, the  $CR$  line will differ between a satisficer and a maximiser as their levels of investment are different.

$$PV = \int_{t=0}^T \frac{(p-w\xi_o^L)Y_o}{(1+i)^t} dt - p_K Y_o \xi_o^K - c_1 Y_o \xi_o^R(1) - c_2 Y_o \xi_o^R(2) \dots (4.26)$$

Where  $c_1$  and  $c_2$  denote the costs per unit of research into capital and labour productivities respectively. The variables  $\xi_o^R(1)$  and  $\xi_o^R(2)$  are defined as,

$$\xi_o^R(1) = \frac{R(1)}{Y_o} = \frac{1}{Y_o} \left[ a \left( \frac{1}{\xi_o^K} \right) \right] \dots (4.27)$$

and

$$\xi_o^R(2) = \frac{R(2)}{Y_o} = \frac{1}{Y_o} \left[ b \left( \frac{1}{\xi_o^L} \right) \right] \dots (4.28)$$

It follows directly from equations (4.26), (4.27) and (4.28) that,

$$\frac{\partial(PV)}{\partial \xi_o^K} = -p_K Y_o + \frac{c_1 a}{(\xi_o^K)^2} = 0 \dots (4.29)$$

$$\frac{\partial(PV)}{\partial \xi_o^L} = -w Y_o + \frac{c_2 b}{(\xi_o^L)^2} = 0 \dots (4.30)$$

$$\frac{\partial(PV)}{\partial Y_o} = p-w\xi_o^L - p_K \xi_o^K - c_1 \xi_o^R(1) - c_2 \xi_o^R(2) = 0 \dots (4.31)$$

where,  $P = \int \frac{P}{(1+i)^t} dt$  and  $W = \int \frac{W}{(1+i)^t} dt$ . There are five equations,

(4.27) - (4.31), in five unknowns, and solutions can be found for

$\xi_o^K$ ,  $\xi_o^L$ ,  $Y_o$ ,  $\xi_o^R(1)$  and  $\xi_o^R(2)$ .

All that is needed here is to note that, from equations, (4.29) and (4.30),

$$\frac{\xi_o^L}{\xi_o^K} = \left( \frac{p_K c_2 b}{w c_1 a} \right)^{\frac{1}{2}} \dots (4.32)$$



which indicates that the ratio of capital to labour on the new vintage will be a function of the relative factor prices, the relative costs of the two areas of research and the relative productivities of research inputs in these two areas. The final first order condition, equation (4.31), states that the profit maximising firm should expand its output from the latest vintage until the net present value of the combined R & D and investments in physical capital fall to zero at the intensive margin.

#### A Link With Hedonic Theory

There is an obvious link here with the hedonic theories, where the price of a particular article (in this case a capital good) is a function of the associated bundle of technical characteristics. As the research costs are spread over the investment in this up-dated vintage, it is as if the price of this vintage has been raised. New technical characteristics are included in the machine such that the resulting reduction in costs (increase in profits) from an additional unit of a particular technical characteristic is equal to the marginal cost of that characteristic - see Kravis and Lipsey (1971).

#### Extending the Model

The model of the defensive or satisficing firm is consistent with the theory developed in Chapter (8). Nevertheless, the model was able to say something about the behaviour of the profit maximiser. In addition, it would be possible to include product as well as process innovations in a theory of this kind. Product innovations would affect the price that the firm could charge, but also the volume of sales and thereby the level and structure of the capital stock. This is obviously an important area for further research.

A. Introduction

Economists have become increasingly aware that problems of aggregation can result in the failure of aggregate production functions to reflect the underlying technology of production. Such fears give micro studies much of their appeal. At a very low level of aggregation extraneous influences should be less important and estimates should directly reflect the micro technologies. Despite the potential insights about the technology of production that research at this level promises to yield, few studies of production have worked at such a detailed level. There are two main causes of this lack of interest: first there are severely restrictive data constraints at the micro level and, second, the good empirical performance of aggregate functions has detracted from the incentive to carry out more detailed studies.

Here we investigate the nature of the micro production relationships that form the building blocks from which more aggregate vintage models are constructed. This chapter attempts to demonstrate that a putty-clay model of the traditional type can be derived directly from these building blocks. The emphasis is on deriving a function that can be estimated, and particular attention is paid to the nature of the function under different ex ante regimes and to the importance of capacity usage. This route to obtaining an aggregate function is followed because the UK data are inadequate to enable each micro function to be estimated individually. Given some ingenuity, a more aggregate putty-clay relationship can be estimated on the basis of published UK data, and the results of this provide insights about the underlying micro technologies.

B. Recent Empirical Work Relevant to Engineering

There are a number of micro studies relevant to the theme of this

chapter that are in general accord with the concept of an integrated production schema. A number of these are discussed in some detail by Johansen (1972, Chapter 8). This section very briefly reviews the much smaller number that draw their evidence from the engineering industries.

An important study by Kurz and Manne (1963) considered the substitution possibilities between various types of metal cutting machine tools, drawing on the extremely detailed data constructed by Markowitz and Rowe (1961). The observations recorded were of output and investment per head associated with each production unit capable of executing a certain group of tasks. Each observation corresponds in principle to an ex post micro function. Kurz and Manne claimed that a number of the possibilities would be used by firms only in an emergency, and, under normal circumstances would be uneconomic. For this reason they devised a system for deleting supposedly inefficient possibilities - omitting those for which investment per head was greater, but output was no higher, than for other units. This censoring rule has been severely criticised by Furubotn (1965), Lave (1966) and Johansen (1972, pp. 191-5). One major criticism was that a higher investment cost may reflect a more durable piece of equipment that, in comparison to its less durable counterparts, is incapable of a greater output in the short run, but is capable of a far greater output in the long run.

Johansen (1972, pp. 191 and 193) claims that the production function fitted to these remaining observations is close to reflecting the ex ante function, and that it would have been even closer had Kurz and Manne fitted a frontier function. However, in so far as the points reflect equipment already in use, the fitted relationship appears to be the best-practice function (or a 'dated' ex ante function) rather than a summary of the current ex ante choices. Information about the capacity

established within the firm, with technical characteristics  $(\xi^1, \xi^2)$ , ie.  $f(\xi^1, \xi^2)$ , is not given in the Kurze and Manne study, and as a result, the data are not sufficient to construct the short-run macro function.

Capital and labour usage were the subject matter of a study by Dudley et al. (1968). This study looked at the periods of idleness of both men and machines within different sections of UK plants (ie. different shops such as machine, press and assembly shops) in four UK engineering industries. The aim was to estimate the potential productivity gains from eliminating underutilisation of factors. Idleness of this type<sup>1</sup> is closely allied to Liebenstein's (1966, 1969) 'X - inefficiency'. Their estimates of the potential increases in productivity appeared to be based on the hypothesis of constant returns to working equipment and labour more intensely. The assumption of constant returns underlies the basic Johansen ex post micro function.

The works of Bell (1972) and Senker and Huggett (1973) were attempts to isolate the implications of technological change for the manpower requirements of the UK engineering industry. Their analyses were essentially qualitative, and they made little attempt to develop an integrated theoretical framework. However, their approach is broadly consistent with a vintage model. The main casual link that enables them to make predictions is shown in Diagram (2.5). Senker, et al. (1975, pp. 86-9) gave evidence that often one man operates a single piece of equipment or a set of installations which suggests that patterns of capital-labour matching may be isolated by observing particular processes in different firms. This approach is adopted in the empirical work reported here.

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1. Not caused by the technical nature of the process, but attributable to management or labour actions.

Senker, et al. (1975, p. 111) were, however, worried that "inter-process links" make it difficult to trace the implications for manpower throughout the firm. While these secondary effects will be more difficult to assess than the primary effects (ie. those associated with manning the new techniques), Senker, et al. (1975, p. 114) were also worried that protracted adjustment to new techniques where there is some choice in the way adjustment occurs may make it necessary to "Abandon...the assumption that there is a relatively fixed and technically determined relationship between a technique and its associated labour force". Rather than dismiss primary relationships out of hand, however, it seems sensible to test for their existence empirically where data exist.

#### C. Data Availability at the micro level

At the level of individual processes or tasks it is extremely difficult to obtain the compatible data essential in the estimation of production relationships. In order to emphasise the problems posed for this study by data constraints, consideration is given to each of the different kinds of information.

##### Output Data

One of the most important problems faced in an empirical study of the putty-clay model is associated with the lack of comprehensive output data at this level of detail. The measure of output required obviously depends on the particular process that we are looking at and, for a given process, we still have to decide what that measure might be. What, for example, is the best measure of output for welding - the number of welds made? Even in this simple case the measure may not be adequate: welds may differ in size and strength. There is, anyway, no published source that records the number of welds carried out by particular firms or industries.

There are two possible ways of avoiding the measurement of output problem. The first is to assume that the output of each process enters the final product in a fixed proportion,

$$Y_x = a_x Y \quad \dots (5.1)$$

where  $x$  denotes the  $x$ th process and  $a$  is a constant. Hence, if

$$Y_x = f_x(K_x, L_x) \quad \dots (5.2)$$

represents the production relationship relevant for the  $x$ th process, the production function which can be estimated is,

$$Y = \frac{1}{a_x} f_x(K_x, L_x) \quad \dots (5.3)$$

The second method is to assume that entrepreneurs adjust the ratio of factor inputs in a cost-minimising way. This transforms the production function, equation (5.2), into employment function form,

$$L_x = g_x(K_x, w_x, r_x) \quad \dots (5.4)$$

and output is made implicit. The theoretical analysis uses this last assumption in order to derive a functional form that can be estimated given current data supplies.

#### Capital Stock

Historically, many studies have found great difficulty in obtaining useful information about the stock of capital available for use in production. There are, however, quite detailed estimates available for the engineering industries' stocks of different types of machine tools in 1961, 1966 and 1971. The Census of Metal Working Machine Tools appears quinquennially and reports estimates of the numbers of machines, categorised by type, industry and age,<sup>2</sup> in the

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2. Other dimensions, such as country of origin, are available, but are not directly relevant to this study.

engineering industries. The importance of the data cannot be doubted. In a study that attempted to provide estimates of the stock of capital, Pyatt (1964b, pp. 24-5) stated that, "Thought of in physical terms, gross investment is simply the number of buildings, machinery and vehicles purchased in a year. Thus, ideally, data on physical gross investment should take the form of a list of assets. These lists, however, would have to be so numerous and so detailed that compiling them on a national scale would be impossible". The information about numbers of machines given in the Census of Metal Working Machine Tools is used in this chapter to consider directly the manning requirements (ie. men-machine ratios) for various tasks.

At this stage it is useful to analyse what information this measure of capital will reveal in a study of this kind. A strong correlation is expected between the number of turners and the number of lathes employed in various industries. The more limited are the activities of turners (ie. the less the turning skill is used as a substitute for other skills) the stronger will be the relationship between the number of men and machines. In testing for substitution between men and machines within the turning process, the regressions seek to isolate and explain variations in the number of machines per worker. This form of substitution can be contrasted with traditional studies where the value of capital is used (which reflects both the number and the value of machines). For this reason, we expect this study to indicate substantially more limited substitution of capital for labour.

There are other problems as well however. The number of machines per man will be influenced by the nature of the production process. One

important characteristic is whether the technology is 'one-off', 'batch' or a 'mass production' process, which will influence the number of machines per man and the variety of tasks performed by any given skill. Given further information, however, it may eventually be possible to pick up this influence by the use of dummies common to a particular technology. A further characteristic which may have an important bearing on the man: machine ratio is the nature and amount of shiftworking. This was considered sufficiently important to devote a separate section to it - see page 5.7.

The data are not comprehensive, and the coverage of capital types leaves much to be desired. Metal working machine tools (particularly cutting and forming) are considered in some detail, but other types of machinery are given little or no coverage (there is more detail in the more recent censuses though, partly because of the advent of new machine types). On the other hand, the list of industries covered enables a minimum of eighteen NLHs (or groups of NLHs) to be distinguished in all census years. By 1971, most of the relevant engineering industries were separately covered.

#### Labour Data

Since 1963 the Engineering Industry Training Board (EITB), in conjunction with the Department of Employment, has collected employment data categorised by skill, sex and industry, for the engineering industries. This NLH level data has been released by the EITB at a level of skill aggregation much higher than that at which it was collected. The Department of Employment on the other hand, has published in its Gazette a very detailed breakdown of skills, but only



for fairly broad industry groups (eight engineering groups). With the exception of one or two skill categories (for instance, welding and metal fabricating) the published information is not sufficiently detailed to be useful in a study of the kind undertaken in this chapter. The EITB was kind enough to release for this study information relating to the small number of skills that can be appropriately matched with machine types.

#### Shiftworking

One variable that has received little attention in the literature on production and employment functions is shiftworking. It is possible for a given piece of equipment to be worked at greatly different intensities depending on the shift system in operation. A given unit of capital may have an average working day of eight hours in a firm that does not use shiftworking or overtime, while in a firm operating a continuous three eight-hour shift system the same machine could be in use (excluding maintenance and repair) for 24 hours a day. This study uses data from the Ministry of Labour Gazette (1965) and the New Earnings Survey (1970).

A major reason why economists have tended to neglect this aspect of the technology of production is the lack of published information. Until recently, the only information was found in two spot observations, relating to 1954 and 1964, published in the Ministry of Labour Gazette (1954, 1965). This source gives detailed MLG information about the nature and prevalence of various shiftworking systems. The New Earnings Survey is the source of data for 1968 and later years. It does not record detailed information about the prevalence of various

shiftworking systems, but gives information about shiftworking premia payments. The Report of the National Board for Prices and Incomes (1970, vol.2, p.118) argues that shift premia data are deficient in so far as people working the normal shift of a shift system are not paid premia. The report notes that, "The effect of this wider definition is that in manufacturing, for example, 25% of adult manual workers are classified as shiftworkers though only 21% had shift premia".

#### Capital Usage

A study of a vintage formulation requires data about capital usage. This chapter concentrates on machine tools and, following Heathfield (1972a), electricity consumption data are assumed to reflect plant and machinery usage. The usage series was calculated from the fuel consumption data which is used in various other parts of the study and constructed as described in detail in Appendix (II). The method is the same as that adopted by Evans (1974), who attempted to estimate the degree of labour hoarding in the engineering industry. Time series regressions are estimated for each MLH, such that

$$E = a + bt + u$$

where  $E$  denotes electricity consumption,  $t$  is a time trend and  $u$  is an error term. The lines are then shifted parallel in an upwards direction (altering the value of  $a$  to  $a'$ ) to pass through the observed  $E_t$  with the largest residual. This revision ensures that  $u_t \leq 0$  for all  $t$ . The values  $\hat{E}_t$  are then calculated such that,

$$\hat{E} = a' + bt$$

and then the usage figures are given by,

$$U = \frac{E_t}{\hat{E}_t}$$

#### Wage Data

The most complicated of the functional forms require quite

detailed information about the price of labour over a very long period. Two alternative measures of the price of labour are wage rates and earnings. If entrepreneurs, when making decisions involving wage information, assume that employees work normal hours and undertake no shiftworking, then wage rates appear to be the appropriate price variables. However, Wabe (1974, p.28) has shown that over much of the post-war period average hours worked have consistently stayed above normal hours; and, in addition, the discussion of shiftworking indicated that it was an important and permanent aspect of the production technology. Hence, employers will generally build an element of overtime and shiftworking costs into their labour price calculations. Thus, average earnings appear to be a better indicator of labour costs than a notional wage rate.<sup>3</sup>

While increasingly detailed information is becoming available, it does not yet exist at a sufficiently high level of industry or skill disaggregation, or over a long enough period. The information required to complement the 1971 machine tool data relates to 'average' or 'typical' earnings in the three decades prior to 1971. The only published earnings data at an MLH level covering this period are found in the Census of Production. The censuses do not provide a continuous series and it was necessary to assume that the 1948, 1958 and 1968 earnings information were 'typical' of each of the three decades. The data used in this study relate to manual workers and exclude the white-collar occupations. The earnings data relates to all manual workers and not to particular skills such as turners, but the data has to serve for the purposes of this study.

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3 . Because of the lack of data, no attempt is made at estimating a 'user cost' of labour. Other labour costs paid by the employer (such as Selective Employment Tax and National Insurance Contributions) are ignored.

There are a number of problems with the data. One minor inconsistency is that the 1948 information refers to G.B., while the 1958 and 1968 data relate to the U.K. The most difficult problem is to reconcile the changes in industrial classification that have taken place between 1948 and 1968. The 1963 census reports the 1958 information on a basis broadly compatible with the 1968 MLHs, but the 1948 information is reported by more aggregate industry groups and the classification is much more difficult to reconcile with the classifications used in later years. By going to the individual census reports, however, information is available about specialisations within each industry group (see Table (6) of each report), and in many cases the subgroups reported match fairly closely the MLHs distinguished in later years. The data available in the 1948 census about subgroups related to total employment and the corresponding wage bill, and not just to manual workers. The manual employment and wage data for the 1948 MLHs were subdivided according to the ratio of workers in each subgroup to total employment, and the ratio of the wage bill of the subgroup to the total MLH wage bill. Those parts of, or whole, 1948 MLHs that were not allocated to 1968 MLHs were put into the appropriate 'not elsewhere specified' groups (339, 349, 369, 389 or 399). In most cases, the resulting levels of employment, wage bills and earnings appeared to be realistic, but the results are tentative and should be treated with caution.

#### Rental Data

The measurement of price per unit of capital raises problems even more difficult than those associated with the labour input. The notion of a rental is adopted as the indicator of price that a firm might use in assessing its investment strategy, and it is defined as the amount a firm would have to pay if it hired a unit of capital for a year. Current rental data could be collected, but this would prove

both difficult and time consuming. In addition, this study requires rental data over three decades. Only price information for machines is available over this period and this is far from ideal.<sup>4</sup>

The initial step in constructing rentals for turning machines was to calculate quality constant price indices over the period 1948-72. The approach is based on the hedonic procedure described in Bosworth (1976b), where prices are regressed on average weight,  $W$ , cumulative patenting activity,  $J$ , and a time trend,  $t$ , for four subcategories of turning machines as well as for the turning group as a whole.<sup>5</sup> The patent term was allowed to lag or lead the dependent variable as there was no a priori information about when an invention would be used relative to the date at which it was patented. The log-linear regression,

$$\log p = a + bt + c \log W + d \log J + u \quad \dots (5.5)$$

gives the most acceptable results and these are reported in Table (5.1).

The functions all have a high explanatory power and all the variables are significant at the 1% level. A note of caution must be sounded, however, because the DW statistic was significant at the 5% level in three of the five cases. In this respect the results were inferior to those reported in the other study - Bosworth (1976b). The main cause was probably the inability to distinguish the  $J$  variable appropriate to each machine type. Table (5.2) below reports the quality constant rates of inflation and compares them with the crude

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4. A 'user cost' measure (i.e. net of tax allowances, etc.) would again be most appropriate, but it did not appear to be possible to calculate a useful series at the level of aggregation used in this study.
  5. The average price and weight information was constructed from data given in the Annual Statement and the patenting variable was taken from the Annual Report of the Comptroller General and Abridgements of Specifications

Table (5.1)

## Hedonic Results for Turning Machines

Regression	Machine Type	Period of P	a	b	c	d	R <sup>2</sup>	F	D <sup>2</sup>
i	All Turning	t + 2	2.3900***	0.0331***	1.0427***	0.0920***	0.969	239.51***	2.0100
ii	Automatics	t + 1	4.6392***	0.0262***	0.9775***	0.2455***	0.996	1942.63***	1.7825
iii	Capstan and Turret	t + 1	5.4654***	0.0376***	0.6147***	0.1382	0.960	169.30***	1.0282 (***)
iv	Other Lathes	t + 1	4.3604***	0.0258*	0.6811***	0.2207*	0.965	196.54***	0.8668 (**) )
v	Numerically Controlled Lathes	-	6.4543***	0.0629	0.8753*	-	0.558	4.16	3.6291 (***)

The levels of statistical significance in this and in subsequent tables are defined by:  
 \*\* significant at 1% level; \* significant at 5% level; + significant at 10% level.

Table (5.2): Post-War Rates of Price Change for Turning Machines

Regression	Machine Type	Quality constant Rate of Inflation	Crude Rate of Price Change
i	All Turning	3.31	10.47
ii	Automatics	2.82	10.82
iii	Capstan and Turret	3.76	5.45
iv	Other Lathes	2.58	4.90
v	Numerically Controlled	6.29	8.34

rate of price change based on a simple exponential time trend.

Quality-constant prices for the five machine categories were evaluated about their geometric mean sample values for the post-war period and values for 1948, 1958 and 1968 were calculated for each of the machine types except the numerical control category, which is a phenomenon of the last decade, and for which only a 1968 price was calculated. The effect of holding quality constant is to raise prices in earlier years relative to later years. The price data were transformed into rentals by dividing them by the estimated life expectancies,  $T$ , of the various types of machines in the different industries. The  $T$  variable was constructed using information from the Census of Metal Working Machine Tools using the approach advocated by Bacon and Eltis (1974). Where centre lathes are studied in the empirical section, the rentals from the 'other lathes' category are used (because centre lathes are by far the largest component of this group). In the case where 'all lathes' are being studied, the rental is constructed as a weighted average of the four types of lathes, where

the weights applied are based on the proportions of the various types held by each MLH.

D. Ex Ante Choices and the Nature of the Ex Post Micro Function

The ex ante function plays a central role in determining the nature of the ex post micro functions. Imperfect knowledge of the ex ante relationship hinders the formulation of realistic ex post functions and, given the data constraints, only the simplest ex ante functions are adopted. By making a number of alternative assumptions about the nature of the ex ante function, the associated ex post micro functions can be derived. The particular cases of ex ante putty and clay are considered in detail below.

As data are available that should make a cross-sectional study feasible, the analysis is developed in terms of cross-sectional functional forms. It is argued that if we can observe a number of production units established at the same time (and hence on the basis of the same technology) but experiencing a variety of relative factor prices, then it should be possible to establish not only the ex post micro functions, but also the ex ante functions from which they were drawn. In the theoretical and empirical analysis that follows, we consider a single production process at a time. This enables us to simplify the notation by suppressing the  $x$  subscript (relating to the  $x$ th production process).

The Ex Ante Clay Case

In this case it is assumed that there is no choice of new technologies. There exists, at each point in time, a single new technology that is the most efficient over the whole range of conceivable factor price ratios. Assuming that there is a single labour skill and capital type used in each production process, the production function can be written,

$$Y_{jv} = \frac{1}{\xi_v^K} K_{jv} = \frac{1}{\xi_v^L} L_{jv} \quad \dots (5.6)$$



where  $j$  denotes the  $j$ th plant, ( $j = 1, \dots, m$ ) and  $v$  the  $v$ th vintage ( $v = 1, \dots, \tau$  - where  $\tau$  is the oldest vintage in use). If output data is not available it can be made implicit in the following way:

$$L_{jv} = \frac{\xi_v^L}{\xi_v^K} K_{jv} = \psi_v K_{jv} \quad \dots (5.7)$$

#### The Ex Ante Putty Case

Traditionally, economic theory suggests that a variety of ex ante choices are possible. The choice of technology by the  $j$ th firm is now determined by the shape of the ex ante function and, assuming cost minimising entrepreneurial behaviour, the actual or expected factor price ratio. We consider just two cases: the Cobb-Douglas function and the more general CES ex ante form.

i. The Cobb-Douglas Function. The ex ante function relating to one such process can then be written,

$$Y_{jv} = A_v K_{jv}^{\alpha} L_{jv}^{\beta} \quad \dots (5.8)$$

where  $A$  is the technical efficiency parameter and the powers  $\alpha$  and  $\beta$  are production coefficients (which reflect capital's and labour's share in output). The associated cost function is,

$$C_{jv} = r_{jv} K_{jv} + w_{jv} L_{jv} \quad \dots (5.9)$$

where  $w$  is the wage rate and  $r$  the rental, while  $C$  denotes the total costs of operating the production unit. If management attempts to minimise costs, the choices of new technologies will be determined by considering the Lagrangian

$$LG = r_{jv} K_{jv} + w_{jv} L_{jv} + \lambda (Y_{jv} - A_v K_{jv}^{\alpha} L_{jv}^{\beta}) \quad \dots (5.10)$$

where  $\lambda$  is the Lagrangian multiplier. From the marginal conditions,

cost-minimising behaviour implies that:

$$L_{jv} = \frac{\beta_v}{\alpha_v} \frac{r_{jv}}{w_{jv}} K_{jv} \quad \dots (5.11)$$

ii. The General CES Function. If the Cobb-Douglas function is replaced by the more general CES form, equation (5.10) becomes:

$$LG = \xi_v (a_v K_{jv}^{-\theta_v} + b_v L_{jv}^{-\theta_v})^{-\frac{1}{\theta_v}} + \lambda (C_{jv}^{-w_{jv}} L_{jv}^{-r_{jv}} K_{jv}^{-r_{jv}}) \quad \dots (5.12)$$

The marginal conditions<sup>6</sup> can be written as the 'side relation'

$$L_{jv} = \left( \frac{b}{a_v} \right)^{\sigma_v} \left( \frac{r_{jv}}{w_{jv}} \right)^{\sigma_v} K_{jv} \quad \dots (5.13)$$

where the elasticity of substitution,  $\sigma_v = \frac{1}{1+\theta_v}$ <sup>7</sup>

#### E. Regression Equations

As rewarding as the linear programming formulation was proved to be for understanding the production problem, there is little hope of obtaining sufficient empirical information to enable a solution to be derived. There is no possibility of setting up a linear programme unless information becomes available about (a) the amounts of labour employed on capital of various types and ages and, (b) about the associated outputs from these production units. This section reports on an attempt to adapt the short-run macro function to a form that can be estimated using regression techniques. The most useful starting point appears to be the constraints of equation (2.3). If the production units could be observed directly, the equation might be modified to:

$$Q^i = \sum_{jv} \xi_{jv}^i \bar{Y}_{jv} U_{jv} = \sum_{jv} \psi_{jv}^i \bar{K}_{jv} U_{jv} \quad \dots (5.14)$$

6. The Nadiri and Rosen (1968) study makes use of cost minimising behaviour to yield a similar functional form, although at a more aggregate level.

7. See Allen (1968, p.53)

where  $\psi^i = \frac{\xi^i}{\xi}$ ,  $U$  is a measure of capacity utilisation and  $Q^i$  is the

quantity of the  $i$ th input used in the industry. Constant returns to the utilisation of existing capacity is assumed, otherwise  $\xi = \xi(U)$  and the equation is severely complicated. This assumption also enables the analysis to switch directly between capacity, capital and labour utilisation given knowledge of  $U$ .

#### Clay Ex Ante Regime

The case of a clay-clay technology is most straightforward. It is assumed that at any given time the various plants in the industry have only a single choice of new technology which is given exogenously. Such a technology must improve on technologies currently embodied in the stock of inputs held by the plants or it would not be adopted. In this case,

$$\psi_{1v} = \psi_{2v} = \dots = \psi_{mv} = \psi_v \quad \dots (5.15)$$

and equation (5.14) can now be written,

$$L = \sum_v \psi_v K_v \quad \dots (5.16)$$

Here  $K$  denotes the input of capital services, which are defined as

$$K_v = \bar{K}_v U_v = \sum_j \bar{K}_{jv} U_{jv} \quad \dots (5.17)$$

and  $L$  denotes the input of labour services which are defined as

$$L = \sum_v L_v = \sum_v \bar{L}_v U_v = \sum_j \sum_v \bar{L}_{jv} U_{jv} \quad \dots (5.18)$$

#### Putty Ex Ante Regime

The more general case is where the ex ante function is characterised by the possibility of substitution between inputs. To obtain the equivalent employment function it is necessary to specify the form of the ex ante function. The Cobb-Douglas function and the more general CES form are separately distinguished below.

i. The Cobb-Douglas Function. Using the 'side relation'<sup>8</sup>, denoted by equation (5.11) above, knowledge of  $w$  and  $r$  enables a weighted capital variable,  $K'_{jv}$ , to be constructed.

$$K'_{jv} = \frac{r_{jv}}{w_{jv}} K_{jv} \quad \dots (5.19)$$

Hence, the employment relationship for the industry can be written,

$$L = \sum_j \sum_v \psi_v K'_{jv} = \sum_v \psi_v K'_v \quad \dots (5.20)$$

where, in this case,  $\psi_v = B_v/\alpha_v$ .

ii. The General CES Function. From the 'side relation' of the more general CES function, shown as equation (5.13) above, a weighted capital variable,

$$K'_{jv} = \left( \frac{r_{jv}}{w_{jv}} \right)^{\sigma_v} K_{jv} \quad \dots (5.21)$$

can be constructed where the elasticity of substitution,  $\sigma_v$ , is known.

Given  $\sigma_v$ , the employment relationship can be written,

$$L = \sum_j \sum_v \psi_v K'_{jv} = \sum_v \psi_v K'_v \quad \dots (5.22)$$

where, in this case,  $\psi_v = \left( b_v/\alpha_v \right)^{\sigma_v}$

#### Aggregation to MLRs

Equations (5.16), (5.20) and (5.22) form the basis for functions which can be estimated cross-sectionally. The constant  $\psi$  is given different

8. See Brown (1966, p.129), Arrow, et al. (1961) and Griliches and Ringstad (1971)

9. In the past vintage theory has often assumed that there is no discrimination in wage payments to workers on different vintages of capital  $w_{j1}^i = w_{j2}^i = \dots = w_{j\tau}^i = w_j^i$ . But the recent disputes about wage payments for manning 'jumbo jets', Concorde and 'advanced passenger trains' are calling this assumption into question.

interpretations according to the nature of the underlying ex ante regime, but in all cases the assumptions made were sufficient to enable summation across firms to take place and thereby to collapse  $\psi_{jv}$  to  $\psi_v$ . However, in so far as the relationship holds across a number of fairly detailed industry groups (for example, MLHs), we may aggregate over firms in each industry and obtain

$$L_j = \sum_v \psi_v K_{jv} \quad \dots (5.23)$$

where  $j$  now refers to the  $j$ th MLH.  $K$  can simply be the total number of machines that complement the skill in question, or the number weighted by the factor price ratio.

#### The Impact of Disembodied Technical Change

The impact of simple disembodied technical change is to modify the coefficients  $\xi^K$  and  $\xi^L$  at the rates  $\mu$  and  $\eta$  % per period. An additional term,  $Ae^{(\nu-\eta)t}$  appears in the labour requirements equation which at any given time appears as a scalar  $B$ . The labour requirements function is now jointly multiplicative and additive of the form,

$$L_j = \sum_v (B\psi_v) K_{jv} = \sum_v \psi_v K_{jv} \quad \dots (5.24)$$

Linear regression techniques allow  $\psi_v$  to be estimated to within a scalar while their relative values are not distorted. Alternatively, different values for  $B$  can be tried, thereby estimating

$$L_j = \sum_v \psi_v (B K_{jv}) \quad \dots (5.25)$$

In the more complicated case where,

$$L_j = \sum_v \psi_v (B_v K_{jv}) \quad \dots (5.26)$$

the separation of  $\psi_v$  and  $B_v$  is even more difficult, but it would permit a

check to be made whether  $B_1 > \dots > B_T$ ,  $B_1 = \dots = B_T$  or  $B_1 < \dots < B_T$ .

#### The Role of Capital Usage

A further problem concerns capital usage. Available stock figures relate to  $\bar{K}$  and not  $K$ . Under the assumption of constant returns to usage<sup>10</sup> we can write

$$K_{jv} = \bar{K}_{jv} U_{jv} \quad \dots (5.27)$$

and

$$L_j = \sum_v \psi_v \bar{K}_{jv} U_{jv} \quad \dots (5.28)$$

The role played by usage and the assumptions we make about the way in which capital is laid up and the returns to usage are crucial if meaningful estimates are to be obtained. If constant returns to scale do not characterise the micro production processes, then, for example,  $\psi_{jv} = \psi(\bar{Y}_{jv} - Y_{jv})$  and  $\psi_{jv}$  is no longer a constant.

Vintage theory suggests that  $U_{jv} = 1$  when the quasi-rent arising from that production unit is non-negative, but that  $U_{jv} = 0$  when it is negative.

The case where inefficient behaviour occurs (ie. where certain less efficient production units continue in operation) severely complicates matters. There is little or no data about  $U_{jv}$  and this may prove of immense importance in trying to derive meaningful estimates. Indeed, even the data about industries ( $U_j = \sum_v U_{jv}$ ) is greatly inadequate.

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10.  $Y_{jv} = \frac{1}{\xi_{jv}^K} (U_{jv} \bar{K}_{jv}) = \frac{1}{\xi_{jv}^L} (U_{jv} \bar{L}_{jv})$

F. Welding: A Pilot Study with Some Lessons

The labour requirements of an individual process have been investigated in a paper by the present author, see Bosworth (1974a, pp. 177-88). Welding and metal fabrication was chosen as the subject of the pilot study because it appeared to be a well defined industrial activity the technology of which would not differ greatly across industries within engineering. In addition, it was a statistically well-documented process for which it was felt that labour skills and capital types could be matched with reasonable accuracy. The functions estimated in the pilot study were simplistic, but the results indicated that this approach could provide an interesting and potentially rewarding link with the Bell (1972) and Senker and Huggett (1973) studies.

The functions were based on a simple clay-clay model (although they were also consistent with a putty-clay world where all industries experienced similar histories of wages and rentals). On the assumption that there was no difference between vintages, the function collapsed to the form which was estimated across industries,

$$L = \rho_0 + \rho_1 K + u \quad \dots (5.29)$$

An attempt was made to take account of the shiftworking problem. It was argued that at a given time an employee will work a single shift, but a unit of capital may be operated on more than one shift. The simplest approach was to adjust the total labour force,  $L$ , to labour employed on the main shift,  $L'$ , and to match this variable against the stock of equipment. This is appropriate as long as all capital is operated on the main shift and there are equal numbers of workers employed on each shift. The new variable was constructed in the following way:

$$L' = L \left( 1 - \frac{1}{2} \frac{L_2}{L} - \frac{2}{3} \frac{L_3}{L} \right) \quad \dots (5.30)$$

where  $L_2$  and  $L_3$  denote labour employed on two- and three-shift systems respectively. The adjustment involves subtracting one half of labour

employed on two-shift and two-thirds of labour employed on three-shift systems.

The results of estimating these two functions for 1966 are reported in Table (5.3) below. The results indicated that, as extra machines are installed, the number of welders required to man them increases by a slightly greater amount.

Table (5.3) Cross-Sectional Welding Results: A Simple Leontief Model

Functional Form	$\rho_0$	$\rho_1$	$\bar{R}^2$	F
i $L = \rho_0 + \rho_1 K$	349.53	1.1430**	0.83	93.96**
ii $L = \rho_1 K$	-	1.0723**	-	-
iii $L' = \rho_0 + \rho_1 K$	248.78	1.0702**	0.84	99.17**
iv $L' = \rho_1 K$	-	1.0911**	-	-

None of the  $\rho$  coefficients is very different from unity and the difference from one-to-one manning is slightly less where shiftworking has been taken into account. Both of the functions that include constant terms have positive intercept values, but, on the basis of estimated t values, in neither case are these significantly different from zero at the 10% level of significance or higher. The adjustment for shiftworking increased the overall fit of the regression very slightly (on the basis of  $\bar{R}^2$  and F), but did not greatly affect the size or significance of the estimated coefficients. The effect of this adjustment was similar in the case of the more complex functional forms which are not reported here.

It became apparent when using the shiftworking data that the adjustment was only very crude. First, the shift-systems were not as straightforward as the calculations imply. The adjustment assumed



continuous two-or three-shift systems, but in practice other types of shiftworking also existed (for instance, a four-crew three-shift system). Second, it was assumed that all equipment (that forms the stock of capital) is operated on the main (normal) shift. Third, the shiftworking data from the Ministry of Labour Gazette referred to manual employees and did not distinguish the nature and prevalence of systems worked by particular occupations. Finally, the survey of shiftworking related to two spot dates, 1954 and 1964, and it was the 1964 information that was used to adjust the 1966 data. This gap of two years may have led to some inaccuracies. It was not possible to interpolate and extrapolate on the basis of the 1954 and 1964 information, because of the intervening (1958) changes in industrial classification.

More complicated functional forms were tested using the 1966 data. In particular, the average age of welding machines,  $V$ , was included as an explanatory variable.

$$L = \rho_0 + \rho_1 K + \rho_2 V + u \quad \dots (5.31)$$

where  $V$  was constructed as a weighted average (formed by weighting the mid-points of the age intervals distinguished in the Census data by the numbers of machines in each interval). Bacon and Eltis (1974), using the same information, have since shown a more appropriate method of constructing  $V$ . A less ad hoc and, from the point of view of economic theory, more easily justified function from economic theory which was estimated, distinguished stocks of capital of various ages directly,

$$L(j) = \rho_0 + \rho_1 K_0 + \rho_2 K_1 + \rho_3 K_2 + u \quad \dots (5.32)$$

where  $K_0$ ,  $K_1$  and  $K_2$  refer to less than 10, 10-20 and greater than 20 years.

