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On Fuzzy b-Subimplicative Ideal

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Abstract: In this paper, we study a new notion of fuzzy subimplicative ideal of a BH-algebra, namely fuzzy subimplicative ideal with respect to an element in BH-algebra is introduced and some related properties are investigated.

Keywords: BH-algebra, fuzzy subimplicative ideal, subimplicative ideal, fuzzy b- subimplicative ideal, Level subset,

1. PRELIMINARIES :

In this section, issdevoted to some basic ordinarycconcepts of BH-algebra, fuzzy ideal, sub-implicative ideal in fuzzy and ordinary, level subset, image and preimage of fuzzy set and homomorphism in BH-algebra, we give some basic concepts about the image of function, the inverse image of a BH-algebra with some remarks in fuzzy sences.

Definition (1.1): [9] A **BH-algebra** is a nonempty set X with a constant 0 and a binary operation * satisfying the following conditions:

i. $x * x = 0, \forall x \in X$.

ii. $x * y = 0$ and $y * x = 0$ imply $x = y, \forall x, y \in X$.

iii. $x * 0 = x, \forall x \in X$.

Remark (1.2): [10] Let X and Y be BH-algebras. A mapping f: X→Y is called a **homomorphism** if $f(x*y) = f(x)*f(y), \forall x, y \in X$. A homomorphism f is called a **monomorphism** (resp., **epimorphism**) if it is injective (resp., surjective). A bijective homomorphism is called an isomorphism. Two BH-algebras X and Y are said to be **isomorphic**, written $X \cong Y$, if there exists an isomorphism f: X→Y. For any homomorphism f: X →Y, the set $\{x \in X : f(x)=0'\}$ is called the **kernel** of f, denoted by ker(f), and the set $\{f(x) : x \in X\}$ is called the **image** of f, denoted by Im(f). Notice that $f(0)=0'$, ∀ homomorphism f.

Definition (1.3): [1] if $\{A_\alpha, \alpha \in \Lambda\}$ is a family of fuzzy sets in X, then :

$$\bigcap_{i \in I} A_i(x) = \inf\{A_i(x), i \in I\}, \forall x \in X.$$

$$\bigcup_{i \in I} A_i(x) = \sup\{A_i(x), i \in I\}, \forall x \in X.$$ which are also **fuzzy sets** in X.

Definition (1.4): [2] Let X and Y be any two sets, A be any fuzzy set in X and f: X→Y be any function. The set $f^{-1}(y) = \{x \in X \mid f(x) = y\}, \forall y \in Y$. The fuzzy set B in Y defined by $B(y) = \begin{cases} \sup\{A(x) \mid x \in f^{-1}(y)\}; & \text{if } f^{-1}(y) \neq \emptyset \\ 0 ; & \text{otherwise} \end{cases}$, $\forall y \in Y$, is called the **image** of A under f and is denoted by f(A).

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Definition (1.5): [2] Let X and Y be any two sets, f: X → Y be any function and B be any fuzzy set in f(A). The fuzzy set A in X defined by: A(x)=B(f(x)), ∀x∈X is called the **preimage** of B under f and is denoted by $f^{-1}(B)$.

Definition(1.6): [6] A fuzzy subset A of a BH-algebra X is said to be a **fuzzy ideal** if and only if:

i. A(0) ≥ A(x), ∀ x∈X.

ii. A(x) ≥ min{A(x*y), A(y)}, ∀ x, y∈X.

Definition(1.7):[5] A nonempty subset I of a BH-algebra X is called **subimplicative** ideal of X if:

i. 0∈I.

ii. ((x*(x*y))*(y*x))*z ∈ I and z ∈ I imply y*(y*x)∈ I, ∀x, y, z ∈ X.

Definition (1.8): [5] Let X be a BH-algebra and b∈X, a fuzzy subset A of X is called a **sub-implicative ideal with respect to an element b (**or briefly, **b-subimplicative ideal)** of X if it satisfies:

i. 0∈I.

ii. (((x*(x*y))*(y*x))*z)*b ∈ I and z ∈ I imply y*(y*x)∈ I, ∀x, y, z ∈ X.

