Conceivability and Haecceitism

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Abstract

This essay aims to redress the contention that epistemic possibility cannot be a guide to the principles of modal metaphysics. I argue that the interaction between the two-dimensional intensional framework and the mereological parthood relation enables epistemic possibilities to target the haecceitistic properties of individuals. I outline the elements of plural logic, and I specify, then, a two-dimensional intensional formula encoding the relation between the epistemic possibility of haecceity comprehension and its metaphysical possibility. I examine the Julius Caesar problem as a test case. I conclude by addressing objections from the indeterminacy of ontological principles relative to the space of epistemic possibilities, and from the consistency of epistemic modal space.

1 Introduction

In this essay, I endeavor to provide an account of how the epistemic interpretation of two-dimensional intensional semantics can be sensitive to *de re* modalities. Let a model, M, be comprised of a set of epistemically possible worlds C; a set of metaphysically possible worlds P; a domain, D, of terms and formulas; binary relations defined on each of C and P; and a valuation function mapping terms and formulas to subsets of C and P, respectively. So, M = ⟨C, P, D, R_C, R_P, V⟩. A term or formula is epistemically necessary or apriori iff it is inconceivable for it to be false ($\Box \iff \neg\Diamond\neg$). A term or formula is negatively conceivable iff nothing rules it out apriori ($\Diamond \iff \neg\Box\neg$). A term or formula is positively conceivable only if the term or formula can be perceptually imagined. According to the epistemic interpretation of two-dimensional intensional semantics, the semantic value of a term or formula can then be defined relative to two parameters, a context and an index. The context ranges over the set of epistemically possible worlds, and the index ranges over the set of metaphysically possible worlds. The value of the term or formula relative to the context determines the value of the term or formula relative to the index. Thus, the epistemically possible value of the term or formula constrains the metaphysically possible value of the term or formula; and so conceivability might, given the foregoing, serve as a guide to metaphysical possibility.

Roca-Royes (2011) and Chalmers (2010; 2011; 2014) note that, on the above semantics, epistemic possibility cannot track the difference between the metaphysical modal profile of a non-essential proposition – e.g., that there is a shooting star – and the metaphysical modal profile of an essential definition, such as a theoretical identity statement – e.g., that water = H2O. Another principle of modal metaphysics to which epistemic possibilities are purported to be insensitive is haecceity comprehension; namely, that $\Box\forall x,y\Box\exists\Phi(\Phi x \iff x = y)$.

The aim of this note is to redress the contention that epistemic possibility cannot be a guide to the principles of modal metaphysics. I will argue that the interaction between the two-dimensional intensional framework and the mereological parthood relation enables epistemic possibilities to target the haecceitistic properties of individuals.

In Section **2**, I examine a necessary condition on admissible cases of conceivability entailing metaphysical possibility in the two-dimensional intensional framework, focusing on the property of super-rigidity. I argue that – despite the scarcity of properties which satisfy the super-rigidity condition – metaphysical properties such as the parthood relation do so. In Section **3**, I address objections to two dogmas of the semantic rationalism underpinning the epistemic interpretation of two-dimensional intensional semantics. The first dogma states that distinctions can be delineated between linguistic intensions and conceptual epistemic intensions, while the second dogma records that there are criteria on the basis of which formal from informal domains, unique to the extensions of various concepts, can be distinguished, such that the modal profiles of those concepts would thus be determinate. I examine the Julius Caesar problem as a test case. I specify, then, a two-dimensional intensional formula encoding the relation between the epistemic possibility of haecceity comprehension and its metaphysical possibility. In Section **4**, I address objections from the indeterminacy of ontological principles relative to the space of epistemic possibilities, and from the consistency of epistemic modal space. Section **5** provides concluding remarks.

2 Super-rigidity

The interaction between mereological parthood and epistemic and metaphysical possibility avoids one crucial issue for the epistemic interpretation of two-dimensional intensional semantics. The issue is that, unless the semanic value for a term is 'super-rigid', i.e. maps to the same extension throughout the sets of epistemic and metaphysical possibilities, the extension of the term in epistemic modal space risks diverging from the extension of the term in metaphysical modal space.

There appear to be only a few expressions which satisfy the super-rigidity condition. Such terms include those referring to the properties of phenomenal consciousness, to the parthood relation, and perhaps to the property of friendship (Chalmers, 2012: 367, 374). Other candidates for super-rigidity are taken to include metaphysical terms such as 'cause' and 'fundamental'; numerical terms such as 'one'; and logical constants such as '$\wedge$' (Chalmers, op. cit.). However, there are objections to each of the other proposed candidates.

Against the super-rigidity of 'fundamental', Fine (2001: 3) argues that a proposition is fundamental if and only if it is real, while Sider (2011: 112, 118) argues that a proposition is fundamental iff it possesses a truth-condition (in a 'metaphysical semantics', stated in perfectly joint-carving terms) for the sub-propositional entities – expressed by quantifiers, functions, predicates – comprising the target proposition. The absolute joint-carving terms are taken to include logical vocabulary (including quantifiers), metaphysical predicates such as mereological parthood, and physical predicates.