The results of estimating both of these functional forms, however, were disappointing. The ad hoc addition of the average age variable failed to improve the fit of the regressions (based on  $\bar{R}^2$  and  $F$  values) and the age variable was not significant. The addition of this variable did

not, however, greatly affect the size or significance of the coefficient on capital. The subdivision of the aggregate capital variable to distinguish a number of vintages was only slightly more successful. The overall fit of the function, measured by  $\bar{R}^2$ , rose approximately 0.83 to 0.94, but the estimated coefficients on capital were not meaningful. The inability to obtain significant and meaningful coefficients was partly the result of extreme multicollinearity between stocks of capital of various vintages. In retrospect we might have expected this in a process as fundamental as welding. Industries which employed a large amount of welding equipment in the past will tend to do so in the present unless anything unusual has happened to the timing of investment or a rapid switch away from welding has occurred in the technology of production. The effect of multicollinearity is to make the estimated coefficients individually unreliable and this makes it impossible to say anything about the nature of embodied technical change. In addition, the problem is singularly difficult to shake off.

The final regressions reported in the welding study distinguished different types of welding machines rather than different vintages of equipment. The approach appeared to be a way of taking technical change into account where it manifested itself in machines of new types (rather than different vintages of existing types). It was felt a priori that multicollinearity might prove less of a problem, but this was not the case. Although the reliability of the estimated coefficients was no greater, the overall fit of the regression was higher than for any of the other forms previously tested.

#### G. Turning Results

Table (5.4) reports the results of estimating equation (5.29) for turning on the basis of the 1966, 1971 and 1966/71 pooled data. Regressions (1)

show the employment of turners as a function of the stocks of lathes in engineering. The slope coefficient is considerably larger in 1971 than in 1966, but both are significant at the 1% level. The overall fit of the function is better in 1971 than in 1966, but the  $F$  statistics are significant at the 1% level in both cases.

Table (5.4) Cross-Sectional Turning Results: A Simple Leontief Model

Regression	$\rho_0$	$\rho_1$	$\bar{R}^2$	$F$	
1966	i	906.740	0.1896**	0.447	15.54**
	ii	1832.550	0.4399**	0.522	19.60**
	iii	259.180	0.7181**	0.545	22.57**
1971	i	-314.906	0.4160**	0.741	75.49**
	ii	-239.208	0.9627**	0.789	98.37**
	iii	-427.153	0.9933**	0.726	70.05**
1966/71	i	571.019	0.2313**	0.532	52.13**
	ii	1623.860†	0.5228**	0.573	60.13**
	iii	- 47.099	0.8103**	0.640	80.88**

The results for this simple function are not as good as in the welding study. One explanation is that the match of labour skill with capital type is less appropriate. Two modifications to the function are tried. First, 'other skilled operatives' are added to 'turners', but while 'turners' tends to understate the number of workers manning lathes, the combined categories form an over-estimate (because some proportion of 'other skilled operatives' man machines other than lathes). The results of modifying the dependent variable in this way are reported as

Table (5.5) Cross-Sectional Turning Results: Some Additional Variables.

Regression	$\rho_0$	$\rho_1$	$\rho_2$	$\rho_3$	$\rho_4$	$\bar{R}^2$	F	
1966	i	-5490.15	0.2641**	-172.371*	478.901	2144.00 <sup>†</sup>	0.661	9.79**
	ii	3070.63	0.5402**	-248.798 <sup>†</sup>	-172.202	6052.70*	0.698	10.83**
	iii	2965.46	0.8047**	-32.491	188.062	1624.97	0.652	8.48**
1971	i	-768.97	0.4025**	9.9471	-28.9872	1689.69**	0.805	27.82**
	ii	-2198.39	0.9406**	23.1182	17.1522	3224.64**	0.831	32.93**
	iii	-2914.92	1.0092**	64.2558*	15.1708	1823.57**	0.824	31.45**
1966/71	i	468.64	0.2317**	-31.5911	-13.0247	2018.39**	0.620	19.35**
	ii	3077.07	0.5205**	-26.7095	-216.630	4543.28**	0.657	22.03**
	iii	-2033.52	0.8617**	16.6544	59.7586	1789.08**	0.758	34.69**

Levels of statistical significance are defined in Table (5.1)

regressions (ii) in Table (5.4). The adjustment raises the value of the slope coefficient and the overall fit of the function is improved in both periods. Second, an attempt is made to specify more closely the type of machine that a turner might man. Turners have traditionally worked on centre lathes and the results of modifying the independent variable in this way are reported as regressions (iii) in Table (5.4). The slope coefficient is higher still. While the overall fit of the function is higher than the alternative formulations in 1966, in 1971 it is inferior to equation (ii).

One further modification is to adjust the capital variable to account for the degree of utilisation (lathes that are not in use will not be manned). The adjustment of  $K$  by an

electricity usage term,  $U$ , does not greatly affect the results. The slope coefficients increase slightly and the overall fit improves marginally, but the changes are not fundamental enough to justify duplicating the results.

The other variables introduced into the function in an ad hoc manner did not have a marked effect on the results. Table(5.5) reports estimates of the following function,

$$L = \rho_0 + \rho_1 K + \rho_2 S + \rho_3 V + \rho_4 D_0 + u \quad \dots (5.33)$$

where  $K$  and  $L$  take the alternative forms suggested above,  $S$  is the percentage of employees working shifts,  $V$  is the average age of capital, and  $D_0$  an industry dummy.  $D_0$  was included because there was a tendency for the residuals to differ between NLHs 331-49 ( $D_0 = 1$ ) and 351-99 ( $D_0 = 0$ ).

The results show that the coefficient on the number of machines,  $\rho_1$ , is again significant at the 1% level.  $D_0$  is fairly consistently different from the overall constant and indicates the need, in the future, to study industry sub-samples. For 1971 shiftworking has the expected positive sign (because the more shifts a given piece of capital

Table (5.6) Cross-Sectional Turning Results: Distinguishing Vintages.

Regression	$\rho_0$	$\rho_1$	$\rho_2$	$\rho_3$	$\rho_4$	$\rho_5$	$\bar{R}^2$	F
1966								
i	987.261	-0.1940	0.3605	0.8002	-157.159*	214.580 <sup>†</sup>	0.620	6.88**
ii	750.276	0.3202	0.9044	0.4056	-263.742 <sup>†</sup>	6071.43*	0.674	8.04**
iii	-497.863	-1.6181	1.670	2.3766*	-11.8406	1855.41	0.635	7.27**
1971								
i	-1418.11 <sup>†</sup>	0.0808	1.4239**	-0.2710	16.5295	1501.23**	0.853	31.11**
ii	-2674.25	-0.7728	3.1646**	0.5884	37.2664	3182.10**	0.860	33.07**
iii	-1865.35	1.9377**	-0.8434	2.1749*	39.7632	1546.90**	0.847	29.82**

Levels of statistical significance are defined in Table (5.1)

works, the more men will be needed to man it) and is significant in regression (iii), but it has a negative sign in 1966. The lack of data about various types of shiftworking and the inability to account for shiftworking carried out by various skills are particularly worrying deficiencies of  $S$ . The insignificant contribution made by  $V$  is not too surprising given the problems associated with its measurement and the ad hoc nature of equation (5.33). Alternatively,  $\rho_3 = 0$  can be interpreted as Hicks neutral embodied technical change ( $\mu = \eta$ ) in a clay-clay world. The overall fit of the model (shown by  $\bar{R}^2$  and  $F$ ) has improved considerably in both periods with the addition of these variables.

Two further modifications to the function are not reported in detail. The adjustment of the capital variable by  $U$  to account for variations in capital usage had little effect on the results, and where  $U$  was included as an additional explanatory variable it was not statistically significant.

The generalisation of equation (5.33), where particular vintages are distinguished, because of the way in which the data is available, appears as

$$L = \rho_0 + \rho_1 K_0 + \rho_2 K_1 + \rho_3 K_2 + \rho_4 S + \rho_5 D_0 + u \quad \dots (5.34)$$

Where  $K_0$  relates to capital of age 0-10 years,  $K_1$  to capital of 10-20 years and  $K_2$  to capital of age 20 or more years. The results of estimating this function for the three alternative pairings of  $K$  and  $L$  are reported in Table (5.6). The pooled data are no longer relevant because the vintages 0, 1 and 2 (i.e. the 0-10, 10-20 and 20+ categories) are not the same for both 1966 and 1971.

The new results gave a slightly lower overall explanatory power for 1966 and a slightly higher one for 1971 than did the model reported in Table (5.5). The coefficients  $\rho_1$ ,  $\rho_2$  and  $\rho_3$  are unstable, only occasionally significant and impossible to interpret in a meaningful way. The reason is not hard to find. Table (5.7) shows that  $K_0$ ,  $K_1$  and  $K_2$

are highly multicollinear. The extreme multicollinearity does not affect the combined contributions of  $K_v$  to the overall explanation of  $L$ .

Equations (5.33) and (5.34) are broadly similar (although equation (5.33) is an ad hoc formulation), and so too are their overall explanatory powers.

Table (5.7) Zero Order Correlation Coefficients for  $K_0$ ,  $K_1$  and  $K_2$  in 1971

	All Lathes		Centre Lathes	
	$K_1$	$K_2$	$K_1$	$K_2$
$K_0$	0.970	0.888	0.942	0.856
$K_1$		0.885		0.890

Both models explain variations in  $L$  better than equation (5.29) does.

Table (5.8) regressions I (i) - (iii), reports the results of estimating,

$$L = \rho_0 + \rho_1 \left(\frac{r}{w}\right)_0^\sigma K_0 + \rho_2 \left(\frac{r}{w}\right)_1^\sigma K_1 + \rho_3 \left(\frac{r}{w}\right)_2^\sigma K_2 + \rho_4 S + \rho_5 D_0 + u \dots (5.35)$$

where  $\sigma = 1.0$ , which is the case of the Cobb-Douglas ex ante function. The results are very similar to those reported in Table (5.6). If anything, they are slightly worse in terms of overall fit - whereas regression (i) is almost identical, regressions (ii) and (iii) are slightly inferior.

Regressions II (i) - (iii) report

$$L = \rho_0 + \rho_1 \left(\frac{r}{w}\right)_0^\sigma K_0 + \rho_2 \left(\frac{r}{w}\right)_1^\sigma K_1 + \rho_3 \left(\frac{r}{w}\right)_2^\sigma (K_2 U) + \rho_4 S + \rho_5 D_0 + u \dots (5.36)$$

for  $\sigma = 1$ , where the only capital laid up is from the oldest vintage. The variable  $(K_2 U)$  is calculated as  $K_2 - (K - KU)$ . Adjustment for the degree of utilisation does not spread to other vintages because  $K_2 > (K - KU)$  in every case. This adjustment corresponds to the traditional vintage model (in a world of monotonically changing factor prices) where the oldest capital is the least efficient (and the least economically viable) and



Table (5.8) Cross-Sectional Turning Results: Cobb-Douglas Ex Ante Functions.

Regression	$\rho_0$	$\rho_1$	$\rho_2$	$\rho_3$	$\rho_4$	$\rho_5$	$\bar{R}^2$	F	
I	i	-1039.85	0.6417	9.5384*	1.7241	4.0341	1460.14**	0.855	31.59**
	ii	-211.06	-10.4802	23.5907*	15.3789*	15.0954	3207.81**	0.852	30.97**
	iii	-1742.73	38.7419	3.3689	6.6921 <sup>†</sup>	50.4571	1614.09*	0.721	14.46**
II	i	-946.71	1.4466	9.7810*	0.5171	2.6032	1400.59**	0.853	31.09**
	ii	-1425.65	-7.9962	25.7778**	12.5797 <sup>†</sup>	0.6864	2921.89**	0.840	28.32**
	iii	-1632.07	39.2272	2.0356	10.5856*	47.3425	1571.84**	0.726	14.77**

Levels of statistical significance are given in Table (5.1)

hence the first to be laid up. The adjustment makes almost no difference to the results.

Given multicollinearity between  $K_v$ , the consistent insignificant contribution of  $S$  and the generally significant contribution of  $D_o$ , the only basis of comparison between regressions where  $\sigma$  changes are  $\bar{R}^2$  and  $F$ . Table (5.9) shows the path of  $\bar{R}^2$  as  $\sigma$  changes from 0.0 to 1.2 in steps of 0.1. The first striking feature is that the  $\bar{R}^2$  are all of roughly the same size. If it were necessary to choose between them, however, the Cobb-Douglas ex ante function would not be selected.

Table (5.9)  $\bar{R}^2$  Coefficients for different  $\sigma$

$\sigma$	$\bar{R}^2$		
	Regression (i)	(ii)	(iii)
0.0	0.853	0.861	0.847
0.1	0.858	0.865	0.874
0.2	0.863	0.868	0.903
0.3	0.866	0.870	0.915
0.4	0.868	0.871	0.897
0.5	0.868	0.871	0.863
0.6	0.868	0.870	0.827
0.7	0.866	0.867	0.780
0.8	0.863	0.863	0.768
0.9	0.859	0.858	0.743
1.0	0.855	0.852	0.721
1.1	0.848	0.845	0.701
1.2	0.842	0.836	0.681

The underlying ex ante function appears to have an elasticity of substitution in the region of 0.3 to 0.6. Although the results are extremely tentative, they indicate that the elasticity of substitution, is, if anything, slightly lower when a more tightly defined machine type (ie. centre lathes) is considered. The difference in  $\bar{R}^2$  as  $\sigma$  varies is also more distinct in the case of centre lathes, ranging from 0.681 (when a CES function with  $\sigma = 1.2$  is assumed) to 0.915 (when  $\sigma = 0.3$ ).

#### H. Conclusions

A function based on putty-clay theory is found to provide a reasonable explanation of the numbers of turners employed by engineering industries. An important conclusion is that the ex ante functions, from which new technologies are chosen, have an elasticity less than unity. Elasticities in the range 0.3 to 0.6 were suggested by the results. Mayor (1969) and Johansen (1972) have pointed to the fact that time-series analyses have tended to produce elasticity estimates clustering around 0.5, whereas cross-sectional studies have produced estimates close to unity. The results contained here bear on the question of "which of these estimates, if any, approximate the actual elasticities" although they relate to the ex ante function and not to an 'average' production relationship. A more tentative conclusion is that where a number of different types of lathes are included - ie. regressions (i) and (ii) - the elasticity is slightly higher than where a specific lathe category is considered. Ex ante substitution appears to arise more through the choice between different types of machines than through adaptations of machines drawn from a particular category.

An important deficiency of the results is that, because of extreme multicollinearity between  $K_0$ ,  $K_1$  and  $K_2$ , it is not possible to

isolate the role played by any particular vintage (summarised by  $\hat{\psi}_0$ ,  $\hat{\psi}_1$  and  $\hat{\psi}_2$ ). In order to learn more about the nature and rate of embodied technological change, the problem of multicollinearity must be broken in future research. Another task for which further theoretical and empirical research is required is that of isolating and testing the significance of disembodied technical changes.

Work at micro level may prove to be particularly rewarding for economists intent on gaining insights into the nature of the technology of production. The principal barrier is the almost complete lack of detailed and accurate information, which is essential to a study of this kind.

## Chapter (6) Aggregate Putty-Clay Functions.

### A. Introduction

The empirical tests of the vintage models at a micro level are severely hampered by data constraints. At a more aggregate level, where particular industrial processes cannot be so easily distinguished, more comprehensive data exists. This information is still not ideal and there remain a number of important problems to resolve. The putty-clay function developed in Chapter (5) is now modified in order to make it as appropriate as possible given the labour and investment data which are available. Particular labour skills and capital types are still distinguished, but the analysis is much more aggregate, being limited by the nature of the investment data available.

### B. Other Author's Work.

There is no great tradition of using investment data in empirical research on production or employment functions. Even authors who are fervent believers in the vintage approach - eg. Johansen (1961) and Pyatt (1963) - have tended to search for ways of formulating their models in terms of more traditional (neoclassical) functional forms. Very few authors appear to have used investment data as an explanatory variable. An important reason for this lack of interest is that investment data, in the form which it is usually published, is deficient for use in testing theoretically rigorous vintage models. Information is available about both the intensive and extensive margins, but the definitions result in data that are far from ideal in a vintage context. In a world that fails to comply with the simple vintage models based on efficient management behaviour, the extremely detailed information that is required about the degree to which the various vintages are

used is not available.

Only occasionally, when specific pieces of information become available, are the problems lessened. Fair (1971), for example, was able to obtain information for the US cement and steel industries about capacity output levels which could be matched with the capacity labour data he was able to construct. In this way, he was able to avoid the problem of separating the short-run determinants of capacity utilisation from the long-run determinants of capital-labour substitution and the demand for capacity. The availability of 'capacity' data enables intermediate vintages to be ignored. On the assumption that only the latest vintage can be purchased at time  $t$ , the Fair study formulated an employment function in first difference terms,

$$\Delta \bar{L}_t = \phi(\Delta Y_t, IS_t) \quad \dots (6.1)$$

Where  $\Delta$  denotes first differences,  $\bar{L}$  signifies full capacity values and  $IS$  denotes the volume of scrapping. A slight revision of the function allows net investment ( $IN$ ) to appear in place of  $\Delta Y$ , but Fair found equation (6.1) better suited to the available data. In practice, an alternative formulation of the model

$$\Delta \bar{L}_t = \phi(\Delta Y_t, \frac{W}{r}) \quad \dots (6.2)$$

is adopted in order to test for substitution between capital and labour. This function is not quite so rigorously linked with an underlying vintage model.

Heathfield (1972b) has developed a vintage model that was tested using time series information about six UK manufacturing industries, one of which was the engineering sector. Given the assumptions of the model, theoretical rigour prevails until,

$$\left(\frac{K}{L}\right)_t = \rho_0 + \rho_1 \sum_{v=1948}^t \frac{IG_v}{K_t} - \rho_2 \sum_{v=1948}^t \frac{IS_v}{K_t} + u_t \quad \dots (6.3)$$

is obtained, but

$$\left(\frac{K}{L}\right)_t = \rho_0 + \rho_1 \sum \frac{IG_v}{K_t} + \rho_2 t + \rho_3 U_t + u_t \quad \dots (6.4)$$

is actually estimated. Here *IG* and *IS* denote gross investment and scrapping respectively, *U* is an electricity based measure of capital usage, *t* a time trend and *u* the error term. Stringent assumptions underlie both models, but the function actually estimated is only loosely connected with a vintage theory.

The final model reviewed here was presented by Lindley (1975) in an attempt to investigate the demand determinants of apprentice recruitment in the UK engineering industry. The intention was to gain an insight about whether apprentice recruitment was determined by the current production situation or whether the number of recruits was influenced by an investment motive. In much the same way as in the Heathfield (1972) model, the relationship between capital and labour is assumed to be of a fixed coefficient kind,<sup>1</sup> but substitution between apprentices and craftsmen (of the traditional log-linear form) is possible. The explanation of apprentice recruitment is formulated initially - see Lindley (1975, p.5) - as a log-linear function in terms of output, relative factor prices, utilisation and lagged apprentice recruitment. An alternative but equivalent formulation that uses capital in place of output - see Lindley (1975, p.16) - is found to be equally useful,

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1. Both Heathfield (1972) and Lindley (1975) adopt the aggregate production functions by analogy with micro theory. Lindley (1975, p.6) points out that the conditions for rigorous aggregation across firms are invariably not met. The Johansen (1972) work emphasises, however, that aggregate functions need not have the same form as the underlying micro technologies and that often the micro-macro analogy will be misleading.

$$L_{1t} = \phi \left( K_t, \frac{w_{1t}}{w_{2t}}, U_t, L_{1t-1} \right) \quad \dots (6.5)$$

where subscripts 1 and 2 refer to apprentices and craftsmen respectively. Towards the end of the paper, however, Lindley (1975, p.19) adopts an ad hoc function,

$$L_{1t} = \phi \left( IG_t, \frac{w_{1t}}{w_{2t}}, U_t, L_{1t-1} \right) \quad \dots (6.6)$$

on the basis that "compared to current output ( $Y_t$ ), gross fixed capital formation deflated by the price of capital goods ( $IG_t$ ) would better reflect the composite pressure of current production and planned production over, say, the next three years, upon the decision to recruit apprentices ( $L_{1t}$ )". The coefficient on  $IG$  is significant at the 1% level and the explanatory power of this function is greater than its more rigorous predecessors. In so far as  $L_1$  and  $IG$  represent activity at the intensive margin, the results can be interpreted as giving some support to the vintage approach of the kind undertaken in this chapter, (but this is not the only possible interpretation).

#### C A Vintage Formulation

The work undertaken in Chapter 5 suggested that a function such as,

$$L = \sum_v \psi_v \bar{K}_v U_v \quad \dots (6.7)$$

could form the basis of a regression equation. The lack of data about the stock of capital in the UK engineering industries suggests that it may be more useful to reformulate the equation in first difference terms and make use of the more detailed investment data that are available. It is assumed that a set of vintages  $\{0, \dots, v-1\}$  are available at time  $t+1$  and  $\{1, \dots, v\}$  at time  $t$ . If a new vintage is introduced in each succeeding period, equation (6.7) can be written,



$$\Delta L_{t+1} = \bar{\Psi}_0 \bar{K}_{0,t+1} U_{0,t+1} + \bar{\Psi}_1 \bar{K}_{1,t+1} \Delta U_{1,t+1} + \dots + \bar{\Psi}_{\tau-1} \bar{K}_{\tau-1,t+1} \Delta U_{\tau-1,t+1} - \bar{\Psi}_{\tau} \bar{K}_{\tau,t} U_{\tau,t} \quad \dots (6.8)$$

where  $\Delta$  denotes first differences, calculated between  $t$  and  $t+1$ .

A number of important assumptions have been made, but a fairly realistic looking equation has evolved. This equation suggests that the change in labour requirements between periods  $t$  and  $t+1$ , is essentially a reflection of:

- a) the coefficient ratios at the intensive and extensive margins;
- b) investment behaviour at the intensive and extensive margins;
- c) the degree of capital usage at the two margins in the appropriate periods:  $U_{t+1}$  and  $U_t$ ;
- d) the change in the degree to which intermediate vintages were utilised between  $t$  and  $t+1$ .

The crucial role played by capital usage in the putty-clay model is now clear. Without detailed data about individual vintages over time, however, there is little chance of estimating the functions given in equations (6.7) and (6.8) as they stand. In fact, we have to at least make some assumptions about the manner in which capital is utilised.

#### Capital Usage

In Johansen's (1972, pp. 13-19) model, the short-run macro function is described as a linear programming problem, the dual of which yields the quasi-rents of the production units. Johansen's model, in common with most putty-clay models, takes the zero quasi-rent as forming the boundary between the utilised and non-utilised production units: those units with positive quasi-rents are utilised and those with negative quasi-rents are laid-up. There are, in most industries, costs and uncertainties associated with the laying-up of certain types of capital

and also with their re-instatement after a lay-up, but these costs may be much more important in the case of tankers<sup>2</sup> than machine tools and they are ignored here.

As capital usage is a key factor in the short-run macro model, it is worth devoting some time to it. The approach reported here begins with the generally adopted economic concept of utilisation, from which more restrictive cases can be derived. The following notation is adopted:  $v = 1, \dots, \tau_t^+$  denote the vintages at time  $t$  which earn a positive quasi-rent;  $v = \tau_t^-, \dots, \tau$  are the vintages which have negative quasi-rents. Hence in general, we have:

$$U_{vt} = 1 \quad \text{for all } v = 1, \dots, \tau_t^+$$

and

$$U_{vt} = 0 \quad \text{for all } v = \tau_t^-, \dots, \tau$$

it is worth emphasising that  $\tau$  may refer to a different vintage in different periods, (i.e. when  $\tau_t^+$  and  $\tau_{t+1}^+$  appear as subscripts, they may denote different vintages of capital).

In practice it is important to assume that the productive importance of vintages with a zero quasi-rent is very small. The relevance of this assumption becomes obvious when it is remembered that such vintages are of indeterminate importance - some members of the vintage may be used and others may not. The number of these units utilised in the industry is known, but the distribution of this productive effort between plants in the industry is indeterminate.

The case being considered is most easily seen when we write out the demand equations for two periods in full:

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2. See Johansen (1972, Chapter 9)

$$L_t = \psi_1 K_1 + \dots + \psi_{\tau_t}^+ \bar{K}_{\tau_t}^+ \quad \dots (6.9)$$

and

$$L_{t+1} = \psi_0 \bar{K}_0 + \dots + \psi_{\tau_{t+1}}^+ \bar{K}_{\tau_{t+1}}^+ \quad \dots (6.10)$$

These equations are based on the usual vintage concept that, in slack periods, the oldest capital is most likely to be laid up. Although, under conditions where the wage/rental ratio is not a monotonically increasing function, this is by no means certain. It is useful to point out that in the equation for period  $t$  the unused vintages, ( $v = \tau_t^- \dots \tau$ ), where positive excess capacity exists, will generally include the vintage about to be scrapped. However, not all vintages which remain unused in this period will be scrapped. So long as their discounted streams of expected future earnings exceed the similarly discounted streams of the costs of operating them, laying them up and re-introducing them, they will be laid-up rather than scrapped.

Equations (6.9) and (6.10) can be written in the difference form used previously:

$$\begin{aligned} \Delta L_{t+1} = \psi_0 \bar{K}_0 - \psi_{\tau} K_{\tau} - (\psi_{\tau_{t+1}}^- \bar{K}_{\tau_{t+1}}^- + \dots + \psi_{\tau-1} \bar{K}_{\tau-1} \\ - \psi_{\tau_t}^- \bar{K}_{\tau_t}^- \dots - \psi_{\tau} \bar{K}_{\tau}) \quad \dots (6.11) \end{aligned}$$

Where the terms in brackets summarise the vintages not in use in the two periods. Equation (6.11) can now be written,

$$\Delta L_{t+1} = \psi_0 \bar{K}_0 - \psi_{\tau} \bar{K}_{\tau} - (\bar{\psi}_{t+1} \sum_{v=\tau_{t+1}}^{\tau-1} \bar{K}_v - \bar{\psi}_t \sum_{v=\tau_t}^{\tau} \bar{K}_v) \dots (6.12)$$

or, defining the summation variables from equation (6.12) as unused capital capacity,  $U^k$ , we have:

$$\Delta L_{t+1} = \psi_0 \bar{K}_0 - \psi_T \bar{K}_T - (\bar{\psi}_{t+1} U_{t+1}^k - \bar{\psi}_t U_t^k) \quad \dots (6.13)$$

under conditions where the technical coefficients are fairly constant,  $\bar{\psi}_{t+1} = \bar{\psi}_t$ , and the final term of equation (6.13) appears as  $\bar{\psi} (\Delta U_{t+1}^k)$ . If the set of utilised vintages formed the whole capital stock in both periods, then the terms which appear in brackets on the right hand side of equation (6.13) are zero. Thus (6.13) reduces to:

$$\Delta L_{t+1} = \psi_0 K_0 - \psi_T K_T \quad \dots (6.14)$$

which is an interesting special case.

#### Short-Run Adjustment

Unfortunately, if  $t$  and  $t+1$  refer to consecutive quarters or even years, estimates of the equilibrium values of the parameters ( $\psi_v$ ) are likely to be distorted by the incomplete adjustment of  $L$  to  $K$ . Although it should be possible to obtain estimates of the adjustment process and the long-run equilibrium values by adding an adjustment mechanism and pooling the time series and cross-sectional dimensions of the data, this study attempts the simpler procedure of reducing the relative importance of the adjustment process by lengthening the gap between  $t$  and  $t+1$  (to say three, five or even seven years). The problem is that, the longer the gap between  $t$  and  $t+1$ , the less homogenous any particular  $K_v$  becomes (where  $K_0 = \sum_{v=t}^{t+1} IG_v$  and  $K_T = \sum_{v=t}^{t+1} IS_v$ ) and  $\psi_v$  appear more as averages than coefficients associated with distinct vintages. It was decided to try a five year gap between  $t$  and  $t+1$ . As labour data were based on mid-year estimates of employment and investment was taken over a calendar year,  $I_{1967}$ , for example, was matched with  $\Delta L_{63,68}$  which, in effect allowed a six month lag between investment and meeting manning targets. A time series of parameter estimates was again obtained by removing the

first year of a sample and adding on the next year available (i.e.

$\Delta L_{63,68}$  ,  $\Delta L_{64,69}$  , .....  $\Delta L_{67,72}$  are matched with  $\begin{matrix} 67 & 68 & 71 \\ E & I, & E & I, & \dots & E & I \\ 63 & 64 & & & & 67 \end{matrix}$

respectively).

#### D Data Availability

UK investment data is far from ideal for estimating even the simplest vintage function. Only three types of investment goods are distinguished (plant and machinery, vehicles and buildings) as well as total investment. Intuition suggested that it might be appropriate to match 'all workers' with total investment, 'craftsmen' with plant and machinery and 'drivers' with vehicles investment. The labour data is again taken from the L7A survey carried out by the Engineering Industry Training Board in conjunction with the Department of Employment (and is recorded in thousands of employees) while the investment data is based on the annual 'sample' and quinquennial 'full' censuses (and is recorded in £ millions).

Data about acquisitions and disposals is available for plant and machinery and for vehicles, but not for buildings until more recent years. MLH data is not published except in quinquennial census years and in the annual censuses from 1970 onwards. Information from the sample censuses can be traced back to 1948 for seven engineering industries which correspond broadly to the 1968 SIC groups. Appendix ( I ) describes how the sample and full censuses can be combined to produce MLH detail annually from 1958 by type of investment good. Although the method of construction leaves much to be desired, the resulting MLH data should adequately reflect the levels of investment undertaken.

The published investment data is in current prices. Ideally,

however, the vintage model requires information about the volume of investment and an attempt to adjust the data for changes in prices must be made. The only reliable price information which is available is the implicit deflator used in the Blue Book. Even this information is inadequate: where constant and current prices are both distinguished (enabling the implicit deflator to be calculated) there is a choice of detail between industry sub-groups or types of investment goods, but not both. This study chooses to use a different deflator for each type of investment good, but whose value is common to all of the engineering industries. There are obvious problems and sources of potential error in not distinguishing a separate index for each industry and type of investment good. This study is forced to adjust acquisitions and disposals by the same index of prices, (disposals being valued in current prices not in original book values), although the index for second-hand machines may differ from that for new equipment, and this may be a further source of measurement error.

The final problem is perhaps even more crucial. Acquisitions (purchases) and disposals (sales) do not correspond to the theoretical concepts of gross investment and scrapping. Acquisitions may include purchases of machines that are not of the latest vintage and disposals may not all be of the vintage being scrapped<sup>3</sup>. (This problem was first raised in Chapter 3). It is probably true to say, however, that acquisitions will tend to be associated with more modern vintages of machines and disposals with older vintages.

Capital usage data based on the consumption of electricity was reported briefly in the previous chapters. An attempt is made here to

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3. It is likely that the machines with positive second-hand value will not have reached the end of their economic working lives. While a machine earning a zero quasi-rent in one industry may command a positive second-hand value (e.g. in an industry with lower labour costs) this seems unlikely to be common because the higher gross profitability of the machine in the new industry must cover the costs of transferring the machine between firms.

match different types of fuel with different types of capital. Usage figures for 'all investment' are based on percentage deviations from linear trends<sup>4</sup> (one for each NLH) estimated from data on total fuel consumption. Usage data for 'plant and machinery' and for 'vehicles' are based on the consumption of 'electricity' and 'all liquid fuels' respectively. The construction of the fuel consumption data used in estimating usage series is described in Appendix ( II ). The method of constructing the usage series has been described in greater detail by Evans (1974).

### E The Results

The results obtained when estimating the simplest function,

$$\Delta L = p_0 + p_1 \Sigma IN + u \quad \dots (6.15)$$

are reported in Tables (6.1), (6.2) and (6.3) for all workers, craftsmen and drivers respectively. The results are not very impressive. The coefficient on net investment is generally insignificantly different from zero and tends to be unstable over time. In addition, the overall fit of the regressions (measured by  $\bar{R}^2$  and F) is not particularly good.

The results for all labour are the most promising. The coefficient on the investment term declines over the period (with the exception of one particularly large jump, which is difficult to explain) and is significant in five out of seven cases (although one of these, 1965-9, has an unexpected sign), and the F statistics are also significant at the 5% level or better in the five cases. The results indicate that an extra £1m of net investment had the effect of creating an extra 173 jobs in engineering in the period 1961-5. This already fairly small impact of investment on overall employment in engineering became negligible by the end of the period.

The results for craftsmen and for drivers are even less impressive.

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4. Linear trends were found to fit at least as well as their exponential counterparts.

Table (6.1) All Labour and Total Net Investment.

Year	$\rho_0$	$\rho_1$	$\bar{R}^2$	F
1961-5	-4.0520	0.1727***	0.32	15.04***
1962-6	-3.2678	0.1086**	0.24	10.67***
1963-7	-1.7296	0.0836*	0.16	6.62***
1964-8	-2.5096	0.0758*	0.13	5.55*
1965-9	20.4484*	-0.4701***	0.61	48.17***
1966-70	-7.7658 <sup>†</sup>	0.0488	0.03	1.84
1970-71	-14.2254 <sup>†</sup>	0.0229	-	0.16

Table (6.2) Craftsmen and Investment in Plant and Machinery.

Year	$\rho_0$	$\rho_1$	$\bar{R}^2$	F
1963-7	2.3210	0.0432	0.02	1.51
1964-8	-0.3692	0.0127	-	0.30
1965-9	-0.2940	0.0144	-	0.33
1966-70	-3.5816	0.0222	-	0.86
1970-71	-7.5991*	0.0673***	0.20	7.62***

Table (6.3) Drivers and Vehicles Investment.

Year	$\rho_0$	$\rho_1$	$\bar{R}^2$	F
1963-7	0.1220	0.1411***	0.32	15.39***
1964-8	0.0759	0.0104	-	0.27
1965-9	0.0149	0.0472***	0.45	25.56***
1966-70	0.0413	0.0288	0.03	1.79
1970-71	0.0347	0.0130	-	0.68

Levels of statistical significance are defined in Table (5.1)



The overall fit of the regression is extremely poor and the slope coefficients are generally insignificant, which makes them extremely difficult to interpret. The direct impact of £1m net investment (i.e. on the labour skill directly associated with that capital type) is to create a small number of jobs in both cases. £1m net investment in plant and machinery creates an extra 43 craftsmen jobs in the 1963-7 period, but 87 jobs in 1967-71. The same additional investment in vehicles produces jobs for 141 drivers in 1963-67, but smaller numbers towards the end of the period.

The inclusion of an extra term for changes in capital usage (based on total fuel for all investment, electricity for plant and machinery and liquid fuels for vehicles investment) in the form,

$$\Delta L = \rho_0 + \rho_1 \Delta IIN + \rho_2 \Delta U + u \quad \dots (6.16)$$

makes little difference to the results overall.  $\Delta U$  is insignificant for every period for craftsmen and drivers, but slightly better for all labour. The results for the aggregate labour group are reported in Table (6.4) below, but even in this case  $\Delta U$  is significant at the 10% level or better in only two cases.

Table (6.4) All Labour, Accounting for Changes in Capital Usage

Year	$\rho_0$	$\rho_1$	$\rho_2$	$\bar{R}^2$	F
1961-5	-3.9380	0.1733**	-0.0562	0.295	7.29**
1962-6	-3.8352	0.1101**	0.2094	0.226	5.37
1963-7	-2.4606	0.0859*	0.2740	0.197	3.43
1964-8	-5.4055	0.0751*	0.5012	0.184	4.36
1965-9	14.7005	-0.4552**	1.2975+	0.636	27.23**
1966-70	-5.2783	0.0499	0.5957+	0.107	2.79
1970-1	-14.5657	0.0251	-0.0553	-	0.10

Tables (6.5) and (6.6) report the results relating to craftsmen and drivers for the slightly more complicated case where net investment is divided into its two component parts: gross investment (IG) and replacement investment (IS). The functional form estimated is,

$$\Delta L = \rho_0 + \rho_1 \text{IG} + \rho_2 \text{IS} + u \quad \dots (6.17)$$

where  $L$  relates to craftsmen and  $I$  is plant and machinery investment or where drivers are matched with vehicles. Separating the component parts of investment improves the results. Only the 1966-70 and 1967-71 craftsmen results are acceptable both in terms of their overall fit and the significance of coefficients, but the 1964-8 and 1965-9 coefficients are also accepted at the 10% level as being significantly different from zero. The coefficients for all of the periods except 1963-7 have the expected sign. The drivers results, on the other hand, give significant  $F$  statistics in four out of the five cases, but the slope coefficients are only really acceptable in 1967-71. The slope coefficients have unexpectedly negative signs on IG and positive signs on IS because of the extremely strong multicollinearity between IG and IS which characterises vehicle investment. Table (6.7) shows the extent of the multicollinearity problem by giving the zero order correlation coefficients for the two cases. The extreme multicollinearity, particularly in the case of vehicles, is shown by zero-order correlation coefficients in excess of 0.99.