Definition(1.9): [5] A fuzzy set A of a BH-algebra X is called a **fuzzy subimplicative ideal** of X if it satisfies:

i. A(0) ≥ A(x) , ∀ x∈X.

ii. A(y*(y*x)) ≥ min {A(((x *(x*y))*(y*x)) *z), A(z)}, ∀ x, y, z ∈ X.

Proposition(1.10):[5] Let X be a BH-algebra. Then every fuzzy sub-implicative ideal of X is fuzzy ideal of X.

Definition(1.11): [7] Let μ be a fuzzy set in X, ∀ α ∈ [0, 1], the set μ_α= { x ∈ X, μ(x) ≥ α } is called a **level subset of A.** Note that, μ_α is a subset of X in the ordinary sense.

Remark(1.12):[suad] Let A be a fuzzy subset of a BH-algebra X and w ∈ X. The set { x∈X | A(w) ≤ A(x)} is denoted by ↑A(w).

2.

3. THE FUZZY SUBIMPLICATIVE IDEAL WITH RESPECT TO AN ELEMENT OF A BH-ALGEBRA.

We define the concept of **a fuzzy sub-implicative ideal with respect to an element of a BH-algebra**. We discuss some properties of this concept and link it with other types of fuzzy ideal of a BH-algebra.

Definition (2.1): Let X be a BH-algebra and b∈X, a fuzzy subset A of X is called a **fuzzy sub-implicative ideal with respect to an element b (**or briefly, **fuzzy b-subimplicative ideal)** of X if it satisfies:

i. i. A(0) ≥ A(x) , ∀ x∈X.

ii. ii. A(y*(y*x)) ≥ min {A((((x *(x*y))*(y*x)) *z)*b), A(z)}, ∀ x, y, z ∈ X.

Example(2.2): Consider the BH-algebra X={0,1,2,3} with the following operation table:

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*	0	1	2
0	0	1	2
1	1	0	2
2	2	2	0

The fuzzy subset A defined by $A(x) = \begin{cases} 0.8 \ ; \ x = 0,1 \\ 0.5 \ ; \ x = 2 \end{cases}$ is a **fuzzy 0-subimplicative ideal** of X.

Theorem (2.3): Let X be a BH-algebra. Then A is a fuzzy subimplicative ideal of X if and only if A is a fuzzy 0-subimplicative ideal of X.

Proof :

Let A be a fuzzy subimplicative ideal of X. Then

i. A(0) ≥ A(x) , ∀ x∈X. [By definition (1.9)(i)]

ii. Let x, y, z ∈X. Then, we have

A(y*(y*x))≥ min{A(((x*(x*y))*(y*x)))*z), A(z)}[By definition(1.5)(ii)]

⇒min{A((((x*(x*y))*(y*x)))*z)*0),A(z)}= min{A(((x*(x*y))*(y*x)))*z), A(z)}

[Since X is a BH-algebra; x*0=x, ∀ x∈X.]

⇒ A(y*(y*x)) ≥ min{A((((x*(x*y))*(y*x))*z)*0),A(z)}

Therefore, A is a fuzzy 0-subimplicative ideal of X.

Conversely,

Let A be a fuzzy 0-subimplicative ideal of X. Then

i. A(0) ≥ A(x) , ∀ x∈X. [By definition (2.1)(i)]

ii. Let x, y, z ∈X. Then A(y*(y*x)) ≥ min{A((((x*(x*y))*(y*x)))*z)*0),A(z)}

[Since A is a fuzzy 0-subimplicative ideal of X. By definition (2.1)(ii)]

⇒ min{A(((x*(x*y))*(y*x))*z)*0), A(z)} = min{A(((x*(x*y))*(y*x))*z), A(z)}

[Since X is a BH-algebra; x*0=x, ∀ x∈X]

⇒ A(y*(y*x)) ≥ min {A(((x*(x*y))*(y*x))*z), A(z)}.

Therefore, A is a fuzzy subimplicative ideal of X.■

Proposition (2.4): Let X be a BH-algebra, b∈X and A be a fuzzy b-subimplicative ideal of X, such that A(b)=A(0). Then A is a fuzzy ideal of X.