Against the super-rigidity of 'cause', Sider (op.cit.: 8.3.5) notes that a causal deflationist might argue that causation is non-fundamental. By contrast, a causal nihilist might argue that causation is non-fundamental as well, though for the distinct reason that there is no causation. So, while both the deflationist and nihilist believe that 'cause' does not carve at the joints – the nihilist can still state that there is a related predicate, 'cause*', such that they can make the joint-carving claim that 'Nothing causes* anything', whereas the deflationist will remain silent, and maintain that no broadly causal locutions carve at the joints.

Against the super-rigidity of 'one', Benacerraf (1965) notes that, in the reduction of number theory to set theory, there must be, and is not, a principled reason for which to prefer the identification of natural numbers with von Neumann ordinals (e.g., $2 = \{\emptyset, \emptyset\}$), rather than with Zermelo ordinals (i.e., an order-type of a well-ordering $2 = \{\{\emptyset\}\}$).[1]

Against the super-rigidity of the logical connective, $\wedge$, the proponent of

[1] Cf. Zermelo (1908/1967) and von Neumann (1923/1967). Well-orderings are irreflexive, transitive, binary relations on all non-empty sets, defining a least element in the sets.

model-theoretic validity will prefer a definition of the constant according to which, for propositions ϕ and ψ and a model, M, M validates $\phi \wedge \psi$ iff M validates ϕ and M validates ψ. By contrast, the proponent of proof-theoretic validity will prefer a distinct definition which makes no reference to truth, according to which $\wedge$ is defined by its introduction and elimination rules: $\phi,\psi \vdash \phi\wedge\psi$; $\phi\wedge\psi \vdash \phi$; $\phi\wedge\psi \vdash \psi$.

Finally, terms for physical entities such as 'tensor field' might have a rigid intension mapping the term to the same extension in metaphysical modal space, and a non-rigid intension mapping the term to distinct extensions in epistemically possible space, such that what is known about the term is contingent and might diverge from its necessary metaphysical profile.[2] That physical terms are not super-rigid might be one way to challenge the soundness of the conceivability argument to the effect that, if it is epistemically possible that truths about consciousness cannot be derived from truths about physics, then the dissociation between phenomenal and physical truths is metaphysically possible (cf. Chalmers, 2010: 151).

Crucially for the purposes of this note, there appear to be no clear counterexamples to the claim that mereological parthood is super-rigid. If this is correct, then mereological parthood in the space of epistemic modality can serve as a guide to the status of mereological parthood in metaphysical modal space. The philosophical significance of the foregoing is that it belies the contention proffered by Roca-Royes (op. cit.) and Chalmers (op. cit.) concerning the limits of conceivability-based modal epistemology. The super-rigidity of the parthood relation ensures that the interaction between the conceivability of mereological

[2] A 'tensor field' is a function from m '1-forms' at a spacetime point, p, and n vectors at p, to the real numbers. A 1-form is a function, ω, s.t. ω maps four vectors to the real numbers, and satisfies the condition that for vectors ≥ 2, μ, τ, and real numbers α and β: $\omega(\alpha\mu + \beta\tau) = \alpha\omega(\mu) + \beta\omega(\tau)$. Cf. Arntzenius (2012): 72.

parthood, which records the existence of haecceities, can serve as a guide to the metaphysical modal profile of haecceity comprehension.

3 Two Dogmas of Semantic Rationalism

The tenability of the foregoing depends upon whether objections to what might be understood as the two dogmas of semantic rationalism can be circumvented.[3]

3.1 The First Dogma

The first dogma of semantic rationalism mirrors Quine's (1951) contention that one dogma of the empiricist approach is the distinction that it records between analytic and synthetic claims. The analogous dogma in the semantic rationalist setting is that a distinction can be drawn between linguistic epistemic intensions – witnessed by differences in the cognitive significance of two sentences or terms which have the same extension, e.g., with $x = 2$, 'x^2' and '2x' – by contrast to conceptual epistemic intensions – e.g., those which denote the properties of phenomenal consciousness. The distinction coincides with two interpretations of two-dimensional intensional semantics. As noted, the epistemic interpretation of two-dimensional intensional semantics takes the value of a formula relative to a context ranging over epistemically possible worlds to determine the extension of the formula relative to an index ranging over metaphysically possible worlds (cf. Chalmers, op. cit.). According to the metasemantic interpretation, a sentence, such as that 'water = H_20', is metaphysically necessary, whereas assertions made about metaphysically necessary sentences record the non-ideal epistemic states of agents and are thus contingent (cf. Stalnaker, 1978, 2004). The first dogma is thus to the effect that there are distinct sets of worlds – sets of non-linguistic

[3] Thanks to xx for the objections.

conceptual possibilities and of linguistic presuppositions, respectively – over which the context ranges in the epistemic and metasemantic interpretations.