The craftsmen regressions appear to give rise to the most reliable estimates of the coefficients. With the exception of 1963-7 they indicate that a given value of investment (IG) creates only one job for approximately every twenty that an equal value of scrapping (IS) loses. In 1966-70, for example, an additional £1m of acquisitions creates 98 jobs, while the same value of scrapping releases 2,549 employees. Multicollinearity may be playing a part in making the estimated coefficients, and hence their relative values, unreliable. Part of the explanation of the

Table (6.5) Craftsmen - Acquisitions and Disposals of Plant and Machinery.

Year	$\rho_0$	$\rho_1$	$\rho_2$	$\bar{R}^2$	F
1963-67	0.6576	0.0118	0.9250	0.016	1.25
1964-68	1.6550	0.0613 <sup>†</sup>	-1.3585 <sup>†</sup>	0.041	1.64
1965-69	2.1543	0.0605 <sup>†</sup>	-1.4221 <sup>†</sup>	0.057	1.91
1966-70	1.4904	0.0978**	-2.5488**	0.305	7.15**
1967-71	-0.0441	0.2208**	-4.1882**	0.534	16.48**

Table(6.6) Drivers - Acquisitions and Disposals of Vehicles.

Year	$\rho_0$	$\rho_1$	$\rho_2$	$\bar{R}^2$	F
1963-7	-0.0607	-0.2156	0.8201	0.357	9.33**
1964-8	0.1100	-0.1472	0.4120	-	0.94
1965-9	0.0186	-0.0600	0.2476	0.501	16.09**
1966-70	-0.0364	-0.1947	0.6632 <sup>†</sup>	0.332	6.96**
1976-71	0.1484	-0.2953**	0.8046**	0.264	5.38**

Table (6.7) Zero-Order Correlation Coefficients -  $r_{IS,IG}$ .

Year	Investment Type Plant and Machinery	Vehicles
1963-7	0.694	0.995
1964-8	0.800	0.995
1965-9	0.738	0.995
1966-70	0.743	0.992
1967-71	0.795	0.996

Levels of statistical significance are defined in Table (5.1)

small 'creation-release' ratio is that, because older machines tend to command a lower price - particularly where transport costs of transferring second-hand machines have to be met out of future earnings, there are more machines scrapped per film than can be purchased for film. The implication is that even if the price index adopted here has adequately coped with translating variations in values to equivalent variations in volume for both new and second-hand machines, it has failed to make the magnitudes of new to second-hand reflect their relative volumes.

#### F Conclusions

This chapter attempts to develop a labour demand equation relevant to the UK engineering industry that can be estimated using the rather limited data available for UK industries. It cannot be denied that some of the assumptions that it proved necessary to make in order to isolate this function were stringent. Nevertheless, the final functional forms - shown above as equations (6.13) and (6.14) - did appear to be both intuitively plausible and empirically testable. The results reported do not provide overwhelming evidence in support of the vintage hypothesis. On the other hand, despite the existence of important theoretical and empirical problems, the results indicate that further research along these lines may well prove rewarding.

The models appear to provide a basis for making forecasts of labour requirements. Given an appropriate pool of data the cross-sectional analysis can be repeated for a number of years. With a sufficiently long time series of estimates of the technology coefficients, explanatory variables might be found which could be used to predict future values of the coefficients. These could then be used in a simulation exercise (with calculated alternative time paths of output, capacity utilisation and investment) to produce conditional forecasts of future manpower requirements. Such an approach would avoid Blaug's (1967, 1970) criticism

of single valued forecasts.

If the approach is to eventually provide models that are accurate descriptions of past changes in employment and useful in forecasting exercises, a number of important data problems must be resolved. Firstly, price indices are needed that relate to different industries, types of investment and distinguish new from second-hand investment goods. Secondly, the data on acquisitions and disposals must be modified. Information is really required separately about purchases of second-hand equipment. Data on disposals should similarly distinguish between sales and scrapping of capital. Thirdly, information about original values as well as second-hand value should be distinguished where they differ. This is particularly important in the case of capital being scrapped because this will generally have no second-hand value. Fourthly, it would be extremely helpful to know the ages of machines being traded or scrapped.

The method of constructing MLH investment data for use in this study was far from ideal. Since 1970, however, information has become available at the MLH level (and even for certain MLH sub-groups). There is some evidence from Tables (6.5) and (6.6) that the results for later years are better than for earlier years, but obviously the improved investment data may not be the only contributory factor.

The MLH fuel data, on which the capital usage series were based, were also based on inadequate data. In this case, the published information does not become more detailed in more recent years. Estimates of capital usage for different types of capital could be made much more accurate given more detail in the fuel consumption series. One problem which must be faced squarely, however, is that usage data is unlikely ever to distinguish the intensity with which particular vintages are worked (except perhaps at a very micro-level). Given the key role which capital usage plays in a vintage model this may prove a crucial deficiency and the inclusion of such data will generally be associated with a lack of theoretical rigour.

There are a number of other improvements that can be made which were not attempted in this chapter. The problems of matching labour skills with machine types was not discussed in detail and in practice requires more experimentation<sup>5</sup>. A related problem is to attempt to trace the secondary employment effects of various investment decisions (i.e. through skill groups not directly linked with that investment good). There are a number of other explanatory variables that have not yet been tested in this chapter (e.g. shiftworking) and, in addition, some simple tests of inter-industry differences (e.g. between SICs) could have been made. Finally, by pooling the time-series and cross-sectional dimensions of the data (or simply by concentrating on the time series dimension), estimates of the short-run adjustment might have been made. This would involve some assumption about the nature of embodied technical change (e.g. exponential), but would have meant that  $\hat{\psi}_0$  and  $\hat{\psi}_t$  were not average values over a five year period. In addition, the larger data base might have reduced the degree of multicollinearity between IG and IS.

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5. See the tests carried out in Bosworth (1976a.).

A. Introduction

The evidence contained in the previous two chapters gave some indication of the existence of direct and fixed links between physical capital and labour employed, at the micro level. Moving up the scale of aggregation, to a level which is commonly employed in other studies, involves a loss of theoretical rigour. In an attempt to retain a measure of the physical stock of capital employed in production, a fuel consumption proxy for the consumption of capital services is adopted in preference to a 'book value of capital' measure. The adoption of a fuel consumption proxy for capital and the extremely aggregate nature of the study imply that the results will throw little if any direct light on the vintage hypothesis.

This chapter therefore reports evidence about the empirical performance of neoclassical production functions, when estimated using data drawn from the UK engineering industry.<sup>1</sup> The functions are aggregate in the sense that they relate to all production processes carried out within an industry: particular capital types and labour skills are not distinguished. This cross-NLH study, however, is concerned with a more homogeneous sample of industries than those studied by other authors - see, for example, Feldstein (1967). Thus, despite the rather aggregate nature of the estimates, certain features of the technology may still be apparent. Sawyer (1971), for example, has argued that the UK industrial groupings used in the compilation of official statistics are based more on the technology of production than any considerations of market structure.

If the neoclassical formulation is accepted as realistic then repeated cross-sectional estimation for a number of consecutive years should enable changes in output caused by a shift in the production function to be isolated from those arising from movements around a given function. Nelson (1973) has argued that it is a major criticism of growth accounting exercises that they have been forced to assume the form of the production function in order to separate changes in output resulting from changes in factor supplies from those caused by changes in technical efficiency. Estimating production functions at different points in time gives rise to a time series of technology parameters, with

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1. Some provisional estimates for this industry were reported in Bosworth (1974, pp. 153-96), but the data has been improved since this earlier study.

the possibility of explaining their past movements and projecting them into the future. If this could be achieved successfully, it could have important implications for planning and forecasting exercises.

The main thrust of the exercise undertaken in this chapter, however, is not an attempt to work within the confines of a neoclassical world, but to provide some evidence of the limitations of the approach. First, the apparently good empirical performance of the neoclassical formulations is shown to be misleading. The performance of the models appears much more suspect where labour productivity (and not the level of output) appears as the dependent variable. The traditional neoclassical variables are not dismissed as irrelevant, but it is argued here, and in the next two chapters, that they are only a part of a much more complex description of the real world. Second, in the chapter which follows, it is argued that fuel consumption is acting as a proxy for the book value of the capital stock. It is then argued, in some detail, that the good fit of the neoclassical functions has more to do with the firm's attempts to balance their books than the underlying technology of production. In this sense it is possible for the vintage and neoclassical (as if) theories to co-exist, as long as it is realised that the former relates to the technology of production and the latter to managerial behaviour.

#### B. Aggregate Production Functions

Neoclassical theoretical and empirical research have become particularly interested in the CES class of functions. The Cobb-Douglas function, which is a special case of the CES class, has obvious practical advantages in estimation because its log-linear form lends itself readily to simple linear regression techniques. The general CES function,

$$Y = \zeta \left[ aK^{-\theta} + bL^{-\theta} \right]^{-1/\theta} \quad \dots (7.1)$$

has combined multiplicative and additive terms that make it extremely difficult to estimate by a simple regression techniques. In equation (7.1),  $\zeta$  is a technical efficiency parameter,  $a$  and  $b$  denote relative factor shares, and  $\theta$  is the return to scale parameter.

The pioneering study by Arrow, *et al.* (1961) estimated the



'side relation',

$$\log \left( \frac{Y}{L} \right) = c + \sigma \log w \quad \dots (7.2)$$

where  $c$  is a constant,  $\sigma$  the elasticity of substitution and  $w$  the wage rate. This function can be obtained by manipulating the production relationship under the assumption of cost minimising behaviour. A major advantage of estimating this functional form is that the elasticity of substitution enters as a first order parameter and there is more chance of estimating it with some precision. However, Brown (1966, p.128-33) has argued that not all of the parameters of the production function appear in the equation, but has suggested that the remaining parameters can be estimated by adopting a stepwise procedure. This involves first estimating the 'side relation' in terms of the equation for the expansion path.

$$\log \left( \frac{K}{L} \right) = d - \sigma \log \left( \frac{r}{w} \right) \quad \text{where } d = \frac{\sigma b}{a} \quad \dots (7.3)$$

Using the results from this equation,  $\sigma$  and  $\frac{b}{a}$  can be estimated. On the assumption that  $b = 1-a$ , and knowing that  $\sigma = \frac{1}{1+\beta}$ , there is sufficient information to construct a new variable,

$$(\hat{KL}) = (aK^{-\beta} + bL^{-\beta}) \quad \dots (7.4)$$

and equation (7.1) can be rewritten as:

$$\log Y = \log \zeta - \frac{\theta}{\beta} \log (\hat{KL}) \quad \dots (7.5)$$

and this enables  $\zeta$  and  $\theta$  to be estimated.

Of course, if we are willing to adopt an iterative procedure (as opposed to a two-stage procedure), it is possible to estimate functions that are more general than CES. Brown (1966, p.135) reports some work by Hilhorst who suggested,

$$Y = \zeta \left[ aK^{\beta} + bL^{\beta} \right]^{1/\beta} \quad \dots (7.6)$$

as a more general formulation in the sense that: it is non-homogeneous where  $\theta_K \neq \theta_L$ ; it is homogeneous of degree  $\frac{\theta}{\theta}$  where  $\theta_K = \theta_L = \theta$ ; and it is linear homogeneous with  $\sigma = \frac{1}{1+\theta}$  where  $\theta_K = \theta_L = \theta$ . The function is estimated by an iterative procedure centering on  $\hat{\theta}_K$  and  $\hat{\theta}_L$ , which Hilhorst found to converge to constant values very rapidly.

The essential problem with the two-stage and the iterative approaches is that they rely on two theories: a theory of the firm and a theory of the technology of production. In Chapter (5) the limitations of the data made it impossible to avoid making an assumption about managerial behaviour, but no such limitations constrain the study at this level of aggregation and other methods of estimating neoclassical functions do exist. A secondary problem is that the introduction of a theory of the firm implies the need for data about factor prices that are often not available at a detailed level. The Hilhorst study is slightly easier in this respect because it only requires information about relative factor shares.

Direct estimation of the technical relationship requires the acceptance of one theory instead of two. The function can be estimated directly by expanding equation (7.1) using Taylor's theorem. The idea was put forward by Brown (1966, pp. 133-4), but his estimates were based on an iterative procedure where each iteration involved a least squares estimate. An advantage of this approach is that, if the iterations produce estimates which converge, the parameter estimates and their standard errors are those of the non-linear equation. Kmenta (1967), however, has produced a version where repeated estimation is avoided. Equation (7.1) is rewritten,

$$\frac{Y}{L} = \zeta L^{\theta-1} \left[ a \left( \frac{K}{L} \right)^{-\theta} + b \right]^{-\theta/\theta} \quad \dots (7.7)$$

and, expansion around  $\theta = 0$ , using Taylor's theorem yields,

$$\log \left( \frac{Y}{L} \right) = a_0 + a_1 \log L + a_2 \log \left( \frac{K}{L} \right) + a_3 \left[ \log \left( \frac{K}{L} \right) \right]^2 \quad \dots (7.8)$$

where:  $a_0 = \log \zeta$ ;  $a_1 = \theta - 1$ ;  $a_2 = \theta a$ ;  $a_3 = -\frac{1}{2} \theta a b \theta$ . The function collapses to the equivalent Cobb-Douglas form when  $a_3$  is insignificantly different from zero. Following Griliches and Ringstad (1971, pp. 7-10) this study adopts equation (7.8) as a direct test of the Cobb-Douglas function.

There are problems with the approach. Griliches and Ringstad (1971, pp.9-10), for example, point to the fact that the expansion was carried out around  $\theta = 0$  (ie.  $\sigma = 1$ ) and the approximation is better the closer the elasticity of substitution is to unity. Unfortunately, the further the elasticity is from unity, the more important the higher order terms (which have been omitted) become. In this case,  $a_3 \neq 0$  may imply production functions outside of the CES class. A second problem arises from the small size of  $a_3$ , which is formed as the product of at least two parameters that are less than unity. The implication is that we are likely to need large samples and an adequate dispersion of  $\left( \frac{K}{L} \right)$  to say anything about the sign and magnitude of  $a_3$ . Thirdly, the parameter estimates of  $a_1$  and  $a_2$  (and by implication  $a$ ,  $b$  and  $\sigma$ ) are not independent of the units by which we measure  $K$  and  $L$ . Griliches and Ringstad (1971, p.10) propose that we evaluate the elasticities at the (geometric) mean levels of the inputs and, in particular, at a level where the geometric means of the sample are equal -

$$\text{ie. } \bar{K} = \bar{L} \text{ and } \log \left( \frac{\bar{K}}{\bar{L}} \right) = \log (1) = 0$$

The data base for this study is relatively large, with information available about 32 MLHs in each year. In addition cross-sectional and time series dimensions of the data can be pooled for some or all of the twelve years for which data is available. The data should be more than adequate to obtain a precise estimate of the coefficient  $a_3$  if the higher order terms make a significant contribution to the explanation of variations in output per head. This study follows Griliches and Ringstad (1971) and interprets the elasticities at the mean levels of the sample. As the alternative techniques for estimating neoclassical functions are much more complicated and time consuming, it was decided to resort to them only if  $a_3$  was statistically significant.

If it had proved necessary to re-estimate the function there were two main alternatives. First, to adopt a non-linear estimation technique, for example, the one used by Leech (1975). The main problem is that it approaches the minimum sum of squared deviations by making use of information about the gradient of the initial curve fitted. With hill-climbing methods the standard statistical tests are not valid and the researcher is forced to use asymptotic tests (ie. those which are true when the sample size grows very large) that are not really applicable to small sample sizes. Second, a simple search procedure that uses alternative values of key parameters in the estimation of other parameters. From equation (7.1), for example, we might try alternative values of  $\beta$  and  $\theta$  in a two dimensional search for the minimum variance. There are two immediate problems with this approach: first, the relevance of searching for the minimum variance; second, the sheer size of the search raises important practical problems.<sup>2</sup>

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2. It is easy to illustrate this point. A search involving 11 values of  $\beta$  with only 3 of  $\theta$  involves 33 regressions for each sample. With 12 different samples, this is a total of 396 regressions. This method of "search over a grid" has been used by Griliches and Ringstad (1971, pp. 79-80).

C. The Basic Formulation, Explanatory Variables and Sources of Data

In the simplest case reported here estimates are obtained on the assumption that all of the MLHs have a common technology of production. The general form of the function can be written,

$$Y_{jt} = Y_t (K_{jt}, L_{jt}, H_{jt}) \quad \dots (7.9)$$

where  $Y$  denotes value added,  $K$  the input of capital services,  $L$  the number of employees,  $H$  the hours which employees work,  $j$  denotes the  $j$ th industry and  $t$  the  $t$ 'th time period. There are both theoretical and practical problems in obtaining meaningful measures of all of the variables which appear in the function.

The Output Variable

This study follows the convention of adopting a value added measure of output. The data is constructed from annual indices of industrial production for the period after 1958. The statistics for individual MLHs are intended to reflect the volume of industrial production over time and hence, no adjustment need be made for changes in prices. Before the data can be used in cross-sectional studies of production, however, weights that reflect the relative magnitudes of the outputs of the various industries have to be applied to the individual index number series. The 1963 Central Statistical Office weights based on the net values of outputs were chosen. The general form of the function adopted does not separately distinguish the input of raw materials and this implies that we should adopt a value added measure of output although this is associated with an important measurement problem - see Klein (1962, p.97). The output data used in this chapter is confidential and is therefore not reported in detail. Over 30 MLHs are distinguished, defined by the 1968 SIC classification to belong to engineering. The information relates to 1961-72 inclusive - a span of twelve years.

The Capital Input

Perpetual inventory estimates of the capital stock have all too

often proved inadequate to the needs of the large and growing interest in the empirical investigation of production. The lack of useful data becomes increasingly obvious at greater levels of disaggregation and yet it is at these levels that some of the more interesting aspects of production become apparent. The inadequacies of the data are not too surprising given the enormous amount of detailed and accurate information that is required to construct traditional measures. The post-war gross stock of capital reported in Pyatt (1964b) and Armstrong (1974) is calculated as the cumulative amount of investment over a period equal to the life expectancy of capital.

The inadequacies of perpetual inventory information have forced researchers to use proxy variables such as time trends - see, for example, Ball and St.Cyr (1966) - and fuel consumption measures - see, for example, Bosworth (1974, pp.165-77) and Moody (1974). The fuel consumption proxy has a certain intuitive appeal and since its use by Foss (1963) has grown considerably in stature. In the recent economics literature, electricity consumption has been used as a proxy for capital services. Heathfield

(1972) compared the consumption of electricity with the stock of capital (the book value of capital was taken to be a measure of the potential supply of capital services). Bosworth (1974, pp 189-90) allocated the SIC value of capital among MLHs in accord with their relative fuel consumptions. Finally, Moody (1974) has argued that the fuel proxy is generally a more useful and accurate measure than its perpetual inventory counterparts, which are often constructed in a roundabout way and generally on the basis of inadequate information. As far as this study is concerned, there is little choice other than to adopt a fuel consumption proxy because the available perpetual inventory estimates relate to much higher levels of aggregation and the time trend proxy is

*inappropriate in this (cross-sectional) context.*

*A fuel based proxy is unlikely to be useful in representing the stock of capital in its role as a measure of the store of wealth. On the other hand, it does appear to be a relevant proxy for the input of capital in the context of a production relationship. In particular, it allows two identical plants, worked at the same capacity, to have the same measure of the capital input even where their factor prices differ - see Solow (1956, p.101). The fuel-based measure of the capital input should enable detailed cross-sectional work to be undertaken for a number of years. The fuel data, which has already been used in this study, is reported in Appendix (II). One important point which must be emphasised is that fuel proxies reflect the consumption of capital services and not the stock of capital. Recent work on employment functions, such as Feldstein (1967) and Craine (1972), have separated employees' hours from numbers employed and the question arises whether the stock of capital and capital usage should be separately distinguished: it is, however the particular human characteristics associated with the length of the working day that make the employment-hours split necessary. One thing is certain, however, where only a fuel proxy is available it is not possible to include both variables in the regression.*

*In a recent paper - see Bosworth (1976c) - it was found that there is a strong relationship between the alternative measures of capital. Two particular tests were made: firstly between the numbers of machine tools and fuel consumption; secondly, between cumulative investment and fuel consumption. The results of these tests are not reported in detail, but there exist strong relationships in both cases and these tend to be closer where particular types of capital are matched with particular fuel categories (ie. total net with all fuels; electricity with plant and machinery; motor vehicle fuels with vehicles). These tests were reassuring in the sense that, even where we expect fuel-based measures to*

be an improvement on the other capital variables in this context, we expect there to be a fairly close relationship between them.

#### The Labour Input

A firm can obviously change its level of activity ceteris paribus by increasing or decreasing the number of people it employs. Annual information about the numbers employed by MLH, distinguishing males and females separately, has been available since 1948. Since 1963, the L7A Survey carried out by the Department of Employment in conjunction with the Engineering Industry Training Board has been the source of this information and, in addition to the existing dimensions, has distinguished occupations. This chapter uses only the total labour input, constructed using information taken from the Historical Abstract and the Labour Yearbooks.

It has long been recognised - see, for example, the work of Douglas (1948) - that the ceteris paribus condition imposed above is invalid. Variations in output can be caused by changes in the length of the working week with a given labour force. The hours variable used in this study is formed as a weighted average of male and female hours, where the weights applied are the relative sizes of male and female employment. The separate elements are aggregated additively (which may not be theoretically rigorous in multiplicative functions) and the marginal products of male and female hours are assumed equal. The information used is from the same source as the labour data.

A third dimension to the input of labour services is the intensity with which the labour force works. It is unlikely that we can obtain any simple summary measure of effort because the influences on this aspect of the labour input are likely to be diverse and not easily quantified. We might expect effort to be related to the number of hours worked, the nature of the shift system, labour bonus schemes, etc. There are physical,



intellectual and mental limits to the performance of labour (the recent work on circadian time may for example, tell us something about the most appropriate type of shift system) as well as organisational and work-place limits (e.g. relationships with superiors and work-mates). These features are unlikely to be introduced explicitly into the production relationship except at the most micro level.

This aspect of the labour input has close connections with Liebenstein's (1966, 1969) work on 'X-efficiency' and Dudley, *et al* (1968) have given evidence (drawn from engineering industries) which suggests that this phenomenon may be empirically important. Hoarding data, see for example Evans (1974, pp. 115-47), may pick up some part of this inefficiency. Hoarding has reached quite large proportions within engineering at various times in the 1960's and early 1970's. The work in this area has emphasised, however, that static inefficiencies may result in dynamic gains - see Evans (1974, pp.123-5) - it may, for example, be less costly to hoard labour in a temporary down-turn than to make someone redundant and obtain a new employee (who may need experience and training) at a later date. Liebenstein's theory of 'inert areas' suggests that competitive forces, the size of firm and other structural variables may have an important bearing on the pressures placed on management (through the degree of accountability to shareholders, the fear of losing a managerial post, etc.) and the ability of management to motivate their workers. These factors may differ between industry groups and, to the extent they do, they may be picked up using industry dummies, sub-dividing the sample by industry group or by the inclusion of structural variables.

#### D. Aggregate Cross-Sectional Results

##### Annual Cross-Sectional Production Functions

This section concentrates on the results obtained from estimating a number of aggregate production functions across all industries within

engineering. The functions are re-estimated for each year of the sample period, (1961 - 1972). The results for the Cobb-Douglas function,

$$Y_{jt} = A_t K_{jt}^{\alpha_t} L_{jt}^{\beta_t} \dots (7.10)$$

are reported in Table (7.1). The explanatory power of the function (measured by  $\bar{R}^2$  and  $F$ ) is high in every year:  $\bar{R}^2$  exceeds 0.9 and the  $F$  statistic is significant at the 1% level in every case, but the results are disappointing in several respects:  $A$  and  $\beta$  are statistically significant at the 1% level, but  $\alpha$  is never significant at this level and only occasionally at the 5% level; the zero order correlation coefficients indicate that there exists strong multi-collinearity between  $K$  and  $L$ , implying that the reported estimates are individually unreliable (comparison with the work of other authors suggests that  $\alpha$  is smaller and  $\beta$  larger than we would have expected and, in addition, they are not stable over time). Although the coefficients  $\alpha$  and  $\beta$  are individually unreliable their combined value,  $\alpha + \beta$ , is unaffected by the problem of multicollinearity and provides a measure of industry level returns to scale ( $\alpha + \beta > 1$  implies increasing,  $\alpha + \beta = 1$  constant and  $\alpha + \beta < 1$  decreasing returns to scale). Increasing returns to scale characterises all of the functions: the combined coefficients are statistically greater than unity at the 10% level of significance in all of the years and at the 5% level in 1970. Increasing returns to scale appear to become greater in magnitude and statistically more significant towards the end of the sample period.

The technical efficiency parameter,  $A$ , is statistically different from zero at the 1% level in all periods. The results reported in Table (7.1), however, give no indication of a simple growth path for  $A$  over time. This can be contrasted with theoretical models, which often assume that the technical efficiency parameter grows steadily (often exponentially) with time.

Table (7.1): Annual Cobb-Douglas Functions

Year	Log A	$\alpha$	$\beta$	( $\alpha + \beta$ )	$R^2$	F	( $R^2$ )
1961	1.8588 **	0.2557 *	0.7977 **	(1.0534)	0.91	155.16 **	(0.078)
2	1.8064 **	0.2207 *	0.8408 **	(1.0615)	0.92	167.67 **	(0.072)
3	1.8647 **	0.2019 *	0.8518 **	(1.0537)	0.93	203.81 **	(0.079)
4	1.7358 **	0.1602	0.9306 **	(1.0908)	0.93	223.04 **	(0.073)
5	1.8225 **	0.1854 *	0.8916 **	(1.0770)	0.94	257.83 **	(0.098)
6	1.7132 **	0.1309	0.9655 **	(1.0964)†	0.94	254.08 **	(0.075)
7	1.7661 **	0.1217	0.9680 **	(1.0897)†	0.94	255.18 **	(0.057)
8	1.8283 **	0.1471	0.9468 **	(1.0939)†	0.94	254.66 **	(0.075)
9	1.8398 **	0.1185	0.9787 **	(1.0972)†	0.94	263.33 **	(0.070)
70	1.6998 **	0.2362 *	0.9080 **	(1.1442) *	0.93	208.92 **	(0.209)
1	1.8502 **	0.0967	0.9998 **	(1.0965)†	0.93	216.30 **	(0.047)
2	2.1135 **	0.3062 *	0.7532 **	(1.0594)	0.89	132.17 **	(0.139)

An obvious generalisation which has been undertaken by other authors is to introduce hours. Until recently, this was achieved by using labour hours as an explanatory variable,

$$Y_{jt} = A_t K_{jt}^{\alpha_t} (L_{jt} H_{jt})^{\beta_t} \quad \dots (7.11)$$

but Feldstein (1967) and Craine (1972) have separated hours as an additional explanatory variable,

$$Y_{jt} = A_t K_{jt}^{\alpha_t} L_{jt}^{\beta_t} H_{jt}^{\gamma_t} \quad \dots (7.12)$$

These two functional forms were estimated using the same data but the results are not reported here because (as we might expect) they show the same traits as those reported in Table (7.1). In particular, strong multicollinearity between K and L and between K and LH make the individual estimates of  $\alpha$  and  $\beta$  unreliable. The labour variable when weighted by hours still makes a significant contribution to the explanation of output and the revised  $\beta$  is similar in size and changes over time in much the same way as it did before. There is nothing in the results from equation (7.11) to indicate that they were an improvement on (7.10). In the function where hours are distinguished separately: H is not statistically significant,  $\gamma$  is unstable and  $\lambda$  is even less tenable than previously. Part of the explanation appears to be that average hours were roughly constant across industries and this makes the separate estimation of  $\lambda$  and  $\gamma$  impossible.

There is a more important aspect of the results which remains hidden in the main body of Table (7.1). It is associated with a problem which has received scant treatment in the literature on production functions. Equation (7.10) is mathematically identical to equation (7.13) below,

$$\frac{Y_{jt}}{L_{jt}} = A_t \left( \frac{K_{jt}}{L_{jt}} \right)^{\alpha_t} L_{jt}^{\alpha_t + \beta_t - 1} \dots (7.13)$$

If the production function is estimated in this form, the coefficients ( $A$ ,  $\alpha$  and  $\beta$ ) are identical to those reported in Table (7.1), but the overall fit of the regression is very much lower (again measured by  $\bar{R}^2$  and  $F$ ). The final (separated column) of Table (7.1) reports the revised values of  $\bar{R}^2$ , which indicate that almost none of the variation in labour productivity across industries has been explained. Whilst labour productivity has a smaller variance across industries than output itself, the proportion of this variance explained by the Cobb-Douglas function has declined.

Griliches and Ringstad (1974, p.64), estimating Cobb-Douglas functions for a cross-section of Norwegian manufacturing establishments, discovered that this transformation reduced their  $R^2$  from 0.94 to 0.35. They argue that the transformation does not affect the real explanatory power of the model, which was quite low. The only way out of this dilemma appears to be to measure the fit when each of the possible alternative transformations is made. When a high explanatory power is obtained for all possible formulations, the overall fit of the model can be said to be good. Here we content ourselves with obtaining a high explanatory power for the labour productivity form which, in practice, proves an extremely difficult task. This form has the advantage that it avoids the tendency of purely scale effects (i.e. large industries tend to employ large amounts of capital and labour) to swamp the effects of variations in labour and capital on the output of industries of a given size.

One obvious reason why little variation in labour productivity has been explained might be that the Cobb-Douglas function is too restrictive and a more general CES form is appropriate. The variables entering the

function were centered on their geometric means (in the way suggested by Griliches and Ringstad) and the Kmenta CES form was then estimated,

$$\frac{Y_{jt}}{L_{jt}} = a_{0t} + a_{1t} \log L_{jt} + a_{2t} \log \left( \frac{K_{jt}}{L_{jt}} \right) + a_{3t} \left[ \log \left( \frac{K_{jt}}{L_{jt}} \right) \right]^2 \dots (7.14)$$

The results are reported in Table (7.2). In no sense were they satisfactory. The  $\alpha$  coefficient ( $a_2$ ) was significant only once at the 5% level or better, although in six of the twelve cases it was significant at the 10% level or better. Returns to scale ( $a_1$ ) were significant only in 1970 and then only at the 5% level. The second order term, ( $a_3$ ), was again significant only in 1970, but in this case at the 1% level. The general insignificance of  $a_3$  gave little hope that it would yield sensible values for the elasticity of substitution. This in fact proved to be the case, with  $\theta < -1$  in seven of the twelve years and  $\sigma$  highly unstable (varying from 1.0810 to 4.0966) in the remaining years. Even in the one case where  $a_3$  was significant,  $\theta$  was less than minus one. It is possible that the samples were not large enough and did not have sufficient variation of  $K$  and  $L$  to allow  $a_3$  to be estimated with sufficient precision.

#### Three Year Pooled Data

The results reported so far suggest that further work on cross-sectional production functions based on data drawn from a single year would probably not be very rewarding.

A lack of variability in the data seems to be the cause both multicollinearity between  $K$  and  $L$  and the lack of precision in estimating  $a_3$ . This may be avoided by pooling the cross-sectional and time series dimensions of the data. It seemed most interesting to increase the sample size by the smallest amount necessary to reduce multicollinearity

Table (7.2): Annual CSS Production Functions

Year	$a_0$	$a_1$	$a_2$	$a_3$	$R^2$	F
1961	1.0284 **	0.0164	0.2029	0.0238	-	0.889
2	1.0158 **	0.0248	0.1924	0.0601	-	0.942
3	1.0077 **	0.0255	0.2375 †	0.1197	-	1.746
4	1.0080 **	0.0693	0.1876 †	0.1085	0.066	1.727
5	1.0326 **	0.0578	0.1781 †	0.0058	0.037	1.401
6	1.0308 **	0.0778	0.1248	0.0164	0.015	1.153
7	1.0101 **	0.0671	0.1166	0.0923	0.013	1.133
8	1.0036 **	0.0753	0.1629†	0.1192	0.061	1.670
9	1.0100 **	0.0750	0.1480	0.0975	0.035	1.370
70	0.9174 **	0.1473 †	0.3911 **	0.4161 **	0.440	9.094 **
1	0.9783 **	0.0878	0.1535	0.1921	0.071	1.788
2	-0.0278	0.0621	0.3200 †	0.0678	0.136	2.631 †

to acceptable levels. The cross-sectional data for three consecutive years were pooled and thus, a moving pool of data was formed (1961-3, 1962-4, ... 1970-2). This procedure still allows the paths of the estimated coefficients to be traced over time, but the coefficients are now averages over three years.

The results of estimating the basic Cobb-Douglas function described by equation (7.13) are reported in Table (7.3). The overall fit of each regression, measured by  $\bar{R}^2$  and  $F$ , again appears to be very good. In addition, the strength of the multicollinearity between capital and labour is diminished sufficiently to allow both  $\alpha$  and  $\beta$  to be significant at the 1% level in all of the samples. Over the period 1961-9,  $\alpha$  falls steadily while  $\beta$  increases, but over the period 1969-72 they return almost to their 1961-3 levels. As in the case of the annual results, other authors' works suggest that  $\alpha$  is slightly smaller and  $\beta$  slightly greater than expected. The estimated returns to scale are of the same order of magnitude as in the annual results. In all but two cases ( $\alpha+\beta$ ) is significantly greater than unity at the 1% level and in the remaining cases it is significant at the 10% and 5% levels. The technical efficiency parameter still fails to grow steadily over time, although its value at the end of the period exceeds its initial value. As in the annual case, Table (7.3) again reports the  $\bar{R}^2$  for the transformed function in the last (separated) column. The  $\bar{R}^2$  are somewhat higher than in the annual cases, but again do not achieve acceptable levels.

The introduction of hours had almost exactly the same consequences as in the annual regressions and the results are not therefore reported in detail. An alternative generalisation is to allow the  $A$  parameter to change from year to year. The results for the simplest Cobb-Douglas function are reported in Table (7.4). The function was estimated using two dummies and a constant, but translated into a form with three dummies



TABLE (7.3):

3 Year Pooled Cobb-Douglas Functions

Year	A	$\alpha$	B	( $\alpha+\beta$ )	$R^2$	F	( $\bar{R}^2$ )
1961-3	1.8426***	0.2258***	0.8304***	(1.0562) †	0.92	555.59***	0.119
2-4	1.8022***	0.1957***	0.8732***	(1.0689)**	0.93	615.91***	0.111
3-5	1.8056***	0.1837***	0.8908***	(1.0745)**	0.94	714.98***	0.119
4-6	1.7565***	0.1598***	0.9285***	(1.0883)**	0.94	779.23***	0.120
5-7	1.7665***	0.1470***	0.9409***	(1.0816)**	0.94	813.55***	0.113
6-8	1.7721***	0.1400***	0.9532***	(1.0932)**	0.94	793.65***	0.111
7-9	1.8112***	0.1356***	0.9585***	(1.0941)**	0.94	801.41***	0.110
8-70	1.7896***	0.1780***	0.9345***	(1.1125)**	0.94	748.70***	0.159
9-71	1.8006***	0.1554***	0.9572***	(1.1126)**	0.94	711.87***	0.143
70-72	1.8676***	0.2084***	0.8929***	(1.1013)**	0.92	543.63**	0.155

Table (7.4)

3 Year Pooled Cobb-Douglas Functions with Dummies

Year	D1	D2	D3	$\alpha$	$\beta$	$(\alpha+\beta)$	$R^2$	F	$(R^2)$
1961-3	1.8302**	1.8305**	1.8624**	0.2237**	0.8326**	(1.0563) <sup>†</sup>	0.921	272.51**	(0.102)
2-4	1.7616**	1.7957**	1.8444**	0.1931**	0.8757**	(1.0688) <sup>†</sup>	0.929	306.19**	(0.105)
3-5	1.7691**	1.8177**	1.8344**	0.1821**	0.8919**	(1.0740) <sup>†</sup>	0.937	354.08**	(0.110)
4-6	1.7469**	1.7645**	1.7595**	0.1591**	0.9290**	(1.0881) <sup>†</sup>	0.941	381.58**	(0.101)
5-7	1.7608**	1.7558**	1.7816**	0.1462**	0.9417**	(1.0879) <sup>†</sup>	0.944	398.86**	(0.095)
6-8	1.7279**	1.7544**	1.8275**	0.1334*	0.9598**	(1.0932) <sup>†</sup>	0.944	399.46**	(0.115)
7-9	1.7517**	1.8251**	1.8575**	0.1292*	0.9644**	(1.0936)	0.944	404.46**	(0.116)
8-70	1.7557**	1.7868**	1.8265**	0.1739**	0.9384**	(1.1123)	0.940	370.79**	(0.151)
9-71	1.7814**	1.8227**	1.7965**	0.1535**	0.9590**	(1.1125)	0.936	349.66**	(0.128)
70-72	1.8745**	1.8549**	1.9205**	0.2079**	0.8943**	(1.1022)	0.920	272.69**	(0.143)

(and no overall constant). The results are almost identical with those reported in Table (7.3), with  $\alpha$  and  $\beta$  of similar size and significance to the previous estimates. The explanatory power was almost the same in the two tables for the basic formulations, although slightly lower for the transformed functions. The dummy variables, however, did not increase from sample to sample. They showed a tendency to increase within any particular sample, but in no case were the second and third year dummies significantly different from an overall constant. For completeness, hours were included along with the time dummies, in the alternative ways already described. The results were, however, almost identical to those reported earlier and are therefore not described in detail.