Proof :

Let A be a fuzzy b-subimplicative ideal of X. To prove A is a fuzzy ideal of X.

i. A(0) ≥ A(x), ∀ x∈X. [Since A is a fuzzy b-sub-implicative ideal of X.]

ii. Let x, y , z ∈X such that

A(x*b)=A((x*0)*b [Since X is BH-algebra; x*0=x]

= A((x*0)*0)*0)*b) [Since X is BH-algebra;x*x=0]

= A(((x*(x*x))*(x*x))*0)*b) [Since X is BH-algebra;x*x=0]

$\Rightarrow$ A(x*(x*x)) $\geq$ min{A(((x*(x*x))*(x*x))*0)*b), A(b)}=A(x*b)

[Since A is a fuzzy b-sub-implicative of X. By definition (2.1)(ii)]

Now, A(x*(x*x)) =A(x*0)=A(x) [Since X is BH-algebra ; x*x=0, x*0=x]

$\Rightarrow$ A(x) $\geq$ min{A(((x*(x*x))*(x*x))*0)*b) ,A(0)}=min{A(x*b) , A(0)}

$\Rightarrow$ A(x) $\geq$ min{A(x*b),A(b)}. [Since A(b)=A(0)]

Therefore, A is a fuzzy ideal of X.■

Remark(2.5): The following example shows that converse of proposition (2.4) is not correct, $\forall b \in X$.

Example (2.6): Consider the BH-algebra X= {0, 1, 2, 3} with the binary operation '*' defined by the following table:

*	0	1	2	3
0	0	0	0	0
1	1	0	0	2
2	2	2	0	1
3	3	3	3	0

The fuzzy subset A defined by $A(x) = \begin{cases} 1 & ; \ x = 0 \\ 0.5 & ; x = 1,2,3 \end{cases}$ is a fuzzy ideal of X, but A is not a fuzzy 0-subimplicative ideal of X.

Since

if x=2, y=1, z=0, then

A(1*(1*2))=A(1*0)=A(1)= 0.5 < min{A((((2*(2*1))*(1*2))*0)*0) ,A(0)}

= min{A((2*2))*(1*2)) ,A(0)}

=min{A(0),A(0)}=A(0)=1.

And A is not a fuzzy 1-subimplicative ideal of X. Since

if x=2, y=1, z=0, then

A(1*(1*2))=A(1*0)=A(1)= 0.5 < min{A((((2*(2*1))*(1*2))*0)*1) ,A(0)}

= min{A(((2*2))*(1*2))*1) ,A(0)}

=min{A(0*1),A(0)}= min{A(0),A(0)}=A(0)=1.

And A is not a fuzzy 2-subimplicative ideal of X. Since

if x=2, y=1, z=0, then

A(1*(1*2))=A(1*0)=A(1)= 0.5 < min{A((((2*(2*1))*(1*2))*0)*2) ,A(0)}

= min{A(((2*2))*(1*2))*2) ,A(0)}

=min{A(0*2),A(0)}= min{A(0),A(0)}=A(0)=1.

And A is not a fuzzy 3-subimplicative ideal of X. Since

if x=2, y=1, z=0, then

A(1*(1*2))=A(1*0)=A(1)= 0.5 < min{A((((2*(2*1))*(1*2))*0)*3) ,A(0)}

$= \min\{A(((2*2))*(1*2))*3), A(0)\}$

$= \min\{A(0*3), A(0)\} = \min\{A(0), A(0)\} = A(0) = 1.$

Therefore, A is not a fuzzy b-subimplicative ideal. $\forall b \in X$.

Proposition (2.7): Let X be a BH-algebra. A be a fuzzy subimplicative ideal of X, $b \in X$ such that A(b)=A(0). Then A is a fuzzy b-subimplicative ideal of X.

Proof:

Let A be a fuzzy subimplicative ideal of X. Then

i. $A(0) \geq A(x),\ \forall\ x \in X$ [By definition (1.9)(i)]

ii. Let x, y, z $\in$X. Then

$A(y*(y*x)) \geq \min\{A((((x*(x*y))*(y*x))*z), A(z)\}$

$\geq \min\{\min\{A(((((x*(x*y))*(y*x))*z)*b), A(b)\}, A(z)\}$

$= \min\{A(((((x*(x*y))*(y*x))*z)*b), A(z)\}$

[Since A is a fuzzy ideal of X. By proposition (1.10) and A(b)=A(0)]

Therefore, A is fuzzy b-subimplicative ideal of X.■

Theorem (2.8): Let X be a BH-algebra. Then a fuzzy ideal A of X satisfying the condition:

$\forall\ x, y \in X;\quad A(y*(y*x)) \geq A((x*(x*y))*(y*x)) \quad (b_1)$

is a fuzzy b-subimplicative ideal of X, where $b \in X$ and A(b)=A(0).