Two examples might be apposite. The physical law that force can be identified by calculating the product of mass and acceleration, $f = ma$, has a distinct linguistic intension than that for a reformulated version of the law, according to which force can be identified by calculating the product of mass and the independently calculated product of acceleration and the second derivative of position, $f = m(d^2x/dt^2)$ (cf. Hicks and Schaffer, 2015: 17). However, '$f = ma$' and '$f = m(d^2x/dt^2)$' have identical ideal conceptual epistemic intensions. Similarly, the linguistic intension for the parthood relation can vary while its conceptual intension remains constant. Thus, the linguistic intension for the sentence that 'the class of renates is a part of the class of cordates' is distinct from the linguistic intension for the sentence that 'the class of entities with kidneys is a part of the class of entities with hearts'. However, the ideal conceptual epistemic intensions for the two thoughts are identical.

If no conditions on the distinctness between linguistic and conceptual epistemic intensions can be provided, then variance in linguistic intension might adduce against the uniqueness of the conceptual intension. Because of the possible proliferation of epistemic intensions, conditions on the super-rigidity of the formulas and terms at issue might thereby not be satisfiable. The significance of the first dogma of semantic rationalism is that it guards against the collapse of conceptual and linguistic epistemic intensions, and thus the collapse of language and thought.

A defense of the first dogma of semantic rationalism might, in response, be proffered, in light of the status of higher-order distributive plural quantification in natural language semantics. Plural quantifiers are distributive, if the individuals comprising the plurality over which the quantifier ranges are conceived of singly, rather than interpreting the quantifier such that it ranges over irreducible collections. Natural language semantics permits plural quantification into both first and second-level predicate position. For all second-order variables ranging over a domain of individuals, one can define a predicate denoting a plurality of individuals at the first level (cf. Rayo, 2006). Subsequently, if there is an individual which satisfies the predicate, then the individual is a part of the extension of the predicate, i.e., the relevant plurality of individuals . For an example in English, the predicate can be interpreted such that it denotes the plurality, 'the books'. Then, an individual satisfies the predicate if and only if it is a part of the plurality of books. It is similarly innocuous in English to avail of plural quantification into second-level predicate position. For all third-order variables ranging over a domain of individuals, one can define a predicate denoting a plurality of individuals at the second level. E.g., the predicate might be interpreted such that it denotes the second-level plurality, the chamber pieces, in the sentence, 'These quartets are among her chamber pieces'. Then, the first-level plurality, the quartets, satisfies the predicate if and only if it is a part of the second-level plurality of chamber pieces over which the third-order plural quantifier has been defined to range (op. cit.).

Advancing to a higher order, for all fourth-order variables ranging over a domain of individuals, one can define a predicate which denotes a plurality of individuals at the third level. However, there are no examples of plural quantification into third-level predicate position in empirical linguistics, despite that examples thereof can be readily countenanced in intended models of formal languages.

The philosophical significance of the foregoing is that it is conceptually innocuous for plural terms of an arbitrary type, i_α, to satisfy a polyadic predicate at $i_{\alpha+2}$th-order which denotes pluralities of type $i_{\alpha+1}$, such that pluralities of the i_αth-type can be defined as parts of pluralities of the $i_{\alpha+1}$th-type. Yet – for polyadic predicates at order > 3 – the above has, as just noted, no analogue in natural language. As follows, higher-order plural quantification might adduce in favor of the first dogma of semantic rationalism, to the effect that linguistic and conceptual epistemic intensions can be sufficiently distinguished.

3.2 The Second Dogma

The second dogma of semantic rationalism mirrors Quine's (op. cit.) contention that another dogma of empiricism is the reduction of the meaning of a sentence to the empirical data which verifies its component expressions. The analogous dogma in the semantic rationalist setting states that individuation-conditions on concepts can be provided in order to distinguish between concepts unique to formal and informal domains. The significance of the second dogma of semantic rationalism is that whether the objects falling under a concept belong to a formal domain of inquiry will subsequently constrain its modal profile.

In the space of epistemic possibility, it is unclear, e.g., what reasons there might be to preclude implicit definitions such as that the real number of the x's is identical to Julius Caesar (cf. Frege, 1884/1980: 56; Clark, 2007) by contrast to being identical to a unique set of rational numbers as induced via Dedekind cuts. It is similarly unclear how to distinguish, in the space of epistemic possibility, between formal and informal concepts, in order to provide a principled account of when a concept, such as the concept of 'set', can be defined via the axioms of the language in which it figures, by contrast to concepts such as 'water', where definitions for the latter might target the observational, i.e. descriptive and functional, properties thereof.