The larger size of the sample (now approximately 96 observations) appears to have significantly reduced the problem of multicollinearity between capital and labour. There is now reason to believe that if the second order terms of the CES function are important then  $a_3$  in the Kmenta form should now be found to be significantly different from zero. The results of estimating the Kmenta form on the pooled information are reported in Table (7.5). The returns to scale term,  $a_1$ , is almost always significant at the 5% level or better. The coefficient  $a_2$  (which is capital's share weighted by the returns to scale,  $\theta_a$ ) is significant at the 1% level in every case. The key coefficient in the Kmenta form,  $a_3$ , is generally insignificantly different from zero and in the cases where it is significant, it is associated with values of  $\theta$  less than -1. The estimated value of  $\sigma$  is almost always unacceptable.

#### All Years Pooled

It seems an obvious step to pool all of the data (32 MLRs for

Table ( 7.5) Three Year Pooled CES Functions:

$$\log \left( \frac{Y}{L} \right) = a_0 + a_1 \log L + a_2 \log \left( \frac{K}{L} \right) + a_3 \left[ \log \left( \frac{K}{L} \right) \right]^2$$

Year	$a_0$	$a_1$	$a_2$	$a_3$	$\bar{R}^2$	F
1961-3	-0.0159	0.0450	0.2371**	0.0755	0.116	4.21**
1962-4	-0.0304	0.0647*	0.2197**	0.0959	0.120	5.28**
1963-5	-0.0203	0.0717*	0.2022**	0.0744	0.124	5.46**
1964-6	-0.0144	0.0871**	0.1675**	0.0387	0.114	5.07**
1965-7	-0.0078	0.0873**	0.1509**	0.0290	0.105	4.73**
1966-8	-0.0192	0.0915**	0.1480**	0.0702	0.113	5.02**
1967-9	-0.0255	0.0915**	0.1484**	0.0922	0.120	5.33**
1968-70	-0.0522 <sup>†</sup>	0.1041**	0.1952**	0.1701**	0.225	10.19**
1969-71	-0.0631 <sup>†</sup>	0.1075**	0.1887**	0.1901**	0.233	10.60**
1970-72	-0.0611 <sup>†</sup>	0.1016**	0.3180**	0.1816**	0.214	9.61**

12 years) and obtain average values of  $\alpha$  and  $\beta$  over the period. The results obtained are reported in Table (7.6). Regression (i) is the simplest Cobb-Douglas function. The values taken by  $A$ ,  $\alpha$  and  $\beta$  are broadly consistent with those found using the smaller (3-year) pools and there are significant increasing returns to scale at the 1% level. The Cobb-Douglas function was re-estimated including a different dummy variable in each year and appears as regression (iv). This innovation only changed the values of  $\alpha$  and  $\beta$  slightly and all of the estimated coefficients are significantly different from zero at the 1% level. The  $\alpha$  coefficient was slightly smaller and  $\beta$  slightly larger when the dummies were included, but the returns to scale were almost unchanged. The individual dummies were all significantly different from zero and those for later years became significantly different from the 1961 value by 1967. The role played by hours in the Cobb-Douglas function is illustrated by regressions (ii), (iii), (v) and (vi). In both forms in which hours can be introduced, they have the effect of reducing  $\alpha$  and raising  $\beta$  slightly although both remain significant at the 1% level. Weighting the labour variable by the number of hours worked does not alter the overall explanatory power of the model. This is also true where hours appear as a separate explanatory variable:  $\gamma$  has the wrong sign in regression (iii) and it is not significant when industry dummies are introduced. The explanatory power of the transformed functions (labour-productivity form), measured by  $\bar{R}^2$  and  $F$ , remains extremely low. The explanation seems to be that hours are roughly constant across industries.

One interesting point is that the technical efficiency parameters,  $D_t$ , now show a strong upward trend over time ( $t = 1, \dots, 12$ ). In the Cobb-Douglas function described by equation (iv),  $D$  rises by 17.63% over the twelve year period. The minor down-turns are associated with years of recession in the engineering industry; they may be a consequence of mismeasuring input usage or they may be a feature of technical change (for example, the result of failing to invest sufficient to overcome

Table (7.6) Twelve Year Pooled Cobb-Douglas Functions

Regression	Functional Form	A	a	B	(a+B)	Y	R <sup>2</sup>	F	(F <sup>2</sup> )											
I.	$Y = AK^aL^b$	1.3244	0.2117	0.8731	(1.0868)	-	0.976	2375.15	(0.147)											
II.	$Y = AK^a(LH)^b$	-1.4654	0.2050	0.8754	(1.0814)	-	0.926	2744.53	(0.132)											
III.	$Y = AK^aL^bH^c$	5.4365	0.2045	0.8953	(1.0998)	-0.9763	0.930	1623.63	(0.161)											
D <sub>1</sub>		D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>	D <sub>8</sub>	D <sub>9</sub>	D <sub>10</sub>	D <sub>11</sub>	D <sub>12</sub>	a	β	(a+β)	Y	R <sup>2</sup>	F	(F <sup>2</sup> )	
IV.	$Y = D_1K^{a_1}L^{b_1}$	1.6744	1.7122	1.7606	1.7772	1.7722	1.7965	1.8664	1.8974	1.9355	1.9131	1.9696	0.1807	0.9063	(1.0870)	-	0.934	398.15	(0.213)	
V.	$Y = D_1K^{a_1}(LH)^{b_1}$	-1.8131	-1.7978	-1.7860	-1.7387	-1.6985	-1.6847	-1.6482	-1.5997	-1.5666	-1.4955	-1.4993	-1.4501	0.1600	0.9212	(1.0812)	-	0.934	374.71	(0.191)
VI.	$Y = D_1K^{a_1}L^{b_1}H^{c_1}$	1.4931	1.4428	1.4621	1.5114	1.5319	1.5311	1.5556	1.6243	1.6594	1.7153	1.6814	1.7426	0.1666	0.9231	(1.0897)	-	0.934	399.08	(0.233)

depreciation).

E. Inter-Industry Variations in Labour Productivity: Some Insights

Engineering forms a much more homogeneous sample than those used by most researchers in the past. Shipbuilding and motor vehicles, for example, have features which make it natural to include them within engineering. Nevertheless, the engineering group is comprised from a number of diverse elements: shipbuilding and electronics produce technically distinct products and require different mixes of capital types and labour skills to make them. An obvious reaction therefore is to look at more detailed sub-groups of engineering and investigate whether the CES functions now perform any better. The study concentrates on the Cobb-Douglas function, the results of which are sufficient to illustrate the conclusions drawn in this Chapter.

Engineering is subdivided into the following industry groups: mechanical engineering; electrical engineering; vehicles; and metal goods not elsewhere specified. Tables (7.7) - (7.10) report results for the basic Cobb-Douglas function for each of these industries in turn, based on the three-year pools of information. Several important conclusions can be drawn. First, all the functions appear to perform well in absolute form:  $\bar{R}^2$  exceeds 0.9 in all of the samples and  $F$  is significant at the 1% level in every case. Second, the smaller sample sizes cause the problem of multicollinearity between  $K$  and  $L$  to reappear. Only mechanical engineering, which has the greatest number of observations, appears to be largely untouched by this problem. This particular industry sub-group has quite different  $\alpha$  and  $\beta$  to the overall values calculated for all engineering. Third, there is some evidence to suggest considerable differences between industries in the returns to scale experienced: vehicles are characterised by significant

Table (7.7) Three Year Pooled: Mechanical Engineering.

Year	$D_1$	$D_2$	$D_3$	$\alpha$	$\beta$	$(\alpha+\beta)$	$\bar{R}^2$	F	$(\bar{R}^2)$
1961-3	2.1508**	2.1126**	2.1390**	0.6930**	0.3700**	(1.0631)	0.914	85.90**	(0.454)
1962-4	2.0429**	2.0717**	2.1677**	0.6694**	0.4063**	(1.0757)	0.930	106.88**	(0.495)
1963-5	2.1168**	2.1168**	2.2140**	0.6826**	0.3842**	(1.0668)	0.948	146.70**	(0.565)
1964-6	2.0570**	2.0013**	2.0038**	0.6869**	0.4163**	(1.1032)	0.949	150.17**	(0.562)
1965-7	1.9646**	1.9668**	1.9431**	0.6929**	0.4193**	(1.1122)*	0.958	182.62**	(0.613)
1966-8	1.9493**	1.9248**	1.9424**	0.7079**	0.4101**	(1.1180)**	0.967	233.48**	(0.696)
1967-9	2.0441**	2.0629**	2.1237**	0.6824**	0.4060**	(1.0884)*	0.966	229.30**	(0.699)
1968-70	2.1847**	2.2477**	2.2586**	0.6050**	0.4488**	(1.0538)	0.953	163.96**	(0.627)
1969-71	2.2475**	2.2649**	2.2603**	0.5127**	0.5335**	(1.0462)	0.934	114.98**	(0.498)
1970-72	2.3395**	2.3241**	2.4009**	0.3957**	0.6257**	(1.0228)	0.909	81.16**	(0.373)



Table (7.8) Three Year Pooled: Electrical Engineering.

Year	$D_1$	$D_2$	$D_3$	$\alpha$	$\beta$	$(\alpha+\beta)$	$\bar{R}^2$	F	$(\bar{R}^2)$
1961-3	2.4113**	2.4535**	2.5017**	0.0006	0.9169***	(0.9175)	0.950	80.96**	(0.001)
1962-4	2.6020**	2.6514**	2.6946**	-0.0210	0.9053**	(0.8843)†	0.958	97.30**	(0.113)
1963-5	2.7099**	2.7537**	2.7598**	-0.0022	0.8760**	(0.8738)*	0.966	120.10**	(0.269)
1964-6	2.6478**	2.6536**	2.6961**	0.0046	0.8917**	(0.8963)	0.965	117.69**	(0.172)
1965-7	2.6813**	2.7242**	2.8016**	-0.0103	0.9000**	(0.8897)	0.947	77.51**	(0.118)
1966-8	2.7874**	2.8667**	2.9430**	-0.0558	0.9284**	(0.8726)	0.941	69.04**	(0.073)
1967-9	2.9000**	2.9763**	3.0285**	-0.0690	0.9338**	(0.8648)	0.950	82.12**	(0.045)
1968-70	3.0251**	3.0778**	3.1233**	-0.0860	0.9395**	(0.8535)†	0.961	106.67**	(0.060)
1969-71	3.2696**	3.3164**	3.3617**	-0.1240*	0.9353**	(0.8113)*	0.960	102.60**	(0.176)
1970-72	3.1679**	3.2147**	3.3368**	-0.1127†	0.9555**	(0.8428)†	0.956	92.49**	(0.265)

Table (7.9) Three Year Pooled: Vehicles.

Year	$D_1$	$D_2$	$D_3$	$\alpha$	$\beta$	$(\alpha+\beta)$	$R^2$	F	$(\bar{R}^2)$
1961-3	0.8862***	0.8740**	0.8556***	0.5692***	0.6718**	(1.2410)**	0.990	353.52***	(0.889)
1962-4	0.8300***	0.8117**	0.8293***	0.5830***	0.6682***	(1.2512)**	0.993	476.36***	(0.903)
1963-5	0.7769***	0.8025**	0.9143***	0.5010**	0.7533***	(1.2543)**	0.992	450.07***	(0.889)
1964-6	0.7074***	0.8320**	0.7803***	0.3469***	0.9231**	(1.2700)**	0.993	512.20***	(0.884)
1965-7	0.8249***	0.7820**	0.8025**	0.1247	1.1431**	(1.2678)**	0.996	882.32**	(0.919)
1966-8	0.7591***	0.7808**	0.9012***	0.1421	1.1916**	(1.2724)**	0.997	1027.04**	(0.932)
1967-9	0.8572***	0.9842**	1.0070**	0.0071	1.2479**	(1.2550)**	0.996	982.69**	(0.921)
1968-70	0.8571***	0.8749**	1.0762**	0.3939*	0.8819**	(1.2758)**	0.981	181.30**	(0.744)
1969-71	0.9661***	1.1587**	0.9900***	0.4064*	0.8479**	(1.2543)**	0.980	174.79**	(0.718)
1970-72	1.4095***	1.2310**	1.4032**	0.3229 <sup>†</sup>	0.8769**	(1.1998)**	0.975	138.81**	(0.597)

Table (7.10) Three Year Pooled: Metal Goods Not Elsewhere Specified

Year	$D_1$	$D_2$	$D_3$	$\alpha$	$\beta$	$(\alpha+\beta)$	$\bar{R}^2$	F	$(\bar{R}^2)$
1961-3	2.5764***	2.5764***	2.5894**	0.2773	0.6023**	(0.8796)	0.903	42.76	(0.006)
1962-4	2.4988***	2.4974***	2.5475**	0.2766	0.6278**	(0.9044)	0.920	55.58	(0.006)
1963-5	2.4330***	2.4824***	2.5154***	0.2645	0.6565**	(0.9210)	0.928	65.69	( - )
1964-6	2.4880***	2.5216**	2.5066**	0.2481	0.6703**	(0.9184)	0.933	70.92	( - )
1965-7	2.6313***	2.6167**	2.6300**	0.2593	0.6307**	(0.8900)†	0.936	74.14	(0.067)
1966-8	2.6550***	2.6672**	2.7313**	0.2940†	0.5881**	(0.8821)*	0.950	95.03	(0.203)
1967-9	2.6459***	2.7073**	2.6819**	0.3493*	0.5421**	(0.8914)*	0.952	101.05	(0.208)
1968-70	2.5657**	2.5434**	2.4699**	0.2741†	0.6518**	(0.9259)	0.957	111.62	(0.086)
1969-71	2.4687**	2.3940**	2.4015**	0.1473	0.7940**	(0.9413)	0.950	96.10	( - )
1970-72	2.6106**	2.6325**	2.2542**	0.3908*	0.5019**	(0.8927)	0.898	45.17	(0.211)

increasing returns to scale, while both electrical and metal goods not elsewhere specified show some signs of decreasing returns to scale. Finally, although the results for the all pooled data are not reported in detail, they show considerable differences in the rates of improvement in technical efficiency.

The overall fit of the function varies considerably. This can be seen most clearly in the form where labour productivity is the dependent variable. The final column of each table reports  $\bar{R}^2$  for the transformed functions. In the case of vehicles, the basic Cobb-Douglas function is almost as good at explaining variations in labour productivity as variations in output. Mechanical engineering shows a similar although not so marked improvement. The results for electrical engineering and metal goods not elsewhere specified are by no means as good, although for some periods there is an improvement over the results for all engineering.

One interpretation of the results is that vehicles and mechanical engineering form samples within which the technology of production is common across NLHs and Cobb-Douglas in form. There is good reason to believe that metal goods not elsewhere specified is a rag-bag of industries that are not grouped together because they have something in common, but because they have nothing to link them with another engineering group. The case of electrical engineering is much less obvious. It contains diverse NLHs which range from one-off heavy electrical capital goods to mass produced consumer goods, but to some extent this is also true of mechanical engineering. This interpretation of the improved results must be treated with caution. It becomes difficult to decide whether it is the more homogeneous technologies or simply the lower degree of variability in the sample which causes the improved empirical performance.

F. Conclusions

Neoclassical production functions fail to provide an explanation of variations in productivity across the broad spectrum of engineering industries. They may, although the evidence is by no means conclusive, be relevant to more tightly defined industry subgroups. The evidence reported in this chapter indicates that the Cobb-Douglas function is better able to explain variations in labour productivity within a given industry group than across groups. One explanation is that a different Cobb-Douglas function is required for each separate technology. An alternative explanation is that the lower level of variation within each industry implies that the Cobb-Douglas function fits because the data approximates an accounting tautology - see Cramer (1969). A number of the possible explanations of the good empirical performance of the Cobb-Douglas function are considered in the next chapter.

## Chapter (8) Neoclassical Production Functions: Fact or Fantasy

### A. Introduction

Economic theory has for some time been living with an important contradiction arising out of the debate about the existence of aggregate neoclassical production functions. In recent years the neoclassicist has been forced increasingly on the defensive. The unfavourable result of the debate about problems of aggregation has caused him to retreat towards micro economics, and the absurdity of the assumption that capital is instantaneously and costlessly malleable has resulted in his interpreting observed points 'as if' they were drawn from a neoclassical world, an assumption that so far lacks justification.<sup>1</sup>

Although the form in which the production relationship is estimated and reported may, in some cases, misleadingly flatter the Cobb-Douglas function - see Chapter 7 - aggregate neoclassical production functions continue to be accepted as statistically accurate descriptions of observed points in the input-output space. The inconsistency between intuitive plausibility and empirical 'fact' forms the subject of this chapter.

In order not to cast the net too widely, a secondary problem concerning variations in the results of neoclassical studies - discussed in Chapter 5 - is avoided as far as possible. The body of empirical evidence has become sufficiently large in recent years to show that different types of data produce different results. Brown (1966, pp.129-130), for example, has noted that the elasticity of substitution between inputs centres around the value of one-half in time series studies, while it is closer to unity in cross-sectional work. Both Brown and Johansen

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1. For the reader who is not convinced of the case against the existence of aggregate neoclassical production functions, see Blaug, (1974, Chapter 2).

(1972) have concluded that different relationships were being investigated in the two cases. Our attention focuses on the cross-sectional case, where, despite the results contained in Chapter 5, the Cobb-Douglas function is by far the most consistent performer.

One can visualize the neoclassicists' reasons for entering the realms of micro theory. If an 'as if' explanation of neoclassical production functions could have been provided at the micro level, it might have been possible to aggregate and produce production functions consistent with those arising from the empirical studies. The neoclassicists, however, have found no explanation of the Cobb-Douglas phenomenon. The 'as if' defence, using a surrogate production function, has been discredited even at the micro level with the realisation of the relevance of the reswitching and reversing phenomena. Even had they been able to find such a justification at the micro level, they would have had to face the problem that the conditions of aggregation are generally so severe as to make the existence of the aggregate Cobb-Douglas function extremely unlikely. In the rush to find safer ground the Cobb-Douglas function has been abandoned even though it continues to yield 'good' empirical results - see Solow (1966, pp. 1259-60). We are left to look at this relic of bygone days and wonder what hypothesis, if any, is being tested - see Blaug (1974, p.18).

A review of the explanations of the Cobb-Douglas relationship is provided in the next section. There can be little doubt that the existing explanations do not give the whole story. The third section of this chapter attempts to provide an alternative avenue of thought combining the more important of the existing explanations. The final section draws some conclusions about how we can interpret the aggregate Cobb-Douglas function.

B. A Review of the Literature

The literature explaining the aggregate Cobb-Douglas result can be divided into three broad groups. First, economists have regarded the relationship as an 'empirical law' in its own right. Second, a number of economists have argued that, although the underlying technology of production may not be Cobb-Douglas, observations drawn from the real world behave 'as if' they are consistent with a neoclassical technology. Finally, it has been argued that the aggregate Cobb-Douglas function arises from features of the data that have little or nothing to do with the underlying technology of production.

'Empirical Laws' and Heroic Aggregation

It was the neoclassicists' hope that the aggregate production function would prove a reflection of the underlying micro technologies. It is at a micro-level, for example, that the economist would be more at ease talking about homogeneous capital and labour inputs. The immediate question was whether micro Cobb-Douglas technologies could be easily aggregated to the sort of functions estimated in the literature.

In reviewing the conclusions of the debate about problems of aggregation we can kill two birds with one stone by moving into a vintage world. It has already been noted that vintage models are intuitively more appealing than are their neoclassical counterparts. A number of them (though by no means all) have assumed the existence of ex ante substitution possibilities in accord with neoclassical principles, and there is some hope that an aggregate production function may, in some way, reflect these underlying technologies. The simplest of these cases involves a putty-putty model of the Solow (1960) and Phelps (1962) variety. Allen (1968, pp. 283-6) investigates this type of model and assumes each ex ante function is Cobb-Douglas.



$$Y_{vt} = A^{gv} K_v^\alpha L_{vt}^{1-\alpha} \quad \dots (8.1)$$

where:  $Y$ ,  $K$  and  $L$  denote output, capital and labour respectively;

$v$  denotes the  $v$ th vintage; and  $t$  the  $t$ th time period,  $t > v$ .

The model is consistent with Solow neutral technical progress at a rate  $g/\alpha$  up to the time when the vintage is first used. Once a production unit is brought into use no further technical change occurs. Allen shows that, if factors are paid their marginal products, an aggregate function of the form

$$Y_t = C^{gt} K_t^\alpha L_t^{1-\alpha} \quad \dots (8.2)$$

will exist so long as capital is measured in terms of equivalent new machines (a surrogate measure),

$$K_t = \int_{v=t-\tau}^t e^{-\frac{g}{\alpha}(t-v)} K_v dv \quad \dots (8.3)$$

The conditions under which an aggregate function, such as equation (8.2), arises are not stressed by Allen. Fisher (1969), however, has followed up the earlier work of Leontief (1947a, 1947b) and has considered the problems of aggregation in some detail. Fisher has shown that, when the underlying production functions are Cobb-Douglas, the Leontief conditions (for the existence of an aggregate production function) are satisfied if the aggregate variables are calculated as geometric means. Fisher further argued, however, that where the aggregate output and labour variables are formed as arithmetic sums (even assuming they are both homogeneous), an aggregate capital variable will exist only if the

underlying production functions exhibit constant returns to scale and capital vintages differ only through the impact of disembodied capital augmenting technical progress. These two features are assumed in the Allen model.

In general, the conditions for the existence of an aggregate production function are rarely met, and it is only rarely that the available statistics are sufficiently detailed to enable geometric means to be constructed. In this case, immense problems are associated with measuring the capital input. In addition, Fisher (1969) and Blaug (1974, pp. 15-16) point to the fact that the conditions for aggregating heterogeneous labour and output are equally stringent. The estimated macro functions that have been reported in the literature cannot be rigorously justified.

Solow, realising the difficulties of rigorously justifying his empirical work, adopted a different standpoint. He claimed that, although more disaggregate models may represent higher orders of approximation to the underlying technology, more aggregate models are equally respectable, because they propose 'empirical laws' of an aggregate type. If such functions represent aggregate laws, then it can be argued that the only tests are those of statistical validity and intuitive plausibility. Tests of this type are compatible with Solow's (1966, pp. 1259-60) prime directive that the aggregate analysis should be treated "as an illuminating parable, or else merely a device for handling data, to be used so long as it gives good empirical results, and to be abandoned as soon as it does not, or as soon as something better comes along." Solow's disagreement with Joan Robinson arose mainly because he believed that she was against useful, although inexact, constructions such as the aggregate Cobb-Douglas function, without providing a better substitute - see Wan (1971, p. 97).

Solow's point is undoubtedly a good one. Economists are only too pleased to make use of 'empirical laws' when, for example, they are attempting to make forecasts. Unfortunately, however, although the function continues to perform well empirically, its intuitive plausibility has been increasingly called into question. The implausible assumptions about the capital input have led to a movement towards models based on heterogeneous capital goods. In addition, a number of economists have argued that particular features of the data cause the Cobb-Douglas phenomenon. This raises the important question of what, if anything, is being tested when an aggregate Cobb-Douglas function is estimated?

#### 'As If': An Inadequate Defence

The neoclassicists' main defence of their empirical results has been to argue that observed points behave 'as if' they are consistent with a neoclassical technology. However, when forced to justify this proposition, they were unable to find any adequate theoretical reasons. In this section we take a look at the two principal surrogate production function models and the Houthakker-Johansen approach.

Surrogate Production Functions. The initial stimulus to the stream of surrogate theory was provided by Joan Robinson (1953-54). It was assumed that there existed a finite number of fixed coefficient technologies (the 'book of blueprints'). Real capital,  $K_L$ , was measured in labour units. Its size was determined by the number of labour units used in its construction, compounded at the relevant rate of interest over the gestation period for that type of capital,  $L_K (1 + i)^t$ .<sup>2</sup> The value of capital, on which the rate of interest ( $i$ ) was hung, was the real capital weighted by the wage rate ( $w$ ). Hence, in the simplest possible case, the accounting tautology for any particular technology was written

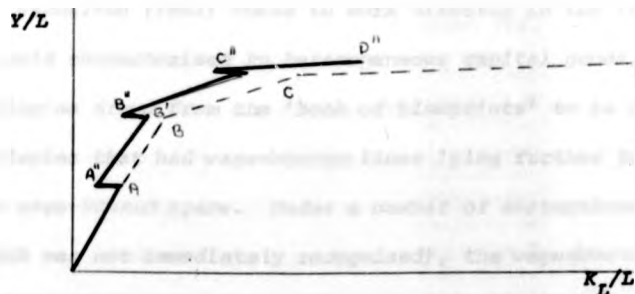
2. This would have been the value of work done in the most profitable alternative activity.

$$Y = i \left[ wL_K (1 + i)^t \right] + wL \quad \dots (8.4)$$

Given  $Y$ ,  $L_K$  and  $L$  for each process, it was then possible to isolate the  $i$  that corresponded to any particular  $w$ . By comparing all of the alternative techniques in the 'book of blueprints', the equipment that yielded the greatest profit was found.

Points on a pseudo-production function were derived by comparing the choices made from the 'book of blueprints' by a number of isolated islands characterised by different wage regimes. The resulting distribution of points in the input-output space formed a linear programming type of function, characterised by horizontal stretches at the points where the function changed slope. The pseudo-production function is represented by the unbroken line in Diagram (8.1). The horizontal stretches were caused by the same valuation of a different piece of capital on two different islands. The obvious implication is that generally the same plant will be given different valuations under different wage-interest regimes.

Diagram (8.1). The Pseudo-Production Function



Solow (1956, p.101) commented on this rather peculiar result, arguing that "from the point of view of production, two identical plants represent two identical plants." Champerowne (1953-54) was also worried about this particular feature of the pseudo-production function and, not

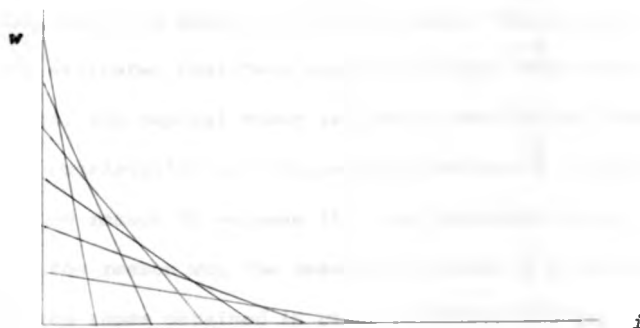
heeding Joan Robinson's warnings,<sup>3</sup> he proceeded to chain-link the capital values and treated the function 'as if' substitution were possible. The result was the more usual linear programming form OABCD in Diagram (8.1). Harcourt (1972, p.32) neatly summarises the result: "In effect Champernowne has removed the 'sigs' - the horizontal stretches - from Joan Robinson's real-factor-ratio . . . ., and changed the slopes of the 'sags' - the upward sloping stretches - so that they now equal the relevant equilibrium values of the 'price' of 'capital'."

As Champernowne astutely realised, chain-linking produced the desired characteristics in the manipulated function only so long as: (a) the particular technique that proved to be the most profitable at a given rate of interest (or range of rates) did not reappear at another rate (or range of rates)- and (b) of two techniques that proved equally profitable at a given rate of interest, the technique with the lower capital intensity and output per head proved the more profitable at a higher rate of interest. Under these assumptions Champernowne was able to translate the production function OABDC into a well-behaved wage-interest frontier.

Samuelson (1962) chose to work directly in the wage-interest space. In a world characterised by heterogeneous capital goods, he allowed technologies drawn from the 'book of blueprints' to be dominated by technologies that had wage-interest lines lying further to the north-east in the wage-interest space. Under a number of assumptions (the stringency of which was not immediately recognised), the wage-interest frontier was shown to be well-behaved, as illustrated in Diagram (8.2)

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3. For example, Joan Robinson (1970, p.103-4) warned that, "To move from one point to another we would have either to rewrite past history or to embark upon a long future."

Diagram (8.2). A Well-Behaved Wage-Interest Frontier

Each wage-rental line can be represented by an equation of the form:

$$w = \frac{1}{\xi^L} - \frac{\xi^K}{\xi^L} i \quad \dots (8.5)$$

where  $\xi^K$  and  $\xi^L$  denote input per unit of output coefficients for capital and labour respectively. Using these equations it is possible to translate directly from the wage-interest to the input-output space, and vice versa, remembering only that as we move around a wage-interest frontier we move from one equation to another. Harcourt (1972, pp.131-44) has demonstrated that the Samuelson model satisfies the four neoclassical postulates: (a) lower rates of profits are associated with higher capital per man; (b) lower rates of profits are associated with higher capital-output ratios; (c) lower rates of profits imply higher sustainable steady-states of consumption per head; and (d) under competitive conditions, the distribution of income between profit receivers and wage earners can be explained by a knowledge of marginal products and factor supplies.

Thus, so long as the appropriate surrogate measure of capital was used ( $K = -L \frac{dw}{di}$ ), the Samuelson approach resulted in a surrogate production function:

$$Y = F(K, L) \quad \dots (8.6)$$

possessing the desired neoclassical properties. Had even this event passed without criticism, the fact would have remained that this result is a long way from being a justification of the aggregate production function estimates that have appeared in the literature. To take just one aspect, the capital stock variables used in the literature are not measured consistently with Samuelson's surrogate variable. In addition, there is no reason to suppose that the surrogate function can throw any light on the reason why the results obtained in time-series studies differ from those obtained in cross-sectional studies. In practice the surrogate production function has been discredited on theoretical grounds. An important deficiency of it is that it is a long-run equilibrium position and no attempt has been made to analyse the way in which the economy adjusts towards this equilibrium. The most telling criticism that has appeared in the literature has concerned the assumption of wage-interest lines.

The wage-interest function becomes concave to the origin when the capital good activity is more capital intensive than the consumption good activity (and vice versa for convexity). This immediately implies the demise of the link with marginal productivity theory. In addition, however, non-linear relationships give rise to the possibility of capital reversing and capital reswitching. These two phenomena can arise if the two assumptions made by Champernowne do not hold. In this case, the unique monotonic relationships between  $w$  and  $K/L$  and between  $i$  and  $K/L$  assumed in Samuelson's parable are no longer valid, and a surrogate production function with neoclassical properties can no longer be isolated. <sup>4</sup>

Capital reversing and reswitching are generally associated with the fact that physical capital can be valued differently under different factor or product price regimes. Reversing and reswitching may prove to be common events in a world of heterogeneous capital goods characterised

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4. See Harcourt (1972, pp 139-149) for a more detailed discussion.

by a variety of time patterns of costs and revenues. It seems all the more surprising, therefore, that Sato (1974, pp. 354-5) should have claimed that, by the inclusion of a technology frontier in this type of model, the neoclassical postulate is "unqualifiedly valid when certain plausible conditions are met by the technology frontier." The Sato approach may play an important role in an integrated production system of the Johansen (1972) kind, but it does not (as Sato claims) provide an all-embracing answer to the 'Cambridge controversy'. The reason is simply that Sato (1974, p. 357) assumes that capital is measured in physical units and this places important aspects of the reswitching and reversing phenomena 'out of bounds'. In fact, dealing in a world of instantaneous capital measured in physical units, the neoclassical postulate was never in any real danger: it is the difference in valuation of capital in different wage-interest regimes that is the underlying cause of the phenomena.

A paper by Pyatt (1963) has suggested that it is possible to move from a putty-clay world<sup>5</sup> to a surrogate production function of the Samuelson kind by a more direct route. Pyatt claims that the main advantage of this approach is that it does not depend on the existence of equilibrium market conditions or of any particular theory of income distribution. It does, however, rely on profit maximising behaviour. From a vintage starting point it works its way to a growth accounting equation similar to the one that appeared in Solow's (1957) pioneering paper,

$$\frac{\dot{Y}}{Y} = \frac{1}{pY} \left( \frac{\dot{K}}{K} \right) + \frac{wL}{pY} \left( \frac{\dot{L}}{L} \right) \quad \dots (8.7)$$

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5. In a later paper, Pyatt (1964) has demonstrated that the same result can be shown for a putty-putty vintage world.



where  $\dot{\phantom{x}}$  denotes time derivatives and  $\kappa$  is a surrogate measure of the capital input, such that:

$$\frac{\dot{\kappa}}{\kappa} = \frac{\dot{pY} - wL}{\Pi} \quad \dots (8.8)$$

where  $\Pi$  denotes total profit. There is no doubt that the empirical results in the literature are not based on such a measure of the capital input and this approach cannot be used as an explanation of the Cobb-Douglas result. Finally, no investigation of the properties of the surrogate production function was undertaken and no evidence exists to suggest that it could provide any insights about the divergence of time-series and cross-sectional results.

The Houthakker-Johansen Approach. An alternative approach that may provide a justification for the neoclassical production function arises from work undertaken originally by Houthakker (1955-56) and extended by Johansen (1972). The basic model assumes the existence of a large number of independent fixed coefficient production units. Each production unit is a substitute for every other unit in the sense that it produces an identical output. The total of such units is the 'region of positive capacity', which is divided into two half-spaces by a separating plane formed by the zero quasi-rent line. The half-space comprised of the economically viable production units is called the 'utilised region'.

For any given zero quasi-rent line, the points within the region can be summed to form  $Y$ ,  $K$  and  $L$  (in the two input cases) where inputs are measured in physical units. Here, in order to provide consistency with the remainder of the analysis, capital is treated as a current input. Johansen, however, finds it more meaningful to assume capital is a special, fixed factor.

The relationship,  $Y = F(K,L)$ , is called the short-run macro function.<sup>6</sup> Problems of aggregation are avoided simply by assuming that a sufficiently large number of input categories are distinguished to ensure that each is internally homogeneous - see Hahn and Matthews (1964, p.110). The exact magnitudes of the aggregate variables are determined by the distribution of established capacity in the 'utilised region'. As the zero quasi-rent line swings or shifts, the 'utilised region' changes, admitting new production units (with different technical characteristics) and evicting existing units. The result is that  $Y$ ,  $K$  and  $L$  change.

Johansen (1972, pp. 52-62) has shown that, if output is held constant but the zero quasi-rent line is allowed to shift, the aggregate production isoquant:

$$\xi^K = g(\xi^L) \quad \dots (8.9)$$

is consistent with the basic neoclassical postulate. That is, as the line swings in the direction of higher wage-rental values, relatively labour intensive units leave the utilised region and relatively capital intensive units enter. Indeed, with full mathematical rigour, Johansen was able to show that  $w$  and  $r$  are the marginal products of this aggregate function:

$$\frac{\partial Y}{\partial K} = r \quad \text{and} \quad \frac{\partial Y}{\partial L} = w \quad \dots (8.10)$$

where the slope of the isoquant is given by:

$$\left. \frac{dL}{dK} \right|_{dY=0} = -\frac{r}{w} \quad \dots (8.11)$$

As an explanation of the Cobb-Douglas empirical phenomenon, however, this type of approach has important deficiencies. First, the short-run macro function exhibits decreasing returns to scale at all levels of

6. It is macro in the sense that aggregation has been undertaken wherever feasible.

activity. Second, the elasticity of substitution between factors is determined by a number of features of the technology: (a) the variety of technologies at the margin; (b) the absolute level of operations; and (c) the amount of production capacity located in a small strip close to the zero quasi-rent line, and there is no immediately apparent reason why the elasticity of substitution should be unity. Finally, the capital input is measured in real terms and this avoids problems of revaluation as the zero quasi-rent line shifts. If capital were measured in value terms, then as the quasi-rent line shifted the distribution of points in the input per unit of output space could change as capital was revalued. Under these circumstances there is no reason why the isoquant should retain the desired neoclassical properties.