Proof:

Let A be a fuzzy ideal of X. Then, we have

i. $A(0) \geq A(x),\ \forall\ x \in X$. [By definition (1.6)(i)]

ii. Let x, y, z$\in$X. Then, we have

$A((x*(x*y))*(y*x)) \geq \min\{A((x*(x*y))*(y*x))*z), A(z)\}$

[Since A is a fuzzy ideal of X. By definition (1.6)(ii)]

$\geq \min\{\min\{A(((((x*(x*y))*(y*x))*z)*b), A(b)\}, A(z)\}$[Since A is a fuzzy ideal of X.]

$= \min\{A(((((x*(x*y))*(y*x))*z)*b), A(z)\}$

[Since $A(((((x*(x*y))*(y*x))*z)*b) = \min\{A(((((x*(x*y))*(y*x))*z)*b), A(b)\}$

and A(b)=A(0)]

$\Rightarrow A(y*(y*x)) \geq \min\{A(((((x*(x*y))*(y*x))*z)*b), A(z)\}$ [By the condition (b1)]

Then A is a fuzzy b-subimplicative ideal of X, A(b)=A(0).■

Theorem (2.9): If X is a BH-algebra of X satisfies the condition:

$$\forall\ x, y \in X \quad ; \quad y*(y*x) = (x*(x*y))*(y*x) \qquad (b2),$$

then every fuzzy ideal of X is a fuzzy b-subimplicative ideal of X, where $b \in X$ and A(b)=A(0).

Proof:

Let A be a fuzzy ideal of X. Then, we have

i. $A(0) \geq A(x),\ \forall\ x \in X$. [By definition (1.6)(i)]

ii. Let x, y, z$\in$X. Then $A((x*(x*y))*(y*x)) \geq \min\{A((x*(x*y))*(y*x))*z), A(z)\}$ [Since A is a fuzzy ideal of X. By definition (1.6)(ii)]

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Now, $A(y*(y*x))=A((x*(x*y))*(y*x))$ [By (b2)]

$\Rightarrow A(y*(y*x)) \geq \min \{A(((x*(x*y))*(y*x))*z), A(z)\}$

$\Rightarrow$ A is a fuzzy subimplicative ideal of X. [By definition (1.9)]

Therefore, A is a fuzzy b-subimplicative ideal of X. [By proposition (2.7)].■

Theorem (2.10) : Let X be a BH-algebra, b ∈ X and A be a fuzzy b-subimplicative ideal of X. Then the set X_A is a b-subimplicative ideal of X.

Proof:

Let A be a fuzzy b-subimp1licative ideal of X. To prove X_A is a b-subimplicative ideal of X.

i. $A(x) = A(0)$.

If x=0, then $0 \in X_A$

ii. Let x, y, z, b ∈ X such that $(((x*(x*y))*(y*x))*z)*b \in X_A$ and $z \in X_A$

$\Rightarrow A((((x*(x*y))*(y*x))*z)*b)=A(0)$ and $A(z)=A(0)$

$\Rightarrow$ by definition of fuzzy b-subimplicative ideal of X , we have

$A(y*(y*x)) \geq \min \{A((((x*(x*y))*(y*x))*z)*b), A(z)\}$

$= \min \{A(0), A(0)\}=A(0)$

$\Rightarrow A(y*(y*x)) \geq A(0)$. But $A(0) \geq A(x)$. [Since A is a fuzzy b-subimplicative ideal of X.]

$\Rightarrow A(y*(y*x))= A(0)$

$\Rightarrow y*(y*x) \in X_A$. Therefore, X_A is a b-subimplicative ideal of X.