The concept of mereological parthood provides a further borderline case. While the parthood relation can be axiomatized so as to reflect whether it is irreflexive, non-symmetric, and transitive, its status as a formal property is more elusive. The fact, e.g., that an order-type is part of the sequence of ordinal numbers impresses as being necessary, while yet the fact that a number of musicians comprise the parts of a chamber ensemble might impress as being contingent.

The Julius Caesar problem, and the subsequent issue of whether there might be criteria for delineating formal from informal concepts in the space of epistemic modality, may receive a unified response. The ambiguity with regard to whether the parthood relation is formal – given that its relata can include both formal and informal objects – is similar to the ambiguity pertaining to the nature of real numbers. As Frege (1893/2013: 161) notes: 'Instead of asking which properties an object must have in order to be a magnitude, one needs to ask: how must a concept be constituted in order for its extension to be a domain of magnitudes [...] a thing is a magnitude not in itself but only insofar as it belongs, with other objects, to a class that is a domain of magnitudes'. Frege defines a magnitude as the extension of a relation on arbitrary domains (op. cit.). The concept of a magnitude is then referred to as a 'Relation', and domains of magnitudes are defined as classes of Relations (162). Bypassing the rational numbers, Frege defines, then, the real numbers as relations on – namely, ratios of – magnitudes; and thus refers to the real numbers as 'Relations on Relations', because the extension of the higher-order concept of real number is taken to encompass the extension of the lower-order concept of classes of Relations, i.e., domains of magnitudes (op. cit.). The interest of Frege's definition of the concept of real number is that explicit mention must be made therein to a domain of concrete entities to which the number is supposed, as a type of measurement, to be applied.

In response: The following implicit definitions – i.e., abstraction principles – can be provided for the concept of real number, where the real numbers are defined as sets, or Dedekind cuts, of rational numbers. Following Shapiro (2000), let F,G, and R denote rational numbers, such that concepts of the reals can be specified as follows: $\forall F,G[\mathbf{C}(F) = \mathbf{C}(G) \iff \forall R(F \leq R \iff G \leq R)]$. Concepts of rational numbers can themselves be obtained via an abstraction principle in which they are identified with quotients of integers – $[\mathbf{Q}\langle m,n\rangle = \mathbf{Q}\langle p,q\rangle \iff n = 0 \wedge q = 0 \vee n \neq 0 \wedge q \neq 0 \wedge m \text{ x } q = n \text{ x } p]$; concepts of the integers are obtained via an abstraction principle in which they are identified with differences of natural numbers – $[\mathbf{D}(\langle x,y\rangle) = \mathbf{D}(\langle z,w\rangle) \iff x + w = y + z]$; concepts of the naturals are obtained via an abstraction principle in which they are identified with pairs of finite cardinals – '$\forall x,y,z,w[\langle x,y\rangle(=\mathbf{P}) = \langle z,w\rangle(=\mathbf{P}) \iff x = z \wedge y = w]$; and concepts of the cardinals are obtained via Hume's Principle, to the effect that cardinals are identical if and only if they are equinumerous – $\forall\mathbf{A}\forall\mathbf{B}\exists R[[Nx\text{: }\mathbf{A} = Nx\text{: }\mathbf{B} \equiv \exists R[\forall x[\mathbf{A}x \rightarrow \exists y(\mathbf{B}y \wedge Rxy \wedge \forall z(\mathbf{B}z \wedge Rxz \rightarrow y = z))] \wedge \forall y[\mathbf{B}y \rightarrow \exists x(\mathbf{A}x \wedge Rxy \wedge \forall z(\mathbf{A}z \wedge Rzy \rightarrow x = z))]]]$.

Frege notes that 'we can never [. . .] decide by means of [implicit] definitions whether any concept has the number Julius Caesar belonging to it, or whether that same familiar conqueror of Gaul is a number or not' (1884/1980: 56). A programmatic line of response endeavors to redress the Julius Caesar problem by appealing to sortal concepts, where it is an essential property of objects that they fall in the extension of the concept (cf. Hale and Wright, 2001: 389, 395).

In order further to develop the account, I propose to avail of recent work in which identity conditions are interpreted so as to reflect relations of essence and explanatory ground. The role of the essentiality operator will be to record a formal constraint on when an object falls under a concept 'in virtue of the nature of the object' (Fine, 1995: 241-242). The role of the grounding operator will be to record a condition on when two objects are the same, entraining a hyperintensional type of implicit definition for concepts which is thus finer-grained and less susceptible to error through misidentification.