The Cobb-Douglas Function: An Empirical Accident

A number of authors have attempted to demonstrate that the Cobb-Douglas function is an empirical accident that is largely unconnected with the technology of production. Brown (1957, pp. 552-57), for example,<sup>7</sup> has produced evidence about a cross-section of Australian industries in 1912. He found that points in the input-output space fell within a narrow tube with a 45° slope to each co-ordinate plane. The explanation for this was that inputs and outputs had changed at such the same rate in all industries. This feature of the data was sufficient to ensure that constant returns to scale existed in the estimated function. In addition, comparison of the accounting identity:

$$p_j Y_j = r_j K_j + w_j L_j \quad \dots (8.12)$$

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7. Similar reasoning is employed by Heathfield (1972b), although, in this case, the explanation turns on the time series trends in the variables. See also Menderhausen (1938).

with a Cobb-Douglas function of the form:

$$Y_j = AK_j^\alpha L_j^{1-\alpha} \quad \dots (8.13)$$

revealed that the estimated coefficients  $\alpha$  and  $1-\alpha$  would reflect factor shares:

$$\alpha = \frac{r_j K_j}{p_j Y_j} \quad \text{and} \quad 1-\alpha = \frac{w_j L_j}{p_j Y_j} \quad \dots (8.14)$$

as long as factor prices did not vary greatly across industries.

The explanation has been revised by Cramer (1969, pp.236-7). He argued that, because the accounting tautology holds, any function fitted to the set of points  $Y, K, L$  will tend to give a good statistical fit. More importantly, however, he argued that, when there was not a great deal of variation in the sample, the log-linear function could be manipulated into linear form:

$$Y_j = \alpha \frac{\bar{Y}}{\bar{K}} K_j + \beta \frac{\bar{Y}}{\bar{L}} L_j + (1-\alpha-\beta)\bar{Y} \quad \dots (8.15)$$

where  $\bar{Y}, \bar{K}$  and  $\bar{L}$  denote geometric sample means. Comparison with the accounting equation indicated that

$$\alpha \frac{\bar{Y}}{\bar{K}} = \frac{r}{p} \quad \text{or} \quad \alpha = \frac{r\bar{K}}{p\bar{Y}}$$

$$\beta \frac{\bar{Y}}{\bar{L}} = \frac{w}{p} \quad \text{or} \quad \beta = \frac{w\bar{L}}{p\bar{Y}}$$

$$(1-\alpha-\beta)\bar{Y} = 0 \quad \text{or} \quad \alpha+\beta = 1 \quad \text{where} \quad \bar{Y} > 0$$

Sufficient variation in the data is essential if the results derived are to be meaningful. Johansen (1972, p. 184) has pointed out that the very act of stratifying data in a technically meaningful way may cut down the degree of variation to a level that is insufficient to reveal the true underlying technology. No one, it appears, has tested the degree of variation that is sufficiently small to cause the Brown-Cramer result. One suspects, however, that the degree of variation in the data used in a number of studies has been considerable and that some additional explanation is required.

Fisher (1971) attempted to isolate the reason why aggregate Cobb-Douglas functions performed well even though they could not be rigorously justified. This involved a large number of simulation experiments, fitting Cobb-Douglas functions to data that, when aggregated, were known to be inconsistent with this technology. The inappropriateness of the aggregate data was ensured by violating the conditions for the existence of an aggregate capital stock.

The estimates were based on time-series information, but the conclusions are nonetheless relevant to this study. Without exception the reported functions generated high  $R^2$ , but this was simply the result of the variables trending over time. Under these circumstances, an aggregate function tends to fit well whether or not it is misspecified. As  $R^2$  was not particularly sensitive and, because interest centred on the explanation of wages, a root mean square measure of the deviation of actual from predicted wages was chosen as a measure of performance.

Over 830 experiments were carried out and the key result established was that the success of the aggregate Cobb-Douglas function was the result of the relative constancy of labour's share. Fisher

(1971, p. 307) noted that, "The point of our results, however, is not that an aggregate Cobb-Douglas function fails to work well when labour's share ceases to be roughly constant, it is that an aggregate Cobb-Douglas function will continue to work well so long as labour's share continues to be roughly constant, even though that rough constancy is not itself a consequence of the economy having a technology that is truly summarised by an aggregate Cobb-Douglas."

The Fisher (1971) paper provides a useful piece of information: that we may expect a Cobb-Douglas function to arise when factor shares are roughly constant, whether or not the technology of production in the economy is Cobb-Douglas. We are left wondering, however, what part is played by the underlying neoclassical technologies. Would the results have been consistent with an aggregate Cobb-Douglas function if the micro technologies had been fixed coefficient? It seems likely, for example, that the further we move away from neoclassical micro technologies, the less certain it is that constant factor shares are sufficient to ensure the good empirical performance of the aggregate relationship.

### C. The Surrogate Production Function: An Alternative View

An attempt is made here to combine some of the more revealing parts of the existing explanations of the good empirical performance of the Cobb-Douglas function. The principal aim is to explain why cross-sectional observations are often consistent with a Cobb-Douglas function exhibiting constant returns to scale. The argument put forward here is that this result arises because of the way in which the inputs have been measured in most empirical studies. It is argued that the peculiar use of the value of capital to represent the capital input - called by Joan Robinson (1970) the "strangest part of the whole affair" - provides a valuable clue to a viable explanation of the neoclassical result.

The starting point is the accounting tautology, which forms the basis of Cramer's (1969) explanation of the Cobb-Douglas result. Only aggregate inputs,  $K$  and  $L$ , are distinguished because it is believed that it is on the basis of accounting information of this kind that firms make their more important decisions. An accounting relationship is assumed to exist for each firm in a particular industry, which is made up from a large number of firms producing identical outputs that are sold in a single product market. The output of the industry is produced by firms in a number of different regions experiencing different factor prices. These assumptions are combined with two further theories about firm behaviour: first that firms attempt to relocate themselves in order to produce at lowest cost per unit of output, and second, that, where relocation does not reduce unit costs sufficiently for firms to remain competitive, the firms in question can undertake defensive innovation.

We adopt a surrogate approach directly in the input-output space rather than in the factor-price space—as in the case of Samuelson's (1962) work. The surrogate arises at the accounting-technology interface under the assumptions made above. In this way we derive a long-run frontier function with neoclassical characteristics. These conditions are, however, consistent with a wide variety of functions, only one of which will be the Cobb-Douglas function.

In order to derive a Cobb-Douglas function from the neoclassical alternatives it is necessary to make a further assumption. Fisher's (1971) study indicated that a set of institutional forces that keep factor shares reasonably constant could give rise to the good empirical performance of the Cobb-Douglas function. In the context of this study the assumption of constant factor shares is sufficient to collapse the wide variety of production functions to the Cobb-Douglas formulation. Whether this assumption is justifiable is a matter for empirical verification.

Allen (1968) and Fisher (1969) have shown that it is a simple matter to aggregate using arithmetic sums only if the underlying neoclassical functions are Cobb-Douglas with constant-returns to scale, where capital inputs differ only through the impact of disembodied technical changes. These conditions can now be shown to be satisfied, allowing aggregation over industries even given the way particular inputs are aggregated in official statistics.

#### The Accounting Identity

Cramer (1969) suggested that we should look more closely at the accounting identity:

$$P_j Y_j = i_j K_j + w_j L_j \quad \dots (8.17)$$

where  $K$  is an accounting measure of the capital stock. His reason was that not only would a large number of functions involving  $Y$ ,  $K$  and  $L$  tend to fit the data well, but, in addition, a lack of variation in the data would cause the coefficients from a log-linear formulation (such as the Cobb-Douglas function) to be equal to factor shares. There is no evidence regarding what degree of variation is required to break this link, but here, for the sake of argument, it is assumed that a great deal of variation does exist.

A number of important assumptions are made. First, all firms belong to the same industry and sell their final produce in a single, highly competitive market. Second, for one reason or another, firms experience a wide variety of wage-interest values. One cause of such diversity may be the existence of production units in a number of different geographical regions. Finally, it is also assumed that factor markets are highly competitive. In the case of regional factor markets we are simply implying that within each market there exist a large number of buyers



(production units) and sellers of factor services, forcing abnormal profits down to zero in long-run equilibrium.

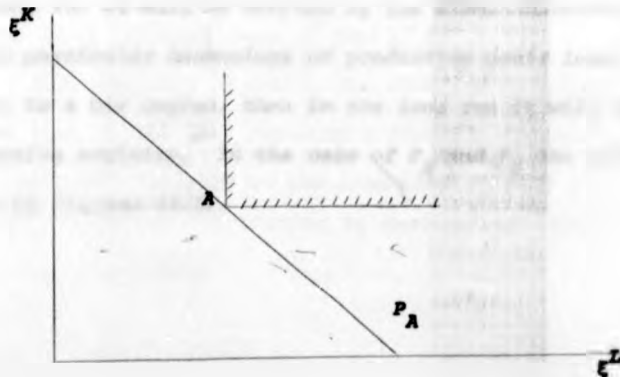
On the basis of these assumptions, we can rewrite equation (8.17) omitting the subscript  $j$  on price. Hence

$$p = i_j \frac{K_j}{Y_j} + w_j \frac{L_j}{Y_j} = i_j \zeta_j^K + w_j \zeta_j^L \quad \dots (8.18)$$

If we draw the price line associated with a given firm,  $F_A$ , in the input per unit of output space, it will appear as  $P_A$  in Diagram (8.3). The price line is denoted by  $P$ , and a point drawn from that line is denoted by  $p$ . Firm  $F_A$  chooses to produce at point  $A$  on the price line  $P_A$  defined by

$p = i_A \zeta_A^K + w_A \zeta_A^L$ . All the points in the rectangular quadrant to the north-east of  $A$  are revealed to be less preferred than point  $A$ . All such points can be attained without having to introduce real technical changes, because they can be reached simply by hoarding more of one or both inputs. Given the factor costs that  $F_A$  faces, all such points would involve higher costs of production per unit of output and would be inconsistent with the accounting identity. We know nothing about any of the points lying between

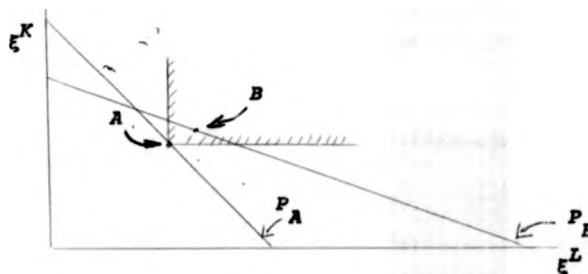
Diagram (8.3) Plant Equilibrium in the Input Per Unit Output Space



the quadrant with corner point A and the horizontal and vertical axes. All we can say is that the set of such points is technically different from the set in the quadrant with corner point A and that a subset of them will be technically superior.

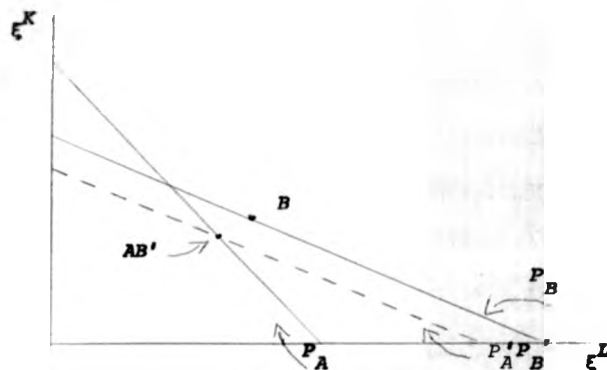
There is nothing to prevent the existence of a plant that has lower technical efficiency (i.e. uses more of both inputs per unit of output) but produces in a region with lower factor costs. Such a firm could still satisfy the accounting identity. Diagram (8.4) illustrates this case where  $P_A = P_B = P$ . In this way it is possible for all points in the input per unit of output space to be feasible.

Diagram (8.4) Existence of Technically Inferior Production Units



Domination and the Long-Run Equilibrium Position

We now impose the condition that, if an existing technology of production would lower the costs of production for a given firm, then in the long run it will be adopted by the firm. Alternatively, if a firm with a particular technology of production could lower its unit costs by moving to a new region, then in the long run it will relocate its productive activity. In the case of  $P_A$  and  $P_B$  the adjustment is as shown in Diagram (8.5).

Diagram (8.5) Towards the Long-run Equilibrium Function

When  $F_B$  adopts the technology of  $F_A$  (or alternatively,  $F_A$  moves its more efficient technology to a region of lower resource costs) the price per unit of output falls from  $p$  to  $p'$ , where  $p > p'$ .

The nature of the equilibrium situation is fairly easy to see. At a given wage-rental ratio there can exist only one price line that corresponds to the combination of the lowest resource costs with the most efficient technology. Where lines of different slopes have points that lie inside other price lines, as indeed point A dominated point B above, further inward shifts in the price lines will occur. Only when all remaining points lie on a single boundary line, analogous to Samuelson's frontier, will all the forces for change disappear. If there exists a sufficiently wide variety of alternative wage/rental ratios, then the frontier takes on the shape of a well behaved neoclassical isoquant similar to that shown in Diagram (8.2) but with axes  $\xi^K$  and  $\xi^L$ .

This function is consistent with neoclassical theory. If we can assume that in the long run such a function evolves, then a single, 'well-behaved' function in the input per unit of output space has been derived. A direct implication is that we can transform the function from its present form

$$\xi^K = g(\xi^L) \quad \dots (8.19)$$

into a production function,

$$Y = f(K, L) \quad \dots (8.20)$$

Returns to Scale in the Surrogate Production Function

What returns to scale are consistent with the surrogate relationship? The model ensures, via its assumption of perfectly competitive factor markets within each region, that factor prices are the same for all production units in a particular region. The nature of the long-run solution ensures that there will be, at the most, only one efficient technology (defined in terms of overall inputs per unit of output -  $(\xi^K, \xi^L)$ ) for each region, which is consistent with the price condition.

Each firm can have a different cost curve in a world where firms are characterised by different heterogeneous sets of capital and labour. The model, however, imposes one common characteristic on the set of possible cost curves. All firms that are to continue in existence in the long-run (i.e. are to be represented by a point on the surrogate function) must produce at the minimum point on their average cost curve and all such minimum points must be at a common price,  $p^*$ .

The model can be demonstrated to be consistent with 'U' shaped firm cost curves. The firm equilibrium positions are shown in Diagram (8.6) and market equilibrium in Diagram (8.7).

Diagram (8.6) Firm Equilibrium Positions

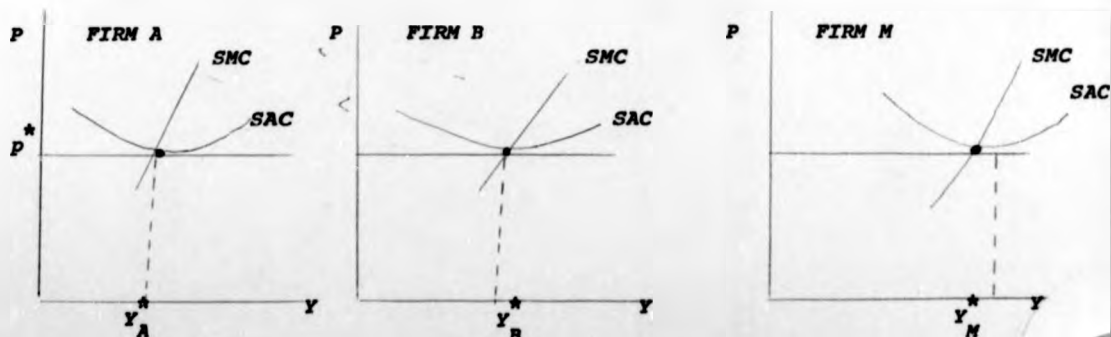
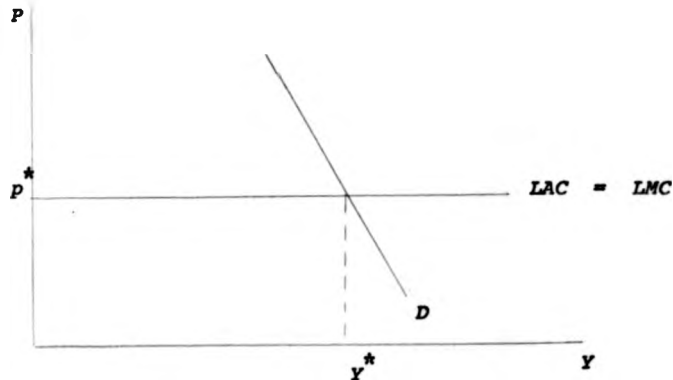


Diagram (8.7) Industry Equilibrium

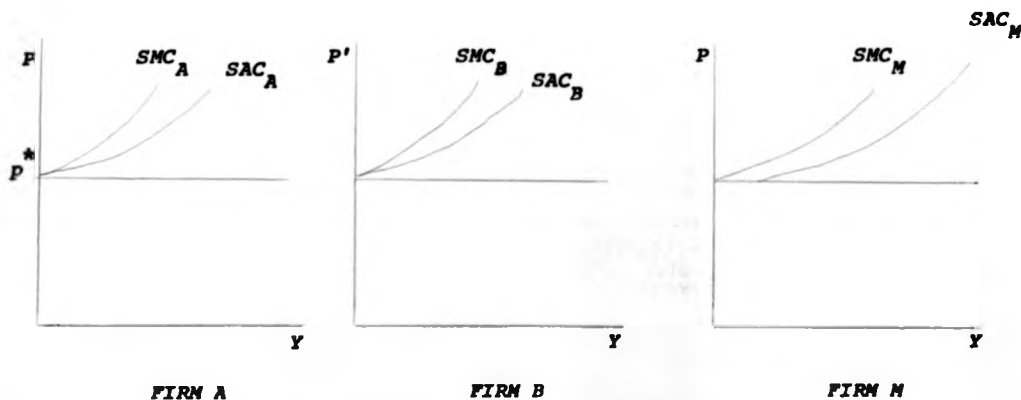
The nature of the long run equilibrium position dictates that no firm possesses the technology and management expertise necessary to enter at a lower price per unit of output than  $p^*$ , and firms attempting to establish production units at a higher price are automatically eliminated by the forces of competition. Hence, the minimum points of the short-run average cost curves, which are all at one common price, define the long-run average cost curve, which is the envelope of all such cost curves and can be represented as a horizontal line, as in Diagram (8.7). The intersection with the demand curve determines the industry output  $Y^*$ . Each firm in the industry is in long-run equilibrium as each firm is equating its marginal cost with its marginal revenue. In long-run equilibrium, each firm is at the point on its cost curve consistent with constant returns to scale.

The model obviously is inconsistent with firms whose cost curves exhibit increasing returns to scale at all levels of output. This would imply that any particular firm could always lower its price by raising its level of production. Taken to its logical conclusion, this implies that a single firm would eventually take over, and cater for the whole of market demand. The set of possible price lines consistent with the surrogate function collapses to a single line. Increasing returns to

scale over all levels of output is obviously inconsistent with perfect competition in both the product and the factor markets.

The case where firms in the industry all experience decreasing returns to scale over the whole range of possible outputs is more bizarre. The degree of peculiarity depends mainly on whether there is a positive minimum scale of operation for the firm. Diagram (8.8) demonstrates what happens when the cost curves emanate from a point on the vertical (price) axis. The industry supply curve (which is again the envelope of all the firms' supply curves) and industry equilibrium can still be represented by Diagram (8.7).

Diagram (8.8) Firm Equilibrium Under Decreasing Returns to Scale



The long-run cost curve is formed as the envelope of all the minimum points on the cost curves of individual firms. But at this price level firms in the industry produce no output. This is a particular case that is inconsistent with long-run equilibrium. In the case where there is a minimum size of firm (perhaps dictated by technical conditions) this peculiar market situation collapses to the more normal, U shaped solution. The number of firms is now dictated by the minimum size of output that firms are willing to produce. The long-run cost curves, can in both cases,

be treated as horizontal and returns to scale are constant.

The result of constant returns to scale is an important one. The existence of this feature is crucial to the ease of aggregation over industries. It cannot be emphasised too strongly, however, that its existence in this model depends on the assumptions that have been made about market structure.

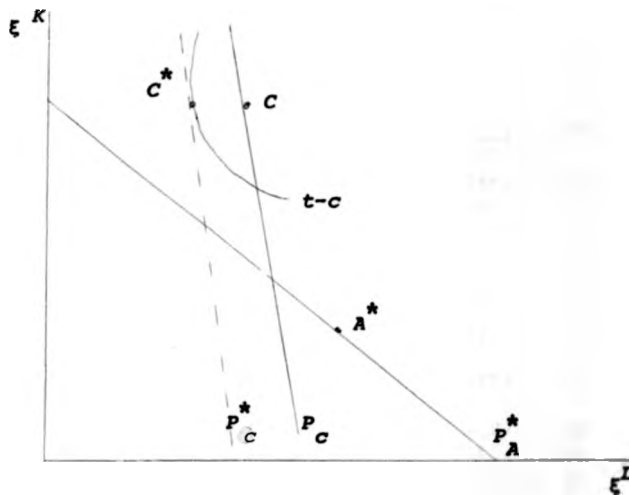
#### The Adjustment Process

Samuelson (1962) was content to establish the nature of a possible long-run equilibrium situation without describing the adjustment process that would enable it to be attained. The adjustment process is more important in the present study, because an explanation of the good empirical performance of the Cobb-Douglas function will depend on the adjustment resulting in points on a number of price lines. There is no reason, however, why the adjustment process, as we have specified it so far (relying solely on domination arising from combining efficient technologies with the cheapest factor supplies), should result in more than one price line, except by chance.

The introduction of technical change alongside the efficient use of existing resources may provide one solution to the problem. If we consider  $F_A$  and  $F_B$  again, we might assume that  $F_A$  has the more efficient technology and the higher quality of management. If  $F_B$  is unable to learn of and adopt the more efficient technology it will go out of business. Let us assume that  $F_C$  can dominate its rivals in a similar way to  $F_A$ . If the resulting price lines are  $P_C$  and  $P_A^*$ , where  $P_C > P_A^*$ , then, if further technical changes are not possible  $F_C$  will be dominated by  $F_A$  and disappear. If, however,  $F_C$  can gain the technical knowledge necessary to shift its price line inwards, and then innovate appropriately, it may yet survive. The manner in which the technical change is made is assumed

to depend on the nature of  $F_C$ 's T-isocost map,  $(t - c)$ , - see Nordhaus (1974). If each member of the T-isocost map is well behaved, equilibrium (based on the cost minimising level of R & D and innovative expenditure) will be at point  $C^*$  in Diagram (8.9), where  $P_C^* = P_A^*$ .

Diagram (8.9) Long-run Equilibrium with Technical Change



A More General Accounting Identity

In principle there is no reason why we cannot insert a more realistic accounting identity. Abnormal short-run profits are introduced, a transitory feature, and these are assumed to be bid away in the long-run. The price line for the  $j$ th firm can now be written:

$$p_j Y_j = i_j K_j + w_j L_j + \Pi_j \quad \dots (8.21)$$

or

$$p = i_j \xi_j^K + w_j \xi_j^L + \pi_j$$

where,  $\pi_j (= \Pi_j/Y_j)$  is profit per unit of output. Rather than set



transitory profits equal to zero in the long run, a normal rate of profit per unit of output can be introduced directly into the function,  $\pi^*$ . Firms will now remain in the industry in the long-run only if  $\pi_j \geq \pi^*$ , but competitive forces ensure that  $\pi_j = \pi^*$ . In this case, the long-run frontier function is simply a shifted version of its former self.

In the short run, profits add a degree of flexibility that was not there previously. For example, (a)  $\Pi > \Pi^*$  may provide an incentive for new firms to enter and for existing firms (earning less profits, i.e.  $\Pi < \Pi^*$ ) to review their current technology and location; and (b) when a firm is losing out to a technically more efficient and/or a better located competitor, a reduction in its profits such that  $\Pi < \Pi^*$  may enable the firm to remain in business, while  $\Pi > 0$  implies that the firm can finance defensive R & D and innovation.

#### Constant Factor Shares and the Cobb-Douglas Function

What we have argued is that, within a particular industry, there may exist forces that move aggregate (i.e. firm level) fixed coefficient technologies,  $(\xi^K, \xi^L)$ , towards a 'well-behaved' isoquant with constant returns to scale. We have, though, not yet succeeded in isolating a Cobb-Douglas function, because a wide variety of functions are consistent with this result. If, however, we add the assumption of institutionally determined constant relative factor shares to our 'well-behaved' function, this is sufficient to produce a Cobb-Douglas function. The realism of the assumption again turns on its empirical validity. If the model described above is correct, then a Cobb-Douglas function with constant returns to scale describes each industry. So long as this is true at some level of aggregation (no matter how detailed), the work of Fisher (1969) indicates that aggregation across industries will generally be possible and an aggregate Cobb-Douglas function will result.

The Derived Function and the Underlying Technology of Production

The movement towards a unique frontier function is associated with the profit maximising behaviour of the firm. The long-run survival of the firm is ensured only by active policies of relocation and defensive R & D and innovation. Firms work on both their factor costs and their technical efficiency in order to ensure their continued existence.

One thing is certain, however, the frontier function is not a purely technical relationship determined independently of factor prices. It cannot, for example, be compared with the ex ante function. The unit of observation is assumed to be the firm and, although the frontier function represents an efficient long-run equilibrium situation, each point on the function represents a particular mix of capital types, vintages and stages of depreciation. It is in a sense a 'best-practice' function, but one intimately connected with factor costs.

We have said very little about the underlying technology of production. The micro technologies can now be fixed coefficient. The only constraint on them is that the stocks of inputs used in production should, when aggregated according to accounting principles, sum to the values that appear in the accounting identity. It is the firm accounts that form the interface between the commercial and technical characteristics of the firm.

D. Conclusions

The review of the literature indicates that the existing theories do not give an adequate explanation of why the Cobb-Douglas function performs well empirically. Aspects of a number of the more important theories were adopted in an attempt to provide a more realistic explanation. It was argued that strategic decisions made by management would be based on aggregate accounting information. This information is in the form of key variables that appear also in traditional work on production functions. The theory developed is based on the assumption

that management attempts to maximise profits in the long-run on the basis of information that appears in the accounting identity. The existence of a Cobb-Douglas function is shown where there is a large degree of variation in the variables. Where this is not the case, the Brown-Cramer result can be expected.

In order to survive in the long run, the firm attempts to reduce factor prices to the lowest level possible even if this implies the relocation of production facilities. In addition, particularly when profits fall below some desired level, the firm is assumed to search for greater technical efficiency in its use of factors of production by adopting existing and developing new techniques. The theory suggests that in periods of prosperity ( $\pi > \pi^*$ ) firms tend to relax their search for greater efficiency and managerial slack appears. In periods of recession ( $\pi < \pi^*$ ) a stimulus appears to lower costs and raise profit levels, resulting in movement towards the long-run equilibrium function. In addition, during periods of economic hardship, the least efficient production units (which are those furthest from the 'best-practice' boundary) will tend to disappear from the system. As economic activity turns upwards, units of the latest vintage (the technologies of which are close to the frontier) will begin to appear in regions of the lowest factor prices. These factors will tend to move the distribution of points towards the 'best-practice' frontier.

The model adopted is broadly in line with Williamson's (1970) view of M-form organisations: subgoals may be followed in the short run and at low levels in the managerial hierarchy, but the overriding long-run consideration is profitability. In the model developed here, top management is assumed to be profit maximising (on the basis of inadequate

accounting information), while production managers 'satisfice' by reducing input per unit of output to acceptable levels (defined by the profit performance of the firm). There is some indication in the work of Piore (1968) and Bell (1972) that the aggregate basis on which decisions are formulated percolates through to induced technical change. Their studies indicate that broad market trends (rather than those associated with particular input categories) affect the pattern of technical change.

The 'best-practice' boundary is a surrogate production function with neoclassical properties. 'Best-practice' in this sense implies a very practical efficient boundary (unlike the Johansen long-run macro function, where all capital is moulded into the very latest vintage) where it is recognised to be efficient to hold capital of different types and ages. Constant factor shares at the micro level is sufficient to collapse the neoclassical class of functions to a Cobb-Douglas function. The existence of micro Cobb-Douglas functions exhibiting constant returns to scale and with capital measured in common (value) units is sufficient to ensure that the aggregate functions have the same form. In this way, the theory developed in this paper gives some explanation of the aggregate relationship that economists often estimate. To call this a production function, however, is extremely misleading. The stocks of physical inputs that constitute the aggregate variables may differ from firm to firm and every firm can be considered as an amalgamation of different types and ages of capital and labour. These physical aspects of production are largely obscured by the value measure adopted in the relationship. The resulting Cobb-Douglas function is consistent with a wide variety of underlying engineering functions.

Chapter (9): Production Functions, Market Structure and Industrial Performance

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A. Introduction

The review in Chapter 8 of the main arguments contained in the current literature stressed the failure to find an acceptable justification for the adoption of a simple neoclassical technology. At the end of the chapter, however, a new surrogate approach was described that resulted in a neoclassical long-run equilibrium function for the industry. The distance of an observed technology from this function is a reflection of the imperfect adjustment of firms to current resource costs and technological possibilities. The results contained in Chapter 7 emphasized the need to find variables other than capital and labour in an explanation of inter-industry variations in labour productivity. The sort of variables required are those that explain why the adjustment toward long-run equilibrium is more rapid and effective in some firms than in others.

In this chapter an attempt is made to explain the causes of deviations from the long-run equilibrium function. It is argued that, while capital and labour play a part in the explanation of inter-industry variations in labour productivity, they do not, by themselves, provide the complete picture. However, the problem of isolating other relevant variables is not wholly resolved here because this is a complicated area of research that so far has not received systematic attention in the literature. The discussion gives some insights about the way in which the theory might develop and some suggestions are made about the sorts of variables that might be relevant.

A number of the more important 'quality' and 'performance' variables are tested in this chapter. Not all of these fit neatly into a neoclassical world (ie. the average ages of capital and labour, output growth and

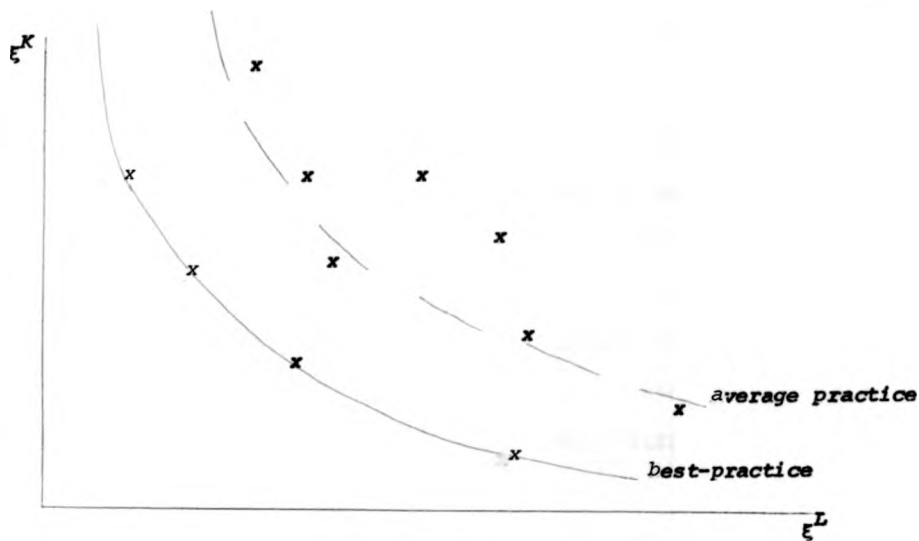
recent investment activity), some relate to aspects of the technology that have been given scant attention in the literature (ie. shiftworking, establishment and firm size) and others (ie. the importance of operations controlled by foreign firms, the levels of concentration, specialisation and exclusiveness) relate more to market structure and industrial performance than to the technology of production.

#### B. The Best Practice Function and Observed Technologies

The best-practice function is the locus of points describing technologies that are both technically and economically efficient. It is conceivable, although extremely unlikely, that a given firm could become technically 'super-efficient' (by over-investing in R & D, shiftworking, numerically controlled machines, etc.), and, thereby, incur higher costs per unit of output than firms on the best-practice function. While such a firm will lie outside of the best-practice function, it seems much more likely that the vast majority of firms will, at any given point in time, have under-invested in technology and therefore lie inside of this function. The more realistic way of treating the best-practice relationship is to view it as a frontier function.

Consideration of a given industry at one point in time will result in observing firms with different levels of technical efficiency. If no firm is technically 'super-efficient' in the sense described above, observations might show one or more firms lying close to or on the frontier, while the majority of such firms lie well inside. On the basis of these assumptions, the best-practice relationship can be found by fitting an envelope function to the available observations.

Firms have already been aggregated to industry groups in the data used by this study. Unless all firms in at least one MLH are technically and economically efficient, then it will not be possible to approximate the best-practice relationship by fitting a frontier function.

Diagram (9.1) The Best-Practice Frontier

The method adopted here follows Griliches and Ringstad (1971) in estimating an average-practice function and explaining deviations from this relationship using 'quality' or 'performance' variables. The problems of using an 'average' function to approximate a best-practice relationship have been discussed, in a slightly different context, in Bosworth (1974).

In the real world of course the best-practice function is unlikely to be static. It was argued in Chapter 8 that defensive relocation and innovation would shift observed technologies inwards until an equilibrium price resulted. Offensive R & D innovation will shift the best-practice function itself inwards. If offensive research is a feature common in industry then the inwards movement of best- and average-practice technologies will form a continuous process. Unless the progressive firms are, at least periodically, halted, enabling the less progressive to 'catch-up', then there is even less reason why the observed data should conform to a neoclassical function.

In a dynamic world at least two theories are needed: first, a theory of how the best-practice technologies move towards higher levels of efficiency; and second, a theory of how the less progressive firms adjust in order to remain competitive. Other theories may also be relevant. If a model of innovation is to be developed, it may, for example, also be necessary to say something about the causes of invention. A dynamic theory of productivity involving invention, innovation and diffusion is an entirely new ball-game.<sup>1</sup> As far as this chapter is concerned, several dynamic summary variables are introduced into the productivity relationship, but a simultaneous system of equations that includes models of R & D, innovation and diffusion is not developed.

#### C. The Determinants of Firm Efficiency

It seems natural to divide the new explanatory variables into two groups: the first dealing with static explanations of deviations from the best-practice function; the second attempting to summarise the rate at which the hypothetical best-practice function shifts inwards. While the static theory takes the best-practice function as given in the long-run, the more dynamic theory allows the position (if not the shape) of the best-practice function to change with the passage of time.

#### Some Variables From a Static World

The variables that intuition suggests may be important can be divided into technical, financial and managerial. Although some variables do not fit neatly into particular categories, the following variables are distinguished:

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1. Work along these lines is currently being undertaken, see Bosworth and Wilson (1976d).



(a) technical and financial - plant size, firm size, the degree of specialisation, age of capital, age of employees and the amount of shiftworking;

(b) managerial - concentration, exclusiveness and foreign control.

Plant Size. Economic theory has put forward the idea that production activities may be characterised by increasing, constant or decreasing returns to scale. The examples of technical economies proposed in the literature are concerned with the sizes of particular pieces of equipment, overcoming indivisibilities or obtaining economies of massed resources. The production relationship, as it was estimated in Chapter 7, shows economies associated with the scale of the industry. It is possible, however, for large MLHs to have a large number of small establishments and for industry level economies to be unrelated to the micro, plant level economies of scale. In order to take the micro-economies of scale into account, an employment based measure of average plant size is included in the regression equation.

Firm Size. In the same way, the potential technical and financial economies of firm size suggest that a measure of average firm size should be included in the regression equation. There are economies of management, R & D and sales promotion that can be achieved as firm size grows. This may be even more true of multiplant firms (ie. in industries where the number of plants and the number of firms diverge) where there are economies of centralised management, R & D and marketing. Of course, the literature suggests that it is possible to allow firms to expand to such a size that diseconomies of scale begin to outweigh the economies - see, for example, Williamson's (1967) discussion of 'control-loss'.

Specialisation. Adam Smith's discussion of the gains from specialisation in the tasks performed by both capital and labour, and the consequent improvements in their productivity, were again about particular micro phenomena. The measures of specialisation within an industry are only

loosely connected with the ideas put forward by Smith. The measure of specialisation generally used in the literature, and incorporated in the productivity relationship here, is the percentage of the industry's output that is the principal product of the industry. The larger the percentage value, the more specialised are the firms in the industry and the greater the potential gains in productivity.

Age of Capital. If the neoclassical function is seen as a long-run equilibrium situation, then, in a vintage world, short-run deviations from this function may be caused by non-optimal age structures of the capital stock. A simple vintage approach suggests that older capital will generally be less well adjusted to current factor prices and of a lower technical efficiency. However, the best-practice function reported in Chapter 8 did not simply describe the most modern technologies, (i.e. it was not an ex ante function). If it can be argued that the optimal situation includes older vintages, it then becomes possible for some firms to be 'super-efficient' and possess technologies lying outside of the best-practice function. In practice, however, the majority of firms are expected to lie inside the function, and, the greater the average age of capital, the further inside the function the technology will lie. Hence, an average age variable is included in the regression equation. The variable used has several deficiencies: first, it relates to the average age of machine tools and not to the stock of capital as a whole; and second, the efficiency of the firm may vary with the age structure of the capital stock for any given average age of capital.

