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Relative and Logarthmic of AI-Tememe Acceleration Methods for Improving the Values of Integrations Numerically of Second Kind

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Abstract: *The aims of this study are to introduce acceleration methods that called relative and algorithmic acceleration methods, which we generally call Al-Tememe's acceleration methods of the second kind discovered by (Ali Hassan Mohammed). It is very useful to improve the numerical results of continuous integrands in which the main error is of the 4th order, and related to accuracy, the number of used partial intervals and how fast to get results especially to accelerate the results got by Simpson's method. Also, it is possible to utilize it in improving the results of differential equations numerically of the main error of the forth order.*

1. INTRODUCTION

There are numerical methods for calculating single integrals that are bounded in their integration intervals.
1.Trapezoidal Rule
2.Midpoint Rule
3.Simpson's Rule

It is called Newton–Cotes formulas.
The study will introduce Simpson's method to find approximate values of single integrals of continuous integrands through using relative and algorithmic acceleration methods, which come within Al-Temem's acceleration seriesof the second kind. We will compare these methods with respect to accuracy and the speed of approaching these values to the real value (analytical) of those integrals.

Let's assume that integration J is:

$$J = \int_{x_0}^{x_{2n}} f(x)dx \ldots \qquad (1)$$

Whereas f(x) is a continuous integrand lies above X axis in the interval $[x_0, x_{2n}]$, and it is required to find approximate value, of J Generally, Newton–Cotes formula for integration (1) can be written in the following form:

$$J = \int_{x_0}^{x_{2n}} f(x)dx = G(h) + E_G(h) + R_G \qquad (2)$$

whereas G (h) represents (Lagrangian – Approximation) the value of integration J, and G refers to the type of the rule, E_G (h) is the correction terms that can be added to G(h) and R_G is the remainder . The Simpson 's rule value G (h) that is referred to by S(h) is:

$$S(h)=\frac{h}{3}[f(a) + 4f(a+h) + 2f(a+2h) + 4f(a+3h) + \cdots + 2f(a+(2n-2)h) + 4f(a+(2n-1)h) + f(b)]$$

and the general formula for E_G(h) is the following:

$$(E_S(h)=E_G(h)) \qquad ({f_{x_{2n}}}^{(5)} - {f_{x_0}}^{(5)})+\ldots\ {f_{x_{2n}}}^{(3)} - {f_{x_0}}^{(3)})+ \frac{h^6}{1512}\ (E_S(h)= \frac{h^4}{180}$$

Fox [1]
So, when integrands of integration is a continuous function and also their derivatives are in each point of integration intervals$[x_0, x_{2n}]$, it is possible to write error formula as:
$J\text{-}s(h)=A_1h^4+A_2h^6+A_3h^8+\ldots.$
whereas $A_1, A_2, A_3, \ldots\ldots$ are constants that their values do not depend on h but on the values of the function derivatives in the end of the integration interval.

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2. AL-TEMEME'S RELATIVE AND LOGARITHMIC ACCELERATION FUNCTIONS OF THE SECOND KIND

We will present the acceleration methods that come within Al-Tememe's series of acceleration and we call it relative and algorithmic accelerations.

It is mentioned above that the error in Simpson's rule can be written as the following:

$$E= A_1h^4+ A_2h^6+\ldots \quad =h^4(A_1+A_2h^2+\ldots\ldots) \qquad (3)$$

$$\cong \frac{h^4}{1+h^2}$$

It is assumed that S(h) is the approximate value of integration in Simpson's rule, so:

$E=J-S(h)\cong \frac{h^4}{1+h^2}$; since $(\frac{h^4}{1+h^2} = h^4 - h^6 + h^8 - \cdots)$ [2]