In his (2015), Fine treats identity criteria as generic statements of ground. By contrast to **material** identity conditions which specify when two objects are identical, **criterial** identity conditions explain in virtue of what the two objects are the same. Arbitrary, or generic, objects are then argued to be constitutive of criterial identity conditions. Let a model, M, for a first-order language, L, be a tuple, where $M = \langle I,\ A,\ R,\ V\rangle$, with I a domain of concrete and abstract individuals, A a domain of arbitrary objects, R a dependence relation on arbitrary objects, and V a non-empty set of partial functions from A to I (cf. Fine, 1985). The arbitrary objects in A are reified variables. The dependence relation between any a and b in A can be interpreted as a relation of ontological dependence (op. cit.: 59-60). Informally, from $a \in A$ s.t. $F(a)$, one can infer $\forall x.F(x)$ and $\exists x.F(x)$, respectively (57). Then, given two arbitrary objects, x and y, with an individual i in their range, '$[(x = i \wedge y = i) \rightarrow x = y]$', such that x and y mapping to a common individual explains in virtue of what they are the same (Fine, 2015).

Abstraction principles for, e.g., the notion of set, as augmented so as to record distinctions pertaining to essence and ground, can then be specified as follows:

- Given x,y, with Set(x) $\wedge$ Set(y): $[\forall z(z\in x \equiv z\in y) \leftarrow_{x,y} (x = y)]$

(Intuitively, where the 'given' expression is a quantifier ranging over the domain of variables-as-arbitrary objects: Given x, y, whose values are sets, it is essential to x and y being the same that they share the same members); and

- Given x,y, with Set(x) $\wedge$ Set(y): $[\forall z(z\in x \equiv z\in y) \rightarrow_{x,y} (x = y)]$

(Intuitively: Given arbitrary objects, x, y, whose values are sets, the fact that x and y share the same members grounds the fact that they are the same).

Combining both of the above directions yields the following hyperintensional, possibly asymmetric, biconditional:

- Given x,y, with Set(x) $\wedge$ Set(y): $[\forall z(z\in x \equiv z\in y) \leftrightarrow_{x,y} (x = y)]$.

A reply to the Julius Caesar problem for real numbers might then avail of the foregoing metaphysical implicit definitions, such that the definition would record the essentiality to the reals of the property of being necessarily non-concrete:

- Given F,G$[\mathbf{C}(F) = \mathbf{C}(G) \leftrightarrow_{F,G} \forall R(F\leq R \iff G\leq R)]$, and
- $\Box\forall X_{X/F}\Box\exists Y\ [\neg E(Y) \wedge \Box(X = Y)]$

(Intuitively: Given arbitrary objects, F,G, whose values are the real numbers: It is essential to the F's and the G's that the concept of the Fs is identical to the concept of the G's iff (i) F and G are identical subsets of a limit rational number, R, and (ii) with E(x) a concreteness predicate, necessarily for all real numbers, X, necessarily there is a non-concrete object Y, to which necessarily X is identical; i.e., the reals are necessarily non-concrete. The foregoing is conversely the ground of the identification.)[4]

[4]Rosen and Yablo (2020) also avail of real, or essential, definitions in their attempt to solve the Caesar problem, although their real definitions do not target grounding-conditions. The need for a grounding-condition is mentioned in Wright (2020: 314, 318).

Heck (2011: 129) notes that the Caesar problem incorporates an epistemological objection: "Thus, one might think, there must be more to our apprehension of numbers than a mere recognition that they are the references of expressions governed by HP [Hume's Principle – HK]. Any complete account of our apprehension of numbers as objects must include an account of what distinguishes people from numbers. But HP alone yields no such explanation. That is why Frege writes: 'Naturally, no one is going to confuse [Caesar] with the [number zero]; but that is no thanks to our definition of [number]' (Gl, 62)".

The condition of being necessarily non-concrete in the metaphysical definition for real numbers – which includes conditions of essence and ground – provides a reply to the foregoing epistemological objection, i.e. the required account, beyond the abstraction principle, of what distinguishes people from numbers.

3.3 Mereological Parthood

The above proposal can then be generalized, in order to countenance the abstract profile of the mereological parthood relation. By augmenting the axioms for parthood in, e.g., classical mereological parthood with a clause to the effect that it is essential to the parthood relation that it is necessarily non-concrete, parthood can thus be understood to be abstract; and truths in which the relation figures would thereby be necessary.

- Given x: $\Phi(x) \land \Box\forall x\Box\exists y\, [\neg E(y) \land \Box(x = y)] \leftrightarrow_x \Gamma(x)$ where
- $\Gamma(x) := x$ is the parthood relation, $<$, which is irreflexive, asymmetric, and transitive, and where the relation satisfies the axioms of classical extensional mereology codified by the predicate, $\Phi(x)$ (cf. Cotnoir, 2014):

Weak Supplementation: $x < y \rightarrow \exists z[(z < y \vee z = y) \wedge \neg\exists w(w < z \vee w = z) \wedge (w < x \vee w = x)]$, and

Unrestricted Fusion: $\forall xx\exists y[F(y,xx)]$,

with the axiom of Fusion defined as follows:

Fusion: $F(t,xx) := (xx < t \vee xx = t) \wedge \forall y[(y < t \vee y = t) \rightarrow (y < xx \vee y = xx)]$