Proposition (2.11): Let $\{A_\alpha \mid \alpha\in\lambda\}$ be a family of fuzzy b-subimplicative ideals of a BH-algebra X. Then $\bigcap_{\alpha\in\lambda} A_\alpha$ is a fuzzy b-subimplicative ideal of X. $\forall b\in X$

Proof: Let $\{A_\alpha \mid \alpha\in\lambda\}$ be a family of fuzzy b-subimplicative ideals of X.

i. Let $x\in X$. Then

$$\bigcap_{\alpha\in\lambda} A(0) = \inf\{A_\alpha(0) \mid \alpha\in\lambda\} \geq \inf\{A_\alpha(x) \mid \alpha\in\lambda\} = \bigcap_{\alpha\in\lambda} A_\alpha(x)$$

[Since A_α is a fuzzy b-subimplicative ideals of X, $\forall\alpha\in\lambda$. By definition (2.1)(i)]

$$\Rightarrow \bigcap_{\alpha\in\lambda} A_\alpha(0) \geq \bigcap_{\alpha\in\lambda} A_\alpha(x)$$

ii.Let x, y, z $\in$X. Then, we have $\bigcap_{\alpha\in\lambda} A_\alpha(y*(y*x)) = \inf\{A_\alpha(y*(y*x)) \mid \alpha\in\lambda\}$

$\geq \inf\{\min\{A_\alpha((((x*(x*y))*(y*x))*z)*b), A_\alpha(z) \mid \alpha\in\lambda\}\}$ [Since A_α is a fuzzy b-subimplicative ideals of X, $\forall\alpha\in\lambda$. By definition (2.1)(ii)]]

= min {inf {A((((x *(x*y))*(y*x))*z)*b), A_α (z) | $\alpha\in\lambda$}}

= min {inf {A_α ((((x *(x*y))*(y*x))*z)*b) | $\alpha\in\lambda$} , inf {A_α(z) | $\alpha\in\lambda$}} = min { $\bigcap_{\alpha\in\lambda} A_\alpha$((((x *(x*y))*(y*x)) *z)*b) , $\bigcap_{\alpha\in\lambda} A_\alpha$ (z)}

$\Rightarrow \bigcap_{\alpha\in\lambda} A_\alpha$(y*(y*x)) ≥ min{{ $\bigcap_{\alpha\in\lambda} A_\alpha$((((x*(x*y))*(y*x))*z)*b), $\bigcap_{\alpha\in\lambda} A_\alpha$(z)}

Therefore, $\bigcap_{\alpha\in\lambda} A_\alpha$ is a fuzzy b-subimplicative ideal of X. ∀b∈X ■

Proposition (2.12): Let {A_α |$\alpha\in\lambda$} be a chain of fuzzy b-subimplicative ideals of a BH-algebra X. Then $\bigcup_{\alpha\in\lambda} A_\alpha$ is a fuzzy b-subimplicative ideal of X. ∀b∈X.

Proof: Let {A_α |$\alpha\in\lambda$} be a chain of fuzzy b-subimplicative ideal of X.

i. Let x∈X. Then $\bigcup_{\alpha\in\lambda} A_\alpha$(0)= sup{ A_α(0) |$\alpha\in\lambda$ }) ≥ sup{ A_α(x) |$\alpha\in\lambda$ } = $\bigcup_{\alpha\in\lambda} A_\alpha$ (x)

[Since A_α is a fuzzy b-subimplicative ideal of X, ∀ $\alpha\in\lambda$. By definition(2.1)(i)]

$\Rightarrow \bigcup_{\alpha\in\lambda} A_\alpha (0) \geq \bigcup_{\alpha\in\lambda} A_\alpha$ (x)

ii. Let x, y, z ∈X. Then, we have $\bigcup_{\alpha\in\lambda} A_\alpha$ (y*(y*x)) = sup {A_α(y*(y*x)) |$\alpha\in\lambda$}

≥ sup {min{A_α ((((x *(x*y))*(y*x)) *z)*b), A_α(z) | $\alpha\in\lambda$}}

[Since A_α is a fuzzy b-subimplicative ideals of X, ∀$\alpha\in\lambda$. By definition (2.1)(ii)]

= min{sup{A_α((((x*(x*y))*(y*x))*z)*b), A_α(z)| $\alpha\in\lambda$}} [Since A_α is a chain]

= min {sup {A_α ((((x *(x*y))*(y*x)) *z)*b) | $\alpha\in\lambda$}, sup{A $_\alpha$(z) | $\alpha\in\lambda$}}