If we assume that we calculate two values for J numerically based on Simpson's rule as $S_1(h_1)$ when $h= h_1$, $S_2(h_2)$, when $h=h_2$, so:

$$J-S_1(h_1) \cong \frac{{h_1}^4}{1+{h_1}^2} \qquad (4)$$

$$J-S_2(h_2) \cong \frac{{h_2}^4}{1+{h_2}^2} \qquad (5)$$

From the equations (4) and (5), we get:

$$A^S{}_{\frac{h^4}{1+h^2}} \cong \frac{({h_2}^4\,(1+{h_1}^2))\,S_1(h_1)-({h_1}^4\,(1+{h_2}^2)\,S_2(h_2)}{{h_2}^4(1+{h_1}^2)-{h_1}^4(1+{h_2}^2)} \qquad (6)$$

The formula (6) is called Al-Tememe's first relative acceleration rule of the second kind that referred to by ($A^S{}_{\frac{h^4}{1+h^2}}$)

Similarly, Al-Tememe's second relative acceleration rule can be written. Because the error E can be written as:

$E=h^4(A_1+A_2h^2+A_3h^4+\ldots)\cong\frac{h^4}{1-h^2}$; since $(\frac{h^4}{1+h^2} = h^4 - h^6 + h^8 - \cdots)$ [2]

Based on the same first acceleration, we get the following:

$$A^S{}_{\frac{h^4}{1-h^2}} \cong \frac{({h_2}^4\,(1-{h_1}^2))\,S_1(h_1)-({h_1}^4\,(1-{h_2}^2)\,S_2(h_2)}{{h_2}^4(1-{h_1}^2)-{h_1}^4(1-{h_2}^2)} \qquad (7)$$

We call the formula (7) as Al-Tememe's second relative acceleration rule of the second kind that referred to by $A^S{}_{\frac{h^4}{1-h^2}}$

Currently, we are going to derive the first algorithmic rule: Since the error is:

$$E(h)=A_1h^4 + A_2h^6 + \ldots = h^2(A_1h^2 + A_2h^4 + \ldots\ldots) \qquad (8)$$

$\cong h^2 \ln(1+h^2)$,since$(\ln(1+h^2)=h^2-\frac{1}{2}h^4+\frac{1}{3}h^6+\ldots.)$ [2]

By the same above method

$$J-S_1(h_1) \cong {h_1}^2\ln(1+{h_1}^2) \qquad (9)$$

$$J-S_2(h_2) \cong {h_2}^2\ln(1+{h_2}^2) \qquad (10)$$

From the equations (9) and (10), we get the following:

$$A^S{}_{\ln(1+h^2)}\cong \frac{\left({h_1}^2\ln(1+{h_1}^2)\right)S_2(h_2)-\left({h_2}^2\ln(1+{h_2}^2)\right)S_1(h_1)}{{h_1}^2\ln(1+{h_1}^2)-{h_2}^2\ln(1+{h_2}^2)} \qquad (11)$$

We call the formula (11) as Al-Tememe's first logorithmic acceleration rule of the second kind that referred to by$A^S{}_{\ln(1+h^2)}$)

Similarly, we can write the second algorithmic acceleration rule of the second kind:

$$A^S{}_{\ln(1-h^2)} \cong \frac{\left({h_1}^2\ln(1-{h_1}^2)\right)S_2(h_2)-\left({h_2}^2\ln(1-{h_2}^2)\right)S_1(h_1)}{{h_1}^2\ln(1-{h_1}^2)-{h_2}^2\ln(1-{h_2}^2)} \qquad (12)$$

Since $\ln(1-h^2)=-h^2+\frac{1}{2}h^4-\frac{1}{3}h^6+\ldots.$ [2]

We call the formula (12) as Al-Tememe's second logorithmic acceleration rule of the second kind that referred to by $A^S{}_{\ln(1-h^2)}$

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3. EXAMPLES:

We will review some integrals that have continuous integrands on the interval integration using relative and algorithmic acceleration methods of Al-Tememe to improve the results numerically:

3.1: $I = \int_0^1 \sqrt{1+x^2}\,dx$ and its analytical value is 1.14779357469631 and it is rounded for 14decimal.

3.2: $I = \int_1^2 ln\sqrt{1+x}\,dx$ and its analytical value is 0.45477125244221 and it is rounded to 14decimal.