Fusions are themselves abstracta, formed by a fusion-abstraction principle. The abstraction principle states that two singular terms – in which an abstraction operator, σ, from pluralities to fusions figures as a subformula – are identical, if and only if the fusions overlap the same locations (cf. Cotnoir, ms). Let a topological model be a tuple, comprised of a set of points in topological space, μ; a domain of individuals, D; an accessibility relation, R; and a valuation function, V, assigning distributive pluralities of individuals in D to subsets of μ:

$M = \langle \mu, D, R, V \rangle$;

$R = R(xx,yy)_{xx,yy \in \mu}$ iff $R_{xx} \subseteq \mu_{xx} \times \mu_{xx}$, s.t. if R(xx,yy), then $\exists o \subseteq \mu$, with $xx \in o$ s.t. $\forall yy \in o R(xx,yy)$, where the set of points accessible from a privileged node in the space is said to be open; and $V = f(ii \in D, m \in \mu)$.[5] Necessity is interpreted as an interiority operator on the space:

$M,xx \Vdash \Box\phi$ iff $\exists o \subseteq \mu$, with $xx \in o$, such that $\forall yy \in o$ $M,yy \Vdash \phi$.

The following fusion abstraction principle can then be specified:

Given $xx,yy,F[\sigma(xx,F) = \sigma(yy,F) \leftrightarrow_{xx,yy} [f(xx,m_1) \cap f(yy,m_1) (\neq \emptyset)]]$.

(Intuitively, given arbitrary objects whose values are the pluralities, xx,yy: It is essential to xx and yy that fusion-abstracts – formed by mapping the pluralities to the abstracta – are identical, because the fusions overlap the same

[5] μ is further Alexandrov; i.e., closed under arbitrary unions and intersections.

nonstationary – i.e., $\neq \emptyset$ – locations. The converse is the determinative ground of the identification.)

The foregoing constraints on the formality of the parthood relation – both being necessarily non-concrete and figuring in pluralities which serve to individuate fusions as abstract objects – are sufficient then for redressing the objections to the second dogma of semantic rationalism; i.e., that individuation-conditions are wanting for concepts unique to formal and informal domains, which would subsequently render the modal profile of such concepts indeterminate. That relations of mereological parthood are abstract adduces in favor of the claim that the values taken by the relation are necessary. The significance of both the necessity of the parthood relation, as well as its being abstract rather than concrete and thus being in some sense apriori, is that there are thus compelling grounds for taking the relation to be super-rigid, i.e., to be both epistemically and metaphysically necessary.

Finally, a third issue, related to the second dogma is that, following Dummett (1963/1978: 195-196), the concept of mereological parthood might be taken to exhibit a type of 'inherent vagueness', in virtue of being indefinitely extensible. Dummett (1996: 441) defines an indefinitely extensible concept as being such that: 'if we can form a definite conception of a totality all of whose members fall under the concept, we can, by reference to that totality, characterize a larger totality all of whose members fall under it'. It will thus be always possible to increase the size of the domain of elements over which one quantifies, in virtue of the nature of the concept at issue; e.g., the concept of ordinal number is such that ordinals can continue to be generated, despite the endeavor to quantify over a complete domain, in virtue of iterated applications of the successor relation, and the concept of real number is such that the reals can continue to be generated via elementary embeddings. Bernays' (1942) theorem states that class-valued functions from classes to sub-classes are not onto, where classes are non-sets (cf. Uzquiano, 2015a: 186-187). A generalization of Bernays' theorem can be recorded in plural set theory,[6] where the cardinality of the sub-pluralities of an incipient plurality will always be greater than the size of that incipient plurality. If one takes the cardinal height of the cumulative hierarchy to be fixed, then one way of tracking the variance in the cardinal size falling in the extension of the concept of mereological parthood might be by redefining the intension thereof (Uzquiano, 2015b). Because it would always be possible to reinterpret the concept's intension in order to track the increase in the size of the plural universe, the intension of the concept would subsequently be non-rigid; and the concept would thus no longer be super-rigid.

One way in which the objection might be countered is by construing the variance in the intension of the concept of parthood as tracking temporal modal properties, rather than metaphysical modal properties. Then, the relation can be necessary while satisfying full S5 – i.e., modal axioms K $[(\Box\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi)]$, T $(\Box\phi \rightarrow \phi)$, and E $(\neg\Box\phi \rightarrow \Box\neg\Box\phi)$ – despite that there can be variations in the size of the quantifier domains over which the relation and its concept are defined. Let $\uparrow$ be an intensional parameter which indexes and stores the relevant formulas at issue to a particular world (cf. Hodes, 1984). The $\downarrow$-symbol is an operator which serves to retrieve, as it were, that indexed information. Adding multiple arrows is then akin to multiple-indexing: The value of a formula, as indexed to a particular world, will then constrain the value of that formula, as indexed – via the addition of the new arrows – to different worlds. Interpreting the operators temporally permits there to be multiple-indexing in the array

[6]See Burgess (2004/2008), for an axiomatization of 'Boolos-Bernays' plural set theory, so named after the contributions of Bernays (op. cit.) and Boolos (1984, 1985). See Linnebo (2007), for critical discussion.

of intensional parameters relative to which a formula gets its value, while the underlying logic for metaphysical modal operators can be S5, partitioning the space of worlds into equivalence classes. Formally:

$\uparrow_1 \forall x \exists \phi \uparrow_2 \exists y[\phi(x) \downarrow_1 \wedge \phi(y) \downarrow_2]$.