= min { $\bigcup_{\alpha\in\lambda} A_\alpha$ ((((x *(x*y))*(y*x)) *z)*b) , $\bigcup_{i\in\Gamma} A_\alpha$(z) }

$\Rightarrow \bigcup_{\alpha\in\lambda} A_\alpha$(y*(y*x)) ≥ min {{ $\bigcup_{\alpha\in\lambda} A$((((x *(x*y))*(y*x)) *z)*b), $\bigcup_{\alpha\in\lambda} A_\alpha$(z)}

Therefore, $\bigcup_{\alpha\in\lambda} A_\alpha$ is a fuzzy b-subimplicative ideal of X. ∀b∈X ■

Proposition(2.13): Let f: (X,*, 0)→(Y,*', 0') be a BH-epimorphism. If A is a fuzzy b-subimplicative ideal of X, then f(A) is a fuzzy b-subimplicative ideal of Y. ∀b∈X

Proof: Let A be a fuzzy b-subimplicative ideal of X. Then

i. Let y ∈Y. Then there exists x ∈ X.

$(f(A))(0')=\sup\{A(x_1) \mid x_1 \in f^{-1}(0')\}$

$=A(0) \geq \sup\{A(x) \mid x \in X\} \geq \sup\{A(x_1) \mid x = f^{-1}(y)\}=(f(A))(y)$

[Since A is a fuzzy b-subimplicative ideal of X. By definition (2.1)(i)]

$\Rightarrow (f(A))(0') \geq (f(A))(y), \ \forall y \in Y.$

iii. Let $y_1, y_2, y_3 \in Y$. Then there exist

$$f(x_1)= y_1,\ f(x_2)= y_2,\ f(z)= y_3,\ f(b)= y_4 \text{ such that } x_1, x_2, z, b \in X$$

$(f(A))(y_2*'(y_2*'y_1))=\sup\{A(x_2*(x_2*x_1)) \mid x_2 * (x_2 * x_1) \in f^{-1}((y_2*'(y_2*'y_1)))\}$

$\geq \min\{\sup\{A((((x_1*(x_1*x_2))*(x_2*x_1))*z)*b), A(z) \mid (((x_1*(x_1*x_2))*(x_2*x_1))*z) * b \in$

$f^{-1}(((y_1*'(y_1*'y_2))*'(y_2*'y_1))*'y_3)*'y_4)\}$ and $z\in f^{-1}(y_3)\}\}$

[Since A is a fuzzy b-subimplicative ideal of X. By definition (2.1)(ii)]

$\geq \min\{\sup\{A((((x_1*(x_1*x_2))*(x_2*x_1))*z)*b) \mid \left(\left((x_1 * (x_1 * x_2)\right) * (x_2 * x_1)\right) * z) * b \in$

$f^{-1}(((y_1*'(y_1*'y_2))*'(y_2*'y_1))*'y_3)*)*'y_4)\}$, $\sup\{A(z) \mid z \in f^{-1}(y_3)\}\}$

$= \min\{((f(A))((f(((x_1*(x_1*x_2))*(x_2*x_1))*z)*b), (f(A))(f(z))\}$

$=\min\{((f(A))(((f(x_1)*'(f(x_1)*'f(x_2)))*'(f(x_2)*'f(x_1)))*'f(z))*'f(b), (f(A))(f(z))\}$

[Since f is an epimorphism. By remark (1.2)]

$= \min\{(f(A))((((y_1*'(y_1*'y_2)) *'(y_2*' y_1)))*' y_3))*'y_4), (f(A))(y_3)\}$

$\Rightarrow f(A))(y_2*'(y_2*'y_1)) \geq \min\{(f(A))((((y_1*'(y_1*'y_2))*'(y_2*'y_1)))*'y_3))*'y_4),(f(A))(y_3)\}$

Therefore, f (A) is a fuzzy b-subimplicative ideal of Y.■

Proposition (2.14):

Let X be a BH-algebra and A be a fuzzy subset of X. Then A is a fuzzy b-subimplicative ideal of X if and only if $A^{\#}(x)=A(x)+1-A(0)$ is a fuzzy b-subimplicative ideal of X, where b∈X.