3.3: $I = \int_0^1 (e^x+1)^{0.5}dx$ and its analytical value is 1.64205578028158 and it is rounded to 14decimal.

4. THE RESULTS

The integrand of integration $I = \int_0^1 \sqrt{1+x^2}\,dx$ is continuous in the integration interval [0,1], and the formula of correction terms of Simpson's rule as above mentioned (equation 3).

We put EPS=10^{-12} (represents the absolute error of the subsequent value- previous value)

We got the results shown in table (1). We got correct values through accelerating $A^s_{\frac{h^4}{1+h^2}}$ and $A_{\frac{h^4}{1-h^2}}$ to 11 decimal when n=28,30,32 while by using Simpson's method without acceleration was correct to 8 decimal when n=32. While by the acceleration$A^s_{\ln(1+h^2)}$ we got the same accuracy when n=24,26,28. Also, we get the same accuracy of acceleration $A^s_{\ln(1-h^2)}$ when n=26,28,30.

The integrand of integration $I = \int_1^2 ln\sqrt{1+x}\;dx$ is continuous in the integration interval [1,2], and the formula of correction terms of Simpson's rule as above mentioned (equation 3).

We got the results shown in table (2). We got correct values through accelerating $A^s_{\frac{h^4}{1+h^2}}$ to 11 decimal when n=18,20,22,24 while by using Simpson's method without acceleration was correct to 8 decimal when n=26. With the acceleration $A^s_{\frac{h^4}{1-h^2}}$, we got the same accuracy when n=26 while the acceleration $A^s_{\ln(1+h^2)}$, save the same accuracy when n=14,16,18. Also, we got the same accuracy for the acceleration $A^s_{\ln(1-h^2)}$ $when\ n = 24, 26.$

The integrand of integration $I = \int_0^1 (e^x+1)^{0.5}$ is continuous in the integration interval [0,1], and the formula of correction terms of Simpson's rule as above mentioned (equation 3).

We get the results shown in table (3). We got correct values through accelerating $A^s_{\frac{h^4}{1+h^2}}$ rounded to 11 decimal when n=24. While the value by using Simpson's method without acceleration was correct to 8 decimal when n=24. But with the acceleration $A^s_{\frac{h^4}{1-h^2}}$, we got the same accuracy when n=18,20,22,24. Also we get the same accuracy for the acceleration $A^s_{\ln(1-h^2)}$ when n=18,20,22 except the acceleration $A^s_{\ln(1+h^2)}$ we get corret values rounded to 12 decimal when n=22.

5. CONCLUSION

We conclude from the mentioned tables that these acceleration methods have the same efficiency and give high accuracy of results in limited number of partial intervals with slight difference.

Table no.(1) to calculate integration $I = \int_0^1 \sqrt{1+x^2}\,dx = 1.14779357469631$ by simpson's rule with the relative and algorithmic acceleration methods of Al-Tememe

n	Values of simpson's rule	$A^S_{\frac{h^4}{1+h^2}}$	$A^S_{\frac{h^4}{1-h^2}}$	$A^s_{\ln(1+h^2)}$	$A^s_{\ln(1-h^2)}$
2	1.14772491956211				
4	1.14778226379088	1.14778681492015	1.14778528190818	1.14778644249624	1.14778567103317
6	1.14779131177394	1.14779363352053	1.14779344071112	1.14779358600185	1.14779348955832
8	1.14779285716189	1.14779358504618	1.14779355961096	1.14779357873727	1.14779356601862
10	1.14779328052368	1.14779357701948	1.14779357142258	1.14779357562711	1.14779357282859
12	1.14779343276188	1.14779357539693	1.14779357372318	1.14779357497989	1.14779357414300
14	1.14779349806165	1.14779357495422	1.14779357434102	1.14779357480129	1.14779357449469
16	1.14779352976621	1.14779357480581	1.14779357454626	1.14779357474104	1.14779357461126
18	1.14779354664315	1.14779357474805	1.14779357462566	1.14779357471750	1.14779357465630