The clause states that, relative to a first temporal parameter in which all of the x's satisfying the sethood predicate are quantified over, there is – relative to a distinct temporal parameter – another element which satisfies that predicate. Crucially, differences in the intensional temporal indices, as availed of in order to record variance, at different times, in the size of the cumulative hierarchy of elements falling in the range of the parthood relation, is yet consistent with the cardinality of the elements in the domain falling in the range of the relation being fixed, such that the valuation of the relation can yet be necessary.

3.4 Summary

In this section, I addressed objections to two dogmas of the semantic rationalism underpinning the epistemic interpretation of two-dimensional intensional semantics. In response to objections to the first dogma – according to which no distinctions can be delineated between linguistic intensions and conceptual epistemic intensions – I noted that higher-order plural terms are conceptually tractable although they have no analogue in natural language semantics. In response to objections to the second dogma – according to which criteria on distinguishing formal from informal domains unique to the extensions of various concepts are lacking, which subsequently engenders indeterminacy with regard to the modal profiles of those concepts – I availed of generic criterial identity conditions, in which it is essential to identical arbitrary representatives of objects that they satisfy equivalence relations which are conversely ground-theoretically determinative of the identification, and further essential thereto that they satisfy the predicate of being necessarily non-concrete. The extensions of indefinitely extensible concepts can further be redefined relative to distinct temporal intensional parameters, despite that the background modal logic for the intensions of the concepts partitions the domain of worlds into equivalence classes, and thus satisfies S5. Thus, parthood can be deemed a necessary, because abstract, relation, despite (i) temporal variance in the particular objects on which the parthood relation is defined; and (ii) variance in the cardinality of the domain in which those objects figure, relative to which the concept's intensions are defined.

When Φ = x$\preceq$xx, $\Box\forall$x,y$\Box\exists\Phi(\Phi$x $\iff$ x = y). By the super-rigidity of the parthood relation, the target two-dimensional intensional formula can, finally, be stated as follows:

If it is epistemically possible that Φx, then it is metaphysically possible that Φx. Formally:

$\forall$c$\in$C,p$\in$P$[\![\Phi x]\!]^{c,p} = 1$ iff $\forall$c'$\in$C,p'$\in$P$[\![\Phi x]\!]^{c',p'} = 1$.

Thus, the epistemic possibility of haecceity comprehension constrains the value of the metaphysical possibility of haecceity comprehension, and – in response to Roca-Royes and Chalmers – there is a case according to which conceivability is a guide to a principle of modal metaphysics.

In the remainder of the paper, I will examine issues pertaining to the determinacy of epistemic possibilities.

4 Determinacy and Consistency

In his (2014), Chalmers argues for the law of excluded middle, such that it is either apriori derivable using the material conditional – i.e. 'scrutable' – that p or scrutable that ¬p, depending on the determinacy of p. Chalmers refers to the case in which p must be determinate, entailing determinate scrutability, as the Hawthorne model, and the case in which it can be indeterminate, entailing indeterminate scrutability, as the Dorr model (259).[7] Chalmers argues that, for any p, one can derive 'p iff it is scrutable that p' from 'p iff it is true that p' (262). However, 'p iff it is scrutable that p' is unrestrictedly valid only on Dorr's, and not Hawthorne's, model (op. cit.).[8]

Chalmers suggests that the relevant notion of consistency might be a property of epistemic possibilities rather than metaphysical possibilities. However, there are general barriers to establishing the consistency of the space of epistemic modality.

One route to securing the epistemic interpretation of consistency is via Chalmers' conception of idealized epistemic possibility. Conceivability is ideal if and only if nothing rules it out apriori upon unbounded rational reflection (2012: 143). The rational reflection pertinent to idealized conceivability can be countenanced modally, normatively, and so as to concern the notion of epistemic entitlement. An idealization is (i) modal iff it concerns what it is metaphysically possible for an agent to know or believe; (ii) normative iff it concerns what agents ought to believe; and (iii) warrant-involving iff it concerns the propositions which agents are implicitly entitled to believe (2012: 63). It is

[7] Cf. Dorr (2003: 103-4) and Hawthorne (2005: sec. 2).