Proof:

Let A be a fuzzy b-subimplicative ideal of X. Then

i. $A^{\#}(0)=A(0)+1-A(0)$

$\Rightarrow A^{\#}(0)=1$. Then $A^{\#}(0) \geq A^{\#}(x), \forall x \in X$

ii. Let x, y, z $\in$X and b$\in$X. Then

$A^{\#}(y*(y*x))=A(y*(y*x))+1-A(0)$

$\geq \min\{A((((x*(x*y))*(y*x))*z)*b),A(z)\}+1-A(0)$

[Since A is a fuzzy b-subimplicative ideal of X. By definition (2.1)(ii)]

$= \min\{A((((x*(x*y))*(y*x))*z)*b)+1-A(0)\ ,\ A(z)+1-A(0)\}$

$\geq \min\{A^{\#}((((x*(x*y))*(y*x))*z)*b))\ ,\ A^{\#}(z)\}$

$\therefore\ A^{\#}(y*(y*x)) \geq \min\{A^{\#}((((x*(x*y))*(y*x))*z)*b),\ A^{\#}(z)\}$

$\Rightarrow A^{\#}$ is a fuzzy b-subimplicative ideal of X.

Conversely,

Let $A^{\#}$ be a fuzzy b-subimplicative ideal of X.

i. Let x$\in$X. Then we have

$A(0)=A^{\#}(0)-1+A(0) \geq A^{\#}(0)-1+A(0)=A(x)$

[Since $A^{\#}$ be a fuzzy b-subimplicative ideal of X. By definition (2.1)(i)]

$\Rightarrow A(0) \geq A(x), \forall x \in X$.

ii. Let x, y, z $\in$X and b$\in$X. Then

$A(y*(y*x)) =A^{\#}(y*(y*x))-1+A(0) \geq \min\{A^{\#}((((x*(x*y))*(y*x))*z)*b),\ A^{\#}(z)\}-1+A(0)$

[Since $A^{\#}$ is a fuzzy b-subimplicative ideal of X. By definition (2.1)(ii)]

$= \min\{A^{\#}((((x*(x*y))*(y*x))*z)*b)-1+A(0)\ ,\ A^{\#}(z)-1+A(0)\}$

$\geq \min\{A((((x*(x*y))*(y*x))*z)*b))\ ,\ A(z)\}$

$\Rightarrow\ A(y*(y*x)) \geq \min\{A((((x*(x*y))*(y*x))*z)*b),\ A(z)\}$

Then A is a fuzzy b-subimplicative ideal of X.■

Proposition(2.15) Let X be a BH-algebra and let w, b ∈ X. If A is a fuzzy b-sub-implicative ideal of X, then ↑A(w) is a b-subimplicative ideal of X .

Proof:

Let A be a fuzzy b-subimplicative ideal of X. Then

i. $A(0) \geq A(x), \forall x \in X$.

[Since A is a fuzzy b-subimplicative ideal of X. By definition (2.1)(i)]

$\Rightarrow A(0) \geq A((w) \Rightarrow 0 \in \uparrow A(w)$

ii. Let x, y, z ∈ X such that $(((x*(x*y)*(y*x))*z)*b \in \uparrow A(w)$ and $z \in \uparrow A(w)$

$\Rightarrow\ A(w) \leq A((((x*(x*y))*(y*x))*z)*b)$ and $A(w) \leq A(z)$

$\Rightarrow\ A(w) \leq \min\{A((((x*(x*y))*(y*x))*z)*b)\ ,\ A(z)\}$

But $A(y*(y*x)) \geq \min\{A((x*(x*y))*(y*x))*z)*b)\ ,\ A(z)\}$

[Since A is a fuzzy b-subimplicative ideal of X .By definition (2.1)(ii)]

$\Rightarrow A(w) \leq A(y*(y*x))$

$\Rightarrow y*(y*x) \in \uparrow A(w)$.

Therefore, $\uparrow A(w)$ is a b-subimplicative ideal of X . ■

Theorem (2.16):

Let X be a BH-algebra, A be a fuzzy ideal of X A(b) =A(0). Then A is a fuzzy b-subimplicative ideal of X if and only if A_α is a b-subimplicative ideal of X, $\forall\ \alpha\in[0, A(0)]$.