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20	1.14779355628900	1.14779357472288	1.14779357466012	1.14779357470721	1.14779357467583
22	1.14779356212304	1.14779357471090	1.14779357467649	1.14779357470230	1.14779357468510
24	1.14779356581830	1.14779357470477	1.14779357468484	1.14779357469979	1.14779357468982
26	1.14779356825041	1.14779357470145	1.14779357468936	1.14779357469843	1.14779357469239
28	1.14779356990386	1.14779357469955	1.14779357469194	1.14779357469765	1.14779357469384
30	1.14779357105954	1.14779357469842	1.14779357469347		1.14779357469471
32	1.14779357188694	1.14779357469773	1.14779357469441		

Table no.(2) to calculate integration $I = \int_1^2 ln\sqrt{1+x}\, dx = 0.45477125244221$ by simpson's rule with the relative and algorithmic acceleration methods of Al-Tememe

n	Values of simpson's rule	$A^s{}_{\frac{h^4}{1+h^2}}$	$A^s{}_{\frac{h^4}{1-h^2}}$	$A^s{}_{\ln(1+h^2)}$	$A^s{}_{\ln(1-h^2)}$
2	0.45474353306039				
4	0.45476939352315	0.45477144594083	0.45477075460014	0.45477127798924	0.45477093008340
6	0.45477087988248	0.45477126128789	0.45477122961408	0.45477125348175	0.45477123763847
8	0.45477113394562	0.45477125361043	0.45477124942886	0.45477125257324	0.45477125048228
10	0.45477120378745	0.45477125270025	0.45477125177693	0.45477125247055	0.45477125200888
12	0.45477122894702	0.45477125251954	0.45477125224293	0.45477125245061	0.45477125231231
14	0.45477123974987	0.45477125247058	0.45477125236913	0.45477125244528	0.45477125239456
16	0.45477124499830	0.45477125245423	0.45477125241127	0.45477125244351	0.45477125242203
18	0.45477124779335	0.45477125244789	0.45477125242762	0.45477125244283	0.45477125243269
20	0.45477124939131	0.45477125244513	0.45477125243473		0.45477125243733
22	0.45477125035801	0.45477125244381	0.45477125243811		0.45477125243954
24	0.45477125097041	0.45477125244314	0.45477125243984		0.45477125244067
26	0.45477125137353		0.45477125244078		0.45477125244128

Table no.(3) to calculate integration $I = \int_0^1 (e^x + 1)^{0.5}\, dx = 1.64205578028158$ by simpson's rule with the relative and algorithmic acceleration methods of Al-Tememe

n	Values of simpson's rule	$A^s{}_{\frac{h^4}{1+h^2}}$	$A^s{}_{\frac{h^4}{1-h^2}}$	$A^s{}_{\ln(1+h^2)}$	$A^s{}_{\ln(1+h^2)}$
2	1.64207587707482				
4	1.64205706754977	1.64205557473032	1.64205607757476	1.64205569688937	1.64205594993757
6	1.64205603563523	1.64205577084206	1.64205579283181	1.64205577626153	1.64205578726083
8	1.64205586119522	1.64205577903323	1.64205578190430	1.64205577974537	1.64205578118102
10	1.64205581344612	1.64205578000567	1.64205578063692	1.64205578016271	1.64205578047834
12	1.64205579628111	1.64205578019887	1.64205578038759	1.64205578024590	1.64205578034026
14	1.64205578891963	1.64205578025124	1.64205578032037	1.64205578026848	1.64205578030304
16	1.64205578534577	1.64205578026873	1.64205578029798	1.64205578027603	1.64205578029066
18	1.64205578344343	1.64205578027552	1.64205578028931	1.64205578027896	1.64205578028586
20	1.64205578235622	1.64205578027847	1.64205578028554	1.64205578028024	1.64205578028377
22	1.64205578169866	1.64205578027987	1.64205578028375	1.64205578028084	1.64205578028278
24	1.64205578128217	1.64205578028059	1.64205578028284		

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