Age of Employees. It is perhaps taking the analogy with physical capital too far, see Blaug (1970, pp. 121-8 ), but it seems natural to test whether the age of the work-force affects its productivity. It might be argued, for example, that over the years, the standard of education has improved and that the topics taught have become more closely orientated

with work-place activities. If this is the case then the younger, more recently trained employees may be expected to be relatively more productive than older workers. On the other hand, it might be argued that the level and quality of education has not altered greatly over time, but that learning by doing - see Arrow ( 1962 ) and Fellner ( 1969 ) is a significant factor in labour productivity. If this is the case, then older workers, with their greater on-the-job experience, should be more productive than their younger counterparts. An average age of labour variable is included in the regression equation without any a priori expectations about its sign.

Shiftworking. The shiftworking variable is included because there may exist a particular type of shiftworking system that, in the light of factor rewards and technical efficiencies, is most efficient. It may pay to employ a certain percentage of the work-force on a shift basis for a number of reasons: maintenance operations are often easier when some of the machines lie idle; specialist numerically controlled machines are often only economic if they are run continuously; smaller amounts of fixed assets (ie. buildings and land) are required per unit of output where a shift system is in operation; and there may be some type and amount of shiftworking that is best suited to the human clock (see the growing science of the human body and the concept of 'circadian time'). If there is an optimal amount of shiftworking within engineering then the inclusion of the percentage of employees working shifts in the regression equation may pick up this effect.

Concentration Ratio. It is often argued that market structure is an important factor in determining the amount of pressure put on management to seek technical efficiency and economic viability. Market power is taken to imply greater profit potential and, because it is associated with less consumer sovereignty, management has the ability to divert

potential profits into non-pecuniary management remunerations.

Liebenstein's (1969) concept of 'inert areas' suggests that the room for manouvre by management is much smaller for firms in competitive markets than for firms with greater monopoly power. The pressure on management to keep X-inefficiency to a minimum depends on other factors as well, for example, the degree to which shareholders attempt to maximise their dividends and their ability to push management in this direction. Static theory suggests that the larger the firm and greater the level of market power the further inside the best-practice function the firm will lie. There are important problems of measuring market power - see Utton (1970) but an attempt is made to take this factor into account by including a five firm concentration ratio (based on employment data) in the regression equation.

Exclusiveness. It is possible for firms in an industry that has its work-force concentrated in the largest five firms to have little or no market power. Apart from the problems associated with using measures of concentration based on employment data and the inadequate nature of concentration ratios, market power also depends on how much of the market demand is met by firms in the industry and how much by firms allocated to other industries or producing from other countries. No attempt is made to take the international aspect into account. This would not only involve attempting to assess the competition from imported goods in UK markets but also the competition faced by UK firms when exporting. The problem of competition from UK firms allocated to other industries can be taken into account by including a measure of exclusiveness - the percentage of the total supply of the product supplied by firms allocated to the industry in question - in the regression equation.

Foreign Control. It has often been argued that foreign controlled UK

operations are more efficient than UK controlled firms - see, for example, Dunning (1966) and Peck (1968). A number of reasons have been put forward: (a) foreign controlled firms employ more capital per worker; (b) they reap the benefits of R & D carried out within the parent company; (c) they employ a higher quality of management. The hypothesis is that industries with a greater element of foreign control should be nearer the best-practice function. The percentage of sales in each industry supplied by foreign controlled firms is included in the regression equation as an explanatory variable. Because the capital-labour ratio is included separately in the equation, this variable will tend to pick up differences in the quality of management and provide a test of whether foreign controlled firms are better managed than UK controlled firms.

#### Some Summary Variables from a Dynamic World

In discussing the sorts of variables that might explain deviations from a static, long-run, best-practice function, it was noted that a number of them had dynamic aspects. The 'foreign control' variable was one example. Caves (1968p.303), for example, has pointed to a reluctance of UK controlled firms to develop modern research establishments and to put innovations into practice. In addition, Schumpeter (1928) and Galbriath (1969) have reasoned that large firm size and high levels of concentration ('bigness and fewness') may provide the ideal environment for rapid technical change.

It was suggested in section B of this chapter that a rigorous approach to taking into account these dynamic factors might involve constructing models of invention, innovation and diffusion. For the purposes of this chapter two summary variables of a dynamic kind are included directly in the regression equation. Their inclusion is a

crude attempt to account for differences in the dynamism of firms in different MLHs. The two variables are: recent output growth and recent investment activity.

Recent Output Growth. Verdoorn's Law is a well established empirical relationship: faster growing firms or industries can expect faster growing productivity. Despite an abundance of empirical support, the underlying causes of the relationship have not yet been established, and much more work needs to be carried out in this area. One particular problem that needs solving concerns the direction of causality: is it the growth in output that causes productivity to change, or is it the productivity growth that raises the demand for the product and thereby the level of output? Even if the direction of causality normally adopted in the literature - from output growth to productivity change - is assumed to be correct the reasons for the existence of the relationship remain complex and obscure.

The explanation that is linked most closely with vintage theory has been proposed by Wabe (1974) and crudely tested by Helps (1974). It is argued that faster growing industries are the heaviest net investors in new capital and are thereby reaping the gains from embodied technical change. By the same token, industries that are contracting are scrapping their most inefficient capital and obtaining increases in productivity from this source. If this explanation is valid, a U-shaped relationship between the rate of growth in output and growth in productivity may be expected. As a first approximation, the absolute rate of growth in output is included in the regression equation as an explanatory variable. This measure constrains the U-shape to be symmetrical. It is expected that the higher the recent growth in output in an industry, the closer the industry will be to the best-practice function.

Recent Investment Activity. Uncertainty about the embodied technical change explanation of the Verdoorn relationship suggested that it would be interesting to include an investment variable directly in the regression equation. If the investment interpretation is correct there should be a strong correlation between the rate of growth of output and the level of investment activity, while the inclusion of the investment variable will not add significantly to the explanatory power of the model. The investment variable was constructed as the absolute sum of recent net investment, normalised for the size of the capital stock (based on the amount of fuel consumed).

#### D. The Specification of the Model

The inclusion of 'quality' and 'performance' variables gives rise to additional problems of measurement, aggregation and specification. Given the problems of a rigorous treatment of the problem, it seems best at this stage, to adopt a heroic stance. Broadly following Griliches and Ringstad (1971), the functions reported below are modifications of the basic Cobb-Douglas form,

$$\frac{Y}{L} = A \left( \frac{K}{L} \right)^{\alpha} L^{(\alpha+\beta-1)} \pi^{\rho} Q_1^{Q_2} Q_3^{Q_4} Q_5^{Q_6} Q_7^{Q_8} Q_9^{Q_{10}} Q_{11}^{Q_{11}} \quad \dots (9.1)$$

where  $Q_1, \dots, Q_{11}$  represent respectively: the average age of capital; average age of labour; percent of employees working shifts; establishment size; firm size; percentage of sales by foreign controlled forms; five firm concentration ratio; specialisation ratio; exclusiveness ratio; recent growth performance; and recent investment activity.

#### E. Sources of Data and Measurement of the Explanatory Variables

Information of the type required here is generally only available from the Census of Production. The estimates reported therefore relate to two annual cross-sections, 1963 and 1968. Only the data on

shiftworking and the average ages of capital and labour are from other sources and refer to different points in time. Shiftworking data is available for the years 1964 and 1968 from the Ministry of Labour Gazette and the New Earnings Survey respectively. Average age data is published for 1961, 1966 and 1971; the age of capital data is from the Census of Metal Working Machine Tools and the age of labour information is from the Census of Population. The shiftworking data was left unchanged as the years complied reasonably with those of the quinquennial censuses but the average age data was interpolated linearly to provide estimates for the census years.

Given the basic theme of the study, it seemed interesting to focus on three variables: the average age of capital, recent output growth and investment activity. All three variables can be given a vintage interpretation. Older capital, for example, will tend to be less efficient and associated with lower levels of labour productivity. The average age of capital is associated with important problems of measurement: its construction, which follows the work of Bacon and Eltis (1974), is based on stringent assumptions (e.g. a constant rate of growth in investment) that are made necessary by the data constraints; the data relates only to machine tools; finally, a given average age may be associated with various distributions of age, which will have different average efficiencies of capital. The output growth variable is linked with the vintage approach in the way described in Chapter 3. In this study the growth phenomenon is assumed to be symmetrical around  $\Delta Y/Y = 0$  and thus  $|\Delta Y/Y|$  is selected as the explanatory variable where the change is calculated over the five years preceding the census ( $\Delta Y_{63-58}/Y_{58}$  and  $\Delta Y_{68-63}/Y_{63}$ ). In a similar way, the recent investment activity variable is constructed as the cumulative sum of net investment over the previous five years deflated by the amount of fuel consumed in the census year



( $\Sigma_{t=58}^{63} IN/F$  and  $\Sigma_{t=63}^{68} IN/F$  ). The variable represents an attempt

to measure the amount of investment activity relative to the existing capital stock.

#### F. Results

The three variables that could be given a vintage interpretation were initially tested separately. To begin with these variables were treated as alternative measures of the same phenomenon and the results are reported in Table (9.1). Although a great deal of the variation in labour productivity is left to be explained, a vintage effect does appear to exist. The average age variable, regression (i) of Table (9.1), has the expected sign (ie. older vintages are less efficient) and comes close to being significant in 1968 when a slightly more reliable measure of age is available, but the additional variable does not radically change the overall fit of the function. Regressions (ii) and (iii), where  $\rho$  relates to output growth and investment activity respectively, show a considerable improvement:  $\bar{R}^2$  is higher and the F statistic is significant at the 1% level in three of the four cases.

Regressions (i) in Table (9.2) show the impact of the eleven 'quality' variables when they all appear together. The overall fit of the function improves on the results reported in Table (9.1) in both years, although the function performs better in 1968 than in 1963.  $\bar{R}^2$  is higher in 1963 than in the case discussed earlier, but the F statistic is only significant at the 5% level. In 1968, however, five of the eleven 'quality' variables are significant at the 1% level and one at the 5% level, while  $\bar{R}^2$  reaches the respectable figure of 0.594 and F is significant at the 1% level. The general significance of these variables in 1968 can be contrasted with the results found by Griliches and Ringstad

(1971).

It was suggested earlier that the average age of capital, output growth and investment activity variables were alternative summary measures of any vintage effects. Treating them as alternatives, regressions (ii), (iii) and (iv) adopt the average age of capital, output growth and investment activity respectively as the summary variables (and exclude the other two alternatives). The effect in every case is to lower  $\bar{R}^2$  and the  $F$  statistic is now only significant in regressions (iii), where the output growth variable appears. The significance of the estimated coefficients are otherwise very similar to those described above. The output growth variable appears to play an extremely important part in explaining cross-industry variations in labour productivity.

Table (9.1) Quality Variables

Regression	Year	A	$\alpha$	$\alpha+\beta-1$	$\rho$	$\bar{R}^2$	F
(i)	1963	3.4285 <sup>*</sup>	0.2937 <sup>†</sup>	0.0510	-0.6024	0.074	1.80
(ii)		1.2470 <sup>**</sup>	0.1968 <sup>*</sup>	0.0622	0.1659 <sup>**</sup>	0.275	4.92 <sup>**</sup>
(iii)		2.7534 <sup>**</sup>	0.0656	-0.3622	0.3897 <sup>**</sup>	0.400	7.43 <sup>**</sup>
(i)	1968	4.0386 <sup>**</sup>	0.1640	0.0867 <sup>†</sup>	-0.8429	0.115	2.30
(ii)		1.4220 <sup>**</sup>	0.1928	0.0797	0.1321	0.118	2.38
(iii)		2.5058 <sup>**</sup>	0.0590	-0.2844 <sup>*</sup>	0.3671 <sup>**</sup>	0.267	4.53 <sup>**</sup>

The most striking difference between the results for 1963 and 1968 is the general significance of the coefficients in 1968, but their insignificance in 1963. While the overall fit of the functions is slightly better in 1968 than in 1963, they are not radically different. Part of the explanation must therefore be found in the higher levels of multicollinearity and the slightly smaller sample size in 1963. A further explanation, that was taken up in more detail in Chapter 8 and requires much more intensive empirical testing, is that the significance of the

Table ( 9.2 )

The Explanation of Labour Productivity, Quality and Performance Variables

Regression	Year	A	a	a+B-1	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	P <sub>6</sub>	P <sub>7</sub>	P <sub>8</sub>	P <sub>9</sub>	P <sub>10</sub>	P <sub>11</sub>	R <sup>2</sup>	F
(i)	1968	0.2692	0.0697	-0.3012	-0.6652	1.3340	-0.0214	0.2519	-0.2776	0.0148	-0.1346	-0.9169	0.8006	0.1314 <sup>†</sup>	0.3407	0.416	2.53 <sup>#</sup>
(ii)		-5.1789	0.1004	0.0207	-0.8057	3.0571	0.0995	-0.0841	0.0350	0.0878 <sup>†</sup>	-0.3121	-1.1507	0.9593	-	-	0.220	1.72
(iii)		-7.3722	0.1023	0.0408	-	3.0648 <sup>#</sup>	0.0931	0.0611	-0.0863	0.0606	-0.2348	-0.5876	0.4190	0.1544 <sup>#</sup>	-	0.375	2.52 <sup>#</sup>
(iv)		4.0042	0.0559	-0.2868	-	-0.1335	0.0284	0.1677	-0.2211	0.9376	-0.0179	-0.5364	0.3686	-	0.3388	0.270	1.94
(i)	1968	-14.4212 <sup>#</sup>	0.3336 <sup>##</sup>	0.1524 <sup>##</sup>	0.9101	0.6389	0.3974 <sup>##</sup>	-1.4342 <sup>##</sup>	1.2468 <sup>##</sup>	0.0916 <sup>##</sup>	-0.0010	2.1452 <sup>#</sup>	0.0088	0.3854 <sup>##</sup>	-0.1819	0.594	4.37 <sup>##</sup>
(ii)		-5.2432	0.1495	0.0509	-0.4136	1.3013	0.2692 <sup>†</sup>	-0.6957 <sup>#</sup>	0.5873 <sup>#</sup>	0.0729 <sup>†</sup>	0.0435	0.6678	-0.0440	-	-	0.262	1.97
(iii)		-9.2783 <sup>†</sup>	0.2584 <sup>##</sup>	0.0078	-	1.1556	0.3098 <sup>##</sup>	-1.0477 <sup>##</sup>	0.9180 <sup>##</sup>	0.0658 <sup>#</sup>	-0.0009	1.3160 <sup>†</sup>	-0.0510	0.2615 <sup>##</sup>	-	0.544	4.25 <sup>##</sup>
(iv)		-4.6757	0.1418	0.0064	-	0.8981	0.2767 <sup>##</sup>	-0.7274 <sup>#</sup>	0.6023 <sup>#</sup>	0.0750 <sup>†</sup>	0.0453	0.5804	0.0302	-	0.0507	0.251	1.91

variables varies over the trade cycle. In boom years (such as 1963 for example), the market structure-industry performance variables may play a relatively small role, but in times of recession (such as 1968) they may become much more important.

Exploring the results for 1968 in slightly more detail reveals that capital and labour both play a significant part in explaining variations in labour productivity. The coefficient on the average age of capital had an unexpected positive sign, but was insignificantly different from zero. The average age of workers did not make a significant contribution although older, more experienced workers were associated with higher levels of productivity.  $\rho_3$  was significant at the 1% level indicating that shiftworking was associated with higher labour productivity. Although firm and establishment size were multicollinear,  $\rho_3$  and  $\rho_4$  were both significantly different from zero. The signs indicated that smaller establishments but larger firms were associated with higher levels of productivity. Industries with higher percentages of foreign controlled firms were significantly more efficient. Higher levels of concentration (based on a five firm concentration ratio) were associated with lower levels of productivity, but not significantly so. Coefficient  $\rho_8$ , associated with the degree of specialisation, was significant at the 5% level with the expected sign, while  $\rho_9$ , associated with the degree of exclusiveness, was insignificant. The output growth variable was significant with the expected sign, but the investment activity variable made an insignificant contribution and had the wrong sign.

For completeness, Table (9.3) gives the zero order correlation coefficients for 1968. While it must be emphasized that we may not be



talking about cause and effect, a number of the correlations appear extremely interesting. (a) The average age of capital is quite strongly correlated with the average age of labour. Industries with older capital stocks also employ a labour force whose age is higher than average. Larger establishments and firms appear to be associated with older capital. Both higher rates of output growth and greater investment activity are associated with younger capital stocks. (b) The foreign control variable is quite strongly correlated with a number of other variables. Industries with a higher degree of foreign ownership have a lower average age of capital and of labour. Such industries are more specialised and have experienced relatively high output growth, but no greater than average investment activity. (c) Higher levels of industrial concentration are found in industries with slightly above average ages of capital and labour. As we might expect, these industries have higher exclusiveness, larger firms and establishments. They are industries where the contribution of foreign firms is low and recent growth experience has been poor.

#### Conclusions

While a great deal more research needs to be done, the results of this chapter enable a number of important conclusions to be drawn. Firstly, although in Chapter 7 the basic-CES functions performed well when explaining inter-industry variations in the level of output, they give an entirely different impression when explaining labour productivity. Secondly, while the performance of the basic functions can be improved by estimating them separately for more tightly defined industry sub-groups, it is impossible to tell whether this is the result of greater similarities

of technology or simply because of the reduction in the variability of the sample. Finally, although much more theoretical and empirical work needs doing, the suggestion is that a more broadly based explanation of variations in productivity (which calls on 'quality' and 'performance' variables) is required. Chapters 8 and 9 are seen as a prelude to such a theory.

Chapter (10) Production Functions - Some Conclusions

Existing theoretical and empirical research has focused on determining the realism of the vintage and neoclassical approaches as a crucial step in the development of a realistic theory of production. It is even more important, however, to delimit the situations where a production function is an appropriate basis for empirical research. Economists have looked no further than the theory of production to explain macro relationships although it is fairly obvious that other factors are at work. While technical relationships can be expected to dominate the micro scene, aspects of market structure and industrial performance can be expected to become increasingly important at higher levels of aggregation.

A cross-sectional approach has been adopted in this study. This is generally thought to be most applicable where long-run equilibrium situations are being considered. Estimation has been undertaken for a number of consecutive years, and this has made it possible to obtain a time series of estimated coefficients. Repeated estimation of this kind allows the stability of the estimated coefficients over time to be observed. In addition, if a sufficiently long time series of coefficients can be obtained, there is the possibility of finding explanatory variables for them. This may enable future values of coefficients to be predicted and may provide the starting point for constructing simultaneous systems. It is an important criticism of most of the time-series work on production and employment functions that models are not re-estimated for subperiods in order to test the stability of the parameters, and that, in making predictions, the estimated coefficients are assumed to remain constant over quite long periods. The empirical exercise undertaken in this book can be viewed as a prelude to a more rigorous forecasting exercise where, given confidence intervals for the predicted coefficients, a range of predictions



can be made, avoiding the criticisms levelled at single-valued forecasts.

Data used in this study were drawn from the U.K. engineering industry. Engineering is a key sector of the economy and is a focal point for any comprehensive economic planning exercise. Planners at the national and at the industry level, aiming to assist market mechanisms to equilibrate supplies and demands of both products and factors, must eventually tread the tortuous paths of the production function literature. Before bodies such as the Engineering Industry Training Board can place any reliance on forecasts when formulating their plans, there must be a greater understanding of the underlying relationships and a more objective view of the ability of the various models to describe and predict.

Undoubtedly a major obstacle in the path of theoretical and empirical improvements is the lack of appropriate data. Although probably more data are available for this than for any other sector, it is by no means adequate. The problem is more acute at the micro level, but it exists also at higher levels of aggregation. The nature and prevalence of shiftworking in the U.K. is largely undocumented. It is one case where the availability of data has deteriorated over time - the more frequent observations that have become available in the 1970s are not so detailed as their less frequent predecessors were. Utilisation is another key variable about which little or nothing is known. The CBI's quarterly survey Industrial Trends can provide some insights; and, while the form taken by the questions posed in the survey is not ideal, these questions have remained unchanged for a long period and the industry detail associated with them has increased considerably with the passage of time. More detailed fuel-consumption data (for the construction of 'Wharton School' indices) and information about the installed rated wattage of plant and machinery (to enable Heathfield<sup>1</sup> type measures to be constructed) may prove even more useful. However, other improvements will be needed before a theoretically rigorous utilisation variable can be derived from available sources of data.

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1. See Heathfield (1972a).

The vintage models are the most affected by data constraints. The models tested in Chapters (5) and (6) were recognised to be simplistic. Certain modifications that intuition suggest would make the models more realistic (i.e. managerial behaviour other than profit maximisation; ex ante functions other than those with classical properties; and different rates of disembodied change for different vintages) are extremely difficult to test empirically because of the wholly inadequate data.

In the light of these data problems, the results detailed in Chapters (5) and (6) are surprisingly good. Those presented in Chapter (5) are perhaps of greatest interest to the theorist. A labour-demand function of vintage form performed best of all the alternatives tested, but because of intense multicollinearity between capital of different ages, it was not possible to establish the nature and rate of embodied technical change. The ad hoc relationship where the average age of capital appeared suggested that the relationship was characterised by Hicks neutral technical change. Perhaps the most important conclusion was that the underlying ex ante function appeared to have an elasticity less than unity, probably somewhere in the region of 0.3 to 0.5. The overall strength of the primary link between a particular labour skill and the type of capital with which it is usually associated suggested that further research along these lines would prove profitable, even if the approach gave no insights about the secondary effects (i.e. on other labour skills). Training boards are often more concerned about particular skills than about overall numbers, and establishing primary relationships of these types may prove valuable. Accurate short-run forecasts of labour requirements might be made using the established links with the available investment-intentions data (i.e. from Business Monitor). It is an admitted criticism of this approach that none of the adjustment mechanisms (which Senker et al. were worried about in their 1975 study) has yet been investigated. In addition, the forecasts, if based on investment-intentions data, would be extremely short-run and therefore of little use in formulating

any major training strategies (where up to five years may be required).

In this respect, the analysis reported in Chapter (6) may prove more appropriate. A great deal of work has been undertaken on investment functions by economists. The existing body of theory might be drawn on to make short- to medium-term forecasts of investment levels. Alternatively, the National Economic Development Office might be able to provide information about possible future rates of investment activity. Positive and normative exercises based on this approach are unlikely, however, to prove satisfactory until the census defines its variables in terms that the economist, rather than the accountant, will find useful (i.e. gross investment, net investment and scrapping). There is a further need for data about second-hand transactions (i.e. purchases of second-hand machines as well as disposals).

The theoretical and empirical chapters emphasise that certain aspects of the vintage model require attention. First, the ex ante function, in particular, needs considerable modification and improvement if it is to play the central role that vintage theory accords it. Second, there is the important question of the role played by ex post modifications to vintages. If this can be shown to be an empirically important phenomenon, it may mean the demise of the simple vintage theory as we now know it and require the researcher to review the entire history of each individual machine. Third, little or nothing is known about the costs of laying up and reintroducing machines. Fourth, it is important (as noted above) to establish the amount of trading in older vintages of equipment, which may severely complicate the ex ante relationship.

While the Johansen study (1972) was quick to emphasise that capital-labour substitution is possible even in a world characterised by fixed-coefficient micro technologies, there are other ways in which substitution can take place. A given machine and crew may be able to undertake diverse tasks. Here the roles of changes in tooling and of machine modifications appear to be extremely important areas in need of further study. In addition,

there are changes in product design - perhaps in the way in which the product is put together (i.e. cast and machined rather than machined and welded). Finally, at a more aggregate level, there are changes in product mix.

The lack of detailed information is not a valid excuse for simply estimating CES functions, or input-output models, to the exclusion of developing vintage theory and empirical work. Yet there is undoubtedly something in the argument (see Barna, 1964, p.125) that we can gain some insights from estimating more simplistic models. Aggregate input-output tables, for example, have revealed a great deal about the structure of the economy and about the links with other economies. In a similar way, CES functions have given some insights into links between firms. The theoretical analysis in Chapter (7) emphasises the danger of falling into the trap of believing these to be descriptions of the underlying technology - they mask diffuse micro production relationships.

If these relationships are not descriptions of the technology but are aggregate empirical laws, it is essential to explain why they exist. This is particularly relevant in the case of the Cobb-Douglas function. While the discussion in Chapter (8) emphasises that the fit of the function has not always been as good as the reported results (i.e.  $\bar{R}^2$  and  $F$ ) have implied, there can be no doubt that the function has often performed well. The explanation developed in Chapter (7) was that market forces may constrain the set of technologies (defined in terms of overall input-output coefficients) that can survive for any given set of product and factor prices to appear 'as if' they are drawn from a neoclassical world. However, any particular aggregate input-output coefficients ( $\xi^K, \xi^L$ ), may be associated with numerous micro technologies.

An explanation of the aggregate 'empirical law' is forced to draw on theories of firm behaviour and market structure as well as on an underlying

theory of the technology of production. It would seem sensible to suggest that the theory of firm behaviour should be one that allows managerial behaviour to modify according to economic conditions, thereby varying between 'satisficing' and profit maximising. The firms that survive in the long run are those best adapted to the economic conditions. Some evidence is given in Chapter (9) in support of this hypothesis (for instance, the industries with a high percentage of sales contributed by foreign-controlled enterprises, which qualitative evidence has suggested to be more competitive, were found to be technically more efficient). Thus, the approach points to a 'survival of the fittest' theory. Firms that are technically inefficient or whose technologies are inappropriate in the light of economic conditions are forced to make changes under their own management, to make adjustments under a new management, or to disappear from the production system entirely.

One further conclusion can be drawn from the rather inept performance of the CES class of functions reported in Chapter (7) and from the relatively more impressive performance of the 'quality' variables in Chapter (9). Not only must the new theory provide links between the theories of technology, the firm and market structure, but also they must be more dynamic. A major criticism of the work undertaken here is that too much attention has been paid to the long-run equilibrium outcome and too little to the means by which it is reached. In terms of the empirical testing of adjustment mechanisms, it may be rewarding to pool the whole body of data (as Nerlove's 1966 study suggests).

The basic conclusions of this study are that if economists are to study production functions they must look at the micro level (more in line with the so-called 'engineering functions'),<sup>2</sup> and to do this they will need much more detailed and accurate data. In addition, if the aggregate CES function is not a reflection of the underlying technology, it is not possible to continue according to its central place when teaching students about production theory. Even so, the function may continue to prove useful to forecasting exercises in so far as it comes as close as any to being an aggregate

2. See for example, Chenery (1949, 1953)

'empirical law', but it may eventually prove to be an even better tool if a viable explanation for it can be found.

Appendix (I) Estimated Investment by Type and Industry within Engineering

The principal aim of this appendix is simply to construct reasonably accurate estimates of investment by MLH for engineering based on published official sources. The investment data is classified by type and covers the period 1958-72, (increasing the coverage before 1958 involves more important changes in classification).

Investment data is available from the annual 'sample' censuses and the quinquennial censuses of production from 1948 onwards. The two sources distinguish the same types of investment goods (i.e. buildings, plant and machinery, and vehicles), but at different levels of industrial aggregation. The sample census distinguishes a maximum of eight industry groups within engineering, while the full census gives information by MLH. It is worth adding that the two censuses are collected on different sampling bases (the annual census uses the company as its sampling unit, while the full census uses the establishment) and this gives rise to some inconsistencies which are discussed in more detail below.

The industry coverage over the period is shown in Table (I.1) below.

Table (I.1)

Census	Industry Group	1958	...	1963	...	1968	1969	1970	1971	1972
Sample	Mechanical Engineering	*	*	*	}	-	*	*	*	*
	Scientific Instruments	*	*	*		-	*	*	*	*
	Electrical Engineering	*	*	*	*	-	*	*	*	*
	Shipbuilding and Marine Engineering	*	*	*	*	-	*	*	*	*
	Motor Vehicles	*	*	*	*	-	*	*	*	*
	Aircraft	*	*	*	*	-	*	*	*	*
	Metal Goods	*	*	*	*	-	*	*	*	*
	Individual MLHs	-	-	-	-	-	*	*	*	
Full	Individual MLHs	*	-	*	-	*	-	-	-	

The appearance of an asterisk (\*) denotes that an observation is available. Sufficient observations appear to exist to allow accurate estimates of investment by type to be made for each of the years 1958-72 where official statistics are not available in this detail. The remainder of this appendix reports an attempt to provide MLH estimates of investment. Their accuracy depends in part on the methodology adopted, but also on the accuracy of the official statistics on which they were based.

The 'Sample' and 'Full' Censuses: Some Inconsistencies

The levels of investment reported by the two sources differ. Take, for example, the 1963 results for shipbuilding and marine engineering (MLH 370) where the 'sample' census indicates a net investment of £10.5 million compared with the £16.0 million reported in the 'full' census. Table (I.2) below gives a comparison of all seven industries for which data are available in 1963.

Table (I.2) Net Investment: 'Sample' versus 'Full'

	'Sample'	'Full'
Mechanical and instrument engineering	112.6	101.3
Electrical engineering	72.6	66.6
Shipbuilding	10.5	16.0
Motor vehicles	79.3	91.4
Aerospace	15.5	11.6
Other vehicles	3.2	1.8
Metal goods not elsewhere specified	43.7	50.9
Total net	337.4	339.6

Consideration of the results given in the table indicates that the differences between the two sets of figures are not all in the same direction. The estimates of total net investment from the two sources for engineering as a whole are very close, 337.4 and 339.6, but this is not true of individual industry groups and the divergences in the figures for the three vehicles groups are particularly large.

There are differences between the two sources which may give rise to the



divergences in the results reported. The annual censuses are based on a sample of units from the industry unlike the quinquennial censuses which used a 100% sample. On the basis of this we might expect the 'full' census to yield a more accurate investment figure although proper design of the questionnaire and selection of the sample should minimize the sampling error of the annual census. The Department of Industry, however, saw the different sampling units as the principal cause of the divergences: the annual censuses were based on returns from "business units" (i.e. company or group of companies) unlike the quinquennial returns from "individual establishments". It can be argued that the establishment basis is more likely to yield accurate estimates and in fact the censuses based on the business unit ceased in 1969. The investment data used in this study were based on MLH estimates that sum to the 'full' census figures.

#### The Method of Construction

The estimates of investment were based on the establishment data, i.e. the 'full' census totals for 1958, 1963 and 1968, and the 'sample' census totals for 1970 and subsequent years. To obtain compatible annual estimates the following procedure was used. Investment by type was obtained from the full census for each of the industry groups that appear in the sample censuses. Hence for 1958 and 1963 it was possible to construct the ratio of the full to the sample census investment values. The values of this ratio for intermediate years (i.e. between 1958 and 1963) were interpolated and the 'sample' census data were up-graded by the appropriate ratios. The years after 1963 (i.e. 1963-7 and 1969) were up-graded by the 1963 ratio of 'full' to 'sample'. This procedure represents an attempt to obtain annual investment data for the eight engineering groups consistent with the intermittent 'full' census observations.

The next step was to construct the proportion of investment carried out by MLHs within each of the eight main industry groups from the 1958, 1963, 1968 and 1970 census detail. These proportions were interpolated for

intermediate years, (i.e. the 1960 proportions were interpolated from the 1958 and 1963 information), and adjusted in order that they summed to unity in each year - a very minor adjustment in practice. These proportions were then used to allocate the adjusted eight industry group information to MLHs. This procedure yields estimates of investment by type and by MLH that are consistent with the 'full' census data.

The only major problem arising from the changes in classification was associated with MLH 390 (engineers small tools and gauges) which changed SIC orders after the 1968 census. In response to this MLH 390 was always handled within the mechanical engineering group (i.e. its original group). MLH 390 was easily subtracted from metal goods not elsewhere specified and added to mechanical engineering in 1970, 1971 and 1972 because MLH detail was available in these years. In 1969, when only the eight industry break-down was available, the size of investment in MLH 390 was estimated by calculating the proportion of MLH 390's investment relative to the metal goods total for 1970 and multiplying this ratio by the metal goods total for 1969. The resulting value was again subtracted from the metal goods not elsewhere specified group and added into the mechanical engineering group.

The results of this estimation procedure are reported in Tables (I.3), (I.4) and (I.5) for the three categories of investment goods used in this study, (i.e. total net investment, plant and machinery, and vehicles).