Proof:

Let A be a fuzzy b-subimplicative ideal of X. To prove A_α is a b-subimplicative ideal of X.

i. Let $x\in A_\alpha$. Then $A(x) \geq \alpha$ [By definition (1.11) of A_α]

But $A(0) \geq A(x)$. [Since A is a fuzzy b-subimplicative ideal of X. By definition(2.1)(i)]

$\Rightarrow A(0) \geq \alpha$.

$\Rightarrow 0\in A_\alpha$.

ii. Let x, y, z, $b\in$ X such that $(((x*(x*y))*(y*x))*z)*b\in A_\alpha$ and $z\in A_\alpha$.

$\Rightarrow A((((x*(x*y))*(y*x))*z)*b) \geq \alpha$ and $A(z) \geq \alpha$ [By definition(1.11) of A_α]

$\min \{ A((((x*(x*y))*(y*x))*z)*b) , A(z) \} \geq \alpha$

But $A(y*(y*x)) \geq \min\{ A((((x*(x*y))*(y*x))*z)*b) , A(z) \}$

[Since A is a fuzzy b-subimplicative ideal of X. By definition (2.1)(ii)]

$\Rightarrow A(y*(y*x)) \geq \alpha \Rightarrow y*(y*x) \in A_\alpha$ [By definition (1.11) of A_α]

Therefore, A_α is a b-subimplicative ideal of X, $\forall\ b\in A_\alpha$.

Conversely,

To prove A is a fuzzy b-subimplicative ideal of X.

i. $0\in A_\alpha$. [By definition (1.8)(i)]. Then $A(0) \geq \alpha = A(x)$, $\forall\ x\in X$.

ii. Let x, y, z $\in$ X such that $\alpha = \min\{ A((((x*(x*y))*(y*x))*z)*b) , A(z)\}$

$\Rightarrow A((((x*(x*y))*(y*x))*z)*b) \geq \alpha$ and $A(z) \geq \alpha$

$\Rightarrow (((x*(x*y))*(y*x))*z)*b \in A_\alpha$ and $z\in A_\alpha$.

$\Rightarrow y*(y*x)\in A_\alpha$ [Since A_α is a b-subimplicative ideal of X. By definition (1.8)(ii)]

$\Rightarrow A(y*(y*x)) \geq \alpha$

$\Rightarrow A(y*(y*x)) \geq \min \{ A((((x*(x*y))*(y*x))*z)*b) , A(z) \}$

Therefore, A is a fuzzy b-subimplicative ideal of X, A(b) =A(0). ■

References

1. D. Dubois and H. Prade, "Fuzzy Sets and Systems", Academic Press. INC. (London) LTD, fifth Avenue, New Yurk, 1980.
2. E. M.kim and S.S. Ahn," On Fuzzy n-fold Strong Ideals of BH-algebras", J. Appl. Math. & Informatics. 30,(No.3) – 4,pp. 665 - 676,2012
3. H. H. Abbass and H. A. Dahham,"Some Types of Fuzzy Ideals with Respect to an Element of a BG-algebra", M.S.c thesis, Kufa University, 2012.
4. H. H. Abbass and H. M. A. Saeed, "The Fuzzy Closed BCH-algebra with Respect to an Element of a BH-algebra", M.S.c thesis, Kufa University, 2011.
5. H. H. Abbass and S. A. Neamah. Saeed, "The Fuzzy Implicative Ideal with Respect to an Element of a BH-algebra", M.S.c thesis, Kufa University, 2014.

6. H. H. Abbass, "The Q-Smarandache Closed Ideal and Q-Smarandache Fuzzy Closed Ideal with Respect to an Element of a Q-Smarandache BCH-agebra", Journal kerbala university,vol.10, No.3 scientific, pp.302-312,2012.
7. M. Ganesh, "Introduction to Fuzzy Sets and Fuzzy Logic", fourth printing, 2009.
8. Q. Zhang, E. H. Roh and Y. B. Jun, "On Fuzzy BH-algebras", J. Huanggang, Normal Univ. 21(3), pp. 14–19, 2001.
9. Y. B. Jun, E. H. Roh and H. S. Kim, "On BH-algebras", Scientiae Mathematicae 1(1), pp. 347–354, 1998.
10. Y. B. Jun, H. S. Kim and M. Kondo," On BH-relations in BH-algebras", Scientiae Mathematicae Japonicae Online, Vol.9, pp. 91–94, 2003.]