[8] Chalmers rejects the epistemicist approach to indeterminacy, which reconciles the determinacy in the value of a proposition with the epistemic indeterminacy concerning whether the proposition is known (op. cit.: 288). Consider, e.g., a color continuum, beginning with a determinate color hue of red and terminating with a determinate color hue of orange. By transitivity, if the determinate hue of red, x, is phenomenally similar to the next point in the continuum, y, and y is phenomenally similar to the next point, z, then x is phenomenally similar to z. However, iterating transitivity would entail that the terminal color hue is red and not orange. Thus, if the culprit in the sorites paradox is the property of transitivity, then the modal axiom which encodes transitivity (namely 4: $\Box\phi \rightarrow \Box\Box\phi$) is false. The epistemic interpretation of the axiom states that if one knows that ϕ, then one knows that one knows that ϕ. Thus, rejecting axiom 4 entails that the cut-off points in a sorites series are knowable, although one cannot know that one knows them. For further discussion, see Williamson (1994).

unclear whether any of (i)-(iii) in the foregoing would either mandate belief in the claim that 'p ∧ it is indeterminate whether p' is true, or explain in virtue of what the conjuncts are consistent. More general issues for the consistency of epistemically possible worlds, even assuming that the idealization conditions specified in (i)-(iii) are satisfied, include Yablo's (1993) paradox, and Gödel's (1931) incompleteness theorems. Yablo's paradox is as follows:

(S1) For all k>1, S_k is false;

(S2) For all k>2, S_k is false;

...

(Sn) For all k>n, S_k is false;

(Sn+1) For all k>n+1, S_k is false.

(Sn) says that (Sn+1) is false. Yet (Sn+1) is true. Contradiction.[9]

Gödel's incompleteness theorems can be thus outlined. Relative to a choice of (i) coding for an ω-complete, recursively axiomatizable language, L – i.e. a mapping between properties of numbers and properties of terms and formulas in L; (ii) a predicate, phi; and (iii) a fixed-point construction: Let phi express the property of 'being provable', and define (iii) s.t., for any consistent theory T of L, there are sentences, p_{phi}, corresponding to each formula, phi(x), in T, s.t. for 'm' := p_{phi},

$\vdash_T$ p_{phi} iff phi(m).

One can then construct a sentence, 'm' := ¬phi(m), such that L is incomplete (the first incompleteness theorem).

Crucially, moreover, L cannot prove its own consistency:

If:

$\vdash_T$ 'm' iff ¬phi(m),

[9]For further discussion, see Cook (2014).

Then:

$\vdash_T$ C $\rightarrow$ m.

So, L is consistent only if L is inconsistent (the second incompleteness theorem).

Another issue concerning the consistency of 'p $\wedge$ it is indeterminate whether p' – let alone the foregoing general issues concerning the consistency of epistemic modal space – is that Chalmers (2009: 102) endorses the indeterminacy of *metaphysical* proposals such as unrestricted fusion and, presumably, the necessity of parthood, with regard to which the epistemic interpretation of consistency would be irrelevant (264).

To redress the issue, the metaphysical indeterminacy of ontological proposals might be treated as in Barnes and Williams (2011), for whom metaphysical indeterminacy consists in there being an unpointed set of metaphysically possible worlds; i.e., a set of metaphysical possibilities, P, such that precisifications concerning the determinacy in the values of the elements of P leave it unsettled which possibility is actual (116, 124). If so, then metaphysical indeterminacy will provide no new objection to the viability of the two-dimensional intensional framework, because the conditions on ascertaining the actuality of the epistemic possibility in the context – relative to which a formula receives a value, and thus crucially determines the value of the formula relative to an index which ranges over metaphysically possible worlds – are themselves indeterminate (cf. Yablo, 2008).

The more compelling maneuver might instead be to restrict the valid apriori material entailments to determinately true propositions; and to argue, against Chalmers's preferred ontological anti-realist methodology, that the necessity of parthood is both epistemically and metaphysically determinately true, if true at all. The (determinate) truth of the proposition might then be corroborated both by the consistency of its augmentation to the logic underlying the intensional semantics, and perhaps in virtue of other abductive criteria – such as strength, simplicitly, and compatability with what is known – on the tenability of the proposal.

5 Concluding Remarks

One of the primary objections to accounting for the relationship between conceivability and metaphysical possibility via the epistemic interpretation of two-dimensional intensional semantics is that epistemic possibilities are purportedly insensitive to modal metaphysical propositions, concerning, e.g., the haecceitistic properties of individuals. In this paper, I have endeavored to redress the foregoing objection, by noting that it relies on an unduly restrictive, propositional view of the elements of epistemic modal space, which ignores the interaction between epistemic possibilities and the nature of higher-order quantification. Further objections, from both the potential indeterminacy in, and inconsistency of, the space of epistemic possibilities, were then shown to be readily answered. In virtue of the super-rigidity of the parthood relation, epistemic modal space can thus serve as a guide to haecceity comprehension principles in modal metaphysics, and thus to *de re* modality.

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