#### Adjustment for Changes in Prices

All of the data used in constructing MLH investment information are in current prices. Unfortunately the official data about the prices of investment goods are inadequate for most purposes. The only fairly reliable indices appear to be the implicit deflators that appear in the Blue Book. These deflators related to gross domestic capital formation. Where they are available for the engineering sector, they are not reported by type of investment good and, where they are reported by type of investment good, they

TOTAL NET INVESTMENT (FULL CENSUS)

MLH	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972
331	1.10	1.05	1.43	1.82	1.78	1.80	2.17	2.20	2.24	2.02	1.90	2.89	4.20	1.90	3.20
332	4.10	4.04	5.60	7.21	7.16	7.30	9.77	11.09	12.62	12.85	13.70	12.19	10.90	11.50	7.60
333	4.40	4.10	5.43	6.72	6.45	6.40	8.31	9.16	10.16	10.09	10.50	11.56	13.20	12.10	13.00
334	8.60	6.22	6.14	5.32	3.16	1.40	1.90	2.18	2.52	2.60	2.80	4.07	5.70	3.20	4.10
335	1.70	1.90	2.87	3.95	4.14	4.40	5.17	5.11	5.03	4.35	3.90	4.47	5.30	6.20	3.70
356	7.10	5.83	6.81	7.44	6.31	5.50	7.99	9.34	11.79	12.68	14.20	13.04	12.20	7.30	6.90
357	3.40	2.94	3.61	4.18	3.76	3.50	4.55	5.03	5.59	5.57	5.80	7.09	8.90	6.20	5.70
358	2.90	2.45	2.91	3.29	2.87	2.60	3.71	4.45	5.33	5.68	6.30	8.18	10.70	8.30	4.10
359	18.30	16.86	22.11	27.16	25.90	25.40	32.63	35.44	38.69	37.84	38.80	35.21	45.70	45.80	40.30
341	11.30	9.39	11.11	12.30	10.59	9.40	2.14	13.31	14.67	14.50	15.00	15.72	17.20	18.60	24.40
342	2.20	1.93	2.43	2.86	2.63	2.50	3.16	3.37	3.62	3.47	3.50	4.42	5.70	6.60	6.40
349	19.30	16.76	20.77	24.16	21.88	20.50	25.03	25.88	26.78	24.78	23.90	29.75	37.80	34.10	29.50
351	7.90	7.70	8.23	10.00	8.81	10.00	14.47	16.12	16.07	16.80	15.72	20.40	18.70	20.40	20.40
352	0.30	0.33	0.38	0.51	0.49	0.60	0.83	0.97	1.13	1.18	1.20	2.28	4.30	3.00	3.80
353	15.30	14.52	15.91	15.15	12.45	13.00	14.93	16.46	15.39	14.95	14.20	13.66	15.40	13.40	15.20
354	5.30	5.77	7.04	7.56	7.11	8.60	10.45	12.20	12.11	12.53	12.70	10.74	10.10	12.40	14.60
363	5.30	5.97	7.52	8.31	8.04	10.00	10.74	10.91	9.26	7.98	6.50	7.55	10.30	11.10	13.50
362	3.10	3.46	4.46	5.05	4.98	6.30	7.36	8.26	7.88	7.82	7.60	7.94	9.80	9.80	58.80
365	3.10	3.46	4.46	5.05	4.98	6.30	7.36	8.26	7.88	7.82	7.60	7.94	9.80	9.80	58.80
364	9.90	10.63	13.27	14.54	13.95	17.20	22.70	28.58	30.37	33.45	35.90	40.53	53.90	40.10	58.80
369	5.70	6.35	8.19	9.23	9.10	11.50	13.82	15.98	15.68	16.05	16.10	17.58	22.70	22.50	17.80
370	21.70	23.21	18.85	16.29	17.14	16.00	19.20	12.30	12.00	6.70	24.90	23.20	28.50	30.20	24.70
380/1	42.80	40.29	57.71	91.99	86.29	90.80	76.40	96.80	107.40	103.60	82.20	126.90	188.40	131.80	97.00
382	2.30	2.05	1.53	1.62	0.90	0.60	1.05	2.38	2.75	1.96	1.60	1.71	2.00	1.70	2.30
383	12.80	15.08	12.60	12.13	12.99	11.60	12.30	11.40	16.40	22.90	30.90	31.40	34.50	17.20	18.90
384	5.00	4.72	3.77	4.24	3.29	1.80	2.55	5.32	5.35	3.34	2.40	2.39	2.60	1.20	1.80
385/9															
391	1.10	1.31	1.52	1.94	1.67	1.60	1.71	1.97	1.93	1.74	1.60	1.86	2.30	2.00	3.70
392	0.60	0.99	1.48	2.33	2.40	2.70	2.74	2.93	2.66	2.19	1.80	1.92	2.20	2.70	1.70
393	4.00	4.50	4.90	5.80	4.59	4.00	4.38	5.14	5.15	4.76	4.50	5.54	7.30	7.40	6.20
394	4.10	4.90	5.70	7.27	6.26	6.00	6.60	7.77	7.82	7.27	6.90	7.57	8.90	9.90	9.10
395	2.10	2.66	3.28	4.43	4.05	4.10	5.24	7.07	8.05	8.36	8.80	9.33	10.60	10.30	7.50
396	1.20	1.35	1.47	1.74	1.38	1.20	2.07	3.37	4.35	4.95	5.60	4.54	3.40	4.10	5.50
399	17.90	22.14	26.67	35.23	31.48	31.30	34.09	39.75	39.64	36.43	34.20	39.93	49.90	50.40	54.40
All Eng.	253.20	251.40	295.70	361.66	334.92	339.60	378.74	430.59	460.48	450.66	456.80	520.24	665.00	563.50	535.10

PLANT & MACHINERY ACQUISITIONS (FULL CENSUS)

	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972
331	0.70	0.65	0.91	1.10	1.09	1.20	1.42	1.46	1.53	1.42	1.40	1.82	2.8	2.10	2.0
332	3.10	2.92	4.20	5.16	5.22	5.80	7.49	8.36	9.55	9.72	10.60	9.46	10.4	10.50	8.6
333	3.30	2.90	3.94	4.46	4.48	4.80	6.15	6.82	7.75	7.84	8.50	8.29	10.1	10.80	11.1
334	6.10	4.32	4.59	4.06	2.80	1.90	2.38	2.58	2.86	2.84	3.0	3.88	3.7	2.6	3.9
335	1.30	1.34	2.06	2.66	2.79	3.20	3.75	3.78	3.89	3.53	3.40	3.82	4.8	5.6	3.9
336	4.50	3.60	4.43	4.73	4.18	4.10	5.05	5.37	5.86	5.68	5.90	6.11	7.9	5.1	5.4
337	2.20	1.79	2.24	2.44	2.20	2.20	2.84	3.16	3.61	3.67	4.00	4.45	6.1	4.1	4.5
338	2.20	1.79	2.28	2.25	2.28	2.30	3.09	3.59	4.25	4.46	5.00	5.78	8.2	6.9	4.5
339	12.20	10.66	14.37	16.72	16.13	17.20	22.30	24.98	28.65	29.26	32.00	30.12	35.3	35.7	37.8
341	7.10	5.81	7.35	8.06	7.33	7.40	9.23	9.95	10.99	10.81	11.40	10.91	13.0	13.9	14.3
342	5.10	3.72	4.11	3.85	2.89	2.30	2.78	2.90	3.11	2.96	3.00	3.42	4.8	5.9	4.7
349	14.90	12.26	15.60	17.21	15.75	16.00	19.38	20.25	21.65	20.57	20.90	22.02	28.9	31.3	30.1
351	4.40	4.26	5.30	5.65	5.25	6.30	8.33	9.47	11.05	11.45	12.70	12.34	16.0	16.4	18.3
352	0.30	0.32	0.41	0.47	0.47	0.60	0.80	0.91	1.05	1.09	1.20	1.86	3.4	4.2	3.6
362	10.50	10.17	11.76	11.12	10.22	10.70	12.03	13.54	13.40	12.49	11.30	12.20	13.4	11.2	13.5
363	3.80	4.27	5.75	6.36	6.89	8.60	9.25	9.88	9.19	7.97	6.60	7.42	8.7	10.6	12.4
364	7.90	8.61	11.29	12.21	12.93	15.80	19.86	24.94	27.55	28.68	29.00	33.52	39.4	33.3	51.4
365	2.10	2.33	3.10	3.39	3.64	4.50	5.20	6.03	6.16	5.96	5.60	5.85	6.2	7.2	7.7
369	3.60	3.93	5.15	5.56	5.88	7.20	8.88	10.96	11.92	12.22	12.20	14.47	17.4	18.6	16.5
370	11.60	12.23	11.05	10.57	12.22	11.20	10.20	7.20	7.20	4.40	11.90	11.20	15.7	19.4	16.3
380/1	29.90	32.41	39.71	65.73	61.72	63.80	60.40	74.40	82.20	89.80	71.80	99.90	162.8	112.0	83.4
382	1.60	1.79	1.46	2.05	2.05	1.20	1.43	1.88	2.69	2.03	1.40	1.70	1.5	2.0	1.7
383	10.10	8.89	12.26	14.34	13.77	10.90	13.00	12.30	20.10	27.80	26.30	37.90	29.3	14.8	19.1
384	1.70	1.56	1.07	1.24	1.03	0.50	0.62	0.83	1.24	0.98	0.70	2.40	1.9	1.8	1.2
385/9	0.70	1.61	1.13	1.36	1.18	0.60	0.85	1.29	2.17	1.89	1.50	1.09	1.6	1.6	2.2
391	0.40	0.88	0.98	1.16	0.96	0.90	0.97	1.03	1.05	0.95	1.00	1.09	1.8	2.0	1.5
392	0.70	1.03	1.03	1.53	1.52	1.70	1.74	1.76	1.59	1.31	1.20	1.25	1.8	2.0	1.5
393	3.20	3.96	4.32	5.04	4.03	3.70	4.03	4.33	4.30	3.90	4.10	4.24	6.0	5.6	5.5
394	2.60	3.51	4.18	5.33	4.66	4.70	5.20	5.68	5.72	5.30	5.70	5.12	6.3	7.3	7.4
395	2.00	2.72	3.25	4.16	3.65	3.70	4.47	5.35	5.89	5.95	7.00	6.61	8.6	8.1	5.7
396	0.90	1.09	1.16	1.31	1.02	0.90	1.35	1.89	2.36	2.66	3.40	2.43	2.1	3.0	3.8
399	11.80	15.19	17.28	21.11	17.69	17.10	19.84	22.84	24.27	23.73	27.00	26.77	36.5	36.6	36.4
All Eng.	177.4	176.29	212.91	258.19	243.49	249.6	282.19	319.18	354.78	363.30	380.4	406.98	523.1	459.2	449.7

PLANT & MACHINERY DISPOSALS (FULL CENSUS)

	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972
331	0.1	0.04	0.03	0.03	0.01	0.0	0.01	0.02	0.04	0.06	0.10	0.07	0.1	0.7	0.1
332	0.2	0.13	0.14	0.23	0.26	0.4	0.41	0.47	0.61	0.64	0.90	0.79	0.1	1.9	1.4
333	0.2	0.12	0.12	0.19	0.20	0.3	0.28	0.28	0.33	0.32	0.40	0.50	1.0	0.8	1.0
334	0.1	0.12	0.17	0.34	0.42	0.7	0.62	0.59	0.64	0.54	0.60	0.26	0.1	0.1	0.1
335	0.1	0.06	0.07	0.12	0.13	0.2	0.23	0.29	0.42	0.47	0.70	0.38	0.3	0.5	0.5
336	0.1	0.06	0.07	0.12	0.13	0.2	0.19	0.17	0.19	0.17	0.20	0.25	0.5	0.4	0.5
337	0.0	0.03	0.06	0.14	0.18	0.3	0.27	0.26	0.29	0.27	0.30	0.23	0.3	0.2	1.0
338	0.0	0.02	0.04	0.04	0.12	0.2	0.19	0.17	0.19	0.17	0.20	0.11	0.1	0.1	0.4
339	0.6	0.35	0.38	0.61	0.66	1.0	1.06	*.20	1.60	1.68	2.40	1.83	2.6	4.1	3.9
341	0.0	0.09	0.11	0.21	0.25	0.4	0.42	0.49	0.65	0.70	1.00	1.15	2.2	1.7	1.2
342	3.7	1.51	1.12	1.20	0.78	0.5	0.43	0.41	0.44	0.37	0.40	0.23	0.2	0.2	1.2
349	0.7	0.39	0.41	0.63	0.68	1.0	1.01	1.10	1.41	1.43	2.00	1.32	1.6	2.9	1.5
351	0.1	0.10	0.14	0.09	0.18	0.3	0.32	0.38	0.52	0.55	0.80	0.40	1.3	1.1	0.8
352	0.0	0.01	0.02	0.02	0.04	0.1	0.08	0.07	0.05	0.02	0.00	0.00	0.1	-	-
356	0.5	0.39	0.73	1.52	2.22	2.1	1.55	1.78	3.51	2.01	0.70	1.35	1.8	1.1	0.9
362	0.1	0.08	0.16	0.30	0.43	0.4	0.43	0.73	2.00	1.62	0.80	0.83	0.5	0.3	0.3
363	0.0	0.00	0.14	0.05	0.08	0.1	0.19	0.41	1.30	1.14	0.60	0.57	0.3	0.5	0.5
364	0.5	0.39	0.72	1.48	2.14	2.0	1.53	1.86	3.81	2.35	0.90	1.83	2.5	4.6	3.1
365	0.0	0.01	0.04	0.14	0.25	0.3	0.26	0.38	0.90	0.67	0.30	0.40	0.4	0.2	0.1
369	0.2	0.13	0.21	0.37	0.43	0.3	0.33	0.55	1.48	1.21	0.60	0.71	0.6	0.9	0.6
370	0.7	0.28	0.72	0.44	0.52	1.5	1.50	1.50	1.20	0.60	0.60	2.10	0.7	1.5	1.5
380/1	1.4	0.65	1.77	1.01	0.95	1.3	3.90	1.40	1.00	2.20	2.90	3.30	2.9	3.2	2.8
382	0.3	0.14	0.11	0.08	0.03	0.1	0.08	0.03	0.05	0.26	0.00	0.00	-	0.1	-
383	0.7	0.87	0.74	1.03	3.14	2.2	2.20	1.50	1.80	1.50	1.20	7.00	2.2	1.2	1.1
384	0.1	0.05	0.05	0.04	0.03	0.1	0.12	0.07	0.20	0.16	0.20	0.20	0.1	0.1	0.1
3385/9	0.1	0.07	0.07	0.07	0.05	0.2	0.20	0.10	0.25	0.18	0.00	0.04	0.1	0.1	0.1
391	0.0	0.01	0.02	0.06	0.05	0.1	0.11	0.02	0.03	0.03	0.00	0.07	0.1	0.1	0.1
392	0.0	0.00	0.00	0.00	0.00	0.0	0.02	0.02	0.03	0.06	0.10	0.07	0.1	0.2	0.5
393	0.1	0.09	0.12	0.20	0.14	0.2	0.28	0.26	0.29	0.41	0.50	0.31	0.3	0.2	0.5
394	0.2	0.17	0.12	0.33	0.21	0.3	0.34	0.26	0.26	0.31	0.30	0.19	0.2	0.1	0.2
395	0.4	0.32	0.26	0.47	0.17	0.3	0.34	0.26	0.26	0.31	0.30	0.23	0.3	0.4	0.4
396	0.0	0.00	0.00	0.00	0.00	0.0	0.02	0.02	0.03	0.06	0.10	0.04	-	0.1	-
399	0.6	0.54	0.57	1.15	0.76	1.1	1.40	1.12	1.20	1.61	1.80	1.52	2.1	1.6	1.9
All Eng.	11.90	7.22	9.40	12.76	15.64	18.2	20.32	18.22	26.97	24.10	22.1	28.61	26.8	31.0	26.0

VEHICLE ACQUISITIONS (FULL CENSUS)

1.8

	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972
331	0.3	0.30	0.36	0.33	0.25	0.3	0.37	0.37	0.38	0.40	0.5	0.60	0.9	0.9	0.9
332	0.5	0.54	0.67	0.68	0.55	0.7	0.87	0.89	0.90	0.96	1.2	1.20	1.5	1.3	1.2
333	0.6	0.62	0.76	0.73	0.58	0.7	0.87	0.89	0.90	0.96	1.2	1.24	1.6	1.6	1.8
334	0.2	0.20	0.24	0.22	0.17	0.2	0.24	0.23	0.23	0.23	0.3	0.27	0.3	0.3	0.3
335	0.3	0.30	0.36	0.33	0.25	0.3	0.37	0.37	0.38	0.40	0.5	0.56	0.8	0.6	0.7
336	0.4	0.33	0.44	0.46	0.38	0.5	0.64	0.67	0.70	0.77	1.0	1.10	1.5	0.8	1.5
337	0.4	0.44	0.56	0.57	0.47	0.6	0.78	0.85	0.89	1.00	1.3	1.32	1.7	1.7	2.1
338	0.1	0.10	0.12	0.11	0.09	0.1	0.14	0.16	0.19	0.21	0.3	0.31	0.4	0.2	0.2
339	2.4	2.56	3.20	3.16	2.55	3.2	3.94	4.05	4.12	4.38	5.5	5.72	7.5	7.7	8.7
341	1.3	1.34	1.64	1.36	1.23	1.3	1.86	1.93	1.99	2.13	2.7	2.81	3.7	4.1	3.8
342	0.1	0.10	0.12	0.11	0.09	0.1	0.13	0.14	0.14	0.15	0.2	0.19	0.2	0.1	0.1
349	1.4	1.43	1.71	1.62	1.25	1.5	1.82	1.84	1.85	1.94	2.4	2.88	4.3	3.8	4.0
351	0.8	0.88	1.05	1.21	0.68	1.0	1.30	1.41	1.50	1.67	2.2	2.46	2.6	2.9	3.6
352	0.0	0.00	0.00	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.0	0.04	0.1	0.1	0.0
361	1.0	1.06	1.38	1.26	1.12	1.3	1.59	1.81	1.43	1.42	1.6	1.76	1.8	1.6	2.1
362	0.3	0.35	0.49	0.49	0.48	0.6	0.73	0.82	0.64	0.63	0.7	0.77	0.8	0.8	1.1
363	0.1	0.15	0.24	0.27	0.29	0.4	0.46	0.48	0.34	0.30	0.3	0.45	0.6	0.5	0.9
364	0.7	0.76	1.00	0.93	0.84	1.0	1.38	1.77	1.55	1.71	2.1	2.73	3.3	2.8	3.3
365	0.5	0.58	0.82	0.82	0.78	1.0	1.27	1.52	1.24	1.29	1.3	1.38	1.1	1.2	1.4
369	0.7	0.76	1.00	0.93	0.84	1.0	1.17	1.29	0.99	0.94	1.0	1.38	1.8	1.5	1.9
370	0.5	0.46	0.79	0.74	0.82	0.9	1.10	1.30	1.10	1.30	0.6	2.30	1.2	0.9	1.0
380/1	1.9	2.23	3.57	2.79	2.79	2.9	3.20	3.50	3.00	3.70	3.8	4.10	6.7	5.7	6.6
382	0.2	0.30	0.24	0.31	0.25	0.2	0.20	0.25	0.25	0.25	0.2	0.19	0.3	0.2	0.2
383	0.6	0.65	1.36	0.96	0.63	0.8	1.10	1.20	1.60	1.50	1.7	1.50	1.1	0.8	1.3
384	0.1	0.16	0.13	0.15	0.13	0.1	0.10	0.13	0.13	0.13	0.1	0.21	0.8	0.3	0.1
385/9	0.2	0.26	0.20	0.21	0.15	0.3	0.38	0.36	0.40	0.45	0.4	0.43	0.4	0.4	0.5
391	0.2	0.22	0.26	0.27	0.27	0.3	0.38	0.42	0.48	0.53	0.3	0.25	0.4	0.5	0.4
392	0.1	0.12	0.15	0.16	0.18	0.2	0.26	0.27	0.29	0.32	0.3	0.25	0.8	0.7	1.0
393	0.4	0.41	0.43	0.39	0.33	0.3	0.39	0.42	0.48	0.53	0.5	0.67	0.8	0.9	1.0
394	0.5	0.56	0.65	0.66	0.65	0.7	0.84	0.83	0.85	0.89	0.8	0.97	1.0	0.9	1.1
395	0.2	0.22	0.26	0.27	0.27	0.3	0.35	0.33	0.35	0.34	0.3	0.37	0.4	0.4	0.4
396	0.2	0.22	0.26	0.27	0.27	0.3	0.38	0.36	0.40	0.45	0.4	0.52	0.6	0.5	0.5
399	3.2	3.75	4.49	4.82	4.89	5.5	6.58	6.30	6.52	6.73	5.9	7.58	8.5	8.5	10.2
All Eng.	20.3	22.56	28.95	27.79	24.52	28.6	34.91	36.87	35.86	38.22	41.6	48.29	58.7	54.3	63.1

## VEHICLES DISPOSALS (FULL CENSUS)

	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972
331	0.1	0.11	0.12	0.11	0.09	0.1	0.13	0.14	0.15	0.15	0.2	0.20	0.3	0.3	0.3
332	0.2	0.25	0.28	0.27	0.23	0.3	0.39	0.40	0.42	0.46	0.6	0.47	0.5	0.6	0.5
333	0.3	0.35	0.37	0.32	0.25	0.3	0.37	0.37	0.38	0.39	0.5	0.49	0.7	0.7	0.8
334	0.1	0.11	0.12	0.11	0.09	0.1	0.12	0.10	0.09	0.1	0.1	0.05	0.0	0.1	0.1
335	0.1	0.11	0.12	0.11	0.09	0.1	0.13	0.14	0.14	0.15	0.2	0.20	0.3	0.3	0.2
336	0.1	0.13	0.16	0.16	0.15	0.2	0.24	0.23	0.24	0.24	0.3	0.32	0.5	0.3	0.4
337	0.1	0.13	0.16	0.16	0.15	0.2	0.25	0.26	0.30	0.31	0.4	0.41	0.6	0.5	0.8
338	0.0	0.00	0.00	0.00	0.00	0.0	0.02	0.03	0.05	0.07	0.1	0.08	0.1	0.1	0.1
339	0.8	0.99	1.15	1.05	0.94	1.2	1.46	1.52	1.52	1.55	2.0	1.93	2.7	3.3	3.6
341	0.4	0.49	0.57	0.52	0.47	0.6	0.72	0.70	0.72	0.71	0.9	0.93	1.4	1.5	1.4
342	0.0	0.00	0.00	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.0	0.00	0.0	0.0	0.0
349	0.5	0.59	0.65	0.65	0.49	0.6	0.70	0.67	0.68	0.65	0.8	0.92	1.5	1.7	1.7
351	0.3	0.40	0.40	0.50	0.30	0.4	0.49	0.50	0.52	0.53	0.7	1.00	1.2	1.2	2.0
352	0.0	0.00	0.00	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.0	0.00	0.0	0.0	0.0
361	0.3	0.39	0.44	0.44	0.40	0.4	0.48	0.50	0.46	0.46	0.5	0.31	0.7	0.6	0.7
362	0.1	0.13	0.17	0.19	0.18	0.2	0.25	0.27	0.26	0.27	0.3	0.16	0.3	0.2	0.3
363	0.0	0.00	0.00	0.00	0.00	0.0	0.02	0.04	0.05	0.07	0.1	0.05	0.1	0.1	0.3
364	0.3	0.39	0.48	0.50	0.47	0.5	0.62	0.69	0.67	0.70	0.8	0.61	1.7	0.9	1.1
365	0.1	0.15	0.21	0.25	0.26	0.3	0.39	0.46	0.46	0.51	0.6	0.25	0.3	0.5	0.5
369	0.2	0.25	0.30	0.32	0.29	0.3	0.34	0.35	0.31	0.29	0.3	0.22	0.6	0.4	0.7
370	0.1	0.22	0.12	0.26	0.28	0.3	0.30	0.30	0.20	0.50	0.2	0.30	0.4	0.3	0.3
380/1	0.7	1.39	2.92	3.74	5.08	1.2	1.30	1.10	1.10	1.60	1.7	1.20	2.5	2.9	2.7
382	0.1	0.11	0.11	0.11	0.11	0.1	0.10	0.10	0.10	0.10	0.1	0.10	0.1	0.1	0.1
383	0.3	0.64	0.51	0.37	0.38	0.2	0.40	0.60	0.40	0.40	0.3	0.60	0.4	0.5	0.5
384	0.0	0.00	0.00	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.0	0.00	0.0	1.5	0.2
385	0.1	0.07	0.05	0.03	0.01	0.0	0.00	0.00	0.00	0.00	0.0	0.00	0.0	1.5	0.2
391	0.1	0.09	0.11	0.09	0.09	0.1	0.12	0.11	0.11	0.12	0.1	0.10	0.1	0.2	0.2
392	0.1	0.09	0.11	0.09	0.09	0.1	0.12	0.11	0.11	0.12	0.1	0.10	0.1	0.2	0.2
393	0.1	0.09	0.11	0.09	0.09	0.1	0.15	0.15	0.17	0.21	0.2	0.20	0.2	0.2	0.3
394	0.2	0.21	0.24	0.24	0.25	0.3	0.36	0.32	0.34	0.35	0.3	0.31	0.3	0.3	0.4
395	0.0	0.01	0.03	0.05	0.07	0.1	0.12	0.11	0.11	0.12	0.1	0.10	0.1	0.1	0.1
396	0.2	0.18	0.17	0.15	0.12	2.0	0.15	0.15	0.11	0.21	0.2	0.20	0.2	0.2	0.2
399	1.2	1.28	1.52	1.54	1.64	2.0	2.49	2.25	2.39	2.59	2.3	2.70	3.1	2.9	4.0
All Eng.	7.2	9.35	11.70	12.34	13.06	10.4	12.73	12.62	12.62	13.92	15.0	14.51	21.0	22.4	24.7



are only available for all manufacturing. This study has made use of the implicit deflators for each type of investment good and the values for all manufacturing industries are reported below in Table (I.6)

Table (I.6) Implicit Deflator - Based on Blue Book Information about  
Gross Domestic Fixed Capital Formation.

Year	All Assets	Plant and Machinery	Vehicles
1961	0.95	0.96	1.04
1962	0.98	0.99	1.04
1963	1.00	1.00	1.00
1964	1.02	1.02	1.00
1965	1.06	1.07	1.02
1966	1.10	1.11	1.04
1967	1.12	1.12	1.05
1968	1.15	1.16	1.10
1969	1.21	1.20	1.14
1970	1.30	1.29	1.21
1971	1.41	1.41	1.34
1972	1.54	1.50	1.43



Appendix (II) Estimates of Fuel Consumption by Industry within Engineering

A. Introduction

There is a noticeable lack of published information about fuel consumption at anything other than a very high level of industrial aggregation. The information that is reported in the Power Digest is constructed mainly from sales data provided by the fuel industries, but this data is insufficiently detailed to be presented on an MLH basis. Nevertheless, a certain amount of information can be obtained from other sources, such as the censuses of production, that can be used in conjunction with the statistics in the Power Digest. Using the available sources of information, this appendix reports the results of an attempt to provide estimates of fuel consumption by type of fuel, for each MLH within the engineering sector. The period covered relates to the 1960s and early 1970s when more detailed data make the calculations reasonably accurate.

The data reported here relates to all types of fuels used to power the capital stock employed by firms allocated to the engineering sector (defined as the 1948 SIC 'engineering and allied trades'). The next section discusses the reconciliation of the fuel types that are reported by the two main sources of data used in this study. Section C similarly defines the industry and regional coverage of the original sources and of the estimates reported in this appendix. It is a major criticism of the existing statistics that they are not reported on a consistent regional and industrial basis for different types of fuel. The original statistics are made consistent with the basic industry group used here (ie. 'engineering and other metal trades') and with a single geographical region (ie. the UK).

Section D of the appendix explains the method that was used to provide estimates of fuel consumption by MLH in 'original' units. Section E describes the translation of fuel consumptions into a common, thermal, unit of measure. A common measure is needed in order to allow aggregation over different types of fuel to be undertaken at various levels of industrial aggregation. Finally, Section F comments about the methodology used and the accuracy of the resulting estimates of fuel consumption.

B. Fuel Types: A Reconciliation by Data Source

It is important to reconcile the fuel types distinguished in the two main sources of information (ie. the Power Digest and the quinquennial census of production) used in this appendix. The two sources distinguish broadly similar fuels. The reconciliation adopted here is reported in Table (II.1).

Table (II.1): Fuel Reconciliation

<u>Fuel Type</u>	<u>Source: Power Digest</u>	<u>Census</u>
Coal	Coal	Coal
Electricity	Electricity	Electricity
Coke	Coke	Coke and other manufactured fuels
Coke breeze		
Other Solid fuel		
Coke oven gas	Gas	Gas
Natural gas		
Town gas		
Creosote	Other liquid fuels	Other liquid fuels
Pitch mixes		
Liquid gas	Petroleum	Derv and motor vehicle fuels
Fuel oil		
Gas/Diesel oil a		
All other petrol b		
Derv and motor vehicle fuels		
Other fuels		Other fuels c

a Excluding derv

b Excluding derv and motor vehicle fuels

c 1968 only

As the table shows, the main problems of comparison fall within the 'petroleum' group. The 'derv and motor vehicle fuel' group from the census appears to correspond fairly closely with the residual left after subtracting 'liquid gas', 'fuel oil' and 'gas and diesel oil' from the Power Digest 'petroleum' group. 'Liquid gas' is a stumbling block as its consumption is not given separately in either the Power Digest or the census. However, its consumption is likely to be extremely small in comparison with the consumption of 'derv and motor vehicle fuel'. Its inclusion within this group in the Power Digest data is unlikely to lead to any real problems of comparison with the corresponding census group.

### C. Consistency in Regional and Industrial Coverage

#### a) Power Digest and Electricity Council Information

Regional Coverage The data available about different types of fuels from these two sources are inconsistent in their regional coverage. Where any industry grouping more detailed than 'engineering and other metal trades' is considered, the geographical scope is as follows: 'electricity' data are given for England and Wales; 'coal', 'other liquid fuels' and 'gas' relate to Great Britain; finally, 'creosote and pitch mixes' and 'coke' are available at the UK level.

It is obviously important to transfer the figures to a common region. In practice it is easier to transform the data to the UK as a region than to G.B. (which fortunately matches the investment data). Information is available from the Power Digest about the consumption of each type of fuel for the UK engineering group as a whole (both in 'original' units and on a 'heat supplied' or 'thermal' basis). The MLH information, about the consumption of various types of fuels, constructed in this appendix is always constrained to sum

to the aggregate consumption totals.

Stocks and consumption. Grossing up in this way also provides a means of adjusting from information about purchases to a consumption basis. Data that give greater industry detail often relate to purchases (which differ from consumption by an element of stock-building or stock-depletion).

Problems of Grossing Up. There are obvious problems with overcoming the difficulties associated with variations in regional coverage and stock-building in this way. The adjustment by region is only valid when the relative importance of the various engineering industries does not differ between regions. In addition, the fuel consumption characteristics of a particular industry must not differ across regions. The first of these two assumptions, in particular, is strong. Evidence contained in the census of production (eg. the regional information given in the summary tables) indicates considerable variation in the relative importance of given industries across regions. There is, however, little that can be done to overcome this problem. In a similar way, the adjustment from purchases to consumption assumes that the propensity to allocate fuel purchases to stocks is constant across industries within the engineering group.

Industry Groups. A further problem arises because the number of industries distinguished within the 'engineering and other metal trades' group differs between fuel types. Information about eight industries is available for 'coal', 'gas and diesel oils' and 'fuel oils'. Five industries are distinguished for 'gas' and 'electricity', but the five are different for the two fuel types. Finally, 'coke' and 'other liquid fuels' data are not given in any more detail than the overall engineering

group. In this appendix, an attempt is made to provide data about different fuels for a consistent region at the MLH level of detail. Reaggregation can then be undertaken to yield information about more aggregate industry groups for each type of fuel, for example, the 1968 SIC Orders or the Cambridge SAM groupings.

b. Census of Production Information

Regional coverage. The regional coverage of the census, for the period considered here, is consistent throughout. The data relates to establishments in the UK.

Stocks and Consumption. The data refers to principal purchases of firms in the industry. For most types of fuels (ie. with the exception perhaps of electricity) it will include an element of stock-building or stock-depletion. No attempt is made to adjust the census information from a purchase to a consumption basis directly, but when integrated with data from the Fuel Digest an implicit adjustment is automatically made (because the latter records consumption data).

Industry Groups. Given the period under consideration, only the 1954, 1963 and 1968 census provide information about fuel purchases at an MLH level. The 1963 Reports give the 1954 fuel figures on the basis of the post 1958 industrial classification, and the differences between the 1963 and 1968 classifications are relatively trivial and easily coped with - see Appendix (III). Hence, the changes in industrial classification do not hinder meaningful comparisons between the censuses. At the greatest level of industrial breakdown, 44 MLHs within engineering are distinguished. The tables reported later in this appendix are based on the

1968 industrial classifications, but are aggregated to the 1963 level of detail.

Adjustment for Partly Reported Information. The census occasionally reports information about fuel purchases of a particular MLH in two parts: (a) a corresponding value and volume; (b) a value with no corresponding volume. The reason is simply that certain establishments in the industry failed to report volume figures. Estimates of the omitted data were calculated using the price paid per unit of fuel for that part of the transaction where both quantity and value were known. Dividing the value element with no corresponding volume by this price yielded an estimate of the unknown volume. The total volume of consumption (in 'original' units) was then found simply by summing the known and estimated parts. Where there was no way of calculating the per unit price paid by establishments in that particular MLH, the corresponding figure for the SIC was used to translate from a given value to an unknown volume.

In addition, there were occasionally no figures (value or quantity) reported for a given fuel purchased by establishments in a particular MLH. However, usually two pieces of information were published which allowed the missing value to be calculated: (a) the total cost of all fuels purchased by the MLH; (b) the total value of all fuels purchased by the MLH other than the one for which data was missing. Simple subtraction yielded an estimate of the value of purchases for the missing fuel. This value was then translated into volume terms using the average price per unit paid for that fuel within the SIC. On the only occasion when data for two fuels were missing for the same MLH, the value was so small that a zero was inserted for each of the missing figures.

Firm Size: Problems with the scope of the sample. The sampling unit adopted by the census is the establishment. In the questionnaire relating to purchases by establishments in the MLH, units employing less than 25 persons were excluded. The results reported in the census therefore relate to 'larger establishments'. Observed fuel consumption in any given MLH will differ from the actual consumption according to the importance of establishments in the industry that employ less than 25 employees. If the coverage of the sample (ie. the proportion of establishments employing 25 workers or more) and the relationship between the fuel consumption in the two size groups of establishments do not vary too greatly across industries, then the problem is reduced, because this study only uses the proportions of fuels consumed in each MLH relative to the total for engineering as a whole. In any case, the sample will include most of the fuel consumed within the industry because engineering is dominated by larger production units. Even if we recognise that there exist certain differences in the proportions of firms in the two size groups, little can be done to appropriately adjust the fuel consumption of each MLH. The only relevant information available appears to be the total cost of raw materials and fuel consumption in the 25 and over units and the total for the industry. However, an adjustment of each type of fuel purchased on this basis is likely to introduce more errors than it removes.

Fuel Consumed in other uses. The fuel consumption data for engineering reported in the census refer almost entirely to those purchases used to power the capital stock of the industry. The one notable exception relates to fuels used in testing products (ie. engine testing in the motor vehicles and aircraft sectors). For most industries the purchases for this purpose are small enough to ignore, but where they are significant, they bias upwards

'derv and motor vehicle fuel' group.

It is difficult to know what, if anything, to do about this. No adjustment was made for two reasons. First, a certain amount of capital stock is used when the firm carries out tests and, in so far as tests relate to current output, this adds to the value of output generated by the plant. Second, it is extremely difficult to exclude fuels used for testing from the Power Digest totals: only the census distinguishes it separately and then only when it is important in magnitude.

D. Calculating fuel consumption by MLH: 1961-72

This section reports, in slightly greater detail, the method by which estimates of fuel consumption by MLH for each type of fuel were obtained. First, where necessary the published figures were grossed up to a UK basis. Second, the proportions of fuel consumed by each MLH were calculated for each census year (1954, 1963 and 1968) and then interpolated or extrapolated to yield a continuous series of proportions for each of the years from 1961 to 1972. The proportions were calculated for the most detailed industry groups for which official sources gave data. No proportion was allowed to become negative: where the calculation indicated a proportion less than or equal to zero, it was constrained to be zero from then on. Finally, these proportions were applied to the corresponding industry totals to yield estimates of fuel consumption by MLH in 'original' units.

Coal consumption. The industry information about coal consumption is drawn from the 'industrial returns' section of the Power Digest. The data relates to coal consumed in eight engineering industries in Great Britain:



mechanical and instrument engineering; electrical engineering; shipbuilding and marine; motor vehicles; aircraft; other vehicles; metal goods not elsewhere specified; and non-ferrous metals. The sample covers almost all of the coal consumed in the industry, although consumers of less than 1000 tons per annum are excluded. This industry level information was grossed up to sum to the 'engineering and other metal trades' coal consumption total, and then the census proportions were applied to each of these eight groups to yield NLH estimates.

Other liquid fuels. The category reviewed here is the census' other liquid fuel' group. 'Gas and diesel oils' and 'fuel oils' are two elements of this group for which data are available for eight industry groups within engineering in the UK. The eight groups covered are identical to those which appeared above for coal consumption.

The other main member of the group is 'creosote and pitch'. In principle, the consumption of this group can be isolated on the same industry basis. The Power Digest gives consumption of 'liquid fuels' in the 'industry returns' section (for firms consuming more than 500 tons per annum). The 'liquid fuel' group includes 'gas and diesel' and 'fuel' oils, as well as 'creosote and pitch mixes'. Unfortunately this information relates to 'engineering' consumption in Great Britain, while the information given separately about gas, diesel and fuel oils relates to UK deliveries.

If deliveries were close to consumption and the Great Britain figures were grossed-up to UK, simple subtraction of gas, diesel and fuel oils from the total 'liquid fuel' group would yield estimates of

'creosote and pitch' consumption by industry. However, the 'creosote and pitch' group is a very small one. (The 45 thousand tons consumed in 1961 shrank to 4 thousand tons in 1972). The data are swamped by inaccuracies of grossing-up and of assuming the volume consumption to be the same as that delivered.

An alternative approach had to be adopted. The UK engineering consumption of 'creosote and pitch' was allocated between the eight industry groups in proportion to their total consumption of gas, diesel and fuel oils. When aggregated together this gave an estimate of UK engineering consumption of 'other liquid fuels' on the basis of the census fuel classification. This information was then divided into MLH consumption using the census proportions for each industry.

Derv and Motor Vehicle Fuels. The calculation of the annual consumption of this group from data given in the Power Digest is extremely difficult. The aggregate 'engineering' group data refer to the consumption of petroleum based fuels, while more detailed industry data on 'gas and diesel' and 'fuel' oils relate to deliveries. To obtain estimates of petroleum consumption, we are forced to assume deliveries and consumption for any given year are one and the same. Subtraction of gas, diesel and fuel oils from 'total petroleum' then yields a fairly crude estimate of consumption of 'derv and motor vehicle fuels'. The MLH proportions in each year for 'engineering and other metal trades' were calculated from the census information and used to allocate the total between MLHs.

Gas. Gas consumption was given in the Power Digest for five engineering groups in Great Britain: engineering and shipbuilding; electrical engineering; vehicles; metal goods not elsewhere specified; and non-ferrous metals. These figures are grossed up to a UK basis, to comply with the aggregate 'engineering' consumption totals. MLH consumption data

were again calculated by applying the census proportions.

Electricity Consumption. The aggregate 'engineering' electricity consumption data given in the Power Digest relate to the UK group 'engineering and other metal trades'. The data include consumption from both primary and secondary sources. The industry level data contained in the annual reports of the Central Electricity Generating Board were grossed up to sum to the UK 'engineering' consumption of electricity total given in the Power Digest. The industries distinguished by the CEGB data were: mechanical, electrical and instrument engineering; shipbuilding and marine; vehicles; metal goods not elsewhere specified; and non-ferrous metals. The electricity data has the largest regional adjustment (from England and Wales to the UK). It is not advisable to make too light of the errors introduced in this way, they may be large for a particular industry group. However, over 90% of UK electricity consumption by industrial concerns took place in England and Wales in 1959 and 1960. The underlying assumptions have been discussed in Section C earlier. The electricity consumption of the broad industry groups were then divided into NLHs. The census information was divided into groups of NLHs on the industry basis of the CEGB data. The proportions of fuel consumed by each NLH within each of these industry groups (ie. there are 5 groups of ratios, where the ratios in each group sum to unity) were then calculated for the three relevant census years. The proportions for inter-census years (1961-62 and 1964-7 inclusive) and for later years (1969-onwards) were interpolated and extrapolated. These proportions for each industry group were applied to the industry totals resulting in estimates of fuel consumption by NLH.

Coke. There is less information about coke than for any other type of

fuel. After reporting coke information by industry for the early 1950s in the Power Digest, the data lapsed to the aggregate consumption of the 'engineering and other metal trades' group at the UK level of regional aggregation. The procedure used in this study was simply to split this aggregate figure according to the interpolated and extrapolated proportions calculated from the 1954, 1963 and 1968 censuses. These estimates can only be claimed to reflect broad trends in relative industry consumptions within engineering; any cyclical and trend movements in coke consumption are represented only in so far as the industry as a whole typifies each component MLH.

#### E. Conversion to Thermal Units

The fuel consumption information is now consistent by regional coverage (ie. the UK) and by industry group (ie. MLHs, according to the 1968 SIC classification, within the 'engineering and other metal trades' group). All that remains to be done before aggregation is undertaken is to convert the figures for each type of fuel into a common unit. Two possible units of measure exist: constant prices and thermal units. The latter was chosen simply because it was a technical measure of consumption, independent of economic variables. Thermal units represent the yields which can be expected from fuels, on the basis of current technology, in the light of their quality in a particular year.

Thermal yields vary considerably across fuel types for a given year, but very much less so for a given fuel over time. Thermal yields per ton of fuel are reported in the Power Digest. In the bases of 'coal', 'gas and diesel oils', 'fuel oils', and 'creosote and pitch mixes', the thermal yields reported were already aggregates. The weights used in the official statistics were the volumes of consumption in the economy and hence, not

the ideal weights for the 'engineering' group. Nevertheless, the fuel split is quite detailed and each is probably internally homogeneous. The greater the degree of homogeneity within each group, the less crucial the weighting becomes. Where further aggregation of the data on thermal yields was necessary to give the fuel groups used in this study, the weights used were the relative amounts of the fuels consumed within the engineering industry. The thermal yields per unit of fuel were then multiplied by the quantities of fuel consumed (in original units) to give the total thermal consumption by type of fuel.

#### F. Conclusions

Tables (II.2) - (II.8) report the thermal consumption of each MLH within 'engineering and other metal trades' for the six census fuel groups and for all fuels as a whole, for each of the years 1961-72 inclusive. This data should prove useful to a variety of economic areas, but a number of important comments must be made about the approach and the accuracy of the results.

i. The fuel groups adopted by the study are those used in the censuses of production - as shown in Table (II.1). In constructing the more detailed data reported in this appendix, information from the Power Digest was also used and there were some problems of reconciling the fuel groups from the two sources. The problems were mainly associated with 'petroleum' and were not thought to detract too greatly from the accuracy of the final results.

ii. The more detailed, by-industry data often related to different regions, and the act of grossing-up to a common region (ie. the UK) involved using a constant multiplier across industries for each year.

Table (11.2) Estimated 'Coal' Consumption By Generating Firm In the UK (M.Therms)

Firm (1968)	Year										
	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971
321	189.6	182.5	189.2	134.9	188.2	120.1	102.5	89.4	70.8	71.0	49.1
331	2.2	2.1	1.9	1.8	1.8	1.7	1.5	1.3	1.2	1.1	0.7
332	8.6	8.9	8.3	7.2	6.9	5.9	4.8	3.9	3.4	2.6	1.6
333	1.4	1.4	1.2	1.2	1.3	1.3	1.2	1.2	1.2	1.0	0.8
334	3.6	3.5	3.2	2.9	2.9	2.6	2.3	2.0	1.8	1.5	1.0
335	10.3	10.2	9.1	8.5	8.7	8.0	7.0	6.3	6.0	5.1	3.6
336	4.3	4.4	4.1	3.5	3.3	2.7	2.1	1.7	1.4	1.0	0.6
337	4.0	3.9	3.4	3.2	3.3	3.1	2.7	2.5	2.4	2.1	1.5
338	2.2	2.1	1.9	1.8	1.9	1.8	1.6	1.4	1.4	1.2	0.8
339	34.3	34.4	31.4	30.9	33.4	32.5	30.0	28.3	28.2	25.1	18.3
341	11.4	10.9	9.6	8.7	8.7	7.9	6.8	5.9	5.5	4.6	3.1
342	35.6	35.9	33.1	28.2	26.2	21.8	17.1	13.5	11.0	7.9	4.5
349	20.1	20.1	18.4	17.6	18.6	17.7	16.0	14.8	14.4	12.6	9.1
351	3.9	3.5	2.9	3.7	4.9	5.6	5.8	6.0	6.5	6.2	4.8
352	0.7	0.7	0.7	0.6	0.7	0.6	0.5	0.5	0.5	0.4	0.3
361	77.0	86.0	83.7	79.1	77.2	71.2	63.6	58.3	51.0	45.0	33.2
362	36.0	39.7	38.1	39.0	41.1	40.7	39.0	38.2	35.6	33.5	26.2
363	16.0	17.2	16.1	14.5	13.4	11.7	9.9	8.5	6.9	5.6	3.8
364	7.5	6.7	4.9	5.1	5.4	5.3	5.1	5.0	4.7	4.4	3.5
365	20.7	24.2	24.5	22.7	21.6	19.4	16.9	15.1	12.8	11.0	7.8
369	12.6	12.2	10.1	9.6	9.5	8.8	7.9	7.3	6.4	5.7	4.3
370	18.8	16.4	13.7	11.7	9.8	6.8	3.8	1.9	1.3	2.1	1.6
381	185.7	203.9	211.7	201.9	203.1	176.0	165.1	165.6	162.2	153.4	133.8
382	13.3	13.7	13.5	13.7	13.7	18.6	18.7	20.0	22.6	24.7	23.9
383	71.3	72.1	70.7	66.1	66.9	62.0	57.6	54.5	49.4	41.4	29.6
384	13.0	11.9	9.5	8.1	8.2	7.0	4.7	4.4	4.8	4.6	3.6
385/9	11.2	9.6	7.2	6.1	6.3	5.3	3.6	3.4	3.6	3.6	2.8
391	3.0	2.4	1.8	1.5	1.4	1.2	0.8	0.7	0.6	0.5	0.4
392	1.0	1.0	0.9	0.8	0.7	0.6	0.4	0.4	0.3	0.3	0.2
393	11.2	11.0	10.1	9.9	10.2	9.9	7.8	7.6	7.7	7.6	7.0
394	17.9	16.9	14.9	14.1	14.0	13.1	10.0	9.4	9.4	9.0	8.0
395	4.6	4.6	4.2	3.9	3.8	3.5	2.7	2.5	2.4	2.3	2.0
396	1.9	1.8	1.5	1.3	1.1	0.9	0.6	0.4	0.3	0.2	0.1
399	47.4	43.9	37.8	34.0	32.3	28.7	21.0	18.8	17.9	16.4	14.0

Table (11.3)

Estimated Consumption of 'Other Liquids' by Engineering Firms in the UK. ('M Tonne').

11.15

MLM (1968)	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971
321	145.3	158.9	176.0	188.9	206.3	199.7	197.7	227.4	244.4	279.3	206.8
331	5.1	5.3	5.6	5.8	6.9	7.1	7.9	8.8	9.3	10.0	9.5
332	15.1	17.5	20.6	20.9	24.2	24.6	26.5	29.1	30.1	31.5	29.2
333	6.8	8.2	10.0	10.5	12.7	13.5	15.2	17.4	18.8	20.6	20.0
334	20.8	18.6	18.4	19.1	22.7	23.7	26.3	29.7	31.5	34.0	32.5
335	7.5	8.3	9.5	10.8	14.0	15.8	18.9	22.8	25.8	29.5	29.8
336	13.2	16.0	19.7	20.5	24.4	25.5	28.4	32.1	34.2	37.0	35.4
337	8.0	9.2	10.8	12.2	15.5	17.2	20.3	24.2	27.1	30.7	30.8
338	9.0	10.5	12.6	11.7	12.3	11.2	10.6	9.9	8.3	6.7	4.2
339	58.3	65.3	74.9	79.4	96.0	101.8	114.9	131.7	142.2	155.7	151.1
341	25.8	28.7	32.8	33.3	38.7	39.3	42.6	46.9	48.6	51.0	47.5
342	10.2	10.4	10.9	11.5	13.9	14.6	16.5	18.8	20.2	22.1	21.3
349	37.3	39.9	43.7	43.4	49.1	48.6	51.2	54.6	54.7	55.5	49.7
351	9.1	7.3	5.3	6.2	8.2	9.4	11.3	13.8	15.8	18.2	18.5
352	11.5	14.2	17.7	17.3	19.3	18.8	19.3	20.1	19.7	19.4	16.7
361	53.9	67.1	81.3	76.7	75.3	75.1	73.3	64.7	61.3	56.3	40.7
362	16.7	19.0	20.9	23.2	26.9	31.9	37.1	39.5	45.6	52.3	48.6
363	12.3	15.5	19.0	18.4	18.6	19.3	19.6	18.3	18.4	18.4	14.8
364	16.0	17.1	17.5	19.3	22.2	26.2	30.3	32.1	37.9	42.3	39.2
365	28.8	36.8	45.8	48.5	53.7	61.0	68.5	70.5	79.0	88.1	79.8
369	19.7	23.7	27.8	31.5	37.1	44.4	52.3	56.2	65.6	75.8	71.0
370	43.2	42.3	45.3	49.9	54.1	53.2	51.7	56.4	54.6	63.4	61.2
381	107.3	164.6	201.6	210.5	230.9	245.6	260.4	289.3	291.7	284.4	262.2
382	3.1	3.1	1.8	2.4	2.8	3.2	3.8	4.0	4.2	4.2	4.0
383	73.5	77.6	87.4	92.8	94.8	93.2	87.6	98.2	109.2	112.1	95.8
384	15.2	15.7	14.3	13.3	13.5	13.3	13.6	13.0	9.6	8.8	5.7
385/9	11.8	12.1	11.0	12.5	15.2	18.3	22.7	26.7	24.8	29.4	25.1
391	4.9	5.5	6.4	6.4	7.0	6.2	6.1	6.1	6.1	5.8	5.0
392	1.3	1.4	1.5	1.9	2.6	2.8	3.3	3.8	4.5	4.9	4.8
393	13.4	13.2	13.5	13.8	15.1	13.6	13.5	13.5	13.8	13.3	11.6
394	14.1	15.8	18.5	19.2	21.4	19.6	19.8	20.2	21.0	20.7	18.5
395	3.7	4.4	5.4	6.0	7.2	7.0	7.6	8.2	9.1	9.5	9.0
396	7.5	8.5	10.1	11.1	13.1	12.7	13.6	14.7	16.1	16.7	15.7
399	103.3	108.2	119.0	124.6	140.0	129.7	132.2	136.3	143.3	143.1	129.3

Table (11.4)

Estimated 'New' and 'Motor Vehicle Fuel' Consumption By Engineering Firms

In the UK (M. Therm)

77.16

M/HT (1968)	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971
321	2.8	0.5	0.8	1.2	1.1	2.9	3.4	4.2	4.6	5.0	5.1
321	0.5	0.1	0.1	0.2	0.2	0.6	0.8	1.1	1.3	1.5	1.6
332	0.9	0.2	0.2	0.4	0.4	1.0	1.3	1.7	2.0	2.3	2.4
333	0.6	0.1	0.2	0.3	0.3	0.7	0.9	1.2	1.4	1.6	1.7
334	0.4	0.1	0.1	0.1	0.1	0.4	0.6	0.8	1.1	1.3	1.5
335	0.7	0.1	0.2	0.3	0.3	0.7	0.9	1.1	1.3	1.4	1.5
336	0.9	0.2	0.3	0.4	0.3	0.9	1.0	1.2	1.3	1.3	1.3
337	0.9	0.2	0.3	0.5	0.5	1.5	2.0	2.7	3.3	4.0	4.5
338	0.2	0.0	0.1	0.1	0.1	0.3	0.5	0.6	0.8	1.0	1.1
339	4.6	0.9	1.3	2.2	2.1	6.3	8.3	10.9	13.1	15.4	17.0
341	3.0	0.6	0.9	1.3	1.2	3.2	3.8	4.6	5.1	5.5	5.5
342	0.5	0.1	0.1	0.1	0.1	0.2	0.3	0.3	0.3	0.2	0.2
349	2.2	0.4	0.6	0.9	0.8	2.3	2.9	3.6	4.1	4.7	5.0
351	1.2	0.2	0.3	0.6	0.5	1.6	2.1	2.8	3.4	4.0	4.4
352	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.2
361	2.2	0.4	0.6	0.5	0.8	2.3	2.8	3.4	3.8	4.2	4.3
362	1.0	0.2	0.3	0.4	0.4	1.0	1.3	1.6	1.8	1.9	2.0
363	0.8	0.2	0.2	0.4	0.3	0.7	0.8	0.9	0.9	0.9	0.7
364	0.7	0.1	0.1	0.2	0.2	0.7	1.0	1.4	1.7	2.1	2.4
365	2.5	0.5	0.8	1.4	1.3	3.7	4.7	6.1	7.2	8.3	9.0
369	1.4	0.3	0.4	0.6	0.6	1.6	2.0	2.5	2.9	3.2	3.4
370	1.7	0.3	0.4	0.6	0.5	1.4	1.6	1.9	2.0	2.1	2.0
381	11.8	2.2	3.2	5.0	4.5	12.3	14.9	18.4	20.4	21.1	21.0
382	0.4	0.1	0.1	0.1	0.1	0.3	0.4	0.5	0.6	0.6	0.7
383	2.1	0.3	0.5	0.8	0.7	2.1	2.6	3.4	4.0	4.6	5.0
384	0.1	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1
385	0.2	0.0	0.1	0.2	0.2	0.9	1.5	2.2	3.0	3.8	4.6
391	0.2	0.0	0.1	0.1	0.1	0.2	0.3	0.3	0.4	0.4	0.4
397	0.1	0.0	0.0	0.1	0.1	0.2	0.2	0.3	0.4	0.5	0.6
393	0.6	0.1	0.2	0.3	0.2	0.6	0.7	0.8	0.9	0.9	0.9
394	1.0	0.2	0.3	0.4	0.3	0.9	1.1	1.3	1.5	1.6	1.6
395	0.7	0.1	0.2	0.3	0.3	0.7	0.9	1.1	1.2	1.3	1.4
396	0.3	0.1	0.1	0.1	0.1	0.3	0.3	0.3	0.2	0.2	0.3
399	6.1	1.1	1.6	2.6	2.3	6.5	8.1	10.1	11.5	12.8	13.4



Table (11.5) Estimated Gas Consumption by Engineering Industries in the UK (M Therms)

MICU (1968)	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971
321	50.0	50.0	54.0	60.0	64.0	68.0	65.0	73.0	79.0	89.0	97.0
321	4.3	4.3	4.3	4.2	4.3	4.7	4.7	4.9	5.1	5.4	5.5
332	6.7	6.7	6.8	6.7	6.9	7.6	7.6	8.2	8.5	9.2	9.6
333	9.4	9.3	9.3	8.9	8.9	9.5	9.4	9.5	9.6	10.1	10.1
334	5.4	5.3	5.2	5.1	5.1	5.6	5.6	5.8	6.0	6.4	6.5
335	8.5	8.3	8.2	7.6	7.2	7.4	6.9	6.6	6.2	6.0	5.5
336	4.2	4.4	4.6	4.6	4.7	5.2	5.3	5.6	5.9	6.4	6.6
337	4.4	4.6	4.8	4.8	5.0	5.6	5.8	6.1	6.5	7.1	7.4
338	3.5	3.6	3.7	3.3	3.1	3.2	2.9	2.7	2.5	2.3	2.0
339	26.8	26.8	27.2	28.2	30.1	34.8	37.0	40.2	43.4	48.8	52.4
341	16.8	17.5	18.4	17.6	17.3	18.5	18.1	18.1	18.1	18.8	18.6
342	8.0	6.8	5.7	5.9	6.2	7.1	7.5	8.0	8.6	9.6	10.2
349	25.7	26.0	26.6	26.1	26.4	29.0	29.3	30.2	31.2	33.5	34.4
351	9.0	9.0	9.2	9.5	10.1	11.7	12.4	13.4	14.5	16.2	17.4
352	0.5	0.5	0.5	0.5	0.5	0.6	0.6	0.6	0.6	0.7	0.7
361	17.0	17.5	18.0	17.7	17.9	16.7	15.2	14.5	14.0	13.6	14.3
362	8.1	8.3	8.5	8.7	9.1	8.8	8.4	8.3	8.4	8.7	9.7
363	2.0	2.0	2.1	2.1	2.3	2.3	2.3	2.4	2.6	2.8	3.2
364	7.5	7.0	6.5	6.5	6.7	6.3	5.9	5.7	5.7	5.7	6.2
365	8.3	9.1	9.9	10.8	12.1	12.4	12.5	13.3	14.3	15.5	16.3
369	12.1	12.1	12.1	13.2	14.9	15.5	15.7	16.7	18.1	19.7	23.3
370	16.7	17.1	17.6	17.1	17.2	18.6	18.6	19.0	19.4	20.6	20.9
381	48.2	50.9	55.1	60.4	65.2	70.9	69.9	73.4	80.4	86.8	110.7
382	7.4	7.1	7.0	6.9	6.6	6.3	5.3	4.8	4.4	3.9	3.9
383	11.4	10.9	10.6	10.8	10.8	10.8	9.8	9.5	9.6	9.5	11.0
384	4.0	4.2	4.4	4.1	3.5	3.0	2.1	1.5	0.8	0.0	0.0
385/8	3.1	2.9	2.8	2.9	3.0	3.0	2.8	2.8	2.8	2.8	3.4
391	4.3	4.2	4.1	4.1	3.8	3.8	3.4	3.2	3.2	3.2	2.9
392	1.9	1.9	2.0	2.0	2.0	2.0	1.9	1.9	2.1	2.2	2.2
393	5.0	4.9	4.8	5.1	4.9	5.0	4.8	4.7	5.1	5.4	5.4
394	16.7	16.3	16.1	16.9	16.4	16.9	16.1	16.1	17.4	18.6	18.8
395	6.6	6.7	6.9	7.4	7.3	7.6	7.4	7.5	8.2	8.9	9.2
396	2.1	1.9	1.8	2.1	2.2	2.5	2.6	2.8	3.3	3.8	4.2
399	61.2	61.0	61.3	66.4	66.3	70.2	68.8	70.8	78.7	86.8	90.3

Table (11.6)

Estimated Electricity Consumed By Engineering Industries in the U.K. (M. Therms)

MWh (1963)	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971
331	1.7	1.8	1.8	2.0	1.9	1.8	1.7	1.7	1.6	1.6	1.4
332	6.8	6.9	7.2	8.0	8.1	8.2	8.1	8.3	8.7	8.9	8.5
333	4.9	5.1	5.4	6.0	6.2	6.4	6.4	6.6	7.1	7.4	7.1
334	4.5	4.4	4.4	4.9	5.0	5.0	5.0	5.1	5.4	5.6	5.4
335	4.6	4.4	4.3	4.8	4.8	4.9	4.9	5.0	5.2	5.4	5.2
336	4.7	5.1	5.7	6.3	6.5	6.6	6.7	6.9	7.3	7.6	7.4
337	4.3	4.6	4.9	5.5	5.6	5.6	5.7	5.7	6.0	6.2	6.0
338	2.0	2.1	2.3	2.6	2.6	2.7	2.7	2.8	3.0	3.1	3.0
339	25.5	26.9	28.9	32.3	32.2	33.8	33.9	34.9	37.1	38.5	37.1
341	13.5	14.0	14.8	16.8	17.6	18.2	18.5	19.4	20.9	22.1	21.6
342	7.2	6.4	5.5	6.5	6.9	7.3	7.6	8.1	8.9	9.6	9.5
349	22.3	22.9	24.0	27.1	28.1	28.9	29.3	30.5	32.8	34.3	33.4
351	7.1	7.5	8.1	9.6	10.5	11.4	12.1	13.1	14.7	16.0	16.1
352	0.5	0.5	0.5	0.6	0.7	0.7	0.7	0.8	0.9	1.0	1.0
361	21.8	22.7	24.1	25.3	24.3	23.0	21.4	20.3	19.7	18.5	15.9
362	11.5	11.8	12.2	14.1	15.0	15.7	16.2	17.2	18.8	20.0	19.8
353	5.7	5.9	6.2	6.4	6.1	5.7	5.2	4.8	4.5	4.1	3.4
364	10.4	10.2	10.2	11.8	12.5	13.2	13.6	14.5	15.8	16.9	16.8
365	14.5	16.3	18.6	21.0	21.2	21.4	22.0	23.2	23.2	24.0	23.0
369	13.6	13.9	14.5	16.2	16.2	17.1	17.2	17.8	19.0	19.8	19.1
370	29.3	31.0	31.1	32.1	33.7	32.8	30.4	31.7	33.2	32.8	30.0
381	71.3	76.9	84.7	94.9	100.6	104.1	106.7	115.0	121.5	127.3	127.0
382	3.2	2.8	2.4	2.7	2.9	3.1	3.2	3.5	3.8	4.1	4.1
383	26.6	26.8	27.5	30.3	31.5	32.0	32.2	34.0	35.3	36.3	35.5
384	3.3	3.4	3.7	3.7	3.4	3.0	2.5	2.2	1.8	1.3	0.7
385/9	3.5	3.5	3.6	3.8	3.8	3.7	3.6	3.6	3.6	3.5	3.2
391	2.9	3.0	3.1	3.2	3.1	3.0	2.9	2.9	2.9	2.9	3.2
392	2.2	2.3	2.5	2.5	2.5	2.4	2.2	2.2	2.2	2.1	1.9
393	7.5	7.7	8.1	8.5	8.6	8.6	8.5	9.0	9.4	9.8	9.6
394	21.7	22.0	22.8	22.8	22.3	21.2	20.0	19.9	19.6	19.2	17.5
395	3.3	3.5	3.8	3.9	3.8	3.7	3.6	3.7	3.7	3.8	3.5
396	2.4	2.3	2.2	2.4	2.6	2.7	2.9	3.1	3.4	3.7	3.8
399	49.1	51.7	55.3	59.8	63.2	65.2	66.6	72.6	77.8	83.9	84.6

Table (11.7)

Estimated 'Coke' Consumption By Engineering Industries in the UK (M. Therms)

MLH (1968)	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971
321	59.4	54.0	55.9	59.2	53.7	47.8	52.2	44.2	54.2	63.4	45.7
331	2.7	2.3	2.3	2.0	1.5	1.1	1.1	0.8	0.8	0.8	0.5
332	7.2	6.6	6.9	7.9	7.7	7.2	8.2	7.2	9.0	10.8	7.9
333	1.9	1.7	1.6	1.4	1.1	0.8	0.7	0.5	0.5	0.5	0.3
334	6.7	5.8	5.7	5.0	3.8	2.9	2.7	1.9	2.0	2.0	1.2
335	9.4	8.2	8.2	7.2	5.4	4.0	3.6	2.5	2.5	2.4	1.4
336	4.2	3.9	4.1	3.7	2.9	2.2	2.1	1.6	1.7	1.7	1.1
337	6.3	5.7	5.9	5.1	3.8	2.8	2.6	1.8	1.8	1.7	1.0
338	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.1
339	36.5	32.3	32.6	29.4	22.9	17.6	16.8	12.5	13.4	13.8	8.8
341	15.8	13.7	13.6	12.4	9.8	7.6	7.4	5.5	6.1	6.4	4.1
342	1.9	1.5	1.3	1.0	0.7	0.5	0.4	0.2	0.2	0.0	0.0
349	12.7	11.2	11.2	10.2	8.0	6.2	5.9	4.5	4.8	5.0	3.2
351	3.2	2.8	2.7	2.4	1.8	1.4	1.2	0.9	0.9	0.9	0.5
352	0.1	0.1	0.1	0.1	0.0	0.1	0.0	0.0	0.0	0.0	0.0
361	10.9	10.1	10.7	9.3	6.9	5.0	4.4	3.0	2.9	2.7	1.4
362	2.4	2.1	2.2	1.7	1.2	0.7	0.5	0.3	0.1	0.0	0.0
363	1.4	1.1	1.0	0.9	0.7	0.5	0.5	0.3	0.4	0.4	0.2
364	2.6	2.1	2.0	1.8	1.5	1.2	1.2	0.9	1.0	1.1	0.7
365	3.0	2.8	3.0	2.7	2.0	1.5	1.4	1.0	1.0	1.3	0.6
369	2.8	2.2	2.0	1.8	1.5	1.2	1.2	0.9	1.0	1.1	0.7
370	18.1	15.7	15.5	13.7	10.5	7.9	7.4	5.3	5.6	5.5	3.4
381	15.6	13.6	13.5	13.5	11.8	10.1	10.7	8.8	10.5	12.0	8.5
382	0.8	0.7	0.6	0.5	0.3	0.2	0.1	0.1	0.0	0.0	0.0
383	11.1	9.6	9.4	8.6	6.7	5.2	5.0	3.8	4.1	4.2	2.7
384	6.4	5.3	5.9	5.1	3.8	2.8	2.5	1.7	1.6	1.5	0.8
385/9	5.5	5.1	5.3	4.3	3.0	2.0	1.6	0.9	0.7	0.3	0.0
391	2.0	1.5	1.5	1.3	1.0	0.7	0.6	0.5	0.5	0.4	0.2
392	0.7	0.7	0.7	0.6	0.4	0.2	0.2	0.1	0.1	0.0	0.0
393	1.5	1.2	1.1	1.0	0.7	0.5	0.4	0.3	0.3	0.3	0.1
394	4.6	4.1	4.1	3.6	2.7	2.0	1.9	1.3	1.4	1.3	0.8
395	0.7	0.6	0.6	0.6	0.4	0.3	0.3	0.2	0.2	0.2	0.1
396	0.8	0.6	0.5	1.1	1.3	1.5	1.9	1.8	2.3	2.9	2.2
399	30.9	26.9	26.6	23.9	18.6	14.3	13.6	10.1	10.6	11.1	7.0

Table (11.8)

Estimated Consumption of all Fuels by Engineering Firms in UK. (M. Therms)

MtH (1968)	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971
331	16.5	15.9	16.0	16.0	16.6	17.0	17.7	18.6	19.3	20.4	19.2
332	45.3	46.8	50.0	51.1	54.2	54.5	56.7	58.4	61.7	65.3	59.2
333	25.0	25.8	27.7	28.3	30.5	32.2	33.8	36.4	38.6	40.0	41.2
334	41.4	38.7	37.0	37.1	39.6	40.2	42.5	45.3	47.8	50.8	48.1
335	41.0	39.5	39.5	39.2	40.4	40.8	42.2	44.3	47.0	49.8	47.0
336	31.5	34.0	38.5	39.0	42.1	43.1	45.6	49.1	51.8	60.0	52.4
337	27.9	28.2	30.1	31.3	33.7	35.8	39.0	43.0	47.1	51.8	51.2
338	17.1	18.4	20.7	19.6	20.1	19.3	18.4	17.5	16.1	14.5	11.2
339	186.0	186.6	186.3	122.4	217.7	226.8	240.9	258.5	277.4	297.3	253.6
341	66.3	85.4	90.1	90.1	93.3	94.7	97.2	100.4	104.3	108.4	100.4
342	63.4	61.1	56.6	53.2	54.0	51.5	49.4	48.9	48.9	49.4	45.7
343	120.3	120.5	124.5	125.3	131.0	132.7	134.6	138.2	143.0	145.6	134.8
349	33.5	31.3	28.5	32.0	36.0	41.1	44.9	50.0	55.8	61.5	61.7
352	13.3	16.0	19.5	19.1	21.2	20.8	21.2	22.1	21.8	21.6	18.9
361	182.8	203.8	218.5	209.0	202.4	193.3	180.7	164.2	152.7	140.3	109.8
362	75.7	81.1	82.2	87.1	93.7	98.8	102.5	105.1	110.3	116.4	106.3
363	38.2	41.9	44.4	42.7	41.4	40.2	38.3	35.2	33.7	32.2	26.1
364	44.7	43.2	41.2	44.7	48.5	52.9	57.1	59.6	65.0	72.5	68.8
365	77.8	89.7	102.6	106.8	111.9	118.4	125.4	128.0	137.5	147.9	138.5
369	62.2	64.4	66.9	72.9	80.3	86.6	96.3	101.4	113.0	125.3	121.8
370	125.8	122.8	123.6	125.1	125.8	120.7	113.5	116.2	116.1	126.5	125.3
381	433.9	512.1	569.8	586.2	616.1	619.0	627.7	670.5	686.7	686.2	675.2
382	28.2	27.5	25.4	28.3	31.3	31.8	32.8	35.5	37.7	36.7	39.3
383	196.0	197.3	206.1	209.4	211.4	205.3	194.8	203.4	211.6	208.0	179.6
384	42.0	41.0	37.8	34.3	32.3	29.1	25.5	22.9	18.7	16.3	16.9
385/9	35.3	33.2	33.0	29.8	28.8	33.2	35.6	39.6	38.5	43.4	39.1
391	17.3	16.7	17.0	16.6	16.4	15.1	14.1	13.7	13.7	13.2	7.2
392	7.2	7.3	7.6	7.9	8.3	8.2	8.2	8.7	9.6	10.0	9.7
393	33.2	38.1	37.8	38.6	47.5	38.2	35.7	36.3	37.2	37.3	34.6
394	76.0	75.3	76.7	77.0	77.1	73.7	68.9	68.0	70.3	70.4	65.2
395	19.6	19.9	21.1	22.0	22.8	22.8	22.5	23.3	25.1	26.0	25.2
396	15.0	15.2	16.2	18.1	20.4	20.6	21.9	23.4	25.6	27.5	26.8
399	298.9	292.8	301.6	311.3	322.7	314.6	310.3	324.1	340.0	354.2	338.8

It is believed, however, that for most industries, the errors introduced were quite small because industrial production is concentrated in England and Wales.

iii. The industry coverage for annual data differed considerably between fuel types, ranging, for example, from eight industries for coal to a single aggregate for 'coke'. As each fuel was treated individually, this in itself caused few problems. However, the fuels which distinguish few separate industries may be distributed fairly accurately across industries at a given point in time, but are unlikely to reflect so accurately the variations over time in a particular industry.

iv. The census of production relates only to establishments employing 25 or more workers, but, because of the way in which the data was used, the underestimation of the absolute amount of fuel consumed does not matter so long as the figures reflect the relative sizes of consumption within engineering. Thus, it is possible that differences in industrial structure might introduce slight errors, but their importance is reduced by the generally large part of fuel consumption which is accounted for by establishments employing 25 or more workers.

v. Fuels consumed for purposes other than powering equipment are likely to be a much greater problem in industries outside of engineering - see Moody (1974, pp.47-8). The problem was apparent, however, in so far as fuels were used for testing. It was argued that either such tests were a part of the production process or carried out as R & D and the inclusion on both counts could be justified.

vi. Finally the weights used to obtain thermal yields per original unit of consumption for each fuel group were in some cases not wholly appropriate. However, fuels that made up each group were fairly homogeneous with regard to their thermal yields and the errors introduced are unlikely to be large.

Appendix (III) Reconciliation of the Industrial Classifications.

Table (III.1) contained in this appendix attempts to provide a reconciliation of the 1958 and 1968 Standard Industrial Classifications for the industry groups that fall within the scope of this study (ie. the engineering industries). Within the engineering sector there are no changes in industrial classification that cannot be overcome by a simple reallocation of MLIs within SICs, at least for the period of interest to this study. I am extremely grateful to G.J. Evans for the preparation of this table.

## RECONCILIATION OF STANDARD INDUSTRIAL CLASSIFICATIONS OF 1958 and 1968 WITH E.I.T.B. INDUSTRY GROUPS

S.I.C. 1958			E.I.T.B. Classification				
Order	M.L.H. No.	Description of M.L.H.	Industry Group No.	Combine Industry Code	Description of M.L.H.	M.L.H. No.	Order
(V) Metal  Manufacture	312*	Steel Tubes	01	I	Steel Tubes	312*	(VI) Metal
	313*	Iron Castings etc.	02		Iron castings etc.	313*	
	321* 322	Light Metals Copper, brass and other base metals	03		Alluminum and alluminum alloys Copper, brass & other copper alloys	321	Manufacture
(VI) Engineering and Electrical Goods	332	Metal-working machine tools	04	II	Metal working machine tools	332	(VII) Mechanical Engineering (XII) Metal Goods N.E.S.
	333	Engineers' small tools and gauges			Engineers' small tools and gauges		
(VII) Shipbuilding and Marine Engineering	370.2	Marine Engineering	05		Marine Engineering	370.2	(X) Shipbuilding and Marine Engineering
(VI) Engineering  and  Electrical  Goods	331*	Agricultural machinery (except tractors)	06	III	Agricultural machinery (except tractors)	331*	(VII) Mechanical Engineering
	334	Industrial engines			Industrial engines	334	
	335	Textile Machinery and accessories			Textile machinery and accessories	335	
	336*	Contractors plant and quarrying machinery			Construction and earth moving equipment	336*	
	337	Mechanical handling equipment			Mechanical handling equipment	337	
	338	Office Machinery			Office Machinery	338	
	339	Other machinery			Pumps, valves and compressors Other machinery	339*	
	341	Industrial plant and steelwork			Industrial plant and steelwork	341	
	342* 349	Ordnance and small arms Other mechanical engineering N.E.S.	Ordnance and small arms Other mechanical engineering N.E.S.	342* 349	Engineering		
	351*	Scientific, surgical and photographic instruments etc.	09	Photographic and document copying equipment Surgical instruments and appliances Scientific and industrial instruments and systems	351* 353*	(VIII) Instrument Engineering	
352	Watches and clocks	Watches and clocks		352			
Electrical   Goods	361	Electrical machinery	10	IV	Electrical machinery	361	(IX) Electrical Engineering
	364	Radio and other electronic apparatus	11		Radio and electronic components Broadcast receiving and sound reproducing equipment	364 365	
	362	Insulated wires and cables			Insulated wires and cables	362	
	363	Telegraph and telephone apparatus	12		Telegraph and telephone apparatus and equipment	363	
	365	Domestic electrical appliances			Electric appliances primarily for domestic use	368	
369	Other electrical goods	Other electrical goods		369			
(VIII)  Vehicles	381	Motor vehicle manufacturing	13	V	Wheeled tractor manufacturing Motor vehicle manufacturing	380 381	(XI) Vehicle
	383*	Aircraft manufacturing and repairing	14	VI	Aerospace equipment manufacturing and repairing	383*	
	382	Motor-cycle, three-wheel vehicle and pedal cycle manufacturing	15	VII	Motor cycle, tricycle and pedal cycle manufacturing	382	
	384	Locomotives and railway equipment			Locomotives and railway track equipment	384	
	385	Railway carriages and wagons and trams			Railway carriages and wagons and trams	385	
387	Propellers, hand-trucks, etc.						



Order	M.L.H. No.	Description of M.L.H.	Industry Group No.	Comline Industry Code	Description of M.L.H.	M.L.H. No.	Order
(IX)	395	Cans and metal boxes	16		Cans and metal boxes	395	(XII)
Metal	391	Tools and implements		VIII	Hand tools and implements	391	Metal
	392	Cutlery			Cutlery, spoons, forks, plated tableware etc.	392	
Goods	393	Bolts, nuts, screws and rivets etc.	17		Bolts, nuts screws, rivets etc.	393	Goods
	394	Wire			Wire	394	
	396*	Jewellery, plate and refining of precious metals			Jewellery and precious metals	396*	
N.E.S.	399	Metal industries N.E.S.			Metal Industries N.E.S.	399	N.E.S.
V	311*	Iron and Steel (General)			Iron and steel (General)	311*	VI
XIV	474*	Shop and office fittings			Shop and office fitting:	474*	XVII
XV	489*	Other printing, publishing, book binding, engraving etc.			Other printing, publishing, book binding, engraving etc.	489*	XVIII
XVI	494*	Toys, games and sports' equipment			Toys, games, children's carriages and sports equipment (Children's carriages 494.2 belong in E.I.T.B. Group 15)	494*	XIX
	499*	Miscellaneous manufacturing industries			Miscellaneous manufacturing industries	499*	
XVIII	500*	Construction	18	IX	Construction	500*	XX
XIX	701*	Railways			Railways	701*	XXII
	707*	Postal services and communications			Postal services and communications	707*	
XX	709*	Miscellaneous transport services & storage			Miscellaneous transport services & storage	709*	XXI
	810.7*	Wholesale Distribution (other non-food goods)			Other wholesale distribution (other non-food goods)	810.7*	
XXII	832*	Dealing in other industrial materials and machinery			Dealing in other industrial materials and machinery	832*	XXV
	879*	Other professional and scientific services			Other professional and scientific services	879*	
XXIII	887*	Motor repairers, distributors, garages and filling stations			Motor repairers, distributors, garages and filling stations	887*	XXVII
XXIV	906*	Local Government Service			Local Government Service	906*	
		M.L.H.s other than those listed above. M.L.H.s not known	19	X	M.L.H.s other than those listed above. M.L.H.s not known		

\* indicates not all M.L.H. covered by E.I.T.B